

Magnetotransport of $\nu=3/2$ composite fermions under periodic effective magnetic-field modulation

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We report magnetotransport measurements around a filling factor $\nu=3/2$ of two-dimensional electron gas under weak unidirectional periodic potential modulation. Composite fermions (CF's) at $\nu=3/2$ which are the electron-hole conjugate of $\nu=1/2$ CF's, show positive magnetoresistance (PMR) and oscillatory magnetoresistance—commensurability oscillation (CO)—for a modulation period of $a=92$ nm. Minima of the CO occur at the positions expected for commensurability with effective magnetic-field modulation, and are consistent with the field-dependent Fermi wave number k_F resulting from field-dependent density of the $\nu=3/2$ CF's. The positions are consistent with k_F for fully spin-polarized CF's, except for one dip which is better explained by a fully spin-unpolarized k_F , smaller by factor $\sqrt{2}$ than its spin-polarized counterpart. With the introduction of in-plane magnetic field by tilting, the PMR shows unexpected asymmetry: it grows both in its magnitude and extent on the lower-field side, while it shrinks on the higher-field side.

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The concept of composite fermions (CF's) is an ingenious scheme that maps the fractional quantum Hall effect (FQHE), which owes its origin to complicated many-body effects, onto the much simpler integer quantum Hall effect.¹ The prototype of a CF is that formed at a filling factor $\nu=1/2$ by attaching two flux quanta to an electron. Away from $\nu=1/2$, the CF's "feel" an *effective magnetic field* $B_{\text{eff}}=B_{\perp}-B_{1/2}$, where B_{\perp} is the field perpendicular to the plane of the two-dimensional electron gas (2DEG). ($B_{\nu}\equiv n_e\Phi_0/\nu$ signifies the field giving the filling ν , with $\Phi_0=h/e$ the flux quantum and n_e the electron density.) Up to the present, quite a few pieces of experimental evidence have been accumulated^{2,3} that show that composite particle can be regarded as a real entity rather than a theoretical expedient. Because of the electron-hole symmetry, CF's are also formed at $\nu=3/2$. Here two flux quanta are attached to a hole from the $\nu=2$ state. Since the number of electrons accommodated in the lowest two spin-split Landau levels is proportional to B_{\perp} , it follows that the density of holes, n_h , hence that of the $\nu=3/2$ CF's, n_{CF} , depends on B_{\perp} as

$$n_{\text{CF}}=n_h=2B_{\perp}/\Phi_0-n_e. \quad (1)$$

This is fundamentally different from the $\nu=1/2$ CF's, which have a fixed density equal to n_e . As a result, the effective magnetic field reads

$$B_{\text{eff}}=B_{\perp}-2n_{\text{CF}}\Phi_0=-3(B_{\perp}-B_{3/2}). \quad (2)$$

Unlike the case of $\nu=1/2$, B_{eff} changes three times as fast as B_{\perp} . A periodic potential modulation is accompanied by a periodic variation in n_e , which is equivalent to a periodic modulation of B_{eff} for CF's. It is then anticipated that CF's in periodic potential modulation behave much like electrons under periodic magnetic-field modulation.^{4,5} (More precisely, CF's are subject to modulation of both potential and B_{eff} . However, the efficacy of the latter by far exceeds that of the former.) Two kinds of characteristic manifestations of the effect of unidirectional periodic modulation of potential and/or magnetic fields on the low-field magnetotransport are

known: positive magnetoresistance (PMR) at low fields with a peak position determined by the magnitude of the modulation,⁶ and oscillatory magnetoresistance originating from the commensurability between the cyclotron radius $R_c=\hbar k_F/e|B_{\perp}|$ and the modulation period a [commensurability oscillation (CO)].⁷ The CO has a characteristic phase factor depending on the type of modulation,^{4,5,8} i.e., the minima appear at

