Pulse duration memory of weakly pinned charge-density waves

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An explanation is proposed for the pulse duration memory (PDM) which certain weakly pinned charge density waves (CDW's) exhibit in their oscillatory response to repetitive pulses of electric field. A steady state is reached in which the phase of the current oscillation at the end of each pulse is practically independent of the pulse duration. Present models of the phenomenon follow Fukuyama, Lee, and Rice (FLR) [Phys. Rev. B **17**, 535 (1978); **19**, 3970 (1979)] in regarding the CDW as an elastic body, and account for the current I_c carried by the CDW always rising as the pulse ends, as in the first observation of PDM, in $K_{0.3}MoO_3$. In numerical studies these models have shown PDM only when the pinning by impurities is strong. In experiments, however, PDM is exhibited by weakly-pinned CDW's, notably in $NbSe_3$, where it differs from that in the models in that I_c is near or past maximum as the pulse ends. In this paper the FLR models are reviewed, and it is concluded that their failure to account for PDM when the pinning is weak, and for certain other features of the experiments, is the result of the CDW being assumed to behave elastically. Alternatives, in which the amplitude of the CDW collapses to allow phase slip, are then considered. A model is developed in which dislocations in the structure of the CDW, generated at Frank-Read sources, allow phase slip between the moving CDW and a layer which remains stationary. With appropriate choices of parameters, the model accounts for the forms of PDM observed in weakly pinned systems, and for other features inexplicable in FLR terms. Evidence of the presence of stationary layers is examined, and it is shown that their disappearance at low temperatures would account for a switching phenomenon, and absence of PDM, seen in some specimens of $NbSe_3$. The relation of the model to the phase organization displayed by the FLR models is also briefly discussed.

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I. INTRODUCTION

Pulse duration memory is one of more remarkable phenomena associated with electrical conduction by incommensurate charge-density waves (CDW's).[1] Such CDW's develop below a critical temperature T_p in certain metals having a chainlike structure, appearing as a spatial modulation of the density of conduction electrons, of the form $\rho_0 + \rho_1 \cos(\boldsymbol{Q}\cdot\boldsymbol{r}+\phi)$. With $\boldsymbol{Q}$ incommensurate with the crystal lattice, the energy of the CDW depends on its position only because impurities and other lattice defects couple to the phase ϕ, which adjusts locally so as to minimize the total energy. The resulting *pinning* of the CDW is said to be *strong* if in equilibrium the potential energy due to individual impurities is near minimum, and *weak* if the rigidity of the CDW allows minimization only of the total energy of regions containing many impurities.

Motion of the CDW, in which a current I_c is carried collectively, is induced by electric fields E stronger than a threshold E_T set by the pinning. Usually I_c contains small oscillatory components, known as *narrow-band noise* (NBN), which correspond to the translation of the CDW through wavelengths, and are attributed to the periodically varying forces which pinning centers exert on the moving CDW. In many specimens, when E is applied as a series of identical pulses and steady conditions have been reached, the phase of the oscillation as each pulse ends appears independent of the pulse length. The CDW appears to anticipate when the pulse is going to end because the duration of previous pulses has been recorded, in the metastable state to which the CDW relaxes when $E=0$. Such a *pulse duration memory* (PDM) was first reported by Fleming and Schneemeyer in $K_{0.3}MoO_3$.[2]

The phase when the pulse ends depends on the circumstances. In $K_{0.3}MoO_3$, I_c was increasing at the end of each pulse in the original observation of PDM, but near a maximum in later studies of more perfect crystals.[3] In $NbSe_3$, PDM in which the pulses end with I_c close to maximum was reported by Okajima and Ido,[4] and recently by Jones *et al.*,[5] but in some specimens at sufficiently low temperature the last authors also observed PDM in which the pulses ended with I_c well past its maximum.[6] A form of PDM in which I_c is decreasing as the pulse ends has also been reported in the spin-density-wave material $(TMTSF)_2PF_6$.[7]

Conventional models of CDW motion, based on the assumption of Fukuyama and co-workers[8] [hereafter referred to as Fukuyama, Lee, and Rice (FLR)] that the CDW behaves elastically in moving over the pinning, have been only partly successful in accounting for PDM. Models of this type, which exhibit PDM with I_c rising as each pulse ends, were described by Tang *et al.*[9] and by Coppersmith and Littlewood.[10] The PDM arises because, at most pinning centers, the phase of the CDW as the pulse ends corresponds to a maximum of potential energy. Such *phase organization* is related to the self-organized critical behavior[11] exhibited by some other nonlinear dynamical systems.

It was recently pointed out by Jones *et al.*[5] that, although these FLR models exhibit PDM, they do not well describe the CDW's in which it has been observed, or account for its various forms. The models assume strong pinning, and this seems to be necessary for them to show PDM, for none has been found in the simulations by Ito[12] and Jones *et al.* of the response to repetitive pulses of FLR models with weak pinning. In $NbSe_3$ the CDW's are known to be very weakly pinned,[13] and the PDM has I_c near (or past) maximum as the pulse ends, and different in several other respects from that in the models.

This paper considers how the PDM of weakly pinned CDW's might arise, and is arranged as follows. Section II examines whether the PDM might, if parameters are chosen suitably, still be accounted for in conventional FLR terms. The strong pinning models are outlined, and the mechanism which there leads to PDM shown to be ineffective when the pinning is weak. Of the other discrepancies with experiment listed by Jones *et al.*, all but one appear to be intrinsic to FLR models. Alternatives are discussed in Sec. III, where (in agreement with Okajima and Ido and Jones *et al.*) it is concluded that PDM involves phase slip in the CDW, which FLR models exclude. The form of this phase slip is then considered, and a model suggested which proves capable, in appropriate conditions, of exhibiting the various observed forms of PDM. The relation of this model to the phenomenon of phase organization is among topics of the concluding discussion of Sec. IV.

II. PULSE DURATION MEMORY IN CONVENTIONAL (FLR) MODELS

A. Review

Soon after it was observed experimentally, Tang *et al.*[9] showed that PDM could be exhibited by a chain of elastically coupled particles moving in a sinusoidal potential, such as was used by Frenkel and Kontorova[14] to model crystal dislocations. The PDM occurs when the coupling between neighbors is weak, making the system analogous to a CDW with strong FLR pinning to regularly spaced impurities. Provided that the system is initially far from equilibrium, a pulsed driving force leads to a phase-organized steady state in which most of the particles are near a potential maximum as each pulse ends. As their velocities, and therefore I_c, are then increasing, the system exhibits PDM, as observed by Fleming and Schneemeyer. It was later shown by Coppersmith and Littlewood[10] that similar (though less pronounced) PDM could be exhibited by a CDW initially in equilibrium, if strongly pinned by impurities distributed randomly in position.

Jones *et al.*[5] compared their observations of PDM in $NbSe_3$ only with the predictions of the model of Tang *et al.* Their numerical simulations differ from experiment in several respects. The wave forms of I_c predicted by the model show evidence of beats, and are rising as the pulse ends, whereas in $NbSe_3$ no beats are observed, and the pulse ends with I_c usually near maximum. In the simulations PDM arose only with strong pinning, yet in $NbSe_3$ the pinning is extremely weak,[13] and developed only after many pulses, whereas experimentally only a few are required. Also listed among the discrepancies was the ''ground-state paradox'' of PDM in the model requiring an initial state far from equilibrium, which is not necessary in experiments. A nonequilibrium initial state is, however, not a requirement for PDM in FLR models generally, as the model of Coppersmith and Littlewood[10] shows.

From this Jones *et al.* concluded, in agreement with Okajima and Ido, that the essentials of PDM lie outside the FLR models. Before accepting this, it is appropriate to enquire whether conflict between the models and experiment is inevitable.

B. FLR models with strong pinning

The origin of PDM in the FLR models may be seen by considering a CDW strongly pinned by a large number N of regularly spaced impurities, which may be assumed, without loss of generality, to have spacing commensurate with $\boldsymbol{Q}$. In equilibrium the phase ϕ_i of the CDW at each impurity i is then zero, and the motion of the ϕ_i is described by equations

$$\frac{d\phi_i}{dt}=E+KC_i-V_i\sin\phi_i, \tag{1}$$

where t denotes time, $C_i=(\phi_{i-1}-2\phi_i+\phi_{i+1})$ will be termed the curvature at i, E is the applied field, and an elastic constant $K\ll 1$ is appropriate to the limit of strong pinning. To simplify matters, the coefficients specifying the motional damping and the charge on which E acts are taken to be 1, and periodic boundary conditions imposed.

