

## Zero-point fluctuations in the ground state of a mesoscopic normal ring

Pascal Cedraschi and Markus Büttiker

Département de Physique Théorique, Université de Genève, 24, quai Ernest Ansermet, CH-1211 Geneva 4, Switzerland

(Received 29 June 2000; published 4 April 2001)

We investigate the persistent current of a ring with an in-line quantum dot capacitively coupled to an external circuit. Of special interest is the magnitude of the persistent current as a function of the external impedance in the zero-temperature limit when the only fluctuations in the external circuit are zero-point fluctuations. These are time-dependent fluctuations that polarize the ring-dot structure and we discuss in detail the contribution of displacement currents to the persistent current. We have earlier discussed an exact solution for the persistent current and its fluctuations based on a Bethe ansatz. In this work, we emphasize a physically more intuitive approach using a Langevin description of the external circuit. This approach is limited to weak coupling between the ring and the external circuit. We show that the zero-temperature persistent current obtained in this approach is consistent with the persistent current calculated from the Bethe ansatz solution. In the absence of coupling our system is a two level system consisting of the ground state and the first excited state. In the presence of coupling we investigate the projection of the actual state on the ground state and the first excited state of the decoupled ring. With each of these projections we can associate a phase-diffusion time. In the zero-temperature limit we find that the phase-diffusion time of the excited state projection saturates, whereas the phase-diffusion time of the ground state projection diverges.

DOI: 10.1103/PhysRevB.63.165312

PACS number(s): 73.23.Ra, 73.23.Hk, 71.27.+a

### I. INTRODUCTION

An interesting aspect of quantum systems is the fact that even at zero temperature there are fluctuations that manifest themselves in observables that do not commute with the Hamiltonian of the system. In mesoscopic systems phase coherence plays an essential role, and it is thus important to ask to what extent coherence is affected by zero-point fluctuations. To investigate this question we consider the *ground state* of a normal mesoscopic ring threaded by an Aharonov-Bohm flux and capacitively coupled to an external circuit. Such a ring exhibits a persistent current that is a direct consequence of phase-coherent electron motion over distances that are large compared to the ring circumference. At zero temperature such a ring interacts with an external circuit (see Fig. 1) only due to zero-point fluctuations. More precisely, the external circuit can, through the generation of zero-point voltage fluctuations, induce polarization fluctuations in the ring, which in turn affect the magnitude of the persistent current. The source of the voltage fluctuations in the external circuit are the resistive elements. We can thus ask: How does the persistent current of the ring depend on the resistive properties of the external circuit? For the system shown in Fig. 1, Ref. 1 provided an answer by mapping a simple model of a ring with a quantum dot and external circuit on the anisotropic Kondo model and using the known Bethe ansatz solutions of this problem.<sup>2</sup> The purpose of this work is to consider the same model and to provide a discussion that is physically more transparent. The discussion given below is, however, limited to the case of weak coupling between the ring and the external circuit.

Investigations of the coherence properties of the ground state of mesoscopic structures are rare and concern mainly superconductors. Hekking and Glazman<sup>3</sup> investigate a thin superconducting loop in an electromagnetic environment; Oshikawa and Zagoskin<sup>4</sup> consider a superconducting grain

coupled to a normal wire. In contrast, investigations on dephasing in systems driven out of the ground state are numerous and there exists a considerable literature. In particular, zero-point fluctuations have been of interest, following recent experimental work by Mohanty, Jariwala, and Webb<sup>5</sup> on weak localization in metallic diffusive wires. These experiments probe a transport state and not the ground state of the system. In the experiments of Mohanty, Jariwala, and Webb it is the weak-localization effect in the linear response conductance of the system which is of interest. Subsequently to these experiments the role of zero-point fluctuations in weak-localization effects has been very much debated<sup>6-9</sup> and a number of works have appeared that suggest that the experiments can possibly be better explained without invoking zero-point fluctuations.<sup>10</sup> Weak localization is a quantum effect that survives ensemble averaging and thus the dephasing rates that count are specific to the fact that we deal with time-reversed trajectories and that an ensemble average has to be performed. The absence of an effect of zero-point fluctuations found in Refs. 7 and 8 might give rise to the mis-

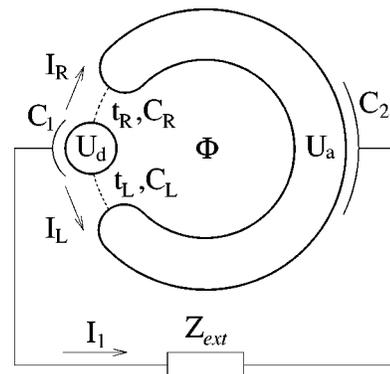


FIG. 1. Ring with an in-line dot subject to a flux  $\Phi$  and capacitively coupled to an external impedance  $Z_{ext}$ .

taken impression that zero-point fluctuations are quite generally without any effect on the coherence properties of a system. The example presented here shows that zero-point fluctuations are clearly important for the coherence properties of the system, although perhaps not for weak-localization phenomena. Below we consider a specific ring (not an ensemble) and ask how the maximum amplitude of the persistent current is affected by zero-point fluctuations.

The effect of a thermal bath on the persistent current has been discussed by Landauer and Büttiker<sup>11</sup> and Büttiker<sup>12,13</sup> within a Debye relaxation approach. In this approach, however, the bath affects essentially only the population of different states, but not the electronic states of the ring itself. As a consequence, at zero temperature, this model exhibits no effect due to zero-point fluctuations: the magnitude of the persistent current is independent of the coupling strength to the bath. Still a different model, introduced by Büttiker,<sup>14</sup> considers a ring coupled via a side branch to a normal electron reservoir. Due to the connection between reservoir and ring a carrier in the ring eventually escapes to the reservoir and is replaced with an incident carrier with a phase that is unrelated to that of the escaping carrier. This model predicts even at zero temperature an amplitude of the persistent current that depends on the coupling strength.<sup>14–18</sup> This effect is not due to zero-point fluctuations but results from the exchange of carriers between the reservoir and the ring. If the ring is coupled to a side branch of finite length, the side branch can nevertheless generate effects that are similar to that of a reservoir, especially if only ensemble-averaged quantities are considered. This is correct only if the side branch has a charging energy that is weak compared to the level spacing.<sup>19</sup> If the charging energy is large compared to the level spacing the side branch has no effect on the ensemble-averaged persistent current.<sup>19</sup> It is thus interesting to ask whether there exist models that are strictly canonical (without carrier exchange with a reservoir) and for which the sample specific persistent current depends nevertheless on the properties of the bath. The model investigated in this work examines a ring that is coupled to the bath only via the long-range Coulomb force.

The mesoscopic ring that we consider (see Fig. 1) is divided into two regions by tunneling barriers. It is a ring with an in-line quantum dot.<sup>20–26</sup> This model allows a simple characterization of the electrostatic potential in terms of only two variables  $U_d$  (for the dot) and  $U_a$  (for the arm) and by the charges  $Q_d$  and  $Q_a$ . The two regions, the dot and the arm of the ring, are coupled via capacitors  $C_1$  and  $C_2$  to the external circuit. The external circuit is described by its impedance  $Z_{ext}$ . The potential at the capacitor  $C_1$  is denoted by  $V_0$ , the charge by  $Q_0$ . Likewise, we write the potential and the charge on the capacitor  $C_2$  as  $V_\infty$  and  $Q_\infty$ .

## II. LANGEVIN EQUATION

To be specific, we consider a purely resistive external impedance,  $Z_{ext} = R$ . The fluctuations generated by this resistor can be represented as a noise source in parallel with the resistor as shown in Fig. 2. The current  $\delta V/R$  through the resistor must be equal to the sum of the current of the noise

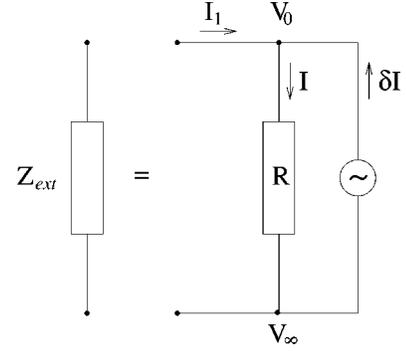


FIG. 2. The external impedance  $Z_{ext}(\omega)$  has been replaced by a resistor  $R$  with a noise source in parallel in order to take quantum fluctuations into account.

source  $\delta I(t)$  and of the current through the ring-dot structure. In terms of the potential difference  $\delta V \equiv V_0 - V_\infty$  across the ring-dot structure we find that the coupling between the ring-dot structure and the external circuit is described by

$$\delta V/R = -C_0 \delta \dot{V} + \frac{C_0}{C_i} \dot{Q}_d + \delta I(t). \quad (1)$$

Here we introduce the capacitance  $C_0^{-1} \equiv C_i^{-1} + C_e^{-1}$ , which is the series capacitance of the *internal* capacitance  $C_i \equiv C_L + C_R$  and the *external* capacitance  $C_e^{-1} \equiv C_1^{-1} + C_2^{-1}$  and the charge on the dot  $Q_d$ . A detailed consideration of the circuit equations leading to Eq. (1) is given in Appendix A. The noise source in parallel to the resistor generates a current noise spectrum,  $\langle \delta I(\omega) \delta I(\omega') \rangle \equiv 2\pi \delta(\omega + \omega') S_{II}(\omega)$ , given by

$$S_{II}(\omega) = \frac{\hbar \omega}{R} \coth\left(\frac{\hbar \omega}{2kT}\right), \quad (2)$$

where  $k$  is the Boltzmann constant and  $T$  is the temperature. To complete the description of this system, we need to investigate the dynamics of the ring in the presence of the fluctuating external voltage  $\delta V(t)$ .

