# Continuous quantum measurement of two coupled quantum dots using a point contact: A quantum trajectory approach

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We obtain the finite-temperature unconditional master equation of the density matrix for two coupled quantum dots (CQD's) when one dot is subjected to a measurement of its electron occupation number using a point contact (PC). To determine how the CQD system state depends on the actual current through the PC device, we use the so-called quantum trajectory method to derive the zero-temperature conditional master equation. We first treat the electron tunneling through the PC barrier as a classical stochastic point process (a quantum-jump model). Then we show explicitly that our results can be extended to the quantum-diffusive limit when the average electron tunneling rate is very large compared to the extra change of the tunneling rate due to the presence of the electron in the dot closer to the PC. We find that in both quantum-jump and quantum-diffusive cases, the conditional dynamics of the CQD system can be described by the stochastic Schrödinger equations for its conditioned state vector if and only if the information carried away from the CQD system by the PC reservoirs can be recovered by the perfect detection of the measurements.

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## I. INTRODUCTION

The origins and mechanisms of decoherence (dephasing) for quantum systems in condensed-matter physics have attracted much attention recently due to a number of studies in nanostructure mesoscopic systems[1–5] and various proposals for quantum computers.[6–9] One of the issues is the connection between decoherence and quantum measurements[10,11] for a quantum system. It was reported in a recent experiment[3] with a ''which-path'' interferometer that Aharonov-Bohm interference is suppressed owing to the measurement of which path an electron takes through the double-path interferometer. A biased quantum point contact (QPC) located close to a quantum dot, which is built in one of the interferometer's arms, acts as a measurement device. The change of transmission coefficient of the QPC, which depends on the electron charge state of the quantum dot, can be detected. The decoherence rate due to the measurement by the QPC in this experiment has been calculated in Refs. 12–16.

A quantum-mechanical two-state system, coupled to a dissipative environment, provides a universal model for many physical systems. The indication of quantum coherence can be regarded as the oscillation or the interference between the probability amplitudes of finding a particle between the two states. In this paper, we consider the problem of an electron tunneling between two coupled quantum dots (CQD's) using a low-transparency point contact (PC) or tunnel junction as a detector (environment) measuring the position of the electron (see Fig. 1). This problem has been extensively studied in Refs. 16–24. The case of measurements by a general QPC detector with arbitrary transparency has also been investigated in Refs. 12–15, 25, and 26. In addition, a similar system measured by a single electron transistor rather than a PC has been studied in Refs. 27,21,19,22,24,28,29, and 30. The influence of the detector (environment) on the measured system can be determined by the reduced density matrix obtained by tracing out the environmental degrees of the freedom in the total, system plus environment, density matrix. The master equation (or rate equations) for this CQD system have been derived and analyzed in Refs. 16 and 14 (here we refer to the rate equations as the first-order differential equations in time for both diagonal and off-diagonal reduced density matrix elements). This (unconditional) master equation is obtained when the results of all measurement records (electron current records in this case) are completely ignored or averaged over, and describes only the ensemble average property for the CQD system. However, if a measurement is made on the system and the results are available, the state or density matrix is a conditional state conditioned on the measurement results. Hence the deterministic, unconditional master equation cannot describe the conditional dynamics of the CQD system in a single realization of continuous measurements that reflects the stochastic nature of an electron tunneling through the PC barrier. Consequently, the conditional master equation

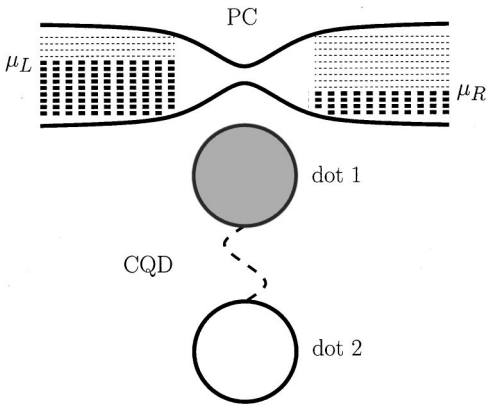


FIG. 1. Schematic representation of two coupled quantum dots (CQD's) when one dot is subjected to a measurement of its electron occupation number using a low-transparency point contact (PC) or tunnel junction. Here $\mu_L$ and $\mu_R$ stand for the chemical potentials in the left and right reservoirs, respectively.

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should be employed. In condensed-matter physics usually many identical quantum systems are prepared at the same time and a measurement is made upon the systems. For example, in nuclear or electron magnetic resonance experiments, generally an ensemble of systems of nuclei and electrons are probed to obtain the resonance signals. This implies that the measurement result in this case is an average response of the ensemble systems. On the other hand, for various proposed condensed-matter quantum computer architectures,[6–9] how to read out physical properties of a single electronic qubit, such as charge or spin at a single electron level, is demanding. This is a nontrivial problem since it involves an individual quantum particle measured by a practical detector in a realistic environment. It is particularly important to take account of the decoherence introduced by the measurements on the qubit as well as to understand how the quantum state of the qubit, conditioned on a particular single realization of measurement, evolves in time for the purpose of quantum computing.

Korotkov[18,20] has obtained the Langevin rate equations for the CQD system. These rate equations describe the random evolution of the density matrix that both conditions and is conditioned by the PC detector output. In his approach, the individual electrons tunneling through the PC barrier were ignored and the tunneling current was treated as a continuous, diffusive variable. More precisely, he considered the change of the output current average over some small time $\tau$, $\langle I\rangle$, with respect to the average current $I_i$, as a Gaussian white-noise distribution. He then updated $\langle I\rangle$ in the density-matrix elements using the new values of $\langle I\rangle$ after each time interval $\tau$. However, treating the tunneling current as a continuous, diffusive variable is valid only when the average electron tunneling rate is very large compared to the extra change of the tunneling rate due to the presence of the electron in the dot closer to the PC. The resulting derivation of the stochastic rate equations is semiphenomenological, based on basic physical reasoning to deduce the properties of the density matrix elements, rather than microscopic.

To make contact with the measurement output, in this paper we present a *quantum trajectory* [31,35–42,28] measurement analysis to the CQD system. We first use the quantum open system approach[31–34] to obtain the unconditional Markovian master equation for the CQD system, taking into account the finite-temperature effect of the PC reservoirs. Particularly, we assume that the transparency of the PC detector is small, in the tunnel-junction limit. Subsequently, we derive microscopically the zero-temperature conditional master equation by treating the electron tunneling through the PC as a classical stochastic point process (also called a *quantum-jump* model).[37,42,28] Generally the evolution of the system state undergoing quantum jumps (or other stochastic processes) is known as a quantum trajectory[31]. Real measurements (for example, the photon number detection) that correspond approximately to the ideal quantum-jump (or point-process) measurement are made regularly in experimental quantum optics. For almost all-infinitesimal time intervals, the measurement result is null (no photon detected). The system in this case changes infinitesimally, but not unitarily. The nonunitary component reflects the changing probabilities for future events conditioned on past null events. At randomly determined times (conditionally Poisson distributed), there is a detection result. When this occurs, the system undergoes a finite evolution, called a *quantum jump*. In reality these point processes are not seen exactly due to a finite frequency response of the circuit that averages each event over some time. Nevertheless, we first take the zero-response time limit and consider the electron tunneling current consisting of a sequence of random $\delta$ function pulses, i.e., a series of stochastic point processes. Then we show explicitly that our results can be extended to the quantum-diffusive limit and reproduce the rate equations obtained by Korotkov.[18,20] We refer to the case studied by Korotkov[18,20] as quantum diffusion, in contrast to the case of quantum jumps considered here. Hence our quantum trajectory approach may be considered as a formal derivation[43] of the rate equations in Refs. 18 and 20. We find in both quantum-jump and quantum-diffusive cases that the conditional dynamics of the CQD system can be described by the stochastic Schrödinger equations[31,35,37,40,42] (SSE's) for the conditioned state vector, provided that the information carried away from the CQD system by the PC reservoirs can be recovered by the perfect detection of the measurements.

This paper is organized as follows. In Sec. II, we sketch the derivation of the finite-temperature unconditional master equation for the QCD system. To determine how the CQD system state depends on the actual current through the PC device, we derive in Sec. III the zero-temperature conditional master equation and the SSE in the quantum-jump model. Then in Sec. IV we extend the results to the case of quantum diffusion and obtain the corresponding conditional master equation and SSE. The analytical results in terms of Bloch sphere variables for the conditional dynamics are presented in Sec. V. Specifically, we analyze in this section the localization rate and mixing rate.[27,21,22] Finally, a short conclusion is given in Sec. VI. The Appendix is devoted to the demonstration of the equivalence between the conditional stochastic rate equations in Refs. 18–20 and those derived microscopically in the present paper.