The system becomes equivalent to the model of Tang *et al.* when the pinning potentials V_i are identical; here they (and therefore E_T) will be set equal to 1. Although the model of Coppersmith and Littlewood provides for randomness in the spacing, pinning potential, and preferred phase of the impurities, its PDM is a result simply of the spatial nonuniformity of the pinning. Here this will be provided by letting $V_i=V_A$ when $i\leq\frac{1}{2}N$ (region A below), and $V_i=V_B$ when $i>\frac{1}{2}N$ (region B), where $V_B>1>V_A\geq K$.

It is supposed that E is applied as a series of pulses of amplitude $E_P>1$ and duration τ, between which the intervals when $E=0$ are sufficient for the system to relax to a stationary (normally metastable) state, in which $\phi_i=\sin^{-1}(KC_i/V_i)$. For any particular metastable state, ϕ_i can be chosen between $-\pi/2$ and $+\pi/2$, and its advance during the next pulse specified as between $2m\pi$ and $2(m+1)\pi$, where the integer m depends on E_P, τ, and C_i. If when the pulse ends $\phi_i>(2m+1)\pi-\sin^{-1}(KC_i/V_i)$, it subsequently relaxes forward to give a total advance $2(m+1)\pi$; otherwise the relaxation is backward, and the total advance $2m\pi$. In either case the advance of ϕ_i resulting from the pulse may be expressed as $2n_i\pi$, where

$$n_i=\mathrm{Int}\left[\frac{\tau}{2\pi}\sqrt{E_F^2-V_i^2}+\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\frac{V_i}{\sqrt{E_F^2-V_i^2}}-\frac{2}{\pi}\right.$$
$$\left.\times\tan^{-1}\left(\frac{V_i}{\sqrt{E_F^2-V_i^2}}-\frac{E_FKC_i}{(V_i+\sqrt{V_i^2-(KC_i)^2})\sqrt{E_F^2-V_i^2}}\right)\right], \tag{2}$$

$E_F=E_P+KC_i$, and as K is small any change in C_i during the pulse is neglected.

In the steady state the same values of the C_i appear repetitively, either in successive metastable states, or in a cycle of q such states, during which the total advance of each ϕ_i is

$2\pi p$, in a form of mode locking between the pulsed field and the translation of the CDW.

The situation most favorable to PDM is when $q=1$; every ϕ_i is then advanced by each pulse by the same amount $2n\pi$, where $n=p$. For given V_i, a range of values of C_i, dependent on E_P and τ, is then consistent with a steady state. PDM arises when a great majority of the C_i are at an extreme of this range: the corresponding n_i are then close to becoming $n\pm 1$, each pulse ends with ϕ_i (modulo 2π) approximating to $\pi-\sin^{-1}(KC_i/V_i)$, giving when $K \ll 1$ a contribution $d\phi_i/dt$ to I_c which is rising, irrespective of the precise value of τ. The pulse ends with the system in the phase organized state mentioned in Sec. II A.

The distribution of the C_i in the steady-state, which governs the wave form of I_c, depends on the arrangement of V_i and on the initial conditions.

1. Spatially uniform pinning

When all the V_i are identical one may, for reasons mentioned below, assume that $q=1$. The n_i then reach a common value

$$n=\mathrm{Int}\left[\frac{\tau}{2\pi}\sqrt{E_P^2-V_i^2}+\frac{1}{2}-\frac{1}{\pi}\tan^{-1}\frac{1}{\sqrt{E_P^2-V_i^2}}\right], \qquad (3)$$

which is obtained by setting $C_i=0$ in Eq. (2), as the steady state with $n_i \equiv n$ is consistent with a range of C_i about zero. This range will be denoted C_1 to C_2, where $C_1<0<C_2$. C_1 and C_2 are available from Eq. (2), whose fractional part approaches 0 or 1 as C_i, respectively, approaches C_1 or C_2. If E_P is not too close to 1, the retention of the first two terms inside the brackets of Eqs. (2) or (3) allows KC_1 and KC_2 to be expressed approximately by

$$KC_a=\left[\left(\frac{\tau_a}{\tau}\right)^2(E_P^2-V_i^2)+V_i^2\right]^{1/2}-E_P, \qquad (4)$$

where $a=1$ or 2, and $\tau_1=(2n-1)\pi/\sqrt{E_P^2-V_i^2}$ and $\tau_2=(2n+1)\pi/\sqrt{E_P^2-V_i^2}$ are the extremes of τ for which Eq. (3) leads to the given value of n. From this, $KC_1\approx -2(E_P-V_i^2/E_P)/(2n-1)$ and $KC_2=0$ when $\tau=\tau_2$; and $KC_1=0$ and $KC_2\approx +2(E_P-V_i^2/E_P)/(2n+1)$ when $\tau=\tau_1$.

The behavior of $d\phi_i/dt$ when C_i is near the limiting values C_1, C_2 is shown in Fig. 1(a). As expected, the pulse ends with $d\phi_i/dt$ (and the contribution to I_c) rising; ϕ_i is then just greater than $(2n-1)\pi-\sin^{-1}(KC_i)$, or just less than $(2n+1)\pi-\sin^{-1}(KC_i)$, according as $C_i=C_1$ or C_2. These are the ''fall forward'' and ''fall back'' cases, respectively noted by Jones *et al.*[5]

As metastable values of C_i between C_1 and C_2 already correspond to steady states, the state exhibiting PDM, with most of the C_i close to the extremes of that range, can be reached only if they are outside it before pulses are applied. Tang *et al.* and Jones *et al.* prepared the initial state by displacing the ϕ_i from equilibrium by large random amounts, leading after relaxation to a metastable state in which sequences of adjacent i have KC_i approaching either $+1$ or -1, which the pinning just renders metastable. Only the simple case shown by curve (i) in Fig. 1(b) need be considered here: sequences of equal length, having KC_i equal to $+1$ and -1, adjoin at sites for which $C_i=0$.

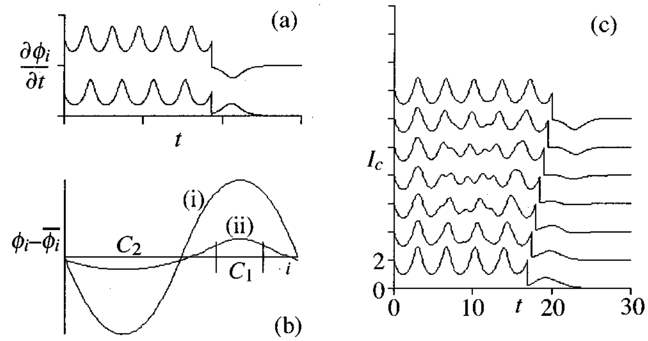

FIG. 1. PDM in a FLR model with spatially uniform strong pinning. (a) $d\phi_i/dt$ in the limiting cases of a pulse causing ϕ_i to advance by $2n\pi$. In the lower trace $C_i=C_1$, in the upper $C_i=C_2$, in both cases with $n=5$. (b) CDW distortions (displacements of ϕ_i from mean $\overline{\phi_i}$) in an initial state far from equilibrium [curve (*i*)] and in the steady state with curvatures $C_1<0$ and $C_2>0$ [curve (*ii*)]. (c) Wave forms of I_c, computed for $E_P=2$, $N=100$, and $K=0.001$.

On applying $E>1$, the differences between the C_i tend to decrease, on account of the term KC_i in $d\phi_i/dt$; when the field is pulsed, a steady state is reached as soon as all the C_i are between C_1 and C_2. It was shown in Ref. 9, and is easily demonstrated by computation, that almost all the C_i are then just within that range, if initially they were far outside it. Since metastable values of ϕ_i are spaced nearly by 2π, these C_i approximate either to the most negative multiple of 2π exceeding C_1, or to the most positive not exceeding C_2. As indicated by curve (*ii*) in Fig. 1(b), the sequences of i for which $C_i\approx C_1$ and $C_i\approx C_2$ are proportional in length to C_2^{-1} and $(-C_1)^{-1}$, respectively (which ensures that C_i remains within the required range at the sites where the sequences adjoin).