First, we consider the effect of the external circuit in linear response. Since we are only interested in the weak-coupling limit, we can choose the ground state of the ring-dot system in the absence of an external circuit as a state around which we can expand. What we need is the linear response relation between  $\delta Q_d(t)$  (the deviation of the charge away from a reference state) as a function of the external voltage  $\delta V(t)$ . Note that both of these variables are in fact operators. Here, we proceed as in the random-phase approximation and use a linear response function to describe the connection between these two quantities. In linear response we can Fourier transform  $\delta Q_d(t)$  and  $\delta V(t)$ . The linear response coefficient gives the increment of the charge on the dot in response to a variation of an external voltage: It is thus a dynamic capacitance that we denote by  $C_d(\omega) = \delta Q_d(\omega) / \delta V(\omega)$ . Similarly, the relation between the charge  $\delta Q_0$  piled up on the external capacitor  $C_1$  and the applied voltage is determined by the overall capacitance of the ring dot-structure vis-à-vis the external circuit, and is

denoted by  $C_\mu(\omega) = \delta Q_0(\omega) / \delta V(\omega)$ . From circuit theory (see Appendix A) it follows that these two capacitances are related:

$$C_\mu(\omega) = C_0 - \frac{C_0}{C_i} C_d(\omega). \quad (3)$$

The noise source (see Fig. 2) sees a total impedance  $Z(\omega)$ , which consists of this capacitance and the resistance:

$$\frac{1}{Z(\omega)} = \left[ \frac{1}{R} - i\omega C_\mu(\omega) \right]. \quad (4)$$

With these response coefficients Eq. (1) leads to

$$\delta V(\omega) = Z(\omega) \delta I(\omega) = [1 - i\omega R C_\mu(\omega)]^{-1} R \delta I(\omega) \quad (5)$$

and the spectrum of the external voltage is thus

$$S_{VV}(\omega) = \frac{R \hbar \omega \coth(\hbar \omega / 2kT)}{1 + \omega^2 [R C_\mu(\omega)]^2}. \quad (6)$$

Our next task is now to find explicit expressions for the capacitances  $C_\mu(\omega)$  and  $C_d(\omega)$ .

### A. The dynamics of the ring

The dynamics of the charge on the dot, determined by  $\hat{Q}_d$ , is in the Heisenberg picture given by

$$\frac{\partial}{\partial t} \hat{Q}_d = \frac{\hbar}{i} [\hat{H}_{ring}, \hat{Q}_d], \quad (7)$$

where  $\hat{H}_{ring}$  is the Hamiltonian of the ring in an applied external potential  $\delta V$ . We assume that the tunneling amplitudes are much smaller in magnitude than the level spacing in the dot and the level spacing in the arm. Moreover, we assume the electrons to be spinless. As discussed in Refs. 20 and 21 a spin singlet appears in the case of electrons with spin, and the tunneling amplitude is enhanced by a factor of  $\sqrt{2}$ . Our choice of parameters excludes the Kondo effect<sup>24</sup> and also if the electron spin is taken into account. As a matter of fact, the Kondo effect appears only at a tunneling amplitude comparable to or larger than the mean level spacing.<sup>25</sup> We note that the number of charge carriers in the ring is conserved. Following Büttiker and Stafford,<sup>20,21</sup> we consider hybridization between the topmost occupied electron level in the arm and the lowest unoccupied electron level in the dot,  $\epsilon_{aM}$  and  $\epsilon_{d(N+1)}$  only. To simplify the notation, we denote the tunneling amplitudes between the levels  $\epsilon_{aM}$  and  $\epsilon_{d(N+1)}$  by  $t_L$  for tunneling through the left junction and by  $t_R$  for tunneling through the right junction, and introduce the total tunneling amplitude

$$\frac{\hbar \Delta_0}{2} = \sqrt{t_L^2 + t_R^2 \pm 2t_L t_R \cos \frac{2\pi\Phi}{\Phi_0}}, \quad (8)$$

where  $\Phi_0 = hc/e$  is the flux quantum, and the sign in front of the cosine depends on the parity of the number of electrons in the ring. The sign is positive, if the number of electrons is odd, and negative if the number is even. In the model intro-

duced above, we are considering only two states, namely the topmost electron being in the dot, which we represent by the vector (1,0), and the topmost electron being in the arm of the ring, written as (0,1). The dynamics of these two states is described by a time dependent Hamiltonian

$$\hat{H}_{ring} = \frac{\hbar \varepsilon(t)}{2} \sigma_z - \frac{\hbar \Delta_0}{2} \sigma_x + \frac{\hbar \nu(t)}{2} \mathbf{1}, \quad (9)$$

where  $\sigma_z$  and  $\sigma_x$  are Pauli matrices, and  $\mathbf{1}$  is the identity matrix. The prefactor  $\hbar \nu(t) = -C_0 \delta V^2(t)$  is a global shift in energy. We split the detuning  $\varepsilon(t)$  into a time-independent and a time-dependent part  $\varepsilon(t) = \varepsilon_0 + \delta\varepsilon(t)$ , with

$$\begin{aligned} \hbar \varepsilon_0 &\equiv \epsilon_{d(N+1)} - \epsilon_{aM} + \frac{e^2(N - N_+ + 1/2)}{C} - \frac{C_e}{C} V_0 \\ &\equiv \frac{e}{C} (Q_{d*} - Q_{d0}), \end{aligned} \quad (10)$$

$$\delta\varepsilon(t) \equiv \frac{e}{\hbar} \frac{C_0}{C_i} \delta V(t). \quad (11)$$

The effective background charge on the dot is  $Q_{d0} = eN_+ + C_e V_0$ , where the first term is a built in background charge and the second term can be externally controlled by applying a static voltage across the ring-dot structure. If coherent tunneling is neglected the ground states of the ring with  $N$  and  $N+1$  carriers are degenerate if the polarization charge is equal to

$$Q_{d*} \equiv e \left( N + \frac{1}{2} \right) + \frac{C}{e} (\epsilon_{d(N+1)} - \epsilon_{aM}). \quad (12)$$

Thus  $Q_{d0} = Q_{d*}$  is the (classical) condition at which the Coulomb blockade is lifted. Quantum mechanically the state in the ring and the state in the arm of the ring are in resonance when  $Q_{d0} = Q_{d*}$  and the persistent current exhibits a peak. Our simple two-level picture is applicable when  $\varepsilon_0$  is small, that is, in the vicinity of a resonance.

We want to find the time evolution of a state  $\psi(t)$ , which we write as

$$\psi = e^{i\chi/2} \begin{pmatrix} \cos \frac{\theta}{2} e^{i\varphi/2} \\ \sin \frac{\theta}{2} e^{-i\varphi/2} \end{pmatrix}, \quad (13)$$

with  $\theta$ ,  $\varphi$  and  $\chi$  real. This is the most general form of a normalized complex vector in two dimensions. In terms of  $\theta$ ,  $\varphi$ , and the global phase  $\chi$ , the time-dependent Schrödinger equation reads

$$\dot{\varphi} = -\varepsilon_0 - \delta\varepsilon(t) - \Delta_0 \cot \theta \cos \varphi, \quad (14)$$

$$\dot{\theta} = -\Delta_0 \sin \varphi, \quad (15)$$

$$\dot{\chi} = -\nu + \Delta_0 \frac{\cos \varphi}{\sin \theta}. \quad (16)$$

Note that Eq. (16) describing the dynamics of the global phase  $\chi$  is special in the sense that it is driven by  $\phi$  and  $\theta$  but it has no back effect on them. Moreover,  $\chi$  is irrelevant for expectation values such as the persistent current or the charge on the dot. It plays an important role, however, in the discussion of phase-diffusion times; see Sec. IV. The system of the two equations for  $\varphi$  and  $\theta$ , Eqs. (14) and (15), is closed by Eq. (1),

$$\dot{Q}_d = \frac{d}{dt} \langle \psi(t) | \hat{Q}_d | \psi(t) \rangle, \quad (17)$$

with  $\psi(t)$  defined above, and

$$\hat{Q}_d = e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + eN_+. \quad (18)$$

We find

$$\dot{Q}_d = e \frac{d}{dt} \langle \psi(t) | \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} | \psi(t) \rangle = -\frac{e}{2} (\sin \theta) \dot{\theta}. \quad (19)$$

Using this result and Eq. (1), gives

$$\delta V = -C_0 R \delta \dot{V} - \frac{RC_0}{C_i} \frac{e}{2} (\sin \theta) \dot{\theta} + R \delta I(t). \quad (20)$$

Equations (14), (15), and (20) form a closed system of equations in which the external circuit is incorporated in terms of a fluctuating current  $\delta I(t)$  and of an Ohmic resistor  $R$ . In the next section, we investigate Eqs. (14), (15), and (20) to find the effect of zero-point fluctuations on the persistent current of the ring.

### B. Expansion around a stationary state

First, let us discuss the stationary states of the system of differential equations, Eqs. (14), (15), and (20) in the absence of the noise term  $\delta I(t)$ . We take  $0 \leq \varphi < 2\pi$  and  $0 \leq \theta < \pi$ . This gives immediately  $\sin \varphi = 0$  and consequently a stationary state has  $\varphi \equiv \varphi_0$ , with  $\varphi_0 = 0$  or  $\varphi_0 = \pi$ . With this it is easy to show that in the stationary state we must have  $\theta \equiv \theta_0$ , with

$$\cot \theta_0 = \pm \frac{\varepsilon_0}{\Delta_0}. \quad (21)$$

The lower sign applies for  $\varphi_0 = 0$ . This is the ground state for the ring-dot system at fixed  $\varepsilon(t) \equiv \varepsilon_0$ , and the upper sign holds for  $\varphi_0 = \pi$ . The energy of the ground state is  $-\hbar\Omega_0/2$ ; thus the global phase is  $\chi_0(t) = \Omega_0 t$ . Here

$$\Omega_0^2 \equiv \varepsilon_0^2 + \Delta_0^2 \quad (22)$$

is the resonance frequency of the (decoupled) two-level system. We also introduce the ‘‘classical’’ relaxation time  $\tau_{RC} \equiv RC_0$ , and a relaxation rate

$$\Gamma \equiv \pi \alpha \frac{\Delta_0^2}{\Omega_0}, \quad (23)$$

which as we shall see in Sec. II E is a relaxation rate due to the coupling of the ring-dot system to the external circuit. Here  $\alpha$  is a dimensionless coupling constant

$$\alpha \equiv \frac{R}{R_K} \left( \frac{C_0}{C_i} \right)^2 \quad (24)$$

with  $R_K \equiv h/e^2$  the quantum of resistance. We will immediately see the usefulness of these definitions.

