

Critical current and Josephson plasma resonance in the vortex glass phase of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

L. N. Bulaevskii,¹ M. B. Gaifullin,² Yuji Matsuda,² and M. P. Maley¹

¹*Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

²*Institute for Solid State Physics, University of Tokyo, Roppongi 7-22-1, Minato-ku, Tokyo 106, Japan*

(Received 31 October 2000; published 16 March 2001)

We calculate the field dependence of the critical current and of the Josephson plasma resonance (JPR) frequency in the macroscopically uniform vortex glass phase in highly anisotropic layered superconductors and in standard pointlike Josephson junctions when the field is applied perpendicular to the junction. In these calculations we assume that, in the single-vortex pinning regime, at low fields B , vortex positions are weakly adjusted to the Josephson coupling and we account only for the adjustment of the phase difference. Our results are in agreement with experimental data for the JPR frequency measured in the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ superconductor in the field range 0.03–0.6 T after field cooling.

DOI: 10.1103/PhysRevB.63.140503

PACS number(s): 74.72.Hs, 74.60.Ge

The Josephson critical current and Josephson plasma resonance (JPR) measurements in the vortex state of highly anisotropic layered superconductors provide direct information on the c -axis correlations of pancake vortices. The interlayer Josephson energy, the c -axis critical current, and the squared c -axis plasma frequency are proportional to the average critical current density $J_0\mathcal{C}$, where J_0 is the Josephson critical current density in zero magnetic field, $\mathcal{C} = \langle \cos \varphi_{n,n+1}(\mathbf{r}) \rangle$, and $\varphi_{n,n+1}(\mathbf{r})$ is the gauge-invariant phase difference between layers n and $n+1$ induced by pancake vortices. Further, \mathbf{r} is the in-plane coordinate, and $\langle \dots \rangle$ means average over thermal disorder and pinning. Thermal fluctuations and uncorrelated pinning cause misalignment of pancake vortices induced by the magnetic field applied along the c axis. This misalignment results in a nonzero phase difference and in the suppression of the Josephson coupling, critical current, and plasma frequency.¹ Thus dependence of \mathcal{C} on the magnetic field provides information on the c -axis correlations of pancakes and allows us to distinguish between vortex phases with different degrees of these correlations.

It is now well established that the phase diagram in the B - T plane of the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ superconductor consists of regions of the vortex liquid at high magnetic fields and temperatures T , of the Bragg glass with almost crystal-like structure at low B and of the vortex glass at high B and low T . Neutron scattering measurements² show that c -axis correlations are weak in the liquid and in the vortex glass phases, while they are significant in the Bragg glass phase.

The field and temperature dependences of \mathcal{C} in the liquid vortex phase of highly anisotropic superconductors like $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ (Bi-2212) are now well understood both experimentally and theoretically; see Ref. 3 and references therein. Here at high fields \mathcal{C} is small because pancake vortices are strongly disordered along the c axis, forming a pancake liquid due to thermal fluctuations. Therefore, \mathcal{C} and deviations of $\cos \varphi_{n,n+1}(\mathbf{r})$ from \mathcal{C} were calculated using Josephson interlayer coupling as a perturbation with respect to thermal energy. At low magnetic fields, near the critical temperature T_c , a liquid of vortex lines is preserved, as in

less anisotropic superconductors; i.e., here deviations of pancakes from straight lines due to thermal fluctuations are smaller than the intervortex distance.^{4,5} Then perturbation theory with respect to pancake displacements was used⁵ to calculate \mathcal{C} . A similar approach was explored also for the vortex crystal phase, which exists below the melting line.⁶ In all these situations good agreement of theoretical results with experimental data for $J_c(B, T)$ (see Ref. 7) and the JPR frequency was obtained.