The resulting PDM is seen in the computed forms of I_c in Fig. 1(c): in these components similar to the lower and upper records of Fig. 1(a) are superposed, with relative amplitudes varying respectively from 1 to 0, and from 0 to 1, as τ increases through the range consistent with n. Beating is evident near the center of the range, where C_1 and C_2 are roughly equal in magnitude. The computer simulations by Jones *et al.*[5] provide further illustrations of the beating phenomenon, and of the variation of C_1 and C_2 with τ, the distribution of C_i and ϕ_i in the metastable state, and the relaxation following the pulse.

2. Spatially nonuniform pinning

Suppose now that V_i takes different values V_A and V_B in regions A and B. When pulses are applied, the differing forces due to the pinning now cause curvatures C_i to develop, even if the system is initially in equilibrium with $C_i \equiv 0$.

For most combinations of E_P and τ, the steady state when K is small is one in which $n_i=n$ for all i, giving mode locking with $p=n$ and $q=1$. Exceptions occur, however, as τ approaches the value at which n would become $n\pm 1$, for

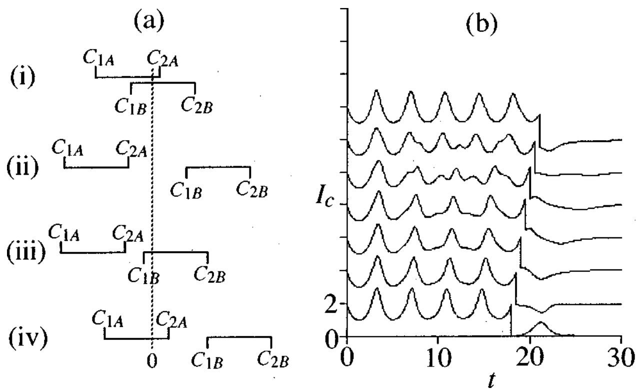

FIG. 2. PDM in a FLR model with nonuniform strong pinning. (a) Ranges of curvature C in cases (i)–(iv) mentioned in the text. (b) Wave forms I_c in case (ii), for $E_P=2$, $N=20$, $V_A=0.5$, $V_B=1.5$, and $K=0.005$.

when the argument of Eq. (2) is close to 0 or 1, changes of $\pm 2\pi$ in C_i may lead to changes of ± 1 in n_i, allowing a subsequent pulse to change C_i by $\mp 2\pi$. Steady states are then possible in which the n_i are successively n or $n+1$, and all reach the same mean value p/q, with $q>1$. When $K \ll 1$, each range of τ in which $q>1$ is approximately $4\pi K$ of the adjacent ranges where $q=1$. Mode locking with $q>1$ is much less likely when all the V_i are equal, because then a steady state having $C_i \equiv 0$ is available, and normally is reached as τ nears the end of the range for given n, so that $n_i \equiv n$ and $q=1$.

In what follows, K is assumed small enough for one to ignore the possibility that $q>1$. The different ranges available to C_i in regions A and B in the steady state can then be found, as before, from Eq. (2) or (4). They will be denoted C_{1A} to C_{2A} in A, and C_{1B} to C_{2B} in B, where $C_{1A}<C_{2A}$ and $C_{1B}<C_{2B}$. As $V_B>V_A$, one has $C_{2B}>C_{2A}$ and $C_{1B}>C_{1A}$. The value of n is that which allows the C_i to have the smallest magnitude in the steady state, implying that $C_{1A}<0<C_{2B}$, $C_{2B}>-C_{2A}$, and $-C_{1B}>C_{1A}$.

The extent to which PDM develops when the system starts from equilibrium depends on the relation of $C_i=0$ to the limiting values just defined. The four possible arrangements are indicated in Fig. 2(a). In case (*i*), when $C_{2A}>0>C_{1B}$, no PDM is possible. The equilibrium state allows $n_i \equiv n$ in both regions, each pulse causes ϕ_i to advance equally, and each new stationary state is an equilibrium state. The situation most favorable to PDM is case (*ii*), when $C_{2A}<0<C_{1B}$. The metastable states then have $C_i<0$ in region A, and $C_i>0$ in region B. To ensure that $n_i=n$ where A and B adjoin, $|C_i|$ there must not be too large, which requires the sum of the C_i in B to be close to that of $-C_i$ in A. In the special case when $C_{1B}=-C_{2A}$, the first metastable state reached has $C_i \approx C_{2A}$ throughout A, and $C_i \approx C_{1B}$ throughout B. Otherwise, if $C_{1B}<-C_{2A}$, one has $C_i \approx C_{2A}$ throughout A, but $C_i \approx C_{1B}$ only in a central part of B, with a larger value $C_i \approx C_{2B}$ in the remainder. An equivalent situation, with C_i close to C_{1B}, C_{2A}, or C_{1A}, occurs when $C_{1B}>-C_{2A}$. Except where the curvature changes sign, all the ϕ_i are then close to a critical value when the pulse ends, leading to PDM, of which examples appear in Fig. 2(b).

Situations intermediate between case (*i*) and (*ii*) arise in case (*iii*), where $C_{2A}<C_{1B}<0<C_{2B}$, and in the equivalent case (*iv*). In case (*iii*), the first metastable state reached has $C_i \approx C_{2A}$ throughout A, but C_i remains 0 in the central part of B, while $C_i \approx C_{2B}$ in the remainder. With ϕ_i at the end of the pulse close to a critical value in A and part of B, the number of sites effective in the PDM is between $\frac{1}{2}N$ and N (ignoring the few where C_i changes sign).

In the computer simulation of their model by Coppersmith and Littlewood,[10] the spatial nonuniformity arose from the randomness of the positions of the impurities. The model, initially in equilibrium, exhibited rather weak PDM: at the end of each pulse I_c was rising, but not at its maximum rate, and only about 70% of the ϕ_i were within 0.2π of a potential maximum. As the model might be expected to display the three types of behavior just described, this response is not surprising. It is also worth noting that spatial nonuniformity of pinning becomes less likely to lead to PDM as E_P increases, for KC_1 and KC_2 obtained from Eq. (4) become less dependent on V_i, making case (*i*) more likely. Experimentally, however, Jones *et al.* found that the PDM of $NbSe_3$ was not degraded by increasing E_P, even beyond $3E_T$.

C. Weak pinning

Consider now the effect on these models of increasing the elastic constant K. An underlying assumption in Sec. II B 1 is that, if the pulse length τ is changed with n constant, the C_i adjust to the new limits C_1 and C_2. That, however, is possible only when the smaller in magnitude of C_1 and C_2, which determines most of the C_i, is sufficiently large compared with 2π. As that quantity does not exceed $(E_P - E_P^{-1})/2Kn$, even with τ central in the range for given n, it appears that one requires $K \ll (E_P - E_P^{-1})/4\pi n$, which for $E_P=2$, $n=5$ becomes $K \ll 0.02$.

The effect of increasing K has been investigated by computation, and is rather less severe than this suggests: the PDM seen when $K=0.001$ [Fig. 1(c)] is only slightly degraded when $K=0.01$. The steady state proves to be modified in two ways, with opposite effects on the PDM. As expected, as the number of metastable values of C_i between C_1 and C_2 decreases, those limits are approached less closely, with C_i taking values which remain unchanged over increasing ranges of pulse length. However, as K increases, the assumption in expressions (2) and (4) that C_i remains constant during the pulse eventually fails: changes in C_i then allow steady states in which C_1 and C_2 are approximated not by a single metastable value of C_i, but by an average of two or three, with one possibly outside those limits. This makes the steady state more able to adjust to changes in τ, and allows PDM with K as large as 0.02, and $C_2 - C_1$ approximately 2π.