Now, we switch on the noise  $\delta I(t)$ . We seek  $\varphi(t)$ ,  $\theta(t)$ ,  $\chi(t)$ , and  $\delta V(t)$  in linear order in the noise current  $\delta I(t)$ . We expand  $\varphi(t)$  and  $\theta(t)$  to first order around the ground state,  $\varphi = 0$  and  $\theta = \theta_0$ . For  $\delta\varphi(t) = \varphi(t) - \varphi_0$ ,  $\delta\theta(t) = \theta(t) - \theta_0$ , we find in Fourier space,

$$-i\omega \delta\varphi = -\delta\varepsilon + \frac{\Omega_0^2}{\Delta_0} \delta\theta, \quad (25)$$

$$-i\omega \delta\theta = -\Delta_0 \delta\varphi, \quad (26)$$

$$-i\omega \delta\varepsilon = \frac{1}{\tau_{RC}} \left[ -\delta\varepsilon - \Gamma \delta\varphi + \frac{e}{\hbar} R \frac{C_0}{C_i} \delta I \right]. \quad (27)$$

We also expand the global phase  $\chi(t)$  around its evolution in the ground state  $\chi_0(t) = \Omega_0 t$ , and define  $\delta\chi(t) = \chi(t) - \chi_0(t)$ . In Fourier space, Eq. (16) becomes

$$-i\omega \delta\chi = \Omega_0 \frac{\varepsilon_0}{\Delta_0} \delta\theta. \quad (28)$$

We note that there is no effect of the global shift in energy,  $\hbar\nu(t)$ , as it is quadratic in the voltage  $\delta V$ , and we are only interested in effects up to linear order in  $\delta V$ .

In the following section, we evaluate the linear response of the ring, described by Eqs. (25) and (26) to an applied external potential  $\delta V(\omega)$ , giving the frequency-dependent capacitance  $C_\mu(\omega)$ .

### C. Capacitance of the ring and impedance

We evaluate  $\delta Q_d(\omega)$  to first order in  $\delta V(\omega)$ , using Eqs. (25) and (26), which gives the dynamic capacitance  $C_d(\omega) = \delta Q_d(\omega) / \delta V(\omega)$ ,

$$C_d(\omega) = C_0 \frac{e^2 / (2C_i)}{\hbar\Omega_0} \frac{\Delta_0^2}{\omega^2 - \Omega_0^2}. \quad (29)$$

The frequency-dependent capacitance as seen from the external circuit reads [see Eq. (3)],

$$C_\mu(\omega) = C_0 \left( 1 - \frac{e^2 / (2C_i)}{\hbar\Omega_0} \frac{\Delta_0^2}{\omega^2 - \Omega_0^2} \right). \quad (30)$$

The frequency-dependent capacitance  $C_\mu$  contains in addition to the geometrical capacitance  $C_0$  a term that arises from the dynamic polarizability of the ring. In the low-frequency regime the polarizability enhances the capacitance,

$$\begin{aligned}
C_\mu(0) &= C_0 \left( 1 + \frac{e^2/(2C_i) \Delta_0^2}{\hbar \Omega_0 \Omega_0^2} \right) \\
&= C_0 + \left( \frac{C_0}{C_i} \right)^2 \frac{2e^2 |t_\pm|^2}{\{ [e(Q_{d0} - Q_{d*})/C]^2 + 4|t_\pm|^2 \}^{3/2}}.
\end{aligned} \tag{31}$$

The zero-frequency-capacitance has already been obtained by Büttiker and Stafford.<sup>20,21</sup> The zero-frequency (electrochemical) capacitance, Eq. (31), can be obtained as a second-order derivative of the grand canonical potential<sup>21</sup> with respect to the voltage applied across the ring-dot system or as a dynamic response to a slowly varying voltage across the ring-dot system. Since the tunneling amplitude  $\Delta_0$  depends periodically on the magnetic flux  $\Phi$  [see Eq. (8)]  $C_\mu(\omega)$  can be modulated by varying  $\Phi$ . The electrochemical capacitance exhibits a peak as the bias  $V_0$  is varied. The peak shows up at resonance,  $Q_{d0} = Q_{d*}$ , corresponding to  $\varepsilon_0 = 0$ . At the same point, there is also a peak in the persistent current. A recent experiment by Deblock *et al.*<sup>27</sup> demonstrated indeed a flux-dependent polarizability of mesoscopic rings.

At high frequencies the polarization cannot follow the external voltage, and the capacitance Eq. (30) is smaller than the geometrical capacitance. At  $\omega = \Omega_0$  the capacitance exhibits a resonance characteristic for dielectric functions. The ring by itself has no damping mechanism; therefore the resonance shows up as a pole. The damping is provided by the dissipation in the external circuit.

From Eqs. (4) and (30), we find

$$\frac{1}{Z(\omega)} = \frac{1}{R} \frac{\omega^2 - \Omega_0^2 - i\omega\tau_{RC}(\omega^2 - \Omega_0^2 - \Gamma/\tau_{RC})}{\omega^2 - \Omega_0^2}. \tag{32}$$

This expression for the impedance ‘‘seen’’ by the noise source contains all the information needed to calculate the various spectral densities.

#### D. Spectral densities

We solve Eqs. (25)–(27) for  $\delta\varphi$  by eliminating  $\delta\theta$  and  $\delta\varepsilon$ , and obtain after some algebra

$$\delta\varphi = \frac{e}{\hbar} \frac{C_0}{C_i} \frac{(-i\omega)Z(\omega)}{\omega^2 - \Omega_0^2} \delta I. \tag{33}$$

It follows immediately that  $\langle \delta\varphi(\omega) \rangle = \langle \delta\theta(\omega) \rangle = 0$  and that  $\langle \delta\varepsilon(\omega) \rangle = 0$ . The spectral densities of  $\delta\varphi(\omega)$ ,  $\delta\theta(\omega)$ , and their cross-correlations can all be expressed in terms of the current noise spectral density. We have  $S_{\theta\theta}(\omega) = (\Delta_0^2/\omega^2)S_{\varphi\varphi}(\omega)$ ,  $S_{\varphi\theta}(\omega) = -i(\Delta_0/\omega)S_{\varphi\varphi}(\omega)$ , and

$$S_{\varphi\varphi}(\omega) = \frac{e^2}{\hbar^2} \left( \frac{C_0}{C_i} \right)^2 \frac{\omega^2 |Z(\omega)|^2}{(\omega^2 - \Omega_0^2)^2} S_{II}(\omega). \tag{34}$$

We will relate the reduction of the persistent current to  $S_{\varphi\varphi}(\omega)$  in Sec. III. While the spectral densities for  $\delta\varphi$  and  $\delta\theta$  are finite at zero frequency, the spectral density of the global phase  $\delta\chi$  is not. We have  $S_{\chi\chi}(\omega)$

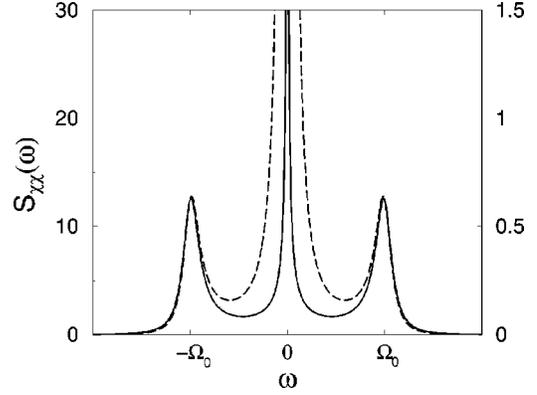


FIG. 3. The spectral function  $S_{\chi\chi}(\omega)$  at finite temperature  $kT = 10\hbar\Omega_0$  (solid line) and at zero temperature (dashed line). The left y axis carries the scale for the finite-temperature curve, the right one carries the scale for the zero-temperature curve. The parameters are the same as in Fig. 4; in addition,  $\varepsilon_0 = \Omega_0/\sqrt{2}$ .

$= (\Omega_0^2 \varepsilon_0^2 / \omega^4) S_{\varphi\varphi}(\omega)$  [see Eq. (28)]; therefore it exhibits a  $\omega^{-2}$  pole in the vicinity of  $\omega=0$  for finite temperatures

$$S_{\chi\chi}(\omega) \sim 2\pi\alpha \frac{2kT}{\hbar} \frac{\varepsilon_0^2}{\Omega_0^2} \frac{1}{\omega} \quad (\omega \rightarrow 0), \tag{35}$$

whereas in the quantum limit  $T=0$  the order of the pole is reduced by 1,

$$S_{\chi\chi}(\omega) \sim 2\pi\alpha \frac{\varepsilon_0^2}{\Omega_0^2} \frac{1}{|\omega|} \quad (\omega \rightarrow 0) \tag{36}$$

(see Fig. 3). These two last results do not affect the persistent current, but they are of great importance in the discussion of the rates of phase diffusion in Sec. IV.