The focus of this paper is the vortex glass state which exists at high fields above the second peak (Bragg-to-vortex glass) transition, i.e., in the fields above ≈ 400 G. Recent JPR measurements in the field cooling (FC) mode revealed that a dramatic change of the JPR frequency ω_{pl} occurs when going across either the Bragg-to-vortex glass or the Bragg-to-liquid transition line.⁸ In the vortex glass (VG) phase vortices are strongly disordered along the c axis as observed in neutron scattering and JPR measurements and discussed in Ref. 9. In the VG phase positions of pancakes depend on history. In the following we will consider a VG phase which is macroscopically uniform. Such a vortex phase may be obtained in the FC mode, while field sweeping at low temperatures leads to a Bean critical state with non-uniform concentration of vortices. In the following we will find \mathcal{C} in the strongly disordered macroscopically uniform vortex glass phase at zero temperature and in the presence of a magnetic field with the component B_z along the c axis, and with the in-plane component B_x , assuming that pancake positions are not adjusted to the Josephson interlayer coupling. We anticipate that such a strongly disordered vortex state exists when the effect of pinning is strong in comparison with that of the intervortex interaction, i.e., in the single-vortex pinning regime below the magnetic field B_b^{2D} , which separates single-vortex and 2D collective pinning regimes.^{4,10} At higher fields $B > B_b^{2D}$, in the 2D collective pinning regime, the effect of pinning on vortex positions diminishes and adjustment of vortex positions to the Josephson coupling may become more important as was discussed in Ref. 10. The estimate for the crossover field B_b^{2D} is $B_b^{2D} = \beta_b(U_{pc}/E_0)(\Phi_0/2\pi\xi_{ab}^2)$, where $U_{pc} = \Phi_0 J_c \xi_{ab}/c$ is the

pinning potential for pancakes, J_c is the in-plane critical current, $E_0 = (\Phi_0/4\pi\lambda_{ab})^2 s$, s is the interlayer spacing, ξ_{ab} is the correlation length, and β_b is a numerical parameter of order 10; see Refs. 4 and 10. For $\xi_{ab} = 30 \text{ \AA}$, $\lambda_{ab} = 1700 \text{ \AA}$, and $J_c = 0.5 \times 10^6 \text{ A/cm}^2$ we estimate $B_b^{2D} \approx 1.2 \text{ T}$.

Let us consider first a general approach³ to find \mathcal{C} . At zero temperature equilibrium vortex positions \mathbf{r}_{iv} and the phase difference $\varphi_{n,n+1}(\mathbf{r})$ are determined by minimization of the total free energy, which consists of the magnetic energy of pancakes, $\mathcal{F}_{em}(\mathbf{r}_{iv})$, the pinning energy $\mathcal{F}_{pin}(\mathbf{r}_{iv})$, the energy of intralayer currents, and the Josephson energy with area density $E_J[1 - \cos \varphi_{n,n+1}(\mathbf{r}, \mathbf{r}_{iv})]$. Here \mathbf{r}_{iv} are the pancake coordinates and $E_J = \Phi_0 J_0 / 2\pi c$. We present the phase difference as a sum of that caused by pancakes positioned at coordinates \mathbf{r}_{iv} when $J_0 = 0$ and that which is caused by Josephson screening currents,

$$\varphi_{n,n+1}(\mathbf{r}, \mathbf{r}_{iv}) = \varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}) + \varphi_{n,n+1}^{(r)}(\mathbf{r}). \quad (1)$$

The first term is singular and is given as

$$\varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}) = \sum_{\nu} [\phi_{\nu}(\mathbf{r} - \mathbf{r}_{n\nu}) - \phi_{\nu}(\mathbf{r} - \mathbf{r}_{n+1,\nu})], \quad (2)$$

where $\phi_{\nu}(\mathbf{r})$ is the polar angle of the point \mathbf{r} . The second term in the phase difference, $\varphi_{n,n+1}^{(r)}(\mathbf{r})$, is regular. The equations to be solved to find the equilibrium vortex positions and regular part of the phase difference are

$$\begin{aligned} & \frac{\partial}{\partial \mathbf{r}_{iv}} [\mathcal{F}_{em}(\mathbf{r}_{iv}) + \mathcal{F}_{pin}(\mathbf{r}_{iv})] \\ & + E_J \int d\mathbf{r} \sin[\varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}) + \varphi_{n,n+1}^{(r)}(\mathbf{r})] \frac{\partial \varphi_{n,n+1}^{(v)}(\mathbf{r})}{\partial \mathbf{r}_{iv}} = 0, \end{aligned} \quad (3)$$

$$\begin{aligned} & \sum_m L(n-m) \nabla^2 \varphi_{m,m+1}^{(r)}(\mathbf{r}) \\ & - \frac{1}{\lambda_J} \sin[\varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}) + \varphi_{n,n+1}^{(r)}(\mathbf{r})] = 0. \end{aligned} \quad (4)$$

Here $\lambda_J = \gamma s$ is the Josephson length, γ is the anisotropy ratio, and $L(n)$ is the inductance of layers, $L(n) = (\lambda_{ab}/2s) \exp(-|n|s/\lambda_{ab})$.