$$2R_c/a=n\pm\frac{1}{4} \quad (n=1,2,3,\dots), \quad (3)$$

where the positive (negative) sign is for the magnetic-field (potential) modulation. Earlier attempts to observe a similar behavior for $\nu=1/2$ CF's (Ref. 9) showed only PMR. PMR is sometimes observed even in a plain 2DEG,¹⁰ albeit much smaller in magnitude, and is attributed to a random potential translated into a random modulation of B_{eff} .¹¹ Therefore, periodicity is not a prerequisite for the appearance of PMR. The observation of CO has been elusive until quite recently.³ Two important requirements must be met in order to observe CO: first, the period a has to be small enough compared with the mean free path L_{CF} of the CF's, so that commensurability is not blurred by scattering; second, the amplitude B_0 of the B_{eff} modulation should not exceed the largest B_{eff} that fulfills Eq. (3), since otherwise cyclotron motion would be inhibited. Corresponding requirements also apply to electrons at low fields, but for CF's the conditions are much harder to meet for the following reasons. First, L_{CF} is much smaller than the electron mean free path L_e . Second, $B_0=(\Delta n_e/n_e)B_{1/2}$ [$B_0=3(\Delta n_e/n_e)B_{3/2}$] for the $\nu=1/2$ ($\nu=3/2$) CF's can be quite large¹² because $B_{1/2}$ and $B_{3/2}$ are typically several T, where Δn_e is the amplitude of the electron-density modulation, and the bar denotes the spatial average of the respective quantity. In the present paper, we report an observation of both PMR and CO for the $\nu=3/2$ CF's. The requirements mentioned above are met by the employment of a short period $a=92$ nm, which is about a factor of 3 smaller than the smallest period used in Ref. 3. Several minima of CO given by Eq. (3) (+ sign) with indices up to $n=3$ are observed, in contrast to Ref. 3, where only $n=1$ structures were seen.

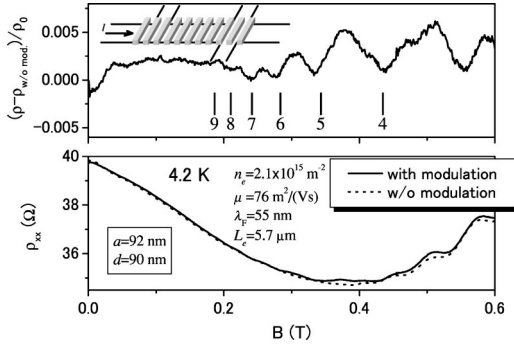


FIG. 1. Lower panel: ρ_{xx} of a 2DEG with and without modulation. Upper panel: the difference of the two traces. Positions given by Eq. (3) (the negative sign) are marked by vertical bars with indices n .

With the introduction of an in-plane field by tilting the sample, PMR takes on an unexpected asymmetry between positive and negative B_{eff} .

Samples are fabricated from GaAs/Al_xGa_{1-x}As 2DEG with a mobility $\mu = 76 \text{ m}^2/\text{Vs}$ (at 4.2 K) and $n_e = 2.1 \times 10^{15} \text{ m}^{-2}$. The heterointerface resides at the depth $d = 90 \text{ nm}$. The 2DEG wafer is defined into a $37\text{-}\mu\text{m}$ -wide Hall bar. Unidirectional potential modulation is introduced by placing a grating made of negative electron-beam resist¹³ on the surface; the strain thus introduced induces a potential modulation via piezoelectric effect.^{14,15} The Hall bar has two sets of voltage probes, so that the longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} of both the section with the grating on top and the adjoining section without grating can be measured. The lower panel of Fig. 1 shows low-field ρ_{xx} at 4.2 K. Traces for both modulated and adjacent unmodulated parts are shown. The CO for electrons is only barely visible at $0.2 \leq B \leq 0.45 \text{ T}$ for the modulated part, attesting to the presence of the modulation and the smallness of its amplitude. [Shubnikov–de Haas (SdH) oscillation is seen at $\geq 0.45 \text{ T}$, which has a $1/B$ period clearly different from the lower field oscillation.] The oscillation appears clearly by taking the difference of the two traces, as shown in the upper panel, with the positions of minima given by Eq. (3) ($-$ sign) denoted by vertical ticks with their indices n . By the analysis of the oscillation amplitude reported elsewhere,¹⁵ the modulation amplitude is evaluated to be $V_0 = 0.015 \text{ meV}$, or 0.2% of the Fermi energy E_F . The small amplitude is attributed mainly to the small value of $a/d (\approx 1)$.

Measurements at higher fields are carried out in a top-loading ^3He - ^4He dilution refrigerator with an *in situ* sample rotation probe, placed in a 17-T superconducting magnet. A standard ac lock-in technique (16 Hz) is used for resistivity measurements. Figure 2 shows ρ_{xx} at fields between $\nu = 2$ and 1 for three different bath temperatures from the base temperature ($\sim 20 \text{ mK}$) up to 350 mK. Since a relatively high excitation current ($I = 100 \text{ mA}$) is employed for the signal-to-noise consideration, the electron temperature can be considerably higher than the bath temperature; the difference can be up to about 50 mK for the base temperature.¹⁶ (For the purpose of the present paper, the qualitative temperature dependence suffices.) As can be seen in the figure, the PMR