#### E. Weak coupling

Within the approach outlined above, we can expect to reproduce the exact result for the reduction of the persistent current only in the limit of weak coupling between the ring-dot system and the external circuit. This means that the *potential* fluctuations of the system and the external circuit are only weakly coupled. Thus weak coupling between the ring and the external circuit is achieved by letting the capacitances coupling the two systems become very small,<sup>28</sup>  $C_1, C_2 \rightarrow 0$ , for a fixed resistance  $R$ . This entails  $C_0 \rightarrow 0$ , thus the coupling strength  $\alpha$  between the ring and the external circuit, Eq. (24), is small against 1. Note that in this approximation the  $RC$  time  $\tau_{RC}$  becomes very small as well.

In the following paragraph, we calculate the poles of  $Z(\omega)$ , in the approximation  $C_0 \rightarrow 0$ . The small parameters in this case are  $\tau_{RC} \propto C_0$  and  $\Gamma \propto C_0^2$ . The equation  $[Z(\omega)]^{-1} = 0$  has one solution behaving as  $C_0^{-1}$ . To leading order in  $C_0$ , it reads

$$\omega_3 = -\frac{i}{\tau_{RC}}. \tag{37}$$

Thus in the weak-coupling limit charge relaxation across the mesoscopic system becomes instantaneous. The other two

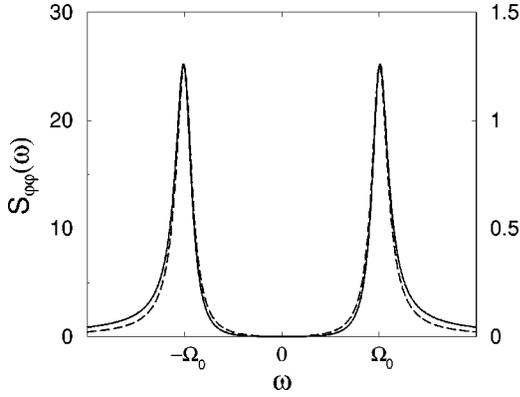


FIG. 4. The spectral function  $S_{\varphi\varphi}(\omega)$  at finite temperature  $kT = 10\hbar\Omega_0$  (solid line) and at zero temperature (dashed line). The left y axis carries the scale for the finite-temperature curve; the right one carries the scale for the zero-temperature curve. The parameters are  $R = 5R_K$ ,  $C_0 = 0.1C_i$ ,  $\Delta_0 = 0.5\Omega_0$ , and  $\Gamma = 0.2\Omega_0$ .

solutions,  $\omega_{\pm}$ , are of zeroth order in  $C_0$ . Up to and including corrections of order  $C_0^2$ , they are

$$\omega_{\pm} = \pm\Omega_0 - i\frac{\Gamma}{2}, \quad (38)$$

with a relaxation rate  $\Gamma$  introduced in Eq. (23). The rate  $\Gamma$  describes the relaxation of perturbations with a frequency close to the eigenfrequency of the isolated ring-dot system. In Sec. IV it is shown that  $\Gamma$  is also related to a phase-diffusion time. The denominator in Eq. (34) can now be written as

$$\frac{(\omega^2 - \Omega_0^2)^2}{|Z(\omega)|^2} = \frac{\tau_{RC}^2}{R^2} \left( \omega^2 + \frac{1}{\tau_{RC}^2} \right) [(\omega - \Omega_0)^2 + (\Gamma/2)^2][(\omega + \Omega_0)^2 + (\Gamma/2)^2]. \quad (39)$$

For frequencies  $\omega\tau_{RC} \ll 1$ , the denominator takes the form

$$\frac{(\omega^2 - \Omega_0^2)^2}{|Z(\omega)|^2} = \frac{1}{R^2} [(\omega - \Omega_0)^2 + (\Gamma/2)^2][(\omega + \Omega_0)^2 + (\Gamma/2)^2]. \quad (40)$$

Since in the quantum limit it is always necessary to introduce a cutoff, Eq. (40) is valid over the entire range of frequencies of interest, if only  $1/\tau_{RC}$  is larger than the cutoff frequency. The spectral density  $S_{\varphi\varphi}(\omega)$  [Eq. (34)], reads therefore

$$S_{\varphi\varphi}(\omega) = \frac{2\pi\alpha\omega^3 \coth(\hbar\omega/2kT)}{[(\omega - \Omega_0)^2 + (\Gamma/2)^2][(\omega + \Omega_0)^2 + (\Gamma/2)^2]}, \quad (41)$$

where the coupling strength  $\alpha$  between the ring and the external circuit is defined in Eq. (24). Note that  $S_{\varphi\varphi}(\omega)$  goes to zero at the origin  $\omega=0$  like  $|\omega|^3$  in the extreme quantum case ( $T=0$ ) and like  $\omega^2$  for finite temperatures; see Fig. 4. For  $|\omega| \gg \Omega_0$ , it behaves like  $|\omega|^{-1}$  for small temperatures and like  $\omega^{-2}$  for large temperatures. Analogous formulas hold for  $S_{\theta\theta}(\omega)$  and for  $S_{\varphi\theta}(\omega)$ . We mention that the spec-

tral density  $S_{\theta\theta}(\omega)$  behaves for small  $\omega$  like  $|\omega|$  at  $T=0$  and goes to a constant for finite  $T$ . All the three spectral densities exhibit peaks of width  $\Gamma$  at  $\pm\Omega_0$ .

The spectral density of the global phase  $S_{\chi\chi}(\omega)$  (Fig. 3) exhibits peaks at  $\omega = \pm\Omega_0$  as well. More important, however, is the fact that it has a pole at  $\omega=0$ , as already pointed out in the previous section.

### III. PERSISTENT CURRENT

In the following, we calculate the persistent current as a quantum *and* statistical average of the operator of the circulating current

$$\hat{I}_c = \begin{pmatrix} 0 & \mathcal{J} \\ \mathcal{J}^* & 0 \end{pmatrix}, \quad (42)$$

where  $\mathcal{J}$  is given by

$$\mathcal{J} = -\frac{e}{\hbar} \left( t_L^2 + t_R^2 \pm 2t_L t_R \cos \frac{2\pi\Phi}{\Phi_0} \right)^{-1/2} \left\{ \pm t_L t_R \sin \frac{2\pi\Phi}{\Phi_0} - i \left[ \frac{C_R}{C_i} t_L^2 - \frac{C_L}{C_i} t_R^2 \pm \frac{C_R - C_L}{C_i} t_L t_R \cos \frac{2\pi\Phi}{\Phi_0} \right] \right\}. \quad (43)$$

Here the first term is a pure particle current contribution, whereas the second term is a consequence of interactions. For the average persistent current of interest here, it is only the first term that contributes. A derivation of Eq. (43) and a discussion of the relationship between  $I_c$  and the magnetization is given in Appendix B.

The expectation value of the persistent current for the state given in Eq. (13) reads

$$I(t) \equiv \langle \psi(t) | \hat{I}_c | \psi(t) \rangle = \frac{1}{2} \text{Re} \mathcal{J}(\sin \theta) e^{-i\varphi}. \quad (44)$$

We are, however, interested in the *statistically averaged* of the persistent current  $\langle I(t) \rangle$ , given by

$$\langle\langle \psi(t) | \hat{I}_c | \psi(t) \rangle\rangle = \frac{1}{2} \text{Re} [ \mathcal{J}(\sin \theta) e^{-i\varphi} ], \quad (45)$$

where the double bracket indicates a quantum and statistical average. Therefore, we have to calculate the correlator  $\langle\langle (\sin \theta) e^{-i\varphi} \rangle\rangle$ . First, we observe that there are no correlations between  $\delta\varphi$  and  $\delta\theta$ ,

$$\langle\langle \delta\varphi(t) \delta\theta(t) \rangle\rangle = 0, \quad (46)$$

as the spectral density  $S_{\varphi\theta}(\omega)$  is an odd function of  $\omega$ . Thus we have

$$\langle\langle (\sin \theta) e^{-i\varphi} \rangle\rangle = \langle\langle \sin \theta \rangle\rangle \langle\langle e^{-i\varphi} \rangle\rangle, \quad (47)$$

to second order in  $\delta I$ , that is, the probability distributions for  $\varphi$  and  $\theta$  are decoupled up to second order in  $\delta I$ . Second, it can be shown that the second-order corrections to  $\varphi$  and  $\theta$  vanish on average. Finally, we assume that the correlations in  $\delta\varphi$  are Gaussian, which allows us to write

$$\langle e^{-i\varphi(t)} \rangle = e^{-i\varphi_0} \langle e^{-i\delta\varphi(t)} \rangle = \left\langle \exp\left(-\frac{\delta\varphi^2(t)}{2}\right) \right\rangle, \quad (48)$$

where we have used  $\varphi_0 = 0$ . In the weak-coupling limit (see Sec. II E) and in the extreme quantum limit,  $T = 0$ , we find for the time-averaged mean square fluctuations

$$\langle \delta\varphi^2(t) \rangle = \int_0^{\omega_c} \frac{d\omega}{\pi} S_{\varphi\varphi}(\omega) \approx 2\alpha \ln \frac{\omega_c}{\Omega_0}, \quad (49)$$

where the cutoff frequency  $\omega_c$  is taken to be larger than all frequency scales (except  $\tau_{RC}^{-1}$ ), and  $\alpha$  is the coupling strength between the ring and external circuit [see Eq. (24)]. In the limit  $\omega_c \gg \Omega_0$ , we can neglect  $\langle \delta\theta^2(t) \rangle = \int_0^{\omega_c} (d\omega/\pi) S_{\theta\theta}(\omega)$  against  $\langle \delta\varphi^2(t) \rangle$ . We insert  $\langle \delta\varphi^2(t) \rangle$  and  $\sin \theta_0 = \Delta_0/\Omega_0$  into  $\langle (\sin \theta) e^{-i\varphi} \rangle$ , and observe that

$$\frac{1}{2} \text{Re} \mathcal{J} = \frac{\hbar c}{2} \frac{\partial \Delta_0}{\partial \Phi}. \quad (50)$$