One can think that these equations may be solved by use of perturbation theory with respect to the Josephson coupling as discussed in Ref. 10. At the first step, vortex positions $\mathbf{r}_{iv}^{(0)}$ may be found by minimizing the pinning energy and the energy of the magnetic interaction. At this stage $\langle \cos \varphi_{n,n+1}(\mathbf{r}, \mathbf{r}_{iv}^{(0)}) \rangle = 0$, because any regular function $\varphi_{n,n+1}^{(r)}(\mathbf{r})$ can be added to $\varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}^{(0)})$. Next, in the first order of perturbation theory, corrections, $\delta \mathbf{r}_{iv}$, to the vortex positions due to the Josephson coupling may be found by solving Eq. (3) with $\varphi_{n,n+1}^{(r)} = 0$. This determines the contribution

$$\mathcal{C}_{vor} = \langle \cos \varphi_{n,n+1}^{(v)}(\mathbf{r}_{iv}^{(0)} + \delta \mathbf{r}_{iv}) \rangle \quad (5)$$

caused by pancakes adjustment to the Josephson interaction. Then, solving Eq. (4) at $\mathbf{r}_{iv} = \mathbf{r}_{iv}^{(0)}$ and $\varphi_{n,n+1}^{(r)} = 0$ in the second (Josephson) term on the right hand side, we determine adjustment of the phase difference to the Josephson coupling at fixed positions of pancakes. This gives the contribution

$$\begin{aligned} \mathcal{C}_{ph} &= \langle \cos[\varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}^{(0)}) + \varphi_{n,n+1}^{(r)}(\mathbf{r})] \rangle \\ &= - \langle \varphi_{n,n+1}^{(r)}(\mathbf{r}) \sin \varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}^{(0)}) \rangle. \end{aligned} \quad (6)$$

Finally, in the first order in Josephson coupling we obtain $\mathcal{C} = \langle \cos[\varphi_{n,n+1}^{(v)}(\mathbf{r}_{iv}^{(0)} + \delta \mathbf{r}_{iv}) + \varphi_{n,n+1}^{(r)}(\mathbf{r})] \rangle \approx \mathcal{C}_{vor}(B) + \mathcal{C}_{ph}(B)$.

The important point is that in the vortex glass phase, Eqs. (3) and (4) have multiple metastable solutions which differ by the vortex positions \mathbf{r}_{iv} and thus by \mathcal{C} . It is primarily the contribution \mathcal{C}_{vor} which depends strongly on history, while \mathcal{C}_{ph} , as we show in the following, depends mainly on B in the case of a macroscopically uniform vortex state with strong c -axis disorder, while its dependence on vortex positions is quite weak. In the following we calculate the contribution \mathcal{C}_{ph} and compare it with experimental data for the plasma frequency measured in Bi-2212 single crystals obtained in the FC mode to see where the contribution \mathcal{C}_{vor} is important.

To find \mathcal{C}_{ph} we need to solve the nonlinear equation (4) for $\varphi_{n,n+1}^{(r)}(\mathbf{r})$ at given vortex positions, i.e., at given $\varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv})$, Eq. (2). For this we use perturbation theory with respect to the Josephson term and find the solution with logarithmic accuracy. In the first approximation the regular part of the phase difference is determined by the positions of pancakes \mathbf{r}_{iv} according to the equation

$$\sum_m L_{nm} \nabla^2 \varphi_{m,m+1}^{(r)}(\mathbf{r}) - \frac{1}{\lambda_J^2} \sin \varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv}) = 0 \quad (7)$$

and \mathcal{C}_{ph} is given by Eq. (6). The solution of Eq. (7) in the Fourier representation is

$$\begin{aligned} \varphi_{n,n+1}^{(r)}(\mathbf{k}, q) &= -\lambda_J^{-2} k^{-2} L^{-1}(q) [\sin \varphi_{n,n+1}^{(v)}(\mathbf{r}, \mathbf{r}_{iv})]_{\mathbf{k}, q}, \\ L^{-1}(q) &= 2(1 - \cos q) + s^2/\lambda_{ab}^2. \end{aligned} \quad (8)$$