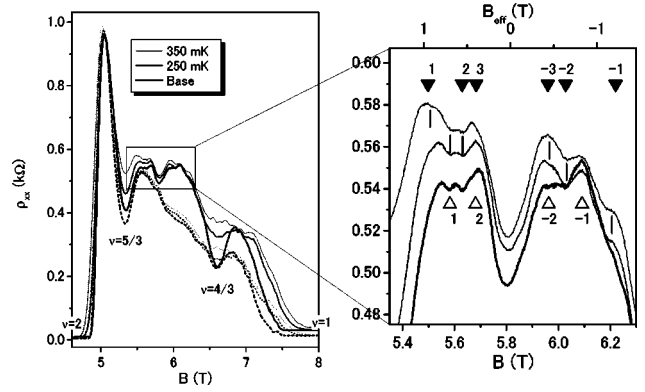


FIG. 2. ρ_{xx} at higher fields for three different (bath) temperatures. Solid and dotted curves are for modulated and unmodulated parts, respectively. A close-up of the vicinity of $\nu = \frac{3}{2}$ for the modulated part is shown in the right panel. Positions of minima calculated by Eq. (3) with a positive sign (Ref. 18), assuming a fully spin-polarized (fully spin-unpolarized) k_F , are shown by solid downward (open upward) triangles with their indices. (The sign of B_{eff} is attached to n for clarity.) The vertical lines between two higher-temperature traces indicate the occurrence of minima, which is replotted in Fig. 3.

emanating from the $\nu = \frac{3}{2}$ filling with the CO on both sides is observed only for the modulated part of the sample. From ρ_{xx} at $\nu = \frac{3}{2}$, the mean free path is calculated to be $L_{\text{CF}} \approx 1.1 \mu\text{m}$ at the base temperature, and to decrease slightly with temperature. Apparently, period $a = 92 \text{ nm}$ is on the verge of the observability limit of the PMR and CO for this value of L_{CF} ; the sample made from an identical 2DEG wafer with a slightly larger period $a = 115 \text{ nm}$ shows only a slight and rather asymmetric dent which might be the incomplete sign of PMR, and no CO is observed at all. The FQHE features at $\nu = \frac{5}{3}$ and $\frac{4}{3}$ diminish rapidly with temperature.¹⁷ By comparison, both PMR and CO are much more robust against temperature, as is the case with the corresponding phenomena for the electrons at low fields. By analogy to electrons,^{8,15} the characteristic temperature describing the damping of the CO of CF is expected to be a factor $ak_F/2$ as large as that for the FQHE—the SdH oscillation of CF's. Since a is small and k_F is about $\sqrt{2/3}$ of the electrons (see below), $ak_F/2 \approx 4.3$ is not so large but is still significantly larger than unity. The PMR extends up to $|B_{\text{eff}}| \sim 0.4 \text{ T}$. The modulation amplitude B_0 is $\sim 35 \text{ mT}$ if one assumes that $\Delta n_e/n_e = V_0/E_F$, deduced at low fields, is also valid at high fields. With this B_0 , it is difficult to explain the observed PMR solely by the channeled orbit,⁶ suggesting that other mechanisms proposed^{19,20} for $\nu = \frac{1}{2}$ CF's is also operative, and/or the modulation amplitude is effectively larger for CF's than for electrons.²¹

The values of B_{eff} giving the minima (or sometimes small dents) are read from the data in Fig. 2 (vertical short lines between 250 and 350-mK traces), and the corresponding values of $2R_c/a$ with $R_c = \hbar k_F/e|B_{\text{eff}}|$ are plotted in Fig. 3 against the indices, which are taken as the integer nearest to the value of $2R_c/a$ attached with the sign of B_{eff} . In the calculation of k_F , spins of CF are assumed to be fully polarized, hence $k_F = \sqrt{4\pi n_{\text{CF}}}$, which varies with field [see Eq.

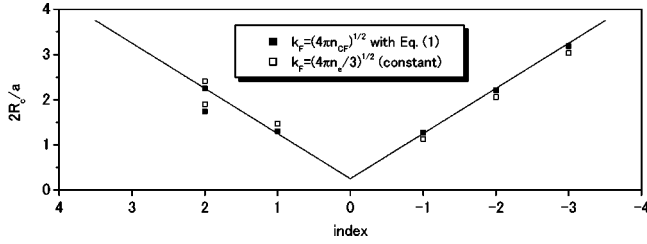


FIG. 3. Plot of $2R_c/a$ corresponding to ρ_{xx} minima, using $k_F = \sqrt{4\pi n_{CF}}$, with n_{CF} given by Eq. (1) (solid squares). The straight lines connect the positions expected from Eq. (3) (the positive sign). $2R_c/a$ calculated assuming that constant k_F at $\nu = \frac{3}{2}$ is also plotted (open squares).