Put together, we obtain the noise-averaged current in the ring

$$\langle I(t) \rangle = -\frac{\hbar c}{2} \frac{\partial \Delta_0}{\partial \Phi} \frac{\Delta_0}{\Omega_0} \left( \frac{\Omega_0}{\omega_c} \right)^\alpha. \quad (51)$$

Equation (51) is a key result of this work. For  $\alpha \ll 1$ , corresponding to weak coupling between the ring and the external circuit, the power law for the persistent current obtained in Eq. (51), as well as the exponent  $\alpha$ , Eq. (24), are the same as those obtained when the external circuit is treated quantum mechanically as well. In a recent work,<sup>1</sup> the authors in collaboration with Ponomarenko have shown that if the external circuit is represented by a transmission line, the persistent current at resonance,  $\varepsilon = 0$ , has for  $\alpha < 1$  the power law behavior

$$I(\varepsilon = 0) \propto \left( \frac{\Delta_0}{\omega_c} \right)^{\alpha/(1-\alpha)}. \quad (52)$$

For a very small coupling parameter,  $\alpha \ll 1$ , the Bethe ansatz result [Eq. (52)], goes over to the power law of Eq. (51). Thus the simplified discussion presented here leads at least in the weak-coupling limit to the same result as that obtained in Ref. 1.

We emphasize that the persistent current is a property of the ground state of a system. In our case, the persistent current is, however, carried by only a part of the system. Due to the coupling to the external circuit this subsystem is subject to fluctuations, which even at zero temperatures suppress the persistent current. If we keep the capacitances fixed, Eqs. (51) and (52) give a persistent current that decreases with increasing external resistance  $R$ . The spectral density of the voltage fluctuations across the ring-dot system increases with  $R$  [see Eq. (6)]. We next characterize the fluctuations of the ring-dot subsystem in more detail.

#### IV. PHASE-DIFFUSION TIMES

In this work we are concerned with the effect of an external circuit on the ground state properties of the system and in

particular on the persistent current of the ring-dot subsystem. In transport experiments that reveal phase-coherent contributions or in experiments that investigate the evolution of an initial state, we can describe the effect of an environment with the help of dephasing rates. On the other hand, it is not *a priori* clear that the coherence properties of the ground state of a system can be equally expressed in terms of dephasing rates. After all the persistent current investigated here exists quite independently of how long we let the system evolve. Here we consider the state in the ring and investigate its evolution away from an initial state. Below we show that this evolution is diffusive at least for times short enough such that we remain within the limit of validity of the calculation.

We have seen in Sec. II E that the dynamics of the ring strongly affects the spectral densities  $S_{\varphi\varphi}(\omega)$  and  $S_{\theta\theta}(\omega)$  in the vicinity of the characteristic frequency  $\Omega_0$ . The characteristic frequency describes the free dynamics of a ring that is disconnected from the external circuit, described by the Hamiltonian  $\hat{H}_{ring}$  [Eq. (9)], without the time-dependent terms  $\delta\varepsilon(t)$  and  $\nu(t)$ ,

$$\hat{H}_0 = \frac{\hbar \varepsilon_0}{2} \sigma_z - \frac{\hbar \Delta_0}{2} \sigma_x. \quad (53)$$

This Hamiltonian can be viewed as describing a spin in a magnetic field of strength proportional to  $(\varepsilon_0^2 + \Delta_0^2)^{1/2} = \Omega_0$ , forming an angle  $\theta_0$  with the  $z$  axis. The angle  $\theta_0$  has been defined in Sec. II B and is related to  $\varepsilon_0$  and  $\Delta_0$  by  $\cot \theta_0 = -\varepsilon_0/\Delta_0$ . The eigenstates of the time-independent Hamiltonian  $\hat{H}_0$  are the *ground state*  $\psi_- = (\cos \theta_0/2, \sin \theta_0/2)$ , with eigenvalue  $-\hbar \Omega_0/2$ , and the *excited state*  $\psi_+ = (-\sin \theta_0/2, \cos \theta_0/2)$  with eigenvalue  $\hbar \Omega_0/2$ . The evolution due to  $\hat{H}_0$  of a spin prepared in a state that is not an eigenstate corresponds to a rotation of the expectation value of the spin about the direction of the magnetic field with a frequency  $\Omega_0$ . Let us now return to the full problem in which polarization fluctuations modify the free evolution of the decoupled system. In the weak-coupling limit, we are interested in the time evolution that is long compared to  $\Omega_0^{-1}$ . Therefore, we switch to the ‘rotating frame’ picture, where the wave function  $\psi(t)$  of the ring, Eq. (13), reads  $\psi_R(t) \equiv \exp(i\hat{H}_0 t/\hbar) \psi(t)$ . We consider the projection  $c_\pm(t) \equiv \langle \psi_\pm | \psi_R(t) \rangle$  of the wave function  $\psi_R(t)$  onto the states  $\psi_\pm$ . If the state  $\psi(t)$  evolves under the influence of the time-independent Hamiltonian  $\hat{H}_0$ , the projections  $c_\pm(t)$  are constant in time. The moduli of the projections averaged over the noise,  $\langle |c_\pm(t)|^2 \rangle$ , are also independent of time if the evolution is determined by  $\hat{H}_{ring}$  which includes the fluctuating potential. The phase of the projections  $c_\pm(t)$ , however, shows diffusive behavior. On the average, we have  $\langle |c_\pm(t) - c_\pm(0)|^2 \rangle \sim t/\tau_\pm$  for sufficiently long times  $t$ . We shall see below that  $\tau_-$  is related to the phase diffusion of the *global* phase  $\chi$ , whereas  $\tau_+$  is related to the diffusion of the *internal* phases  $\varphi$  and  $\theta$  [see Eq. (13)]. It is a known feature of two-level systems that they exhibit two distinct dephasing times.<sup>29,30</sup> The difference between  $\tau_-$  and  $\tau_+$  is particularly pronounced in the low-temperature limit. The phase breaking

time  $\tau_-$  diverges at zero temperature, whereas the time  $\tau_+$  saturates to a finite value for temperatures  $kT$  below  $\hbar\Omega_0$ .

We expand the wave function  $\psi_R$  to first order in  $\delta\varepsilon$ , that is, to first order in  $\delta\varphi$ ,  $\delta\theta$ , and  $\delta\chi$ ,

$$\begin{aligned} \psi_R(t) = & \left[ 1 + i \left( \frac{\delta\chi(t)}{2} - \frac{\varepsilon_0}{\Omega_0} \frac{\delta\varphi(t)}{2} \right) \right] \psi_- \\ & + \left[ \frac{\delta\theta(t)}{2} - i \frac{\Delta_0}{\Omega_0} \frac{\delta\varphi(t)}{2} \right] e^{i\Omega_0 t} \psi_+. \end{aligned} \quad (54)$$

From this expression, we immediately obtain the projections  $c_-(t) = 1 + i[\delta\chi(t)/2 - (\varepsilon_0/\Omega_0)\delta\varphi(t)]$  and  $c_+(t) = (1/2)[\delta\theta(t) - i(\Delta_0/\Omega_0)\delta\varphi(t)]\exp(i\Omega_0 t)$ . As the averages  $\langle\delta\varphi^2(t)\rangle$ ,  $\langle\delta\theta^2(t)\rangle$ , and  $\langle\delta\chi^2(t)\rangle$  as well as all the ‘‘crossed’’ averages of the type  $\langle\delta\varphi(t)\delta\theta(t)\rangle$  are constant in  $t$ , the moduli  $\langle|c_\pm(t)|^2\rangle$  are indeed independent of time. The mean squared displacement of the projections  $c_\pm(t)$  read, respectively,

$$\langle|c_-(t) - c_-(0)|^2\rangle = \int \frac{d\omega}{2\pi} \sin^2 \frac{\omega t}{2} \left[ S_{\chi\chi}(\omega) + \frac{\varepsilon_0^2}{\Omega_0^2} S_{\varphi\varphi}(\omega) \right], \quad (55)$$

$$\begin{aligned} \langle|c_+(t) - c_+(0)|^2\rangle = & \int \frac{d\omega}{2\pi} \sin^2 \frac{\omega t}{2} \\ & \times \left[ S_{\theta\theta}(\omega + \Omega_0) + \frac{\Delta_0^2}{\Omega_0^2} S_{\varphi\varphi}(\omega + \Omega_0) \right]. \end{aligned} \quad (56)$$

The long time behavior of Eq. (55) is dominated by the frequencies near  $\omega=0$ . The spectral density  $S_{\varphi\varphi}(\omega)$  vanishes like  $\omega^2$  for finite temperatures or even like  $|\omega|^3$  in the zero-temperature limit. The spectral density  $S_{\chi\chi}(\omega)$ , however, has a pole at  $\omega=0$ . For finite temperatures the pole is of order  $\omega^{-2}$  [see Eq. (35)], entailing a long time behavior of the type  $\langle|c_-(t) - c_-(0)|^2\rangle \sim t/\tau_-$ , with a characteristic phase-diffusion time

$$\tau_- = \frac{\hbar}{2\pi\alpha kT} \frac{\Omega_0^2}{\varepsilon_0^2}. \quad (57)$$

We stress again that  $\tau_-$  determines the diffusion time of the global phase  $\chi$  as it is given in terms of the spectral function  $S_{\chi\chi}(\omega)$ . In the quantum limit  $T=0$ , the phase diffusion time  $\tau_-$  diverges. Furthermore, it follows from Eq. (57) that  $\tau_-$  is *tunable*, since  $\varepsilon_0$  may be varied by an external dc bias. In particular, at resonance  $\varepsilon_0=0$ , the phase-diffusion time  $\tau_-$  diverges for any temperature.