## II. UNCONDITIONAL MASTER EQUATION FOR THE CQD AND PC MODEL

The appropriate way to approach quantum measurement problems is to treat the measured system, the detector (environment), and the coupling between them microscopically. Following from Refs. 16,18 and 20, we describe the whole system (see Fig. 1) by the following Hamiltonian:

$$\mathcal{H}=\mathcal{H}_{CQD}+\mathcal{H}_{PC}+\mathcal{H}_{coup}, \tag{1}$$

where

$$\mathcal{H}_{CQD}=\hbar[\omega_1 c_1^\dagger c_1+\omega_2 c_2^\dagger c_2+\Omega(c_1^\dagger c_2+c_2^\dagger c_1)], \tag{2}$$

$$\mathcal{H}_{PC}=\hbar\sum_k\ (\omega_k^L a_{Lk}^\dagger a_{Lk}+\omega_k^R a_{Rk}^\dagger a_{Rk})$$

$$+\sum_{k,q}\ (T_{kq}a_{Lk}^\dagger a_{Rq}+T_{qk}^* a_{Rq}^\dagger a_{Lk}), \tag{3}$$

$$\mathcal{H}_{coup}=\sum_{k,q} c_1^\dagger c_1(\chi_{kq}a_{Lk}^\dagger a_{Rq}+\chi_{qk}^* a_{Rq}^\dagger a_{Lk}). \tag{4}$$

$\mathcal{H}_{CQD}$ represents the effective tunneling Hamiltonian for the measured CQD system. For simplicity, we assume strong inner and interdot Coulomb repulsion, so only one electron can occupy this CQD system. We label each dot with an index 1,2 (see Fig. 1) and let $c_i$ ($c_i^\dagger$) and $\hbar\omega_i$ represent the electron annihilation (creation) operator and energy for a single electron state in each dot, respectively. The coupling between these two dots is given by $\hbar\Omega$. The tunneling Hamiltonian for the PC detector is represented by $\mathcal{H}_{PC}$ where $a_{Lk}$, $a_{Rk}$ and $\hbar\omega_k^L$, $\hbar\omega_k^R$ are, respectively, the fermion (electron) field annihilation operators and energies for the left and right reservoir states at wave number $k$. One should not be confused by the electron in the CQD with the electrons in the PC reservoirs. The tunneling matrix element between states $k$ and $q$ in left and right reservoir, respectively, is given by $T_{kq}$. Equation (4), $\mathcal{H}_{coup}$, describes the interaction between the detector and the measured system, depending on which dot is occupied. When the electron in the CQD system is close to the PC (i.e., dot 1 is occupied), there is a change in the PC tunneling barrier. This barrier change results in a change of the effective tunneling amplitude from $T_{kq}\rightarrow T_{kq}+\chi_{kq}$. As a consequence, the current through the PC is also modified. This changed current can be detected, and thus a measurement of the location of the electron in the CQD system is effected.

The total density operator $R(t)$ for the entire system in the interaction picture satisfies

$$\dot{R}_I(t)=-\frac{i}{\hbar}[H_I(t),R_I(0)]$$

$$-\frac{1}{\hbar^2}\int_0^t dt'[H_I(t),[H_I(t'),R_I(t')]]. \tag{5}$$

The dynamics of the entire system is determined by the time-dependent Hamiltonian:[44]

$$H_I(t)=\sum_{k,q}(T_{kq}+\chi_{kq}c_1^\dagger c_1)a_{Lk}^\dagger a_{Rq}e^{i(\omega_k^L-\omega_k^R)t}+\text{H.c.}, \tag{6}$$

where we have treated the sum of the tunneling Hamiltonian parts in $\mathcal{H}_{PC}$ and $\mathcal{H}_{coup}$ as the interaction Hamiltonian $\mathcal{H}_I$, and H.c. stands for Hermitian conjugate of the entire previous term. By tracing both sides of Eq. (5) over the bath (reservoir) variables and then changing from the interacting picture to the Schrödinger picture, we obtain[31–33] the finite-temperature, Markovian master equation for the CQD system:

$$\dot{\rho}(t)=-\frac{i}{\hbar}[\mathcal{H}_{CQD},\rho(t)]+\mathcal{D}[\mathcal{T}_+ +\mathcal{X}_+ n_1]\rho(t)$$

$$+\mathcal{D}[\mathcal{T}_-^* +\mathcal{X}_-^* n_1]\rho(t), \tag{7}$$

where $\rho(t)=\mathrm{Tr}_B R(t)$ and $\mathrm{Tr}_B$ indicates a trace over reservoir variables. In arriving at Eq. (7), we have made the following assumption and approximations: (a) treating the left and right fermion reservoirs in the PC as thermal equilibrium free-electron baths, (b) weak system-bath coupling, (c) small transparency of the PC, i.e., in the tunnel-junction limit, (d) uncorrelated and factorizable system-bath initial conditions, (e) relaxation time scales of the reservoirs being much shorter than that of the system state, (f) Markovian approximation, (g) $|eV|,k_BT\ll\mu_{L(R)}$, and (h) energy-independent electron tunneling amplitudes and density of states over the bandwidth of $\max(|eV|,k_BT)$. Here $k_B$ is the Boltzmann constant, $T$ represents the temperature, $eV=\mu_L-\mu_R$ is the external bias applied across the PC, and $\mu_L$ and $\mu_R$ stand for the chemical potentials in the left and right reservoirs, respectively. In Eq. (7), $n_1=c_1^\dagger c_1$ is the occupation number operator for dot 1. The parameters $\mathcal{T}_\pm$ and $\mathcal{X}_\pm$ are given by

$$|\mathcal{T}_\pm|^2=D_\pm=2\pi e|T_{00}|^2 g_L g_R V_\pm/\hbar, \tag{8a}$$

$$|\mathcal{T}_\pm+\mathcal{X}_\pm|^2=D'_\pm=2\pi e|T_{00}+\chi_{00}|^2 g_L g_R V_\pm/\hbar, \tag{8b}$$

where $D_\pm$ and $D'_\pm$ are the average electron tunneling rates through the PC barrier in positive and negative bias directions at finite temperatures, without and with the presence of the electron in dot 1, respectively. Here the effective finite-temperature external bias potential $eV_\pm$ is given by the following expression:

$$eV_\pm\equiv\frac{\pm eV}{1-\exp[\mp eV/(k_BT)]}. \tag{9}$$

$T_{00}$ and $\chi_{00}$ are energy-independent tunneling amplitudes near the average chemical potential, and $g_L$ and $g_R$ are the energy-independent density of states for the left and right fermion baths. Note that the average electron currents through the PC barrier is proportional to the difference between the average electron tunneling rate in opposite directions. Hence, the average currents $eD=e(D_+-D_-)$ and $eD'=e(D'_+-D'_-)$, following from Eqs. (8) and (9), are temperature independent[45,46] at least for a range of low temperatures $k_BT\ll\mu_{L(R)}$. In addition, the current-voltage characteristic in the linear response region $|eV|\ll\mu_{L(R)}$ is of the same form as that for an Ohmic resistor, though the nature of charge transport is quite different in both cases.

We have also introduced, in Eq. (7), an elegant superoperator[37,28,47–49] $\mathcal{D}$, widely used in measurement theory in quantum optics. Physically the ''irreversible'' part caused by the influence of the environment in the unconditional master equation is represented by the $\mathcal{D}$ superoperator. Generally superoperators transform one operator into another operator. Mathematically, the expression $\mathcal{D}[B]\rho$ means that superoperator $\mathcal{D}$ takes its operator argument $B$, acting on $\rho$. Its precise definition is in terms of another two superoperators $\mathcal{J}$ and $\mathcal{A}$:

$$\mathcal{D}[B]\rho=\mathcal{J}[B]\rho-\mathcal{A}[B]\rho, \tag{10}$$

where

$$\mathcal{J}[B]\rho=B\rho B^\dagger, \tag{11}$$

$$\mathcal{A}[B]\rho=(B^\dagger B\rho+\rho B^\dagger B)/2. \tag{12}$$

The form of the master equation (7), defined through the superoperator $\mathcal{D}[B]\rho(t)$, preserves the positivity of the density matrix operator $\rho(t)$. Such a Markovian master equation is called a Lindblad[50] form.