The effect when K is larger than this, and $C_2 - C_1$ less than 2π, is seen in Fig. 3(a), which shows $I_c(t)$ when $K=0.05$ and $E_P=2$, for τ in the range giving $n=5$. The PDM is now lost, although in the steady state the C_i need not all be zero, even when the relevant limit is less than 2π, as it may

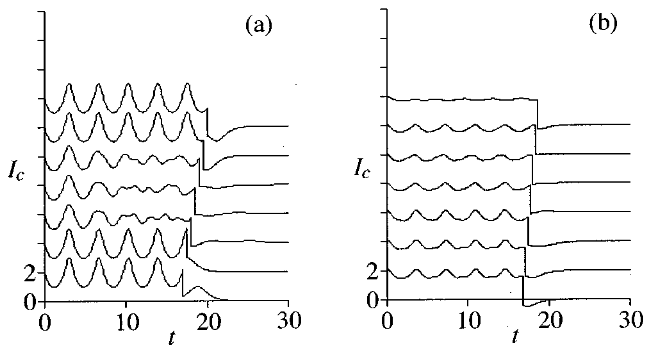

FIG. 3. The effect of weak pinning on FLR models. (a) Wave forms of I_c in the conditions of Fig. 1(c), but with $K=0.05$. (b) Response to pulses of a weakly pinned FLR system, with $E_P=2$, $N=100$, and $K=0.5$. The pulse lengths τ cover the range in which each pulse displaces the CDW by five wavelengths.

be approximated by an average C_i. However, the nonzero C_i become concentrated in short sequences of the i (the ϕ_i are somewhat analogous to the atomic displacements in a Frenkel-Kontorova dislocation), and these concentrations do not survive in the steady state as τ approaches either end of its range for given n, when all C_i become zero. As K increases further, nonzero C_i occur only for τ in a diminishing central part of its range, outside which the form of $I_c(t)$ during the pulse is independent of its duration.

This explains why, as Jones *et al.* found by simulation, the FLR model of Sec. II B 1 exhibits PDM only when K is sufficiently small. It is not difficult to see that the same applies to the FLR model of Sec. II B 2. Although the C_i need not be near zero, their ranges (C_{1A} to C_{2A}, and C_{1B} to C_{2B}) are reduced, and become less than 2π with K still less than 0.02. For larger K, C_i reach values practically independent of τ, again precluding PDM.

In the general FLR model,[8] impurities are distributed randomly rather than being spaced regularly, and usually in two or three dimensions rather then one. As this randomness of pinning clearly will not enhance phase ordering, the general FLR model can hardly show PDM when the simplified versions above do not. As PDM in those models requires K to be less than about 0.02, it is to be expected that values even smaller are required in the general case.

Such small values of K are inconsistent with weak pinning. In weak pinning the elastic coupling induces correlation between the phases ϕ_i at neighboring sites, the preferred values of which are distributed randomly. The onset of weak pinning may be taken to be when the correlation between adjacent sites [i.e., the average of $\cos(\phi_i-\phi_{i+1})$] reaches e^{-1}, which happens when $K\approx 0.2$ in the one-dimensional case. It is not necessary to consider values of K much greater than this, even though the extreme of weak pinning is represented by $K\gg 1$, since i may always be redefined to represent the Lee-Rice domains, and the correlation in ϕ_i between adjacent domains is of the order of e^{-1}. As $K\approx 0.2$ is an order of magnitude greater than allows PDM even in the idealized models discussed above, it is concluded that no PDM can be exhibited by a weakly pinned elastic CDW. Examples of the response of a weakly pinned FLR model to repetitive pulses, showing no evidence of PDM, appear in Fig. 3(b).

D. Comparison with experiment

As well as appearing only with strong pinning, the PDM in Sec. II B differs from experiment in the further respects listed by Jones *et al.*,[5] and already mentioned in Sec. II A. Whether these discrepancies are unavoidable with FLR models is now of interest.

While there are signs of it in the original observation of PDM in $K_{0.3}MoO_3$, the beating phenomenon in Fig. 1(c) has never been observed in $NbSe_3$. Beating is an accompaniment of PDM in both the models in Sec. II B: it arises because the concentration of C_i near two or more limiting values causes different ϕ_i to advance during the pulse approximately by $(2n-1)\pi$ and $(2n+1)\pi$. This association of beats with PDM is to be expected in FLR models generally.

The number of pulses required to establish PDM in the experiments is usually small: values ranging from one or two (Ref. 2) to ten (but in one case 200) (Ref. 4) have been reported. In simulations the number has commonly been of the order of 10^4, but no estimates have been given of how small it might be in favourable circumstances. The number depends on the relative displacements of different ϕ_i needed to convert the initial into the steady state, and on the change effected by each pulse. The arbitrary choice of initial state prevents a clear comparison of the model of Sec. II B 1 with experiment. In the model of Sec. II B 2, initially in equilibrium, the relative displacement to be established is $C_a(N/4)^2$, where $C_a>2\pi$ is a representative limiting curvature which, as it establishes a common mean velocity in regions A and B, is of the order of $\Delta V/2K$, where $\Delta V=V_B-V_A$. The first pulse, if E_P is a few times greater than V_B and V_A, leads to a relative displacement roughly $\Delta V\tau(V/\sqrt{E_P^2-V^2})$, where $V=(V_B+V_A)/2\approx 1$. The number of pulses needed to establish the steady state is then expected to be a few times $N^2(E_P^2-1)/(64\pi Kn)$ which, as $C_a>2\pi$ implies that $K<(E_P^2-1)/(4\pi nE_P)$, will be at least a few times greater than $N^2E_P/16$. Computations of the development of PDM show this to be correct in order of magnitude: in calculations with $E_P=1.5$, and with $N=6$ and $K=0.003$ (respectively the smallest and largest values for clear PDM), the steady state was reached, on average, after about 20 pulses. It is clear from this that, while not inconsistent with every observation, the idealized model cannot account for the appearance of PDM after a few (or even ten) pulses. More general FLR models, in effect with larger N, will fail more spectacularly.

The remaining discrepancy between the models and experiment is in the behavior of I_c as the pulse ends. The models predict I_c then to be rising, which follows from the steady state being such that C_i just ensure that n_i have the value n, rather than $n+1$ or $n-1$. Such a marginal stability of a strongly pinned system clearly cannot account for the PDM observed in $NbSe_3$, where the pulse ends with I_c at, or past, its maximum. From this, and Sec. II C, it may safely be concluded that the essential physics of PDM in real crystals lies outside the FLR models.

III. PULSE DURATION MEMORY: MODELS BEYOND FLR'S

A. General considerations

The failure of FLR models to account for the PDM actually observed is presumably a result of their neglect of inelastic behavior in the CDW. That PDM might involve amplitude collapse was suggested by Okajima and Ido,[4] to explain their finding that during each pulse the CDW advances by an integral number of wavelengths. They proposed that processes of phase slip, in which local collapse is followed by recovery with phase changed by 2π, enable the CDW to move past centers of unusually strong pinning, and that the phase slip at different centers is synchronised by means unknown. An alternative suggested by Jones *et al.* [5] is that the phase slip is that which adds or removes CDW wave fronts near the current terminals. This proceeds through the motion of dislocations in the CDW structure,[15] and it is suggested that the ordering of these leads to PDM.

The arrangement of dislocations near the terminals can, however, be relevant to PDM only in very short specimens. A notable feature of the response of longer specimens to pulsed fields is that the amplitude of the NBN is much larger than when the field is applied continuously.[5] This is to be expected of a CDW composed of nearly independent Lee-Rice domains:[8] when induced to leave the metastable state, where their energy is minimized, different domains make oscillatory contributions to I_c which at first are nearly in phase, but become dephased as motion proceeds. For such a CDW to exhibit PDM, the domain motion throughout the moving CDW must become adjusted to the pulse length. As the motion is determined by local conditions, ordering of dislocations near the terminals can hardly lead to PDM in specimens many domains long.

Local conditions far from the terminals depend on the pinning and applied field, included in the FLR model, and also on any macroscopic stress arising when the mean velocity is spatially nonuniform. Such stress is inevitably present when motion is sustained only in part of the crystal, for it drives the phase slip needed where mobile and stationary regions of CDW adjoin. Stationary regions exist in almost all experiments on $NbSe_3$, as motion is confined to a central segment of the ribbonlike crystal, between current terminals on its sides. In many crystals the motion in the central segment does not extend over its entire cross section, so that a layer stretching between the terminals also remains stationary; evidence of this is presented in Sec. IV.