The long time behavior of Eq. (56), on the other hand, is determined by the frequencies near  $\Omega_0$ . In the vicinity of this characteristic frequency,  $S_{\varphi\varphi}(\Omega_0 + \omega)$  as well as  $S_{\theta\theta}(\Omega_0 + \omega)$  show a  $\omega^{-2}$  behavior *at finite as well as at zero temperature*, which is cut off by the relaxation rate  $\Gamma$ , defined in Eq. (23) at very small frequencies  $\omega \sim \Gamma \ll \Omega_0$ . In summary, we have, for  $|\omega| \ll \Omega_0$

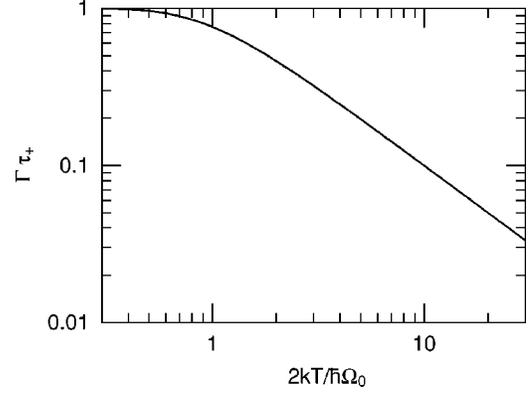


FIG. 5. The characteristic phase diffusion time  $\tau_+$  normalized by the decay rate  $\Gamma$ , defined in Eq. (23), as a function of  $2kT/\hbar\Omega_0$  in a logarithmic scale. For high temperatures,  $\tau_+$  is inversely proportional to the temperature  $T$ . For low temperatures,  $kT \ll \hbar\Omega_0$ , it saturates at a value  $\tau_+ = \Gamma^{-1}$ .

$$S_{\varphi\varphi}(\Omega_0 + \omega) \approx \frac{2\pi\alpha\Omega_0 \coth(\hbar\Omega_0/2kT)}{\omega^2 + (\Gamma/2)^2} \quad (58)$$

and  $S_{\theta\theta}(\Omega_0 + \omega) \approx (\Delta_0^2/\Omega_0^2)S_{\varphi\varphi}(\Omega_0 + \omega)$ . The time evolution of Eq. (56) for times much larger than the inverse of the characteristic frequency  $\Omega_0$ , yet smaller than the inverse of the relaxation rate  $\Gamma$ , is therefore linear in time with a characteristic time  $\tau_+$ , where

$$\tau_+ = \frac{1}{\Gamma} \tanh \frac{\hbar\Omega_0}{2kT}. \quad (59)$$

Note that Eq. (59) holds for finite temperatures as well as in the quantum limit. The phase-diffusion time  $\tau_+$  is inversely proportional to  $T$  at high temperatures,

$$\tau_+ = \frac{1}{\Gamma} \frac{\hbar\Omega_0}{2kT} \quad (kT \gg \hbar\Omega_0), \quad (60)$$

just as the other characteristic time  $\tau_-$ . In the low-temperature or quantum limit, however, it *saturates* to a value

$$\tau_+^{(0)} = \frac{1}{\Gamma} \quad (kT \ll \hbar\Omega_0) \quad (61)$$

(see also Fig. 5). The crossover from high-temperature behavior to the quantum limit behavior takes place at  $kT \sim \hbar\Omega_0$ . We point out that  $\tau_+$  is related to the spectral densities  $S_{\varphi\varphi}(\omega)$  and  $S_{\theta\theta}(\omega)$  of the internal phases  $\varphi$  and  $\theta$ . This indicates that there is a relation between dephasing (at finite temperature) and the reduction of the persistent current at zero temperature, which is determined by  $S_{\varphi\varphi}(\omega)$ .

We point out that Eq. (55) and Eq. (56) do not hold for arbitrarily long times. In reality the mean square displacements  $\langle|c_\pm(t) - c_\pm(0)|^2\rangle$  are bounded since the wave function  $\psi_R(t)$  is normalized to 1. The fact that  $\langle|c_-(t) - c_-(0)|^2\rangle$  [Eq. (55)] grows without bounds is an artifact of the linearization of Eqs. (14)–(16) and (20).

In summary, we find two characteristic phase-diffusion times  $\tau_-$  and  $\tau_+$ , related to the projection of the equilibrium

state  $\psi(t)$  onto the ground state and the excited state. The time  $\tau_-$ , associated with the projection on the ground state, is related to the loss of coherence in the global phase  $\chi$  and diverges as the temperature goes to zero. The time  $\tau_+$ , on the other hand, characterizes the loss of coherence in the internal phases  $\varphi$  and  $\theta$ , and saturates to a finite value in the limit of zero temperature. This indicates that even at zero temperature, coupling to the external circuit causes excitations of the ring into the excited state. In the zero-temperature limit these excitations still decay in a finite time.

## V. CONCLUSION

We have investigated the persistent current in a normal metal ring coupled capacitively to a resistive external circuit using a Langevin equation approach. We have shown that the quantum fluctuations in the external circuit suppress the persistent current at zero temperature, thus confirming an earlier Bethe ansatz result.<sup>1</sup> Within the same framework, but at finite temperature, we have derived two characteristic phase-diffusion times  $\tau_-$  and  $\tau_+$ , which are the phase-diffusion times of the projections of the wave function of the ring to the ground state and the excited state, respectively, of a ring that is disconnected from the external circuit. We show that  $\tau_+$  is related to the spectral densities of the ‘‘internal’’ phases  $\varphi$  and  $\theta$  of the wave function of the ring, which are also responsible for the reduction of the persistent current. While  $\tau_-$  diverges in the zero-temperature limit,  $\tau_+$  saturates to a finite value.

The phase-diffusion rates are not needed to obtain the persistent current, the quantity of principal interest. In fact for small systems a direct calculation of the quantity of interest, the persistent current, the conductance, or its fluctuations, without first calculating a dephasing rate, is preferable. In very small systems the dephasing rate is itself a sample specific quantity that is characterized by a distribution.<sup>31</sup> Sample specific rates are clearly no longer a concept that is that useful. On the other hand, the phase-diffusion rates calculated above clearly demonstrate that due to the interaction with the bath the state of the ring-dot system acquires an overlap with the excited state of the decoupled ring-dot system. This overlap, taken as an initial state, decays with time even in the zero-temperature limit with a rate determined by  $\tau_+$ . To obtain this result, we require only that the combined coupled system (ring-dot and external circuit) is in an equilibrium state and not each system separately as is sometimes assumed.<sup>32</sup>

In this work we have used the term ‘‘phase-diffusion times’’ in distinction to ‘‘phase breaking times.’’ In the ground state considered here polarization fluctuations reduce the amplitude of the persistent current but still leave eventually a coherent state in place. This is very different from situations in which an initial coherent superposition evolves into a final incoherent state.<sup>33</sup>

Persistent currents have played a mayor role in the development of mesoscopic physics and continue to be a subject of considerable interest. For a brief review of the status of the persistent current measurements we refer the reader to Ref. 34. While for semiconducting rings theory and experi-

ments seem to agree, for metallic diffusive rings, the fact that theory predicts a persistent current that is considerably smaller than that found in experiments, remains. In metallic diffusive samples one is thus lead to search for effects which enhance rather than reduce the persistent current. It would be very intriguing to think that the polarization fluctuations of an environment could give rise to a persistent current above that found in the decoupled system, for instance, in a situation when the ring arm and dot are not at resonance. For the model investigated here the effect of the bath is, however, always a suppression. In contrast, nonequilibrium radiation has been shown to enhance the persistent current in metallic diffusive samples.<sup>10</sup> Even in a purely classical system nonequilibrium noise can generate circulating currents.<sup>35</sup>

An experimental investigation of the role of polarization fluctuations either for the persistent currents or for the conductance of a mesoscopic system would clearly be very interesting. As shown in our work, bringing nearby gates to a mesoscopic conductor should affect the coherence properties of the system. An experimental situation in which the coupling to gates could be tuned would lead to an additional parameter that would permit introduction of another dimension into the investigation of phase breaking. In fact, a similar investigation for an array of Josephson junctions above a backgate has already been performed.<sup>36</sup>

In semiconductors mesoscopic systems are already formed with the help of gates.<sup>37</sup> Their role in the dephasing of open conductors is discussed in Ref. 31. While the voltage at gates is maintained with high precision at low frequencies, at higher frequencies their impedance is finite. For an experimental investigation of the role of nearby gates on the dephasing process the characterization of the impedance (dynamical conductance) over the relevant range of frequencies is needed. We hope that the future will bring us experiments that combine high precision dynamical conductance measurements with the investigation of the coherence properties of mesoscopic systems.