To demonstrate the equivalence between the master equation (7) and the rate equations derived in Ref. 16, we evaluate the density matrix operator in the same basis as in Ref. 16 and obtain

$$\dot{\rho}_{aa}(t) = i\Omega[\rho_{ab}(t) - \rho_{ba}(t)], \tag{13a}$$

$$\begin{aligned}\dot{\rho}_{ab}(t) = {} & i\varepsilon\rho_{ab}(t) + i\Omega[\rho_{aa}(t) - \rho_{bb}(t)] \\ & - (|\mathcal{X}_T|^2/2)\rho_{ab}(t) + i\,\mathrm{Im}(\mathcal{T}_+^*\mathcal{X}_+ \\ & - \mathcal{T}_-^*\mathcal{X}_-)\rho_{ab}(t).\end{aligned} \tag{13b}$$

Here $\hbar\varepsilon = \hbar(\omega_2 - \omega_1)$ is the energy mismatch between the two dots, $\rho_{ij}(t) = \langle i|\rho(t)|j\rangle$, and $\rho_{aa}(t)$ and $\rho_{bb}(t)$ are the probabilities of finding the electron in dot 1 and dot 2, respectively. The rate equations for the other two density matrix elements can be easily obtained from the relations: $\rho_{bb}(t) = 1 - \rho_{aa}(t)$ and $\rho_{ba}(t) = \rho_{ab}^*(t)$. Compared to an isolated CQD system, the presence of the PC detector introduces two effects to the CQD system. First, the imaginary part of the product of $\mathcal{T}_+^*\mathcal{X}_+ - T_-^*\mathcal{X}_-$ [the last term in Eq. (13b)] causes an effective temperature-independent shift in the energy mismatch between the two dots. Here $(\mathcal{T}_+^*\mathcal{X}_+ - T_-^*\mathcal{X}_+ = \mathcal{T}_-^*\mathcal{X}_-) = \mathcal{T}^*\mathcal{X}$ is a temperature-independent quantity where $\mathcal{T} = \mathcal{T}_+(0)$, i.e., $\mathcal{T}_+$ and $\mathcal{X}_+$ evaluated at zero temperature, respectively. Second, it generates a decoherence (dephasing) rate

$$\Gamma_d = |\mathcal{X}_T|^2/2 \tag{14}$$

for the off-diagonal density matrix elements, where $|\mathcal{X}_T|^2 = |\mathcal{X}_+|^2 + |\mathcal{X}_-|^2$. We note that the decoherence rate comes entirely from the effect of the measurement revealing where the electron in the CQD's is located. If the PC detector does not distinguish which of the dots the electron occupies, i.e., $\mathcal{X}_\pm = 0$, then $\Gamma_d = 0$. The rate equations in Eq. (13) are exactly the same as the zero-temperature rate equations in Ref. 16 if we assume that the tunneling amplitudes are real, $T_{00} = T_{00}^*$ and $\chi_{00} = \chi_{00}^*$. In that case, the last term in Eq. (13b) vanishes and $\Gamma_d = \mathcal{X}^2/2 = (\sqrt{D'} - \sqrt{D})^2/2$. Actually, the relative phase between the two complex tunneling amplitudes may produce additional effects on conditional dynamics of the CQD system as well. This will be shown later when we discuss conditional dynamics. Physically, the presence of the electron in dot 1 raises the effective tunneling barrier of the PC due to electrostatic repulsion. As a consequence, the effective tunneling amplitude becomes lower, i.e., $D' = |\mathcal{T} + \mathcal{X}|^2 < D = |\mathcal{T}|^2$. This sets a condition on the relative phase $\theta$ between $\mathcal{X}$ and $\mathcal{T}$: $\cos\theta < -|\mathcal{X}|/(2|\mathcal{T}|)$.

The dynamics of the unconditional rate equations at zero temperature was analyzed in Ref. 16. Here, following from Eqs. (14), (8), and (9), we find that the temperature-dependent decoherence rate due to the PC thermal reservoirs has the following expression:

$$\frac{\Gamma_d(T)}{\Gamma_d(0)} = \frac{e(V_+ + V_-)}{eV} = \coth\left(\frac{eV}{2k_BT}\right). \tag{15}$$

As expected, $\Gamma_d(T)$ increases with increasing temperature, although the average tunneling current through the PC is temperature independent[45,46] for the same range of low temperatures $k_BT \ll \mu_{L(R)}$. This temperature dependence of the decoherence rate is in fact just the temperature dependence of the zero-frequency noise power spectrum of the current fluctuation in a low-transparency PC or tunnel junction.[51] The CQD system weakly coupled to another finite-temperature environment beside the PC detector was discussed in Ref. 20. However, the influence of the finite-temperature PC reservoirs on the CQD system, presented here, was not taken into account. The finite-temperature decoherence rate of a one-electron state in a quantum dot due to charge fluctuation of a general QPC has been calculated in Ref. 13. In Ref. 26, the temperature-dependent decoherence rate for a two-state system caused by a QPC detector has been discussed specifically in the context of the measurement problem.

## III. QUANTUM-JUMP, CONDITIONAL MASTER EQUATION

So far we have considered the evolution of the reduced density matrix when all the measurement results are ignored, or averaged over. To make contact with a single realization of the measurement records and study the stochastic evolution of the quantum state, conditioned on a particular measurement realization, we derive in this section the quantum-jump, conditional master equation at zero temperature.

The nature of the measurable quantities, such as accumulated number of electrons tunneling through the PC barrier, is stochastic. On average, of course, the same current flows in both reservoirs. However, the current is actually made up of contributions from random pulses in each reservoir, which do not necessarily occur at the same time. They are indeed separated in time by the times at which the electrons tunnel through the PC. In this section, we treat the electron tunneling current consisting of a sequence of random $\delta$-function pulses. In other words, the measured current is regarded as a series of point processes (a quantum-jump model).[37,42,28] The case of quantum diffusion will be analyzed in Sec. IV.

Before going directly to the derivation, we discuss some general ideas concerning quantum measurements. If the system under observation is in a pure quantum state at the beginning of the measurement, then it will still be in a pure conditional state after the measurement, conditioned on the result, provided no information is lost. For example, if the initial normalized state is $|\psi(t)\rangle$, the unnormalized final state given the result $\alpha$ at the end of the time interval $[t, t+dt)$ of the measurement becomes

$$|\tilde{\psi}_\alpha(t+dt)\rangle = M_\alpha(dt)|\psi(t)\rangle, \tag{16}$$

where $\{M_\alpha(t)\}$ represents a set of operators that define the measurements and satisfies the completeness condition

$$\sum_\alpha M_\alpha^\dagger(t) M_\alpha(t) = 1. \tag{17}$$

Equation (17) is simply a statement of conservation of probability. The corresponding unnormalized density matrix, following from Eq. (16), is given by

$$\tilde{\rho}_\alpha(t+dt)=|\tilde{\psi}_\alpha(t+dt)\rangle\langle\tilde{\psi}_\alpha(t+dt)|=\mathcal{J}[M_\alpha(dt)]\rho(t), \tag{18}$$

where $\rho(t)=|\psi(t)\rangle\langle\psi(t)|$ and the superoperator $\mathcal{J}$ is defined in Eq. (11). Of course, if the measurement is made but the result is ignored, the final state will not be pure but a mixture of the possible outcome weighted by their probabilities. Consequently, the unconditional density matrix can be written as

$$\rho(t+dt)=\sum_\alpha \tilde{\rho}_\alpha(t+dt)=\sum_\alpha \Pr[\alpha]\rho_\alpha(t+dt), \tag{19}$$

where $\Pr[\alpha]=\mathrm{Tr}[\tilde{\rho}_\alpha(t+dt)]$ stands for the probability for the system to be observed in the state $\alpha$, and $\rho_\alpha(t+dt)=\tilde{\rho}_\alpha(t+dt)/\Pr[\alpha]$ is the normalized density matrix at time $t+dt$.

Now we proceed to derive the quantum-jump, conditional master equation in the following. Only two measurement operators $M_\alpha(dt)$ for $\alpha=0,1$ are needed for a measurement record that is a point process. For most of the infinitesimal time intervals, the measurement result is $\alpha=0$, regarded as a *null* result. On the other hand, at randomly determined times, there is a result $\alpha=1$, referred as a *detection* of an electron tunneling through the PC barrier. Formally, we can write the current through the PC as

$$i(t)=edN(t)/dt, \tag{20}$$

where $e$ is the electronic charge and $dN(t)$ is a classical point process that represents the number (either zero or one) of tunneling events seen in an infinitesimal time $dt$. We can think of $dN(t)$ as the increment in the number of electrons $N(t)$ in the drain in time $dt$. It is this variable, the accumulated electron number transmitted through the PC, which is used in Refs. 16, 27, and 22. The point process is formally defined by the conditions on the classical random variable $dN_c(t)$:

$$[dN_c(t)]^2=dN_c(t), \tag{21}$$

$$E[dN_c(t)]=\mathrm{Tr}[\tilde{\rho}_{1c}(t+dt)]=\mathrm{Tr}\{\mathcal{J}[M_1(dt)]\rho_c(t)\}=\mathcal{P}_{1c}(t)dt. \tag{22}$$

Here we explicitly use the subscript $c$ to indicate that the quantity to which it is attached is conditioned on previous measurement results, the occurrences (detection records) of the electrons tunneling through the PC barrier in the past. $E[Y]$ denotes an ensemble average of a classical stochastic process $Y$. Equation (21) simply states that $dN_c(t)$ equals either zero or one, which is why it is called a point process. Equation (22) indicates that the ensemble average of $dN_c(t)$ equals the probability (quantum average) of detecting electrons tunneling through the PC barrier in time $dt$. In addition, $dN_c(t)$ is of order $dt$ and obviously all moments (powers) of $dN_c(t)$ are of the same order as $dt$. Note here that the density matrix $\rho_c(t)$ is not the solution of the unconditional reduced master equation, Eq. (25a). It is actually conditioned by $dN_c(t')$ for $t'<t$.