The distributions of strain in these situations, and the paths taken by the dislocations whose motion effects the phase slip, are shown in Fig. 4. In Fig. 4(a), where the central segment moves as a whole, longitudinal stress induces the formation of edge dislocation loops in the CDW near the terminals; the climb of one such loop to the crystal surface near each terminal allows the CDW between to advance by one wavelength. In Fig. 4(b) a layer of CDW between the terminals is also stationary, and the stress is mainly shear except close to the terminals. The shear stress leads to the formation of pairs of opposite edge dislocations, whose glide to the terminals and subsequent climb to the surface again allows the CDW to advance through one wavelength.

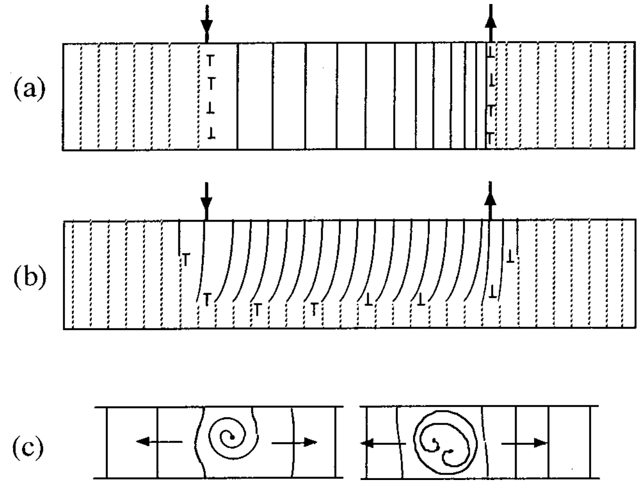

FIG. 4. Strain distributions, dislocation paths, and Frank-Read sources. In (a) and (b) wave fronts in moving and stationary regions of the CDW are shown as continuous and broken lines, respectively. The symbols ⊥ and **T** represent edge dislocations; the vertical line indicates the incomplete wave front near the dislocation. In (a), motion (from left to right) of the CDW between the terminals is made possible by the climb to the surface of edge dislocation loops, generated by longitudinal stress. In (b), motion of a layer of CDW is made possible by the glide toward the terminals, and subsequent climb, of dislocations generated by a Frank-Read source on the boundary of the stationary layer. Two forms of Frank-Read source are shown in (c); the lines represent dislocations at the interface of the moving and stationary layers.

Two mechanisms are available for the generation of dislocations by stress. The random process of thermal nucleation has been discussed at length[16,17] in the case of Fig. 4(a), and in principle may occur also in Fig. 4(b), but predicted rates of phase slip have been much less than those observed. The alternative is generation by Frank-Read (FR) sources, which proceeds deterministically at a rate dependent on the local stress. In Fig. 4(a), where the FR sources create or destroy wave fronts of CDW's, they tend to be removed by the motion, though a pair operating at identical rates near each terminal might survive indefinitely.[17] Although this has led to their neglect in favor of thermal nucleation, there is no reason why FR sources on the boundary of the stationary layer in Fig. 4(b), and based on edge dislocations within it, should not be permanent.[18] Examples of such sources, which provide phase slip in which wave fronts are conserved, are shown in Fig. 4(c): under shear stress, dislocations of mixed edge and screw types, permanently present on the boundary, generate pairs of opposite edge dislocations.

In both Figs. 4(a) and 4(b), the situation most favorable to PDM is when every field pulse generates the same number n of dislocation pairs, enabling the moving CDW to advance in phase everywhere by $2n\pi$. Such generation, being nonrandom, requires a FR source or sources, and is likely only in Fig. 4(b). A model of PDM in that case is considered below. The model accords with Okajima and Ido's view that the CDW relies on phase slip to escape from pinning: here phase slip is effected by the glide of dislocations along the interface with the stationary layer, whose supply by the FR source provides a means of synchronization. The stationary layer is not necessarily subject to increased pinning: its sustained motion may be prevented by an absence of the dislocations which would enable it to gain or lose wave fronts near the terminals, for those generated by the FR source add or remove wave fronts only of the moving layer.

B. A model of PDM involving phase slip

1. Basic assumptions

In modeling the situation of Fig. 4(b), the regions of longitudinal strain near the terminals are ignored, and the mobile CDW in the sheared region treated as one dimensional. Its phase ϕ_i is sampled in N cells i along the chain direction, which are coupled elastically to corresponding cells of the ''phase-slip plane'' where the mobile region adjoins the stationary layer. The phases ϕ_i are assumed to satisfy equations

$$\frac{d\phi_i}{dt}=E+KC_i-V_i\sin(\phi_i-\beta_i)-K_sS_i, \tag{5}$$

where randomness is introduced through the preferred phases β_i distributed randomly between $-\pi$ and $+\pi$, with $V_i=1$; E, K and C_i have meanings as in Eq. (1), and I_c is taken as the mean of $d\phi_i/dt$ with respect to i.

The final term in Eq. (5) represents the coupling to the stationary layer: $S_i=\phi_i-\psi_i$, where ψ_i denotes the phase in cell i of the phase-slip plane, and K_s is an elastic constant dependent on the shear modulus and thickness of the mobile region. The glide of a dislocation through the cell i advances ψ_i (assumed initially to be zero) by 2π.

The dislocations are assumed to be generated by a single FR source in cell s of the phase-slip plane, driven by shear stress, of which K_sS_s is the one-dimensional representation (where several sources operate, they adjust so as to generate dislocation pairs in the same plane, at a net rate which is that of the most active). Except in its immediate vicinity, where S_s affects S_i elastically in about $\sqrt{(K/K_s)}$ cells, the source influences $d\phi_i/dt$ only by enabling ψ in i and neighboring cells to advance. It is assumed N is such that I_c is dominated by cells far from the source, which then need not appear explicitly in Eq. (5). In the steady state the response to the pulsed field then depends on the numbers n_p of dislocation pairs generated by successive pulses, and also on the direction from which the S_i approach their eventual values, which depends on when ψ_i begins to increase. Calculation is simplified by supposing that all ψ_i (now written ψ) advance simultaneously, making n_p advances of 2π during each pulse, after a delay during which d_p advances would otherwise have occurred.

Many factors influence the possible values of n_p and d_p. Operation of the FR source is expected to require stress in excess of some threshold, and also to be resisted by pinning, and by damping arising from the difficulty of inducing glide normal to the chain axis.[15] A further complication is that the rate at which dislocations appear will not respond immediately to changes in shear stress. The rate varies both as the glide velocity, and as the number of turns in the spiral structure of Fig. 4(c), if that is large. While both are expected to be proportional to stress[19] once steady conditions have been reached, the pitch of the spiral takes time to adjust to a new value. The stress driving the source, though represented in the steady state by K_sS_s, is due to displacement of the CDW next to the phase slip layer, and will rise more rapidly than K_sS_s when the field pulse E_P is first applied to the system in equilibrium. While this may allow dislocations to appear almost immediately, the finite glide velocity delays the advance of ψ, which may occur before or after S_i can reach its eventual value.

It is clear from this that, while d_p may allow the steady value of S_i to be approached from either direction, the pinning and damping of the source ensure that, in the steady state, n_p restricts the rate of increase of ψ to be less than that of ϕ_i would have been if not coupled to the stationary layer. This leads to the development of the shear stresses K_sS_i, which equalize the mean rates of increase of the ϕ_i and ψ as the displacement of the CDW becomes large. The difference between the actual rate of advance of the ϕ_i (and ψ), and that which would apply were there no coupling to the stationary layer, is conveniently expressed as an effective field δE, such that the average I_c induced by a steady field E is the same as would be induced by $E-\delta E$ if the stationary layer were not present. The shear stresses K_sS_i, which in this case are independent of the approach to the steady state, are then equivalent to δE.

2. Strong pinning

The PDM is simplest when the pinning is strong ($K \ll 1$). Except in very small ranges of τ, the randomness associated with β_i may be ignored, and the steady state is one in which each pulse creates the same number n of dislocation pairs, advancing all ϕ_i by $2n\pi$. The stresses K_sS_i, of the same order as δE, now depend on how the steady state was approached. It will be assumed that the system is initially in equilibrium, so that in the absence of randomness C_i and S_i are zero. During the motion, as neighboring ϕ_i advance simultaneously, C_i then remains zero and may be ignored.