*Note added in proof:* Our phase-diffusion rates  $\tau_-$  and  $\tau_+$  can be related to the relaxation rate and the dephasing time given by Grifoni, Paladino and Weiss.<sup>39</sup> To find the dephasing rate we write the time evolution of the state  $\Psi_R(t)$  [see Eq. 54] with the help of an overall phase  $\eta(t)$  in the form  $\Psi_R(t) = \exp(-i\eta(t))\Psi_R(0)$ . For times scales over which  $\eta$  remains small we have  $\Psi_R(t) = (1 - i\eta(t))\Psi_R(0)$  or  $\Psi_R(t) - \Psi_R(0) = -i\eta(t)\Psi_R(0)$ . The scalar product of  $\Psi_R(t) - \Psi_R(0)$  with itself is  $|\Psi_R(t) - \Psi_R(0)|^2 = |\eta(t)|^2$  and its expectation value is thus just  $\langle |\eta(t)|^2 \rangle = \langle |c_-(t) - c_-(0)|^2 \rangle + \langle |c_+(t) - c_+(0)|^2 \rangle$ . The dephasing rate is  $1/\tau = \langle |\eta(t)|^2 \rangle / 2t$  and is therefore given by

$$1/\tau = 1/2 \tau_+ + 1/2 \tau_- = \frac{\Gamma}{2} \coth \frac{\hbar \Omega_0}{2kT} + \frac{\pi \alpha kT}{\hbar} \frac{\epsilon_0^2}{\Omega_0^2}. \quad (62)$$

The dephasing rate found here for the ground state of our ring-dot system is identical with the dephasing rate found for the loss of phase coherence of non-equilibrium states in spin-boson systems on short and intermediate time scales<sup>39</sup> [see Eq. (4.5)].

### ACKNOWLEDGMENTS

This work was supported by the Swiss National Science Foundation.

### APPENDIX A: COULOMB INTERACTIONS AND DISPLACEMENT CURRENTS

In this Appendix, we briefly discuss the electrical coupling of the ring and the external circuit. The fluctuations of the charge in the ring and the external circuit are not arbitrary but are connected by the flow of a displacement current between the ring and the external circuit. We can express the coupling between the ring and external circuit in terms of the displacement current. Since the coupling between these subsystems is crucial, we present first a discussion of the displacement currents.

In our model the charges and potentials are related via

$$Q_0 = C_1(V_0 - U_d), \quad (\text{A1})$$

$$Q_d = C_1(U_d - V_0) + C_i(U_d - U_a), \quad (\text{A2})$$

$$Q_a = C_2(U_a - V_\infty) + C_i(U_a - U_d), \quad (\text{A3})$$

$$Q_\infty = C_2(V_\infty - U_a), \quad (\text{A4})$$

where we have introduced the parallel capacitance  $C_i \equiv C_L + C_R$ . In the following, we shall also need the external (serial) capacitance  $C_e^{-1} \equiv C_1^{-1} + C_2^{-1}$ , as well as the parallel and the serial total capacitances  $C \equiv C_i + C_e$  and  $C_0^{-1} \equiv C_i^{-1} + C_e^{-1}$ . The whole structure, that is, the ring together with the external circuit is charge neutral. As a matter of fact, it follows from Eqs. (A1)–(A4) that  $Q_0 + Q_d + Q_a + Q_\infty = 0$ . The ring and the external circuit taken separately, however, do not need to be neutral. The total charge of the ring  $Q_d + Q_a$  is balanced by the charge on the external capacitors  $C_1$  and  $C_2$ ,  $Q_0 + Q_\infty = -(Q_d + Q_a)$ , as follows from Eqs. (A1)–(A4). As the ring and the external circuit do not exchange particles, the charges on the ring and in the external circuit are conserved, and it is more convenient to consider the deviations from a reference state. We denote these deviations by  $\delta Q_d$  on the dot and by  $\delta Q_a$  on the arm, and similarly by  $\delta Q_0$  and  $\delta Q_\infty$  for the external circuit. They obey the relations  $\delta Q_d + \delta Q_a = \delta Q_0 + \delta Q_\infty = 0$ . For time derivatives, it is of course irrelevant whether we consider total charges or deviations.

The current flowing out from the capacitor  $C_1$  is a pure displacement current

$$I_1 = -\dot{Q}_0 = C_1 \frac{\partial}{\partial t} (U_d - V_0), \quad (\text{A5})$$

whereas the currents flowing through the right and the left junction, respectively, are particle currents  $I_{L/R}^p$  augmented by displacement currents

$$I_L = I_L^p + C_L \frac{\partial}{\partial t} (U_d - U_a), \quad (\text{A6})$$

$$I_R = I_R^p + C_R \frac{\partial}{\partial t} (U_d - U_a). \quad (\text{A7})$$

The particle currents are related to the charge on the dot by  $I_L^p + I_R^p = -\dot{Q}_d$ . At each node we have  $I_1 + I_L + I_R = I_2 - (I_L + I_R) = 0$ , and thus  $I_1 + I_2 = 0$ . These equations correspond to the law of current conservation at the nodes of an electrical network.

We can express all electrical quantities of interest in terms of the external bias  $\delta V \equiv V_0 - V_\infty$  and the charge deviation,  $\delta Q_d$ . We combine Eqs. (A2) and (A3) and the condition  $\delta Q_d + \delta Q_a = 0$  to obtain the potential difference inside the ring:

$$U_d - U_a = \frac{C_e}{C} \delta V + \frac{\delta Q_d}{C}. \quad (\text{A8})$$

The charge deviation  $\delta Q_0$  on the capacitor  $C_1$  is found from Eqs. (A1), (A4), and (A8),

$$\delta Q_0 = C_0 \delta V - \frac{C_0}{C_i} \delta Q_d, \quad (\text{A9})$$

where we have used the identity

$$\frac{C_e}{C} = \frac{C_0}{C_i}, \quad (\text{A10})$$

whence we obtain for the current flowing in the external circuit

$$I_1 = \frac{C_0}{C_i} \dot{Q}_d - C_0 \delta \dot{V}. \quad (\text{A11})$$

In the circuit containing a current noise source (see Fig. 2), the current  $I_1$  flowing out of the circuit augmented by the current  $\delta I$  coming from the noise source must equal the current through the resistor  $I = \delta V/R$ , namely,

$$\delta V/R = I_1 + \delta I. \quad (\text{A12})$$

Together with Eq. (A11), this gives Eq. (1).

### APPENDIX B: THE CIRCULATING CURRENT

The equilibrium persistent current  $I$  is a quantum and statistical average that can be obtained from the derivative of the free energy  $I = -c \partial F / \partial \Phi$ . Some discussion is required, if we are, as in Ref. 1, concerned with current fluctuations. Naively, we might want to investigate the fluctuations of the persistent current by considering the second-order derivative of the thermodynamic potential. However, such a procedure works only if the observable of interest commutes with the Hamiltonian. Moreover, as pointed out above, the true and physically relevant currents is the total current (particle current plus displacement current). Whereas the displacement current needs not to be considered as long as we are interested in average quantities only, this is not true, if we consider fluctuations of the current. For the model considered here, we can derive expressions for the current operators that are particle operators weighted according to the distribution

of the geometrical capacitances of the system.

Consider first for a moment the electrically isolated loop. In this case the capacitances to the exterior circuit are  $C_1 = C_2 = 0$ . From Eq. (A2) we find  $Q_d = C_i(U_d - U_a)$  and thus we can also write the currents through the left and right junctions, Eqs. (A6) and (A7), in terms of the charge on the dot as,  $I_L = I_L^p + (C_L/C_i)dQ_d/dt$  and  $I_R = I_R^p + (C_R/C_i)dQ_d/dt$ . Using particle conservation,  $I_L^p + I_R^p - dQ_d/dt = 0$ , to eliminate the time derivative of the charge we find immediately that the current *circulating* in this loop is  $\hat{I}_c \equiv \hat{I}_L = -\hat{I}_R$  with

$$\hat{I}_c = \frac{C_R \hat{I}_L^p - C_L \hat{I}_R^p}{C_i}, \quad (\text{B1})$$

where  $C_i = C_L + C_R$ . The circulating current is thus in general not determined by the particle currents but by an average of these currents weighted according to the Coulomb interaction (capacitance ratios). Expressions of this type are familiar from the dynamic transport through double barriers, but seem to be novel for persistent currents. We emphasize that Eq. (B1) does not mean that the particle currents can now be calculated from the noninteracting problem. The dynamic particle currents depend on the self-consistent potential distribution. For an illustration of this point we refer the reader to Ref. 38 where dynamic current noise spectra for double barriers are compared based on calculations using the particle currents of the noninteracting problem and calculations using the particle currents of the interacting problem.

Next let us consider the system coupled to the external circuit. In this case  $C_1 > 0$  and  $C_2 > 0$ . We can proceed as above. We first express the time derivative of potential difference  $U_d - U_a$  in terms of the time derivative of the charge on the dot and the current  $I_1 = dQ_0/dt = C_1 d(V_0 - U_d)/dt$  [see Eq. (A1)]. Using current conservation  $I_1 = I_L + I_R$  and particle conservation  $I_L^p + I_R^p - dQ_d/dt = 0$ , we can again eliminate the time derivative of the charge  $Q_d$  and find for the currents through the left and right barrier,

$$\hat{I}_L = \hat{I}_c + (C_L/C_i)I_1, \quad (\text{B2})$$

$$\hat{I}_R = -\hat{I}_c + (C_R/C_i)I_1, \quad (\text{B3})$$

with  $\hat{I}_c$  as given in Eq. (B1). The current  $I_1$ , which can be induced with the help of the external circuit, is divided into the two branches of the ring-dot sample also according to a capacitance ratio.

What is observed in a measurement of the magnetization? It is important to note that the external circuit (see Fig. 1) also forms a current loop and depending on the geometry of the circuit also contributes to the fluctuation of the magnetization. Suppose, the circuit of Fig. 1 is a planar structure in the  $x$ - $y$  plane. Let the ring with the in-line dot enclose an area  $A_r$  and let the external circuit (excluding the ring) enclose an area  $A_e$ . For this circuit the magnetic moment can be viewed as being generated by a current  $I_L$  enclosing the area  $A_1$  and by a current  $I_1$  enclosing the area  $A_e$ . The magnetic moment is thus  $m = (1/c)(I_L A_r - I_1 A_e)$ . Of course the topology of the current distribution matters: If we consider the external circuit to be above the ring-dot structure instead of below as shown in Fig. 1, the magnetization is  $m = (1/c)(-I_R A_r + I_1 A_e)$ . In these two circuits the external circuit gives magnetization contribution into opposite directions. The average magnetization of the two circuits is just  $m = (1/c)I_c A_e$ . A third circuit that permits us to investigate the magnetization of the ring and the external circuit separately is a structure in which the ring lies in the  $x$ - $y$  plane and the external circuit say in the  $x$ - $z$  plane. Then the magnetization is  $m_z = (1/c)I_c A_r$  and  $m_y = (1/c)I_1 A_e$ .  $I_c$  appears as the most natural generalization of the equilibrium persistent current. In Ref. 1 we have investigated the fluctuations of  $I_c$ .