The stochastic conditional density matrix at a later time $t+dt$ can be written as

$$\rho_c(t+dt)=dN_c(t)\frac{\tilde{\rho}_{1c}(t+dt)}{\mathrm{Tr}[\tilde{\rho}_{1c}(t+dt)]}+[1-dN_c(t)]\frac{\tilde{\rho}_{0c}(t+dt)}{\mathrm{Tr}[\tilde{\rho}_{0c}(t+dt)]}. \tag{23}$$

Equation (23) states that when $dN_c(t)=0$ (a null result), the system changes infinitesimally via the operator $M_0(dt)$ and hence $\rho_c(t+dt)=\rho_{0c}(t+dt)$. Conversely, if $dN_c(t)=1$ (a detection), the system goes through a finite evolution induced by the operator $M_1(dt)$, called a *quantum jump*. The corresponding normalized conditional density matrix then becomes $\rho_{1c}(t+dt)$. One can see, with the help of Eq. (20), that in this approach the instantaneous system state conditions the measured current [see Eq. (22)], while the measured current itself conditions the future evolution of the measured system [see Eq. (23)] in a self-consistent manner. It is straightforward to show that the ensemble average of the conditional density matrix equals the unconditional one, $E[\rho_c(t)]=\rho(t)$. Tracing over both sides of Eq. (19) for $\alpha=0,1$, we obtain

$$\mathrm{Tr}[\tilde{\rho}_{0c}(t+dt)]=1-\mathrm{Tr}[\tilde{\rho}_{1c}(t+dt)]. \tag{24}$$

Then taking the ensemble average over the stochastic variables $dN_c(t)$ on both sides of Eq. (23), replacing $E[dN_c(t)]$ by using Eq. (22), and comparing the resultant equation with Eq. (19) completes the verification.

Next we find the specific expression of $\tilde{\rho}_{1c}(t+dt)$ and $\tilde{\rho}_{0c}(t+dt)$ and derive the conditional master equation for the CQD system measured by the PC. If a perfect PC detector (or efficient measurement) is assumed, then whenever an electron tunnels through the barrier, there is a measurement record corresponding to the occurrence of that event; there are no ''misses'' in the count of the electron number. As a result, the information lost from the system to the reservoirs can be recovered using a perfect detector. Here we assume a zero-temperature case for the efficient measurement. At finite temperatures, the electrons can, in principle, tunnel through the PC barrier in both directions. But experimentally the detector might not be able to detect these electron tunneling processes on both sides of the PC barrier. This may result in information loss at finite temperatures. Hence, at zero temperature the unconditional master equation (7) reduces to

$$\dot{\rho}(t)=-\frac{i}{\hbar}[\mathcal{H}_{CQD},\rho(t)]+\mathcal{D}[\mathcal{T}+\mathcal{X}n_1]\rho(t) \tag{25a}$$

$$=-\frac{i}{\hbar}[\mathcal{H}_{CQD}-i\hbar(\mathcal{F}^*\mathcal{X}-\mathcal{F}\mathcal{X}^*)n_1/2,\rho(t)]+\mathcal{D}[\mathcal{X}n_1+\mathcal{T}+\mathcal{F}]\rho(t), \tag{25b}$$

$$\equiv \mathcal{L}\rho(t), \tag{25c}$$

where $\mathcal{D}$ is defined in Eq. (10). Here $\mathcal{F}$ is an arbitrary complex number,[48,49] while we are using $\mathcal{T}$ and $\mathcal{X}$ to represent, respectively, the quantities $\mathcal{T}_+$ and $\mathcal{X}_+$ in Eq. (8) evaluated at zero temperature.

Requiring that the ensemble average of the conditioned density matrix $E[\rho_c(t+dt)]=\rho(t+dt)$ satisfies the unconditional master equation (25) leads to

$$\tilde{\rho}_{0c}(t+dt)+\tilde{\rho}_{1c}(t+dt)=(1+dt\mathcal{L})\rho_c(t). \tag{26}$$

Here we have explicitly used the stochastic Itô calculus[52,53] for the definition of time derivatives as $\dot{\rho}(t)=\lim_{dt\to 0}[\rho(t+dt)-\rho(t)]/dt$. This is in contrast to the definition $\dot{\rho}(t)=\lim_{dt\to 0}[\rho(t+dt/2)-\rho(t-dt/2)]/dt$, used in another stochastic calculus, the Stratonovich calculus.[52,53] Recall that Eq. (22) indicates that $E[dN_c(t)]/dt$ equals the average electron tunneling rate through the PC barrier. From Eq. (8), the electron tunneling rates are $D=|\mathcal{T}|^2$ when $n_1=0$ and $D'=|\mathcal{T}+\mathcal{X}|^2$ when $n_1=1$. From Eq. (22) we thus have the correspondence

$$\mathrm{Tr}[M_1(dt)\rho_c(t)M_1^\dagger(dt)] = \mathrm{Tr}\{\rho_c(t)[\mathcal{T}^*+n_1\chi^*][\mathcal{T}+n_1\chi]\}dt. \tag{27}$$

Also, for Eq. (26) to reproduce the master equation (25b) we must have[48,49]

$$M_1(dt)=\sqrt{dt}(\mathcal{X}n_1+\mathcal{T}+\mathcal{F}) \tag{28}$$

for some arbitrary complex number $\mathcal{F}$. By inspection of Eq. (27) we must have $\mathcal{F}=0$, so that

$$\tilde{\rho}_{1c}(t+dt)=\mathcal{J}[\mathcal{T}+\mathcal{X}n_1]\rho_c(t)dt. \tag{29}$$

Substituting Eq. (29) into (22) yields

$$E[dN_c(t)]=\mathrm{Tr}[\tilde{\rho}_{1c}(t+dt)]=[D+(D'-D)\langle n_1\rangle_c(t)]dt, \tag{30}$$

where $\langle n_1\rangle_c(t)=\mathrm{Tr}[n_1\rho_c(t)]$. The remaining part, except the jump of Eq. (29), on the right hand side of Eq. (26) in time $dt$, corresponds to the effect of a measurement giving a null result on $\rho_c(t)$:

$$\tilde{\rho}_{0c}(t+dt)=\rho_c(t)-dt\times\left\{\mathcal{A}[\mathcal{T}+\mathcal{X}n_1]\rho_c(t)-\frac{i}{\hbar}[\mathcal{H}_{CQD},\rho_c(t)]\right\}, \tag{31}$$

where $\mathcal{A}$ is defined in Eq. (12). The corresponding measurement operator is

$$M_0(dt)=1-dt[(i/\hbar)\mathcal{H}_{CQD}+(1/2)(\mathcal{T}^*+\mathcal{X}^*n_1)(\mathcal{T}+\mathcal{X}n_1)]. \tag{32}$$

Finally, substituting Eqs. (29), (31), (24), and (30) into Eq. (23), expanding, and keeping the terms of first order in $dt$, we obtain the stochastic master equation, conditioned on the observed event in time $dt$:

$$d\rho_c(t)=dN_c(t)\left[\frac{\mathcal{J}[\mathcal{T}+\mathcal{X}n_1]}{\mathcal{P}_{1c}(t)}-1\right]\rho_c(t)+dt\left\{-\mathcal{A}[\mathcal{T}+\mathcal{X}n_1]\rho_c(t)+\mathcal{P}_{1c}(t)\rho_c(t)+\frac{i}{\hbar}[\mathcal{H}_{CQD},\rho_c(t)]\right\}, \tag{33}$$

where

$$\mathcal{P}_{1c}(t)=D+(D'-D)\langle n_1\rangle_c(t). \tag{34}$$

Note that $dN_c(t)$, from Eq. (30), is of order $dt$. Hence terms proportional to $dN_c(t)dt$ are ignored in Eq. (33). Again averaging this equation over the observed stochastic process by setting $E[dN_c(t)]$ equal to its expected value, Eq. (30), gives the unconditional, deterministic master equation (25a). Equation (33) is one of the main results in this paper.

So far we have assumed perfect detection or efficient measurement. In this case, the stochastic master equation for the conditioned density-matrix operator (33) is equivalent to the following stochastic Schödinger equation (SSE) for the conditioned state vector:

$$d|\psi_c(t)\rangle=\left[dN_c(t)\left(\frac{\mathcal{T}+\mathcal{X}n_1}{\sqrt{\mathcal{P}_{1c}(t)}}-1\right)-dt\left(\frac{i}{\hbar}\mathcal{H}_{CQD}+\frac{(\mathcal{T}^*+\mathcal{X}^*n_1)(\mathcal{T}+\mathcal{X}n_1)}{2}-\frac{\mathcal{P}_{1c}(t)}{2}\right)\right]|\psi_c(t)\rangle. \tag{35}$$

This equivalence can be easily verified using the stochastic Itô calculus[52,53]

$$d\rho_c(t)=d(|\psi_c(t)\rangle\langle\psi_c(t)|)=[d|\psi_c(t)\rangle]\langle\psi_c(t)|+|\psi_c(t)\rangle d\langle\psi_c(t)|+[d|\psi_c(t)\rangle][d\langle\psi_c(t)|], \tag{36}$$

and keeping terms up to order $dt$. Since the evolution of the system can be described by a ket state vector, it is obvious that an efficient measurement or perfect detection preserves state purity if the initial state is a pure state. In this description of the SSE, the quantum average is now defined, for example, as $\langle n_1\rangle_c(t)=\langle\psi_c(t)|n_1|\psi_c(t)\rangle$. The unconditional density-matrix operator is equivalent to the ensemble average of *quantum trajectories* generated by the SSE, $\rho(t)=E[|\psi_c(t)\rangle\langle\psi_c(t)|]$, provided that the initial density operator can be written as $\rho(0)=|\psi_c(0)\rangle\langle\psi_c(0)|$.