Such a solution becomes incorrect at small k (large distances in the ab plane), because we linearized Eq. (4) in our perturbation approach. To find the distance at which perturbation theory fails we calculate $\langle [\varphi_{n,n+1}^{(r)}(\mathbf{r}) - \varphi_{n,n+1}^{(r)}(0)]^2 \rangle$ using the perturbation result to see at what distance the correction due to Josephson coupling becomes large. We obtain

$$\begin{aligned} & \langle [\varphi_{n,n+1}^{(r)}(\mathbf{r}) - \varphi_{n,n+1}^{(r)}(0)]^2 \rangle \\ &= \frac{1}{2\pi^2 \lambda_J^4} \int d\mathbf{k} k^{-4} Y(\mathbf{k}) [1 - \cos(\mathbf{k}\mathbf{r})], \end{aligned} \quad (9)$$

$$\begin{aligned} Y(\mathbf{k}) &= \int \frac{dq}{2\pi} L^{-2}(q) \langle [\sin \varphi_{n,n+1}^{(v)}(\mathbf{r})]_{\mathbf{k}, q} \\ & \times [\sin \varphi_{n,n+1}^{(v)}(\mathbf{r})]_{-\mathbf{k}, -q} \rangle. \end{aligned} \quad (10)$$

The function $Y(\mathbf{k})$ is nonzero at $k \rightarrow 0$ and its characteristic scale we denote by $1/l_\varphi$. Then $\langle [\varphi_{n,n+1}^{(r)}(\mathbf{r}) - \varphi_{n,n+1}^{(r)}(0)]^2 \rangle$ increases as $(rl_\varphi/\lambda_J^2)^2 \ln(r/l_\varphi)$ at large r . The distance where perturbation theory results become invalid is $R = \lambda_J^2/l_\varphi$. The situation here is similar to the perturbation theory treatment of the vortex lattice in the presence of disorder as discussed by Larkin and Ovchinnikov.¹¹ Strictly speaking, perturbation expansion for Eq. (4) does not work because we used a two-dimensional (2D) solution $\varphi_{n,n+1}^{(v)}$ for the total phase difference as the first step, while the actual solution should be three dimensional. The difference is large at large distances where screening by Josephson currents changes dramatically the two-dimensional solution. However, due to the weak logarithmic nature of the divergence, we still can use perturbation theory, but with a cutoff $R = \lambda_J^2/l_\varphi$ at large distances.

From Eqs. (6) and (8) we get

$$C_{ph} = \frac{1}{\lambda_J^2} \int_{r < R} d\mathbf{r} S_v(\mathbf{r}) \ln \frac{R}{r}, \quad (11)$$

where $S_v(\mathbf{r})$ is given as

$$S_v(\mathbf{r}) = \exp[-F_v(r)] \cos(2\pi s B_x y / \Phi_0). \quad (12)$$

The function $F_v(r)$ is connected to the density correlation function³

$$K(\mathbf{r}) = \langle [\rho_n(\mathbf{r}) - \rho_{n+1}(\mathbf{r})][\rho_n(0) - \rho_{n+1}(0)] \rangle \quad (13)$$

by the relation

$$F_v(r) = \int d\mathbf{R} d\mathbf{R}_1 K(\mathbf{R} - \mathbf{R}_1) \beta(\mathbf{r}, \mathbf{R}) \beta(\mathbf{r}, \mathbf{R}_1). \quad (14)$$

Here $\rho_n(\mathbf{r}) = \sum_v \delta(\mathbf{r} - \mathbf{r}_{iv})$ is the pancake density and $\beta(\mathbf{r}, \mathbf{R}) = \phi_v(\mathbf{r}/2 - \mathbf{R}) - \phi_v(-\mathbf{r}/2 - \mathbf{R})$. The function $K(\mathbf{r})$, Eq. (13), depends only on the density correlations inside the layer and between neighboring layers.

For strong c -axis disorder the characteristic scale of the functions $K(\mathbf{r})$ and $S_v(\mathbf{r})$ is of order of the intervortex distance $a = (\Phi_0/B)^{1/2}$ in the ab plane. Then we get with logarithmic accuracy at $B_x = 0$

$$\frac{J_c(B_z)}{J_c(0)} = \frac{\omega_{pl}^2(B_z)}{\omega_{pl}^2(0)} = C_{ph}(B_z) = \frac{C_0 B_J}{B_z} \ln \frac{B_z}{B_J}, \quad (15)$$