All the above expressions are based on linear relations between currents, charges, and potentials. All the above expressions are thus valid also for operators. We now present a specific expression for the operator of the circulating current used in Ref. 1. Let us consider the particle current operators  $\hat{I}_L^p$  through the left and  $\hat{I}_R^p$  through the right tunnel barrier. In terms of the Hamilton operators  $\hat{H}_L$  and  $\hat{H}_R$  responsible for the tunneling across the left and the right tunnel barrier, respectively, they read  $\hat{I}_{L/R}^p = -i/\hbar[\hat{H}_{L/R}, \hat{Q}_d]$ . In other words, they are equal to the decrease of the charge on the dot per unit of time through the right and left junction. For the two-level system, in a basis in which the Hamiltonian is real [see Eq. (9)] we have

$$\hat{I}_L^p = -\frac{e}{\hbar} \frac{\pm t_L t_R \sin(2\pi\Phi/\Phi_0)\sigma_x + [t_L^2 \pm t_L t_R \cos(2\pi\Phi/\Phi_0)]\sigma_y}{\sqrt{t_L^2 + t_R^2 \pm 2t_L t_R \cos(2\pi\Phi/\Phi_0)}}, \quad (\text{B4})$$

$$\hat{I}_R^p = \frac{e}{\hbar} \frac{\pm t_L t_R \sin(2\pi\Phi/\Phi_0)\sigma_x - [t_R^2 \pm t_L t_R \cos(2\pi\Phi/\Phi_0)]\sigma_y}{\sqrt{t_L^2 + t_R^2 \pm 2t_L t_R \cos(2\pi\Phi/\Phi_0)}}. \quad (\text{B5})$$

Using the particle current operators in Eq. (B1) leads to the operator given by Eq. (42). With the help of this operator we can investigate the average persistent current and the fluctuations [see Eq. (7) of Ref. 1].

- <sup>1</sup>P. Cedraschi, V. V. Ponomarenko, and M. Büttiker, *Phys. Rev. Lett.* **84**, 346 (2000).
- <sup>2</sup>Details of the calculations leading to the results of Ref. 1 can be found in P. Cedraschi and M. Büttiker, *Ann. Phys. (N.Y.)* **289**, 1 (2001) and P. Cedraschi, Ph.D. thesis, University of Geneva (2000).
- <sup>3</sup>F. W. J. Hekking and L. I. Glazman, *Phys. Rev. B* **55**, 6551 (1997).
- <sup>4</sup>M. Oshikawa and A. M. Zagoskin, *Superlattices Microstruct.* **25**, 1177 (1999).
- <sup>5</sup>P. Mohanty, E. M. Q. Jariwala, and R. A. Webb, *Phys. Rev. Lett.* **78**, 3366 (1997).
- <sup>6</sup>D. S. Golubev and A. D. Zaikin, *Phys. Rev. Lett.* **81**, 1074 (1998).
- <sup>7</sup>I. L. Aleiner, B. L. Altshuler, and M. E. Gershenson, *Waves Random Media* **9**, 201 (1999).
- <sup>8</sup>I. L. Aleiner, B. L. Altshuler, and M. E. Gershenson, *Phys. Rev. Lett.* **82**, 3190 (1999).
- <sup>9</sup>D. S. Golubev and A. D. Zaikin, *Phys. Rev. Lett.* **82**, 3191 (1999); A. D. Zaikin and D. S. Golubev, *Physica B* **280**, 453 (2000).
- <sup>10</sup>V. E. Kravtsov and B. L. Altshuler, *Phys. Rev. Lett.* **84**, 33 (2000); Y. Imry, H. Fukuyama, P. Schwab, *Europhys. Lett.* **47**, 608 (1999); A. Zawadowski, J. von Delft, and D. C. Ralph, *Phys. Rev. Lett.* **83**, 2632 (1999); I. L. Aleiner, B. L. Altshuler, Y. M. Galperin, cond-mat/0010228 (unpublished).
- <sup>11</sup>R. Landauer and M. Büttiker, *Phys. Rev. Lett.* **54**, 2049 (1985).
- <sup>12</sup>M. Büttiker, in *SQUID'85, Superconducting Quantum Interference Devices and their Applications*, edited by H. D. Hahlbohm and H. Lübbig (Walter de Gruyter, Berlin, New York, 1985), pp. 529–560.
- <sup>13</sup>M. Büttiker, *Ann. N.Y. Acad. Sci.* **480**, 194 (1986).
- <sup>14</sup>M. Büttiker, *Phys. Rev. B* **32**, 1846 (1985).
- <sup>15</sup>H.-F. Cheung, Y. Gefen, and E. K. Riedel, *IBM J. Res. Dev.* **32**, 359 (1988).
- <sup>16</sup>P. A. Mello, *Phys. Rev. B* **47**, 16 358 (1993).
- <sup>17</sup>M. V. Moskalets, *Physica E (Amsterdam)* **4**, 111 (1999).
- <sup>18</sup>M. T. Liu and C. S. Chu, *Phys. Rev. B* **61**, 7645 (2000); *Solid State Commun.* **114**, 167 (2000).
- <sup>19</sup>P. Cedraschi and M. Büttiker, *J. Phys.: Condens. Matter* **10**, 3985 (1998).
- <sup>20</sup>M. Büttiker and C. A. Stafford, *Phys. Rev. Lett.* **76**, 495 (1996).
- <sup>21</sup>M. Büttiker and C. A. Stafford, in *Correlated Fermions and Transport in Mesoscopic Systems*, edited by T. Martin, G. Montambaux, and J. Trân Thanh Vân (Editions Frontières, Gif-sur-Yvette, 1996), pp. 491–500.
- <sup>22</sup>R. Kotlyar and S. Das Sarma, *Phys. Rev. B* **55**, R10 205 (1997).
- <sup>23</sup>P. Sandstroem and I. V. Krive, *Phys. Rev. B* **56**, 9255 (1997).
- <sup>24</sup>H.-P. Ecker, H. Johannesson, and C. A. Stafford, *J. Low Temp. Phys.* **118**, 475 (2000); cond-mat/0010101 (unpublished).
- <sup>25</sup>K. Kang and S.-C. Shin, *Phys. Rev. Lett.* **85**, 5619 (2000); I. Affleck and P. Simon, cond-mat/0012002 (unpublished).
- <sup>26</sup>Aharonov-Bohm conductance oscillations in open rings with in-line quantum dots have found considerable attention following A. Yacoby, M. Heiblum, D. Mahalu, and H. Shtrikman, *Phys. Rev. Lett.* **74**, 4047 (1995). As an entry to the more recent literature we refer the reader to T. Taniguchi and M. Büttiker, *Phys. Rev. B* **60**, 13 814 (1999); G. Hackenbroich, cond-mat/0006361 (unpublished).
- <sup>27</sup>R. Deblock, Y. Noat, H. Bouchiat, B. Reulet, and D. Mailly, *Phys. Rev. Lett.* **84**, 5379 (2000).
- <sup>28</sup>Weak coupling gives rise to a small coupling coefficient in the Coulomb energy if it is expressed in terms of potentials. If the Coulomb energy is written in terms of the polarization charges weak coupling corresponds to a divergent coupling term in the Coulomb energy.
- <sup>29</sup>L. Tian, L. S. Levitov, C. H. van der Wal, J. E. Mooij, T. P. Orlando, S. Lloyd, C. J. P. M. Harmans, and J. J. Mazo, in *Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics*, edited by I. O. Kulik and R. Ellialtioglu (Kluwer, Netherlands, 2000), pp. 429.
- <sup>30</sup>U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 2000).
- <sup>31</sup>M. Büttiker, in *Quantum Mesoscopic Phenomena and Mesoscopic Devices*, edited by I. O. Kulik and R. Ellialtioglu (Kluwer, Netherlands, 2000), pp. 211.
- <sup>32</sup>U. Gavish, Y. Levinson, and Y. Imry, *Phys. Rev. B* **62**, R10 637 (2000).
- <sup>33</sup>A. O. Caldeira and A. J. Leggett, *Phys. Rev. A* **31**, 1059 (1985).
- <sup>34</sup>P. Mohanty, *Ann. Phys. (Leipzig)* **8**, 549 (1999).
- <sup>35</sup>M. Büttiker, *Z. Phys. B: Condens. Matter* **68**, 161 (1987); Ya. M. Blanter and M. Büttiker, *Phys. Rev. Lett.* **81**, 4040 (1998).
- <sup>36</sup>A. J. Rimberg, T. R. Ho, C. Kurdak, J. Clark, K. L. Campman, and A. C. Gossard, *Phys. Rev. Lett.* **78**, 2632 (1997).
- <sup>37</sup>A. G. Huibers, J. A. Folk, S. R. Patel, C. M. Marcus, C. I. Duruoz, and J. S. Harris, Jr., *Phys. Rev. Lett.* **83**, 5090 (1999).
- <sup>38</sup>Ya. M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 1-166 (2000).
- <sup>39</sup>M. Grifoni, E. Paladino, and U. Weiss, *Eur. Phys. J. B* **10** 719 (1999).