The interpretation[37] for the measured system state conditioned on the measurement, in terms of gain and loss of information, can be summarized and understood as follows. In order for the system to be continuously described by a state vector (rather than a general density matrix), it is nec-

essary (and sufficient) to have maximal knowledge of its change of state. This requires perfect detection or efficient measurement, which recovers and contains all the information lost from the system to the reservoirs. If the detection is not perfect and some information about the system is *untraceable*, the evolution of the system can no longer be described by a pure state vector. For the extreme case of zero efficiency detection, the information (measurement results at the detector) carried away from the system to the reservoirs is (are) completely ignored, so that the stochastic master equation (33) after being averaged over all possible measurement records reduces to the unconditional, deterministic master equation (25a), leading to decoherence for the system. This interpretation highlights the fact that a density-matrix operator description of a quantum state is only necessary when information is lost irretrievably. The purity-preserving, conditional state evolution for a pure initial state and gradual purification for a nonpure initial state have been discussed in Refs. 18–20 and 23 in the quantum-diffusive limit.

## IV. QUANTUM-DIFFUSIVE, CONDITIONAL MASTER EQUATION

In this section, we extend the results obtained in the previous section and derive the conditional master equation when the average electron tunneling current is very large compared to the extra change of the tunneling current due to the presence of the electron in the dot closer to the PC. This limit is studied and called a ''weakly coupling or responding detector'' limit in Refs. 18 and 20. Here, on the other hand, we will refer to this case as quantum diffusion in contrast to the case of quantum jumps. In the quantum-diffusive limit, many electrons, $(N>[(D'+D)/(D'-D)]^2\gg 1)$, pass through the PC before one can distinguish which dot is occupied. In addition, individual electrons tunneling through the PC are ignored and time averaging of the currents is performed. This allows electron counts, or the accumulated electron number, to be considered as a continuous variable satisfying a Gaussian white-noise distribution. In Refs. 18 and 20 a set of Langevin equations for the random evolution of the CQD system density-matrix elements conditioned on the detector output was presented, based only on basic physical reasoning. In this section, we show explicitly, under the quantum-diffusive limit, that our microscopic approach reproduces[43] the rate equations in Refs. 18 and 20.

In quantum optics, a measurement scheme known as homodyne detection[31,47,48] is closely related to the measurement of the CQD system by a weakly responding PC detector. In both cases, there is a large parameter to allow the photocurrent or electron current to be approximated by a continuous function of time. We will follow closely the derivation of a smooth master equation for homodyne detection given in Ref. 48 (sketched first by Carmichael[31]) for the CQD system.

There are two ideal parameters $\mathcal{T}$ and $\mathcal{X}$ for the CQD system. In the quantum-diffusive limit, we assume $|\mathcal{T}| \gg |\mathcal{X}|$, which is consistent with the assumption, $(D+D') \gg (D'-D)$, made in Refs. 18 and 20 for the weakly coupling or weakly responding PC detector. Consider the evolution of the system over the short-time interval $[t,t+\delta t)$. We relate the three parameters, $\mathcal{X}$, $\mathcal{T}$, and $\delta t$ in our problem as $|\mathcal{X}|^2\delta t \sim \epsilon^{3/2}$, where $\epsilon=(|\mathcal{X}|/|\mathcal{T}|)\ll 1$. This scaling is chosen so that in time $\delta t$, the number of detections (electron counts) with dot 1 being unoccupied scales as $\delta N \sim |\mathcal{T}|^2\delta t \sim \epsilon^{-1/2}\gg 1$. However, the extra change in electron number detections due to the presence of the electron in dot 1 scales as $|\mathcal{X}|^2\delta t \sim \epsilon^{3/2}\ll 1$. To be more specific, the average number of detections, following Eq. (30), up to order of $\epsilon^{1/2}$ is

$$E[\delta N(t)]=|\mathcal{T}|^2\delta t[1+2\epsilon\cos\theta\langle n_1\rangle_c(t)], \tag{37}$$

where $\theta$ is the relative phase between $\mathcal{X}$ and $\mathcal{T}$. The variance in $\delta N$ will be dominated by the Poisson statistics of the current $eD=e|\mathcal{T}|^2$ in time $\delta t$. Since the number of counts in time $\delta t$ is very large, the statistics will be approximately Gaussian. Indeed, it has been shown[47] that the statistics of $\delta N$ are consistent with that of a Gaussian random variable of mean given by Eq. (37) and the variance up to order of $\epsilon^{-1/2}$ is $\sigma_N^2=|\mathcal{T}|^2\delta t$. The fluctuation $\sigma_N^2$ is necessarily as large as expressed here in order for the statistics of $\delta N$ to be consistent with Gaussian statistics. Thus, $\delta N$ can be approximately written as a continuous Gaussian random variable:[52,53]

$$\delta N(t)=\{|\mathcal{T}|^2[1+2\epsilon\cos\theta\langle n_1\rangle_c(t)]+|\mathcal{T}|\xi(t)\}\delta t, \tag{38}$$

where $\xi(t)$ is a Gaussian white noise characterized by

$$E[\xi(t)]=0, \quad E[\xi(t)\xi(t')]=\delta(t-t'). \tag{39}$$

Here $E$ denotes an ensemble average and $\delta(t-t')$ is a delta function. In stochastic calculus,[52,53] $\xi(t)dt=dW(t)$ is known as the infinitesimal Wiener increment. In Eq. (38), the accuracy in each term is only as great as the highest-order expression in $\epsilon^{1/2}$. But it is sufficient for the discussions below.

Although the conditional master equation (33) requires $dN_c(t)$ to be a point process, it is possible, in the quantum-diffusive limit, to simply replace $dN_c(t)$ by the continuous random variable $\delta N_c(t)$, Eq. (38). This is because each jump is infinitesimal, so the effect of many jumps is approximately equal to the effect of one jump scaled by the number of jumps. This can be justified more rigorously as in Ref. 47. Finally, expanding Eq. (33) in power of $\epsilon$, substituting $dN_c(t)\to\delta N_c(t)$, keeping only the terms up to the order $\epsilon^{3/2}$, and letting $\delta t\to dt$, we obtain the conditional master equation

$$\begin{aligned}\dot{\rho}_c(t)=&-\frac{i}{\hbar}[\mathcal{H}_{CQD},\rho_c(t)]+\mathcal{D}[\mathcal{T}+\mathcal{X}n_1]\rho_c(t)\\&+\xi(t)\frac{1}{|\mathcal{T}|}[\mathcal{T}^*\mathcal{X}n_1\rho_c(t)+\mathcal{X}^*\mathcal{T}\rho_c(t)n_1-2\,\mathrm{Re}(\mathcal{T}^*\mathcal{X})\\&\times\langle n_1\rangle_c(t)\rho_c(t)].\end{aligned} \tag{40}$$

Thus the quantum-jump evolution of Eq. (33) has been replaced by quantum-diffusive evolution, Eq. (40). Following the same reasoning in obtaining the SSE, Eq. (35), for the

case of quantum-jump process, we find the quantum-diffusive, conditional master equation (40) is equivalent to the following diffusive SSE:

$$d|\psi_c(t)\rangle = \left[ dt\left( -\frac{i}{\hbar}\mathcal{H}_{CQD} - \frac{|\mathcal{X}|^2}{2}[n_1 - 2n_1\langle n_1\rangle_c(t) + \langle n_1\rangle_c^2(t)] - i\,\mathrm{Im}(\mathcal{T}^*\mathcal{X})n_1 \right)\right] + \xi(t)dt\frac{1}{|\mathcal{T}|}\{\mathcal{T}^*\mathcal{X}n_1 - \mathcal{X}^*\mathcal{T}\langle n_1\rangle_c(t)\}|\psi_c(t)\rangle. \tag{41}$$

This equivalence can be verified using Eq. (36) and keeping terms up to order $dt$. Note, however, in this case[52,53] that terms of order $\xi(t)dt$ are to be regarded as the same order as $dt$, but $[\xi(t)dt]^2=[dW(t)]^2=dt$.