where $B_J = \Phi_0/\lambda_J^2$ and $C_0 = a^{-2} \int d\mathbf{r} S(\mathbf{r})$ depends on pancake positions but is of order unity anyway in the case of strong disorder. This result differs by the logarithmic factor from that found in Ref. 10, Eq. (18), for the case when the adjustment of pancakes to the Josephson coupling is neglected. Taking into account the B_x component in the same way as in Ref. 3, we obtain

$$\frac{\omega_{pl}^2(\mathbf{B})}{\omega_{pl}^2(0)} = C(\mathbf{B}) = \frac{B_J l_\varphi^2}{B_z a^2} \ln \left(\frac{B_z a^2}{B_J l_\varphi^2} \right) f \left(\frac{2\pi s B_x l_\varphi}{a \sqrt{\Phi_0 B_z}} \right), \quad (16)$$

$$f(b) = 2\pi \int_0^\infty dx x \exp[-F_v(x)] J_0(bx),$$

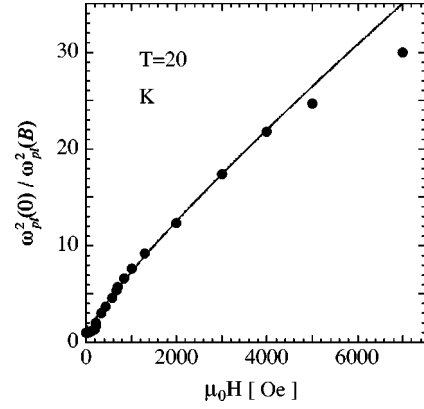


FIG. 1. Dependence of the plasma frequency, $\omega_{pl}(B)$, on magnetic field in Bi-2212 single crystal at 20 K in the field cooling mode. The solid line is the dependence, Eq. (11), with $C_0 = 0.6$ and the parameter $B_J = 20$ G estimated from the zero-field plasma frequency $\omega_{pl}/2\pi = 125$ GHz.

where l_φ characterizes the length scale of the correlation functions $K(\mathbf{r})$ and $S_v(r)$, while $J_0(x)$ is the Bessel function. This gives Eq. (15) at $l_\varphi^2 = C_0 a^2$ and $B_x = 0$. The small parameter for our perturbative approach is a^2/λ_J^2 and thus results are valid at $B \gg B_J$.

The experiments were performed on a slightly underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ single crystal ($T_c = 82.5$ K) with dimensions $1.2 \times 0.5 \times 0.03$ mm³ grown by the traveling floating zone method. The magnetization measurement by a superconducting quantum interference device (SQUID) magnetometer showed a clear magnetization step in the high-temperature regime which can be attributed to the first-order melting transition of the vortex lattice. This transition terminates at $T_{cp} \approx 40$ K and the step is followed by the second magnetization peak located at ~ 230 Oe. The JPR is measured by sweeping the microwave frequency ω continuously from 20 GHz to 180 GHz using backward-wave oscillators in magnetic fields applied parallel to the c axis.¹² For this crystal $\omega_{pl}(0) = 125$ GHz at $T = 0$, corresponding to the out-of plane London penetration length $\lambda_c = c/\sqrt{\epsilon_c} \omega_{pl} \approx 0.011$ cm, taking the high-frequency dielectric constant $\epsilon_c = 11$. Then, taking the in-plane penetration length λ_{ab} in the interval 1700–2000 Å, we obtain the anisotropy parameter $\gamma = \lambda_c/\lambda_{ab}$ (550–650) and B_J in the interval 19–27 G.

All experiments have been performed under the FC condition. In the FC mode, the system is in equilibrium or at worst is trapped in a metastable state below the irreversibility line. We expect that such a state should be much closer to equilibrium compared to the state obtained in the field sweeping condition (FS). In fact, while the resonance frequency below the irreversibility line did not change at all with time in more than 48 h in the FC mode, it increases gradually with time in the FS mode. Moreover, the resonance lines in the FS mode are much broader than those in the FC mode, indicating a quite spatially inhomogeneous vortex state typical of the Bean critical state.

The results of our measurements together with theoretical curve at $C_0 = 0.6$ and $B_J = 20$ G are shown in Fig. 1. We see that in the field interval 0.03–0.6 T agreement is quite good,

but at higher fields deviations become noticeable, indicating a change of the correlation function $K(\mathbf{r})$ and/or adjustment of pancakes to Josephson coupling at high B , which lead to a slight increase of \mathcal{C} with B in comparison with our model. Measurements of the JPR frequency as a function of the parallel field component may distinguish between these two cases. The field 0.6 T, above which deviations become noticeable, is in reasonable agreement with our estimate for the field B_b^{2D} separating single-vortex and 2D collective pinning regimes.