In the metastable states reached between pulses, the S_i consistent with a steady state lie between limits S_1 and S_2, analogous to the range C_{1B} to C_{2B} of the C_i in Sec. II C. As before, PDM requires the S_i (which are close to multiples of 2π) to be near one or other limit through the relevant range of τ. This is achievable if $S_2-S_1 \gg 2\pi$, which requires $K_s \ll 1$.

Which limit, if any, is reached depends on the initial S_i and on the delay d_p. If d_p is small, then as the system starts with $S_i=0$, PDM is possible only if $S_1>0$, since otherwise $S_i=0$ provides a steady state. This requires δE to be large enough for S_i to increase during the first pulse by at least $(2n+1)\pi$, to reach a metastable value $S_i \geqslant 2\pi$. The increase continues until S_i reaches the first metastable value beyond S_1, provided that d_p is small enough for ψ to ad-

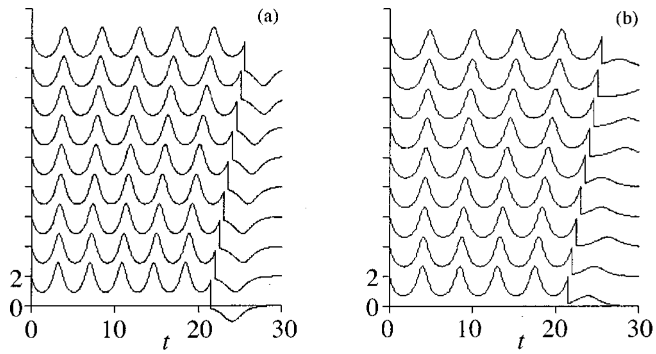

FIG. 5. PDM in the present model, with strong pinning. Wave forms of I_c, computed for $E=2$, $\delta E \approx 0.2$, and $K=K_s=0.001$ are shown for τ corresponding to $n=5$. The system starts from equilibrium, with d_p such that in the steady state $S=S_1$ in (a), and $S=S_2$ in (b).

vance before that value is exceeded. The value closest to S_2 is reached if d_p is large enough for S_i to exceed S_2 before ψ begins to advance.

When $S_i \approx S_1$, ϕ_i (modulo 2π) just fails to reach $\pi + \sin^{-1}(K_s S_1)$ as the pulse ends, and thereafter relaxes backward to the greatest extent possible, giving a net advance $2n\pi$ from each pulse. With $S_i \approx S_2$, the comparable ϕ_i just passes $-\pi + \sin^{-1}(K_s S_2)$, and achieves the net advance $2n\pi$ by relaxing forward to the greatest extent. In either case the pulse ends with I_c rising, as in Fig. 5. While in this respect the PDM is similar to that in Figs. 1 and 2, it differs in that no beats are present, as all the ϕ_i behave similarly, and also in that the direction of relaxation following the pulse does not depend on τ. These differences may be absent, however, if S_i are distributed between S_1 and S_2, as can happen if the glide velocity provides appropriate d_p in different regions of the system.

3. Weak pinning

The computations of Ito,[12] and those on which Fig. 3(b) is based, show that in the FLR model a weakly pinned CDW is translated through an integral number of wavelengths by each pulse only when τ lies within ranges amounting to about 30% of the whole. Otherwise, q pulses, where $q>1$, are needed to translate the CDW by an integral number p of wavelengths. A consequence of this for the present model is that, even in the steady state, the numbers n_p of dislocation pairs generated by successive pulses are not necessarily equal.

The behavior of a FR source coupled to a weakly pinned CDW depends on several factors. Suppose the CDW to extend in the transverse direction by many times its phase coherence length; the motion of the small region which is directly coupled to the FR source and phase-slip plane is then effectively determined by that of the remainder. If the source responds immediately to displacement of the adjacent CDW, then in the steady state it generates a total of p dislocation pairs whenever the bulk CDW moves through p wavelengths, so that if this requires $q>1$ pulses the n_p are not all equal. Although p/q depends on the shear stress, which is influenced by the source, the stress adjusts only over many pulses, with the result that p/q is integral over ranges of τ which, relative to the whole, are the same as in the FLR case. If, however, the FR source responds slowly, effectively to a time-averaged shear stress, the variation of n_p is reduced, and the ranges over which p/q is integral are increased (the ranges of integral p/q are also increased if the transverse dimension of the CDW is comparable with the coherence length, as its motion closely follows developments in the phase-slip plane, but PDM is not then expected).

The PDM of weakly pinned systems has been modeled using Eq. (5), choosing values of n_p consistent with the field E_P and duration τ of the pulses, and with the properties of the FR source summarized in δE and an assumed range over which p/q is integral. As discussed in Sec. II C, weak pinning can be modeled by a wide range of the elastic constant K, from 0.2 to the extreme $K \gg 1$, if ϕ_i is interpreted appropriately. Here the lower value will be used, so that the cells i may be regarded as Lee-Rice domains, whose relaxation to metastable states is substantially independent of that of their neighbors. It will be seen below that, to avoid restricting the availability of metastable states on which PDM depends, K_s needs to be smaller than $\delta E/2\pi$. Otherwise its value is not critical, as it refers to shear on a scale set by the thickness of the mobile region, rather than by the impurity separation, which would be relevant to the pinning strength; in particular, $K_s \ll 1$ is entirely compatible with weak pinning.

It is convenient to represent distributions of $\Phi_i = \phi_i - \beta_i$ (modulo 2π) as in Fig. 6: the radial width of the shaded area expresses the distribution function $D(\Phi)$ of Φ_i, and their advance under an applied field appears as a rotation, whose angular velocity contains a component $-V_i \sin \Phi_i$ due to the pinning. The effect of applying a steady field $E>E_T$ to a system unrestricted by coupling to a stationary layer, and initially in equilibrium, is shown in Figs. 6(a)–6(c). The concentration of D around zero in the equilibrium state [Fig. 6(a)] disperses as motion proceeds, so that the distribution passes through Fig. 6(b) and eventually becomes uniform, except for the effects of velocity modulation by the pinning, as in Fig. 6(c). The process leads to a gradual decay of the oscillatory part of I_c. When the field is removed, then to a fair approximation the Φ_i relax forward or backward, according as they are greater or less than π, until a metastable state is reached. It is easy to see, and is demonstrated by computation, that when the metastable state is near equilibrium, as in Fig. 6(f), the total relaxation is backward to the greatest extent in Fig. 6(d), when $D(\pi/2)$ has its maximum value, and forward to the greatest extent in Fig. 6(e), when $D(3\pi/2)$ is maximum. These situations correspond respectively to a minimum and to a maximum of I_c.

The way in which one might expect the PDM in Fig. 5 to be modified when the pinning is weak is now clear. Suppose first that δE is small and positive, and that the n_p all take the same value n (so that $p=n$, $q=1$), in which situation PDM is most likely. As when the pinning is strong, the S_i adjust until a steady state is reached in which each pulse advances every ϕ_i by $2n\pi$. The mean value of S_i (now denoted S) has a range of possible values consistent with metastable states: these are distributed almost uniformly, with roughly N values