Our conditional master equation by its derivation is formulated in terms of Itô calculus, while the stochastic rate equations in Refs. 18 and 20 are written in a Stratonovich calculus form.[52,53] In contrast to the Stratonovich form of the rate equations, it is easy to see that the ensemble average evolution of our conditional master equation (40) reproduces the unconditional master equation (25a) by simply eliminating the white-noise term using Eq. (39). To show that our quantum-diffusive, conditional stochastic master equation (40) reproduces the nonlinear Langevin rate equations obtained semiphenomenologically in Refs. 18 and 20, we evaluate Eq. (40) in the same basis as for Eq. (13) and obtain

$$\dot{\rho}_{aa}(t) = i\Omega[\rho_{ab}(t) - \rho_{ba}(t)] - \sqrt{8\Gamma_d}\rho_{aa}(t)\rho_{bb}(t)\xi(t), \tag{42a}$$

$$\dot{\rho}_{ab}(t) = i\varepsilon\rho_{ab}(t) + i\Omega[\rho_{aa}(t) - \rho_{bb}(t)] - \Gamma_d\rho_{ab}(t) + \sqrt{2\Gamma_d}\rho_{ab}(t)[\rho_{aa}(t) - \rho_{bb}(t)]\xi(t). \tag{42b}$$

In obtaining Eq. (42), we have made the assumption of real tunneling amplitudes (i.e., $0_\pi$) as in Refs. 16, 18 and 20 in order to be able to compare the results directly. We have also set $\mathcal{X}=\sqrt{2\Gamma_d}$. Again, the ensemble average of Eq. (42) by eliminating the white-noise terms reduces to Eq. (13). To further demonstrate the equivalence, we translate the stochastic rate equations of Refs. 18 and 20 into the Itô formalism and compare them to Eq. (42). This is carried out in the Appendix. Indeed, Eq. (42) is equivalent to the Langevin rate equations in Refs. 18 and 20 for the ''ideal detector.''

## V. ANALYTICAL RESULTS FOR CONDITIONAL DYNAMICS

To analyze the dynamics of a two-state system, such as the CQD system considered here, one can represent the system density-matrix elements in terms of Bloch sphere variables. The Bloch sphere representation is equivalent to that of the rate equations. However, some physical insights into the dynamics of the system can sometimes be more easily visualized in this representation. Denoting the averages of the operators $\sigma_x$, $\sigma_y$, $\sigma_z$ by $x$, $y$, $z$, respectively, the density-matrix operator for the CQD system can be expressed in terms of the Bloch sphere vector $(x,y,z)$ as

$$\rho(t) = [I + x(t)\sigma_x + y(t)\sigma_y + z(t)\sigma_z]/2 \tag{43a}$$

$$= \frac{1}{2}\begin{pmatrix} 1+z(t) & x(t)-iy(t) \\ x(t)+iy(t) & 1-z(t) \end{pmatrix}, \tag{43b}$$

where the operators $I$ and $\sigma_i$ are defined using the fermion operators for the two dots:

$$I = c_2^\dagger c_2 + c_1^\dagger c_1, \tag{44a}$$

$$\sigma_x = c_2^\dagger c_1 + c_1^\dagger c_2, \tag{44b}$$

$$\sigma_y = -ic_2^\dagger c_1 + ic_1^\dagger c_2, \tag{44c}$$

$$\sigma_z = c_2^\dagger c_2 - c_1^\dagger c_1. \tag{44d}$$

It is easy to see that $\mathrm{Tr}\rho(t)=1$, $I$ is a unit operator, and $\sigma_i$ defined above satisfies the properties of Pauli matrices. In this representation, the variable $z(t)$ represents the population difference between the two dots. Especially, $z(t)=1$ and $z(t)=-1$ indicate that the electron is localized in dot 2 and dot 1, respectively. The value $z(t)=0$ corresponds to an equal probability for the electron to be in each dot.

The master equations (25a), (40), and (33), can be written as a set of coupled stochastic differential equations in terms of the Bloch sphere variables. For simplicity, in this section we assume that the tunneling amplitudes are real, i.e., $0=\pi$, and we set $|\mathcal{T}|=\mathcal{T}$ and $\mathcal{X}=\sqrt{2\mathcal{T}_d}$. By substituting Eq. (43a) into Eq. (25a), and collecting and equating the coefficients in front of $\sigma_x$, $\sigma_y$, $\sigma_z$ respectively, the unconditional master equation under the assumption of real tunneling amplitudes is equivalent to the following equations:

$$\frac{d}{dt}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} -\Gamma_d & -\varepsilon \\ \varepsilon & -\Gamma_d \end{pmatrix}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -2\Omega z(t) \end{pmatrix}, \tag{45a}$$

$$\frac{dz(t)}{dt} = 2\Omega y(t). \tag{45b}$$

Similarly for the quantum-diffusive, conditional master equation (40), we obtain

$$\frac{dx_c(t)}{dt} = -\varepsilon y_c(t) - \Gamma_d x_c(t) - \sqrt{2\Gamma_d}z_c(t)x_c(t)\xi(t), \tag{46a}$$

$$\frac{dy_c(t)}{dt} = \varepsilon x_c(t) - 2\Omega z_c(t) - \Gamma_d y_c(t) - \sqrt{2\Gamma_d}z_c(t)y_c(t)\xi(t), \tag{46b}$$

$$\frac{dz_c(t)}{dt} = 2\Omega y_c(t) + \sqrt{2\Gamma_d}[1 - z_c^2(t)]\xi(t). \tag{46c}$$

Again the $c$ subscript is to emphasize that these variables refer to the conditional state. It is trivial to see that Eq. (46) averaged over the white noise reduces to Eq. (45), provided

that $E[x_c(t)]=x(t)$ as well as similar replacements are performed for $y_c(t)$ and $z_c(t)$. The analogous calculation can be carried out for the quantum-jump, conditional master equation (33). We obtain

$$dx_c(t)=dt\left(-\varepsilon y_c(t)-\frac{(D'-D)}{2}z_c(t)x_c(t)\right)-dN_c(t)\times\left(x_c(t)\frac{2\Gamma_d-(D'-D)z_c(t)}{2D+(D'-D)[1-z_c(t)]}\right), \tag{47a}$$

$$dy_c(t)=dt\left(\varepsilon x_c(t)-2\Omega z_c(t)-\frac{(D'-D)}{2}z_c(t)y_c(t)\right)-dN_c(t)\left(y_c(t)\frac{2\Gamma_d-(D'-D)z_c(t)}{2D+(D'-D)[1-z_c(t)]}\right), \tag{47b}$$

$$dz_c(t)=dt\left(2\Omega y_c(t)+\frac{(D'-D)}{2}[1-z_c^2(t)]\right)-dN_c(t)\times\left(\frac{(D'-D)[1-z_c^2(t)]}{2D+(D'-D)[1-z_c(t)]}\right). \tag{47c}$$

As expected, by using Eq. (30), the ensemble average of Eq. (47) also reduces to the unconditional equation (45).

Next we calculate the localization rate, at which the electron becomes localized in one of the two dots due to the measurement, using Eqs. (46) and (47). Obviously, the stochastic, conditional differential equations provide more information than the unconditional ones do. In the unconditional case Eq. (45), the average population difference $z(t)$ between the dots is a constant of motion ($[dz(t)/dt]=0$) when $\Omega=0$. However, if the present model indeed describes a measurement of $n_1=c_1^\dagger c_1$ (in other words the position of the electron in the dots), then in the absence of tunneling $\Omega=0$, we would expect to see the conditional state become localized in one of the two dots, i.e., either $z=1$ or $z=-1$. Indeed, for $\Omega=0$, we can see from the conditional equations (46c) and (47c) that $z_c(t)=\pm 1$ are fixed points. We can take into account both fixed points by considering $z_c^2(t)$. Hence it is sensible to take the ensemble average $E[z_c^2(t)]$ and find the rate at which this deterministic quantity approaches one. Applying Itô calculus[52,53] to the stochastic variable $z_c^2(t)$, we have $d(z_c^2)=2z_c dz_c+dz_c dz_c$. Let us first consider the case for the quantum-jump equations. Using Eqs. (47c) and (30) and the fact that $dN_c^2(t)=dN_c(t)$, we find that

$$E[dz_c^2(t)]=E\left[\frac{(D'-D)^2[1-z_c^2(t)]^2}{4D+2(D'-D)[1-z_c(t)]}\right]dt. \tag{48}$$

If the system starts in a state which has an equal probability for the electron to be in each dot then $z_c(0)=z(0)=0$. In this case, the ensemble average variable $z(t)$ would remain to be zero since $[dz(t)/dt]=0$ when $\Omega=0$. However if we average $z_c^2$ over many quantum trajectories with this initial condition then we find from Eq. (48) that for short times [by setting $z_c(t),z_c^2(t)\approx 0$]

$$E[z_c^2(\delta t)]\approx\frac{(D'-D)^2}{2(D'+D)}\delta t=\gamma_{\rm loc}^{\rm jump}\delta t. \tag{49}$$

That is to say, the system tends toward a definite state (with $z_c=\pm 1$ so $z_c^2=1$) at an initial rate of

$$\gamma_{\rm loc}^{\rm jump}=\frac{(D'-D)^2}{2(D'+D)}=\frac{(\sqrt{D'}+\sqrt{D})^2}{(D'+D)}\Gamma_d. \tag{50}$$

Similarly for the case of quantum diffusion, using Eqs. (47c) and (39) and the fact that $[\xi(t)dt]^2=[dW(t)]^2=dt$, we find $E[dz_c^2(t)]=E[2\Gamma_d[1-z_c^2(t)]^2]dt$. Applying the same reasoning for obtaining Eq. (49), we find $E[z_c^2(\delta t)]\approx 2\Gamma_d\delta t=\gamma_{\rm loc}^{\rm diff}\delta t$. This implies that the localization rate in this case is $\gamma_{\rm loc}^{\rm diff}=2\Gamma_d$. This is consistent with the result of localization time, $t_{\rm loc}\sim(1/\gamma_{\rm loc}^{\rm diff})$, found in Ref. 18 in the quantum-diffusive case. As expected, Eq. (50) in the quantum-diffusive limit, $\mathcal{T}\gg\mathcal{X}$ or $(D+D')\gg(D'-D)$, reduces to $\gamma_{\rm loc}^{\rm jump}\to 2\Gamma_d=\gamma_{\rm loc}^{\rm diff}$. The rate of localization is a direct indication of the quality of measurement. It is necessarily as large as the decoherence rate since a successful measurement distinguishing the location of the electron on the two dots would destroy any coherence between them.