The same approach allows us to calculate the Josephson coupling energy in a standard pointlike Josephson junction in the presence of a magnetic field applied perpendicular to the junction. One can assume that the positions of Abrikosov vortices in the electrodes are determined mainly by uncorrelated pinning inside the electrodes and that these vortices do not adjust to weak Josephson coupling. Then, the Josephson energy is $E_J = (\Phi_0 J_0 / 2\pi c) \mathcal{C}_{ph} S$ where \mathcal{C}_{ph} is given by Eq. (15) when the area S of the junction is of order λ_J^2 , while for $S \ll \lambda_J^2$ one should replace $\ln(B/B_J)$ with $\ln(S/\Phi_0)$ in Eq. (15). The results of such an approach are in agreement with measurements of the activation energy of resistivity in Nb pointlike junctions at low temperatures.¹³ The activation en-

ergy for phase slips leading to dissipation in pointlike junctions is $E_J(B)$. Using Eq. (15) we fit the dependence of the activation energy, $E_J(B)$, shown in Fig. 3 of Ref. 13, with parameter $C_0 = 1.4$ at $B \gg B_J$. For these samples $B_J \approx 0.15$ G, $\lambda_J \approx 12$ μm , and $S \approx \lambda_J^2$.

In summary, we have calculated the field dependence of the interlayer critical current and of the JPR frequency in the macroscopically uniform vortex glass state with strong c -axis disorder. Our main result, Eq. (15), gives a good description of experimental data for the field dependence of the JPR frequency observed in the FC mode below 0.6 T in a Bi-2212 single crystal. This means that in this field interval the positions of pancakes are very weakly adjusted to the Josephson coupling even in the field cooling mode. We associate this interval with the single-vortex pinning regime. We show that this approach describes also the field dependence of the Josephson critical current in standard pointlike Josephson junctions when the magnetic field is applied perpendicular to the junction.

We thank A.E. Koshelev and V.M. Vinokur for useful discussions. The work in Los Alamos was supported by the U.S. DOE.

¹L.N. Bulaevskii, M.P. Maley, and M. Tachiki, Phys. Rev. Lett. **74**, 801 (1995).

²R. Cubitt and E.M. Forgan, Nature (London) **365**, 407 (1993).

³A.E. Koshelev, Phys. Rev. Lett. **77**, 3901 (1996); A.E. Koshelev, L.N. Bulaevskii, and M.P. Maley, *ibid.* **81**, 902 (1998); A.E. Koshelev and L.N. Bulaevskii, Phys. Rev. B **60**, R3743 (1999); Y. Matsuda, M.B. Gaifullin, K. Kumagai, K. Kadowaki, and T. Mochiku, Phys. Rev. Lett. **75**, 4512 (1995); A.E. Koshelev, L.N. Bulaevskii, and M.P. Maley, Phys. Rev. B **62**, 14 403 (2000).

⁴G. Blatter *et al.*, Phys. Rev. B **54**, 72 (1996).

⁵L.N. Bulaevskii, A.E. Koshelev, V.M. Vinokur, and M.P. Maley, Phys. Rev. B **61**, R3819 (2000).

⁶A.E. Koshelev and L.N. Bulaevskii, Physica C **341-348**, 1503 (2000).

⁷T. Yasuda, T. Uchiyama, T. Fukami, T. Aomine, and S. Takano, Supercond. Sci. Technol. **9**, A170 (1996).

⁸M.B. Gaifullin, Yu. Matsuda, N. Chikumoto, J. Shimoyama, and K. Kishio, Phys. Rev. Lett. **84**, 2945 (2000).

⁹A.E. Koshelev and V.M. Vinokur, Phys. Rev. B **57**, 8026 (1998).

¹⁰A.E. Koshelev, L.I. Glazman, and A.I. Larkin, Phys. Rev. B **53**, 2786 (1996).

¹¹A.I. Larkin and Yu.N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

¹²M.B. Gaifullin, Yu. Matsuda, N. Chikumoto, J. Shimoyama, K. Kishio, and R. Yoshizaki, Phys. Rev. Lett. **83**, 3928 (1999).

¹³D.H. Kim, K.E. Gray, and J.H. Kang, Phys. Rev. B **45**, 7563 (1992).