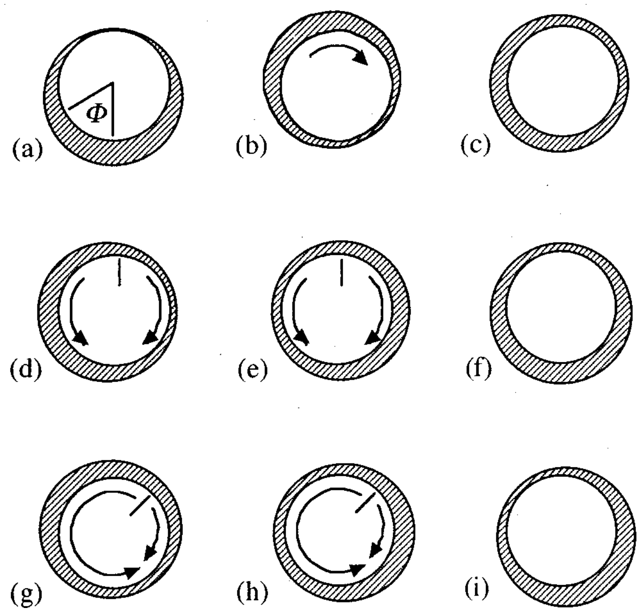

FIG. 6. States of the present model with weak pinning. The radial thickness of the shaded area represents the distribution function $D(\Phi_i)$ defined in the text. The equilibrium state is shown by (a), the motion of the CDW is represented by (b), and (c) shows the steady state reached after prolonged continuous motion. States reached in a pulsed field are shown by (d)–(i). When δE is small, state (d) gives maximum backward relaxation (indicated by the arrows), and state (e) maximum forward relaxation, to the metastable state (f). When δE is large the corresponding states are respectively (g), (h), and (i).

per interval 2π, up to $S \approx E_T/K_s$, where $E_T \approx 0.6$ when $K = 0.2$. If the system starts from equilibrium and d_p is small, S increases in successive metastable states, to reach in the steady state a value S_1 just sufficient to prevent the total advance of the ϕ_i resulting from each pulse from exceeding $2n\pi$. As the relaxation following each pulse is then backward to the maximum extent, the state of the system when the pulse ends is as in Fig. 6(d), and I_c is near minimum. If d_p is sufficiently large, S rises beyond its eventual value S_2 in the steady state, which is then just low enough to ensure that the ϕ_i advance by $2n\pi$ as a result of each pulse. The relaxation is then forward to the maximum extent, as in Fig. 6(e), and the pulse ends with I_c near maximum.

This behavior is modified when δE, and therefore K_sS in the steady state, is large. The metastable state is then as in Fig. 6(i), and the distribution of D which gives the greatest backward relaxation after the pulse ends is as in Fig. 6(g), where I_c is past its minimum. The forward relaxation is greatest, and I_c is past its maximum, in Fig. 6(h).

4. Computer simulations (weak pinning)

These expectations have been confirmed by computation. Provided that the pinning is weak ($K \approx 0.2$ or greater), and K_s small enough for S in the steady state to be at least a few times 2π, the values chosen for these parameters have little effect on the form of the PDM.

Figure 7 shows the wave form of I_c for pulses of various lengths τ, taking integral and nonintegral values of p/q corresponding to small δE. The behavior when the delay d_p is small is shown in Fig. 7(a). When p/q is integral ($n_p = n = 5$), the steady state reached from equilibrium has $S = S_1$, and the pulse ends, as expected, with I_c close to minimum. Figure 7(b) shows I_c when d_p is large enough to give a steady state with $S = S_2$. The pulse now ends with I_c close to maximum.

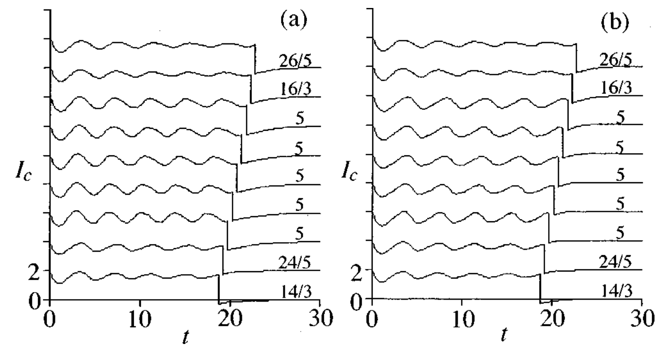

FIG. 7. PDM in the present model with weak pinning, when δE is small (0.2). Wave forms of I_c are shown for a range of pulse lengths τ, with the values of n (or of p/q, where nonintegral) indicated, when $E_P = 2$, $N = 100$, $K = 0.2$, and $K_s = 0.002$. The system was initially in equilibrium, with d_p such that in (a) the advance of ψ commenced before S reached S_1, and in (b) was delayed until after S exceeded S_2. Values of K_sS in the steady state range from 0.04 to 0.21 in (a), and from 0.13 to 0.25 in (b).

Although clearest when the n_p have a common value n, PDM does not vanish immediately p/q ceases to be integral. When p/q is slightly less than n, then for most pulses $n_p = n$, but a few give $n_p = n-1$. These pulses allow S to increase from the values between S_1 and S_2 which would correspond to steady states if all n_p were equal to n. As a result, and irrespective of d_p, a steady state is not then reached until S is somewhat greater than S_2, and takes in turn q values, increasing when $n_p = n-1$ and decreasing toward S_2 when $n_p = n$. The wave form of I_c, averaged over q pulses, then ends the pulse close to maximum. When p/q is slightly greater than n, S between S_1 and S_2 is reduced by the occasional pulses for which $n_p = n+1$ rather than n. In the steady state, again irrespective of d_p, S takes q values not far below S_1, and I_c is close to minimum when the pulse ends.

This is evident in Fig. 7, in the records for which p/q departs from integral n by $\frac{1}{5}$ and, to a lesser extent, by $\frac{1}{3}$. In either case the wave form appears independently of d_p.

The form of the PDM when δE is large is shown in Fig. 8 (the pulse lengths are much as in Fig. 7, but the principal value of n_p is now 3 rather than 5). As expected, the pulse now ends with I_c substantially beyond its minimum or maximum.

C. Comparison with experiment

The present model provides an explanation of many of the experimental features of PDM which are inconsistent with FLR models. These are summarized below.

(i) The model exhibits PDM even when the CDW is weakly pinned.

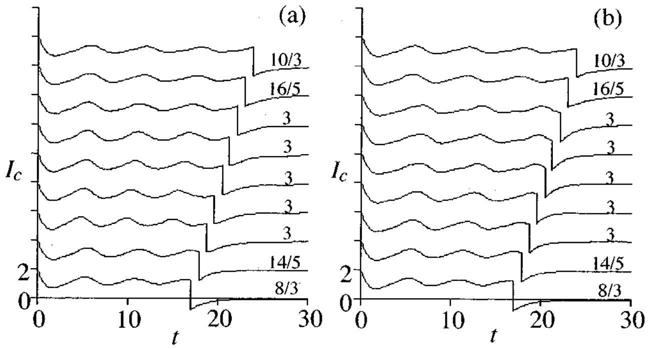

FIG. 8. As Fig. 7, except that δE is larger (0.7); K_sS ranges from 0.45 to 0.55 in (a), and from 0.57 to 0.64 in (b).

(ii) The various observed forms of PDM can be accounted for, without requiring a nonequilibrium initial state, by assigning appropriate values to the parameters describing the supply of dislocations by the FR source. PDM in which the pulses end with I_c close to maximum is explicable if a sufficient delay d_p is present before phase slip occurs far from the source. This form of PDM is seen in $NbSe_3$ at 50 K.[5] However, as the temperature is reduced, in those specimens still exhibiting PDM, the pulses end with I_c to an increasing extent past its maximum.[6] The model associates such behavior with increasing δE, which might be expected if, in the FR source, the glide perpendicular to the chain direction is thermally activated. The form of PDM first seen in $K_{0.3}MoO_3$,[2] where I_c increases as the pulses end, is accounted for if d_p is small and δE large. In $(TMTSF)_2PF_6$,[7] it is not clear whether I_c is near a minimum, or merely decreasing, as the pulse ends; small d_p and δE would account for the former possibility, and large d_p and δE for the latter.

(iii) The model exhibits PDM in which no beats appear in the wave form of I_c, as is observed in $NbSe_3$. The beats observed in Ref. 2, and the near-constancy of I_c until the end of the pulse approaches in Ref. 7, are perhaps the result of the CDW moving in many independent layers, so that components having different n are superposed.