The above localization rates are related to the signal-to-noise ratio for the measurement and can be obtained intuitively as follows. Consider the electron with equal likelihood in either dot so that $z_c(0)=z(0)=0$. For the case of quantum diffusion, the electron tunneling current through the PC obeys Gaussian statistics. Recall in Sec. IV that the mean of the probability distribution of the number of electron detections through the PC is given by Eq. (37) and its variance takes the form $\sigma_N^2=\mathcal{T}^2\delta t$ in time $\delta t$. If the electron is in dot 1, then the rate of electrons passing through the PC is $\mathcal{T}^2-2\mathcal{T}\mathcal{X}$; if it is in dot 2, then the rate is $\mathcal{T}^2$. One may define the width of the probability distribution as the distance from the mean when the distribution falls to $e^{-1}$ of its maximum value. For a Gaussian distribution, the square of the width is twice the variance. The above two probability distributions will begin to be distinguishable when the difference in the means of the two distributions is of order the square root of the sum of twice the variances (square of the widths) at time $\tau$. That is, when

$$|(\mathcal{T}^2-2\mathcal{T}\mathcal{X})\tau-\mathcal{T}^2\tau|\sim\sqrt{2\mathcal{T}^2\tau+2\mathcal{T}^2\tau}. \tag{51}$$

Solving this for $\tau$ gives a characteristic rate: $\tau^{-1}\sim\mathcal{X}^2=2\Gamma_d$. This is just the $\gamma_{\rm loc}^{\rm diff}$ discussed above. For the case of quantum jumps, the statistics of the electron counts through the PC can be approximated by Poisson statistics. For a Poisson process at rate $\mathcal{R}$, the probability for $N$ events to occur in time $t$ is

$$p(N;t)=\frac{(\mathcal{R}t)^N}{N!}e^{-\mathcal{R}t}. \tag{52}$$

The mean and variance of this distribution are equal and given by $E[N]=\mathrm{Var}(N)=\mathcal{R}t$. In the quantum-jump case from Eq. (30), if the electron is in dot 1 then the rate of electrons passing through the PC is $D'$. If the electron is in dot 2, then the rate is just $D$. Requiring the difference in means of the two probability distributions, $p(N,\tau)$, being of the order of the square root of the sum of twice the variances at time $\tau$ yields

$$|D'\tau - D\tau| \sim \sqrt{2D'\tau + 2D\tau}. \tag{53}$$

Solving this for $\tau^{-1}$ yields a characteristic rate which is the same as $\gamma_{\mathrm{loc}}^{\mathrm{jump}}$ defined in Eq. (50).

A similar conclusion is reached in Refs. 27,21, and 22. The measurement time, $t_{\mathrm{ms}}$, in Refs. 27,21, and 22 is roughly the inverse of the localization rate given here. However, there the condition for being able to distinguish the two probability distributions is slightly different from the condition discussed here. The measurement time[27,21,22] is denoted as the time at which the separation in the means of the two distributions is larger than the sum of the widths, i.e., the sum of the square roots of twice the individual variance rather than the square root of the sum of twice the variance. If this condition is applied here, instead of Eqs. (51) and (53), we have

$$|(\mathcal{T}^2 - 2\mathcal{T}\mathcal{X})t_{\mathrm{ms}} - \mathcal{T}^2 t_{\mathrm{ms}}| \geqslant \sqrt{2\mathcal{T}^2 t_{\mathrm{ms}}} + \sqrt{2\mathcal{T}^2 t_{\mathrm{ms}}} \tag{54}$$

for the quantum-diffusive case and

$$|D' t_{\mathrm{ms}} - D t_{\mathrm{ms}}| \geqslant \sqrt{2D' t_{\mathrm{ms}}} + \sqrt{2D t_{\mathrm{ms}}} \tag{55}$$

for the quantum-jump case. We find from Eqs. (54) and (55) that the inverse of the measurement time $t_{\mathrm{ms}}$ is the same for both quantum-diffusive and quantum-jump cases, and is equal to the decoherence rate:

$$t_{\mathrm{ms}}^{-1} = \frac{\mathcal{X}^2}{2} = \frac{(\sqrt{D'} - \sqrt{D})^2}{2} = \Gamma_d = \tau_d^{-1}, \tag{56}$$

where $\tau_d = (1/\Gamma_d)$ is the decoherence time. This is in agreement with the result in Refs. 21 and 22. Our condition shows, on the other hand, the different localization rates for the quantum-jump and quantum-diffusive cases. This is consistent with the initial rates obtained from the ensemble average of Bloch variable, $E[z_c^2(\delta t)]$.

There is another time scale denoted as mixing time, $t_{\mathrm{mix}}$, discussed in Refs. 27,21, and 22. It is the time after which the information about the initial quantum state of the CQD's is lost due to the measurement-induced transition. This transition arises because of the nonzero coupling $\Omega$ term in the CQD Hamiltonian, which does not commute with the occupation number operator of dot 1 (the measured quantity) and thus mixes the two possible states of the CQD system. Below we estimate the mixing time using the differential equations for the Bloch variables. It is expected that effective and successful quantum measurements require $t_{\mathrm{mix}} \gg t_{\mathrm{ms}} \sim t_{\mathrm{loc}} \sim \tau_d$. In other words, the readout should be achieved long before the information about the measured initial quantum state is lost. In terms of different characteristic rates, we have, in this case, the relation $\Gamma_d \sim \gamma_{\mathrm{loc}} \sim t_{\mathrm{ms}}^{-1} \gg \gamma_{\mathrm{mix}}$, where $\gamma_{\mathrm{mix}} = (1/t_{\mathrm{mix}})$ represents the mixing rate. For finite $\Omega$, the rate at which the variables $x(t)$ and $y(t)$ relax can be found from the real part of the eigenvalues of the matrix in the first term on the right hand side of Eq. (45a). This gives the decay (decoherence) rate $\Gamma_d$ for the off-diagonal variables, $x(t)$ and $y(t)$. The variable $z(t)=0$ represents an equal probability for the electron in the CQD's to be in each dot. Hence the rate at which the variable $z(t)$ relaxes to zero corresponds to the mixing rate,[22] $\gamma_{\mathrm{mix}}$. Under the assumption of $\Gamma_d \gg \gamma_{\mathrm{mix}}$ for effective measurements, the variables $x(t)$ and $y(t)$ therefore relax at a rate much faster than that of the variable $z(t)$. As a result, it is valid to substitute the steady-state value of $y(t)$ obtained from Eq. (45a) into $\dot{z}(t)$, Eq. (45b), to find the mixing rate. Consequently, we obtain

$$\frac{dz(t)}{dt} = -\frac{4\Omega^2\Gamma_d}{\Gamma_d^2 + \varepsilon^2} z(t) = -\gamma_{\mathrm{mix}} z(t). \tag{57}$$

It is easy to see that the mixing rate Eq. (57) vanishes as $\Omega \to 0$. Finally, the self-consistent requirement for the assumption $\Gamma_d \gg \gamma_{\mathrm{mix}}$ yields, from Eq. (57), the condition $\Omega \ll (\sqrt{\Gamma_d^2 + \varepsilon^2}/2)$. The mixing rate Eq. (57) is in agreement with the result found in Ref. 22 under a similar required condition.[54]

## VI. CONCLUSION

We have obtained the unconditional master equation for the CQD system, taking into account the effect of finite temperature of the PC reservoirs under the weak system-environment coupling and Markovian approximations. We have also presented a *quantum trajectory* approach to derive, for both quantum-jump and quantum-diffusive cases, the zero-temperature conditional master equations. These conditional master equations describe the evolution of the measured CQD system, conditioned on a particular realization of the measured current. We have found in both cases that the dynamics of the CQD system can be described by the SSE's for its conditional state vector provided that the information carried away from the system by the PC reservoirs can be recovered by perfect measurement detection. Furthermore, we have analyzed for both cases the localization rates at which the electron becomes localized in one of the two dots when $\Omega = 0$. We have shown that the localization time discussed here is slightly different from the measurement time defined in Refs. 27, 21, and 22. The mixing rate at which the two possible states of the CQD's become mixed when $\Omega \neq 0$ has been calculated as well and found in agreement with the result in Ref. 22.

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## APPENDIX: EQUIVALENCE OF STOCHASTIC RATE EQUATIONS IN DIFFERENT CALCULUS FORMS

In this appendix, we translate the stochastic rate equations of Refs. 18 and 20, written in terms of Stratonovich calculus into the Itô calculus formalism.[52,53] Although the translation was sketched and the result was stated in Ref. 18, for completeness, we fill in the calculation steps using our notation here. Equations (11) and (12) of Ref. 18 in the Stratonovich calculus formalism are rewritten in terms of our notation as follows:

$$\dot{\rho}_{bb}(t)=i\Omega[\rho_{ba}(t)-\rho_{ab}(t)]+4\sqrt{\frac{\Gamma_d}{S_I}}\rho_{bb}(t)\rho_{aa}(t)\times\left(-\sqrt{S_I\Gamma_d}[\rho_{aa}(t)-\rho_{bb}(t)]+\sqrt{\frac{S_I}{2}}\xi(t)\right), \tag{A1}$$

$$\dot{\rho}_{ba}(t)=-i\varepsilon\rho_{ba}(t)+i\Omega[\rho_{bb}(t)-\rho_{aa}(t)]+2\sqrt{\frac{\Gamma_d}{S_I}}[\rho_{aa}(t)-\rho_{bb}(t)]\times\left(-\sqrt{S_I\Gamma_d}[\rho_{aa}(t)-\rho_{bb}(t)]+\sqrt{\frac{S_I}{2}}\xi(t)\right)\rho_{ba}(t), \tag{A2}$$

where we have substituted the notation used in Ref. 18 to $H/\hbar\rightarrow\Omega$, $\varepsilon/\hbar\rightarrow\varepsilon$, and expressions for $\mathcal{R}$ and $\Delta I$ in terms of $\Gamma_d$ and $S_I$ using Eqs. (10) and (2) of Ref. 18. Specifically, we have set $\Delta I=-2\sqrt{S_I\Gamma_d}$. In addition, the white noise $\xi(t)$ in Ref. 18 has spectral density $S_\xi=S_I$, which implies $E[\xi(t)\xi(t')]=(S_I/2)\delta(t-t')$, different from our definition, Eq. (39). Hence, the replacement $\xi(t)\rightarrow\sqrt{S_I/2}\xi(t)$ has been employed. Moreover, since an ideal detector is assumed, $\gamma_d$ is set to zero for Eq. (12) of Ref. 18. Note finally that the electron operator indices in the CQD should be interchanged. For example, the electron annihilation operator $c_2$ in Ref. 18 should be $c_1$ in our notation. As a result, $\rho_{11}(t)$ in Ref. 18 is rewritten as $\rho_{bb}(t)$, and $\rho_{12}(t)$ as $\rho_{ba}(t)$ here.

As pointed out in Ref. 18, to translate Eqs. (A1) and (A2) into the Itô formalism, one needs to add the term[52,53] $(F/2)\times(dF/d\rho_{ij})$ for each rate equation $\dot{\rho}_{ij}(t)$, where $F$ is the factor before $\xi(t)$ in each equation, respectively. Note that the factor $S_I/2$ appearing in front of the term needed to be added for the translation in Ref. 18 is set to 1 here. This is because of the different definitions of the stochastic white-noise variables $\xi(t)$ in both cases, discussed above. To be more specific, $F=\sqrt{8\Gamma_d}\rho_{bb}(t)\rho_{aa}(t)$ for Eq. (A1) in our notation. By using the relation $\rho_{aa}(t)=1-\rho_{bb}(t)$, it is easy to find the derivative $dF/d\rho_{bb}=\sqrt{8\Gamma_d}[\rho_{aa}(t)-\rho_{bb}(t)]$. As a consequence, the term needed to be added to Eqs. (A1) is

$$4\Gamma_d\rho_{bb}(t)\rho_{aa}(t)[\rho_{aa}(t)-\rho_{bb}(t)], \tag{A3}$$

which exactly cancels the first term inside the large parenthesis in the second term of Eq. (A1). Hence the resultant equation for Eq. (A1) in Itô form is just Eq. (42a) with an overall minus sign in front of it $[\dot{\rho}_{bb}(t)=-\dot{\rho}_{aa}(t)]$. As for Eq. (A2), it is easy to find that $F=\sqrt{2\Gamma_d}[\rho_{aa}(t)-\rho_{bb}(t)]\rho_{ba}(t)$. In order to carry out the derivative with respect to $\rho_{ba}(t)$, one needs the expression of Eq. (8) of Ref. 18 to relate diagonal elements, $\rho_{bb}(t)$ and $\rho_{aa}(t)$, to $\rho_{ba}(t)$. We then obtain

$$\frac{dF(t)}{d\rho_{ba}(t)}=\sqrt{2\Gamma_d}\left[\frac{2[\rho_{aa}(t)-\rho_{bb}(t)]^2-1}{\rho_{aa}(t)-\rho_{bb}(t)}\right]. \tag{A4}$$

Thus the terms needed to be added into Eq. (A2) are

$$2\Gamma_d[\rho_{aa}(t)-\rho_{bb}(t)]^2\rho_{ba}(t)-\Gamma_d\rho_{ba}(t). \tag{A5}$$

The first term in Eq. (A5) exactly cancels the term in the square brackets inside the large parentheses in the last term of Eq. (A2). Therefore the resultant equation for Eq. (A2) in Itô form is equal to the complex conjugate of Eq. (42b) $[\dot{\rho}_{ba}(t)=\dot{\rho}^*_{ab}(t)]$. This completes our demonstration of the equivalence.

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[43] Recently, the authors received a copy of a paper, Ref. 24, from Korotkov, in which a somewhat similar derivation to the authors' approach for his Langevin rate equations is presented.
[44] In obtaining Eq. (6), we have neglected in the interaction picture the time dependence of the electron number operator in dot 1 due to the tunneling term $\Omega$ in the CQD's, $c_1^\dagger(t)c_1(t)\rightarrow c_1^\dagger c_1$. This becomes exact when $\Omega=0$. This is reasonable if $\hbar\sqrt{\varepsilon^2+4\Omega^2}\ll\max(|eV|,k_BT)$. Here $\hbar\varepsilon=\hbar(\omega_2-\omega_1)$ is the energy mismatch between the two dots, $k_B$ is the Boltzmann constant, $T$ represents the temperature, $eV=\mu_L-\mu_R$ is the external bias applied across the PC, and $\mu_L$ and $\mu_R$ stand for the chemical potentials in the left and right reservoirs, respectively. This assumption of small $\Omega$ or $\sqrt{\varepsilon^2+4\Omega^2}$, made in Refs. 18 and 20 and implicitly in Ref. 16, can be understood as follows. In deriving the master equation, we assume the electron tunneling amplitudes and density of states are almost constant over some bandwidth $\Delta\omega$ where tunneling may occur. Since the bandwidth $\hbar\Delta\omega$ is roughly in the order of magnitude of $\max(|eV|,k_BT)$, this is a good approximation if $|eV|,k_BT\ll\mu_{L(R)}$. We also assume the weak system-bath coupling, which implies the average electron tunneling rates, Eq. (8), $\hbar D,\hbar D'\ll\max(|eV|,k_BT)$. If $\Omega$, or more precisely the internal characteristic frequency of the CQD system $\sqrt{\varepsilon^2+4\Omega^2}$, is smaller than, or comparable to, $D,D'$, then it is acceptable to use Eq. (6). On the other hand, if $\sqrt{\varepsilon^2+4\Omega^2}$ is much larger than $D,D'$, then we have to establish the condition to use Eq. (6). In general, for finite $\Omega$, the electron tunneling through the PC may occur at different frequencies $\omega_k^L=\omega_k^R$ and $\omega_k^L=\omega_k^R\pm\sqrt{\varepsilon^2+4\Omega^2}$. In order for these frequency-dependent electron tunneling amplitudes through the PC to be almost constant over the bandwidth $\Delta\omega$ as in the derivation for the master equation in the text, it is necessary to require $\hbar\sqrt{\varepsilon^2+4\Omega^2}\ll\max(|eV|,k_BT)$. If $\sqrt{\varepsilon^2+4\Omega^2}$ is not small enough as mentioned above, one instead has to include in Eq. (6) the time-dependent phases $\exp(\pm i\sqrt{\varepsilon^2+4\Omega^2}t)$ and, beside the original term proportional to $c_1^\dagger c_1$, the dynamically generated terms via $\mathcal{H}_{CQD}$, such as terms proportional to $c_2^\dagger c_2$, $c_1^\dagger c_2$, and $c_2^\dagger c_1$. As a consequence, the electron tunneling rates through the PC Eq. (8), for example, may change and depend on the value of the internal characteristic frequency of the CQD system, $\sqrt{\varepsilon^2+4\Omega^2}$.
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