(iv) The PDM can be established by fewer pulses in the present than in the FLR models. The number required depends on the mean S to be established, and the amount by which S changes as a result of each pulse. As weak pinning in effect provides S with a continuum of metastable values, S_1 and S_2 need not be much larger than 2π to be followed by S as τ varies. When E_P is substantially greater than E_T, S_1, and S_2 are of the order of $\delta E/K_s$, and if d_p is small the first pulse changes S by about $2\pi n\,\delta E/E_P$. From this, the steady state is approached after a number of pulses of the order of $E_P/2\pi n K_s$, or about 6 if $E_P=2$, $n=5$, and $K_s=0.01$. In computer simulations using these values the steady state was reached typically after up to ten pulses, though after three pulses the PDM usually was close to its final form. Roughly the same number of pulses establishes the steady state with $S=S_2$ when $d_p\approx 2(S_2/2\pi)$. Thus the development of PDM after a few pulses seems explicable.

(v) In their study of PDM in $NbSe_3$, Okajima and Ido found that the advance of phase during each pulse was always close to $2n\pi$, with n an integer. This conflicts both with the present and the FLR models if the pinning is strong: in the present model the advance is either $(2n-1)\pi$ or $(2n+1)\pi$, depending on d_p, and in the FLR models varies between those limits, depending on τ. In each case relaxation after the pulse ensures that the total advance is $2n\pi$. In the present model with weak pinning, the relaxation following the pulse depends on $D(\Phi)$ when the pulse ends. When I_c is near maximum, and $S=S_2$ in the steady state, the distribution is as in Fig. 6(e). The relaxation, although forward to the maximum extent, depends on how uniform D has become as a result of the motion: the mean phase advance after the pulse cannot exceed $\pi/2$, but is usually much smaller, and vanishes if the distribution is uniform (and may be negative when E_P is near threshold, as $D(\pi/2)$ may then exceed $D(3\pi/2)$ even when the latter has its greatest value). In Fig. 7(b), the phase advance after the pulse varies between -0.07π and $+0.06\pi$ as τ increases through the range appropriate to $n=5$. Thus the advance during the pulse is within 1% of $2n\pi$, and entirely consistent with Okajima and Ido's observations.

(vi) In the experiments by Jones *et al.*[5] on $NbSe_3$, PDM was observed in ranges of τ amounting to about 60% of the total. This is much greater than the 30% of the total in which, in the FLR model with weak pinning, each pulse translates the CDW through an integral number of wavelengths. In the present model the translation per pulse is determined by the FR source, and may be expected to be an integral number of wavelengths over wider ranges than in the FLR model. The model can therefore accommodate the ranges over which PDM is seen, especially as it allows PDM when the translation per pulse departs slightly from an integral number of wavelengths.

A feature of the PDM noted by Okajima and Ido which is not exhibited by the model is the response to changes in the pulse length τ. The PDM in question is that in which the pulse ends with I_c near maximum, which in the model requires $S=S_2$. If, when the steady state has been reached, τ (and therefore S_2) is reduced, then in the model S decreases to the new S_2 within a few pulses. On the other hand, if τ is increased, S already corresponds to a steady state, and no means is available for it to increase to the new value of S_2 without first returning to equilibrium, or otherwise interrupting the advance of ψ. Experimentally, however, the PDM adjusts to an increase in τ in fewer pulses than are required for adjustment to a decrease.

This discrepancy between the model and experiment (the only one so far apparent) would be explicable if dislocations generated by the FR source were occasionally to be lost. Each loss allows the system to reach a new steady state, with greater S, until the new value of S_2 is reached. As no steady state is available beyond S_2, further losses increase S only temporarily, and if they occur infrequently the system for practical purposes remains in a steady state with $S\approx S_2$, as when p/q is slightly less than an integral value n. To continue indefinitely without macroscopic strain becoming infinite, the loss of dislocations would have to be associated with the addition or removal of wave fronts of the mobile region. One way in which a suitable loss of dislocations might occur is through annihilation with dislocations of opposite type, provided by the expansion to the phase slip plane of loops nucleated thermally within the bulk, perhaps where local strain develops around obstacles to the motion. The macroscopic longitudinal stress, which develops in the CDW through the obstruction of its motion by the terminals, would ensure that any expanding loops would be of the appropriate type to annihilate with dislocations generated by a FR source roughly midway between them.

IV. DISCUSSION

The consistency of the model with the various manifestations of PDM makes any evidence that layers of CDW remain fixed while the rest is in motion of some interest. X-ray studies have detected stationary layers in some specimens of $NbSe_3$,[19] and also shear strains[20] in the adjacent moving regions, of magnitude quite sufficient to lead to PDM (for which S need correspond to displacements only of a few wavelengths). The different regions are associated with steps in the crystal thickness, which occur in most specimens. Whether they are present in specimens showing clear NBN oscillations, in which PDM is studied, is less certain.[13]

The possibility that stationary layers result from surface pinning in $NbSe_3$ is remote in the case of the broad (*bc*) faces of the crystals, for it is clear that the tendency of E_T to increase with decreasing crystal thickness is a result of the pinning being effectively two dimensional,[13] with Lee-Rice domains extending between opposite faces. Stationary layers may however arise on the narrow (*ab*) faces if the adjacent domains contain appropriate dislocations, as these extend only over a small part of the crystal width. As mentioned in Sec. III A, an absence of suitable dislocations to allow wave front addition and removal near the terminals is sufficient to render such layers stationary. They will also be subject to increased pinning through their proximity to the surface, as the adjustment of the CDW phase to impurities costs less elastic energy than in the bulk, and perhaps also by the accumulation there of impurities.

An assumption that stationary layers are present in most crystals makes it possible to explain certain pinning effects of mobile In impurities,[21] and also to account quantitatively for the voltage absorbed in sustaining phase slip near current terminals, by attributing it to the climb of dislocations reaching the terminal regions from FR sources elsewhere.[18] If the present model is accepted, then PDM provides further evidence of stationary layers. It is interesting that Jones *et al.*[6] found no PDM when, in some specimens of $NbSe_3$ at low temperature, the CDW conduction appeared suddenly (and hysteretically) as E_T was exceeded, in the phenomenon known as *switching*. PDM was seen at low temperature in nonswitching specimens, and in switching specimens at temperatures high enough for no switching to occur. This suggests that switching may be associated with the disappearance of the FR sources which allow a stationary layer to coexist with the moving CDW.

Several features of the switching phenomenon reported by in Ref. 22 are in accord with this suggestion. The sudden increase in E_T as switching appears follows from the need to depin the previously stationary layer and, as no FR source is available, to nucleate dislocation loops at the ends of the moving segment of CDW (i.e., near the terminals or, in the crystals studied in Ref. 22, at phase-slip centers detected between them). As expected, the increase in E_T is reduced when the crystal is shortened, so that the moving segment extends to its ends and no phase slip is required. The disappearance of the FR sources which leads to switching may be due to an increase in elastic modulus of the CDW as the temperature falls: the dislocation structure becomes unstable when its elastic energy becomes too great relative to the pinning. This, if one assumes that impurities adsorbed by the crystal surface contribute to the pinning, and so inhibit switching by preventing the FR sources from disappearing, would explain why switching becomes less common as specimens age, but can be restored by revealing new surfaces by cleaving or chemical treatment.

Finally, the relation between PDM in the present model, and the phenomenon of phase organization merits comment. Phase organization is exhibited by the models discussed in Sec. II, which when $K \ll 1$ reach steady states in which, as the pulse ends, almost all the ϕ_i are close to positions of maximum potential energy. This situation, where practically all the degrees of freedom have energy near maximum, is described[9,10] as minimally stable, since many metastable states are accessible to the system through slight disturbance.

While this ensures that the same steady state (or its equivalent) can be reached from a wide range of initial states, such minimal stability is not essential to PDM. It is sufficient for the system to have a single degree of freedom, provided that it possesses a large number of metastable states (an example is the single-particle system, showing a form of PDM, discussed by Coppersmith[23]). The present model in effect has a single degree of freedom,[24] characterized by the macroscopic shear strain, but with weak pinning has a quasicontinuous distribution of metastable states. The stability of the steady state is then better described as marginal than as minimal. Phase organization is present only as an average over many domains, and as the steady state is reached through a sequence of metastable states determined by the pinning and the FR source, the number accessible through slight disturbance is unimportant. Although Jones *et al.*[5] suggest that a reduced abundance of metastable states underlies the disappearance of PDM when the field E between pulses is close to $\pm E_T$ rather than zero, this seems more likely to be a result of incomplete relaxation to the metastable state.

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