(1)]. For comparison, the values of $2R_c/a$ calculated with a constant $k_F = \sqrt{4\pi n_e/3}$, using n_{CF} at the exact filling $\nu = \frac{3}{2}$, are also plotted. Straight lines in Fig. 3 indicate the theoretically expected positions given by Eq. (3) (+ sign), demonstrating that the observed minima are consistent with the B_{eff} modulation and with the field-dependent k_F , except for one dip with the index of 2 that fails to fall on the line (occurring at $B_{\text{eff}} \approx 0.7$ T). The exceptional dip has $2R_c/a \approx 1.74$, and seems at first glance consistent with Eq. (3) with a negative sign. However, it is difficult to attribute this dip to commensurability of CF's with the direct potential modulation for the following two reasons. First, the potential and the B_{eff} modulations are, due to the origin of the latter, expected to be in phase. For in-phase modulation, the positions of minima shift from Eq. (3) according to the relative amplitude of the two types of modulations, rather than showing independent minima corresponding to each type.^{4,8} Second, the oscillation amplitude for the direct potential modulation is calculated to be orders of magnitude smaller than that for the B_{eff} modulation if one applies the theory for electrons:⁸ the ratio is $[(V_0/E_F)(1/B_0)\Phi_0(k_F/2a)]^2 \approx \pi/(12a^2 n_e)$ and is about ~ 0.015 for the present sample. In Fig. 3, we have assumed a fully polarized spin for the CF's. However, the spin polarization of CF's is still an issue of controversy, especially for $\nu = \frac{3}{2}$. Geometric resonance with a surface acoustic wave²² defines k_F consistent with (almost) the full spin polarization of the $\nu = \frac{3}{2}$ CF's, while the angular-dependent magnetotransport coincidence measurement²³ suggests polarization ratio of about 5:3. In general, for a partially polarized system with a population ratio $f_1:f_2$ ($f_1 + f_2 = 1$), k_F is defined for each branch as $k_{F,i} = \sqrt{4\pi n_{CF} f_i}$ ($i = 1$ and 2), and, in principle, commensurability for each branch can take place independently.²⁴ In the other extreme case of fully unpolarized spins, $f_1 = f_2 = 0.5$, and both branches have an identical $k_F = \sqrt{2\pi n_{CF}}$ which is a factor of $\sqrt{2}$ smaller than the fully polarized case. With the use of this fully unpolarized k_F , $2R_c/a = 1.74$ reduces to 1.23, and conforms with the theory for the B_{eff} modulation. (Also see the right panel of Fig. 2.) This suggests, if taken literally, the coexistence of fully spin-polarized and fully spin-unpolarized phases. The data would be more readily understandable if we could find a set of f_1 and f_2 that fulfills the condition $f_1 + f_2 = 1$ and explains observed minima. However, the n th minima of the dominant branch ($f_i \geq 0.5$) should appear somewhere between the two

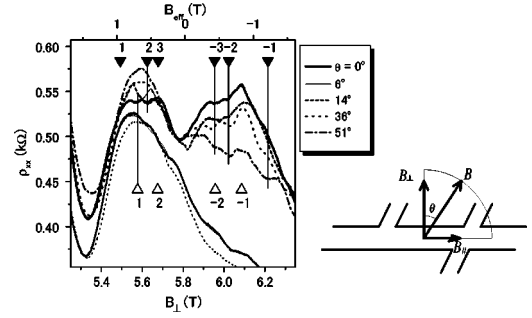


FIG. 4. Resistivity (at base temperature) with several tilt angles θ defined in the right panel. Solid and open triangles are the same as in Fig. 2. Traces for the unmodulated part are also shown.

extreme cases, i.e., fully spin polarized ($f = 1$) and fully spin unpolarized ($f = 0.5$), shown in the right panel of Fig. 2, and therefore cannot explain any of the minima observed. Obviously, more studies are necessary to clarify this puzzling result.

In order to explain the relative magnitude of the observed oscillatory features, several factors should be taken into account. As a general trend, the oscillation amplitude decreases with decreasing $|B_{\text{eff}}|$. This explains the weakness of the minima ± 3 (a $+3$ minimum is even difficult to observe) and the absence of oscillation with higher indices. However, dips for the indices ± 1 are less conspicuous than those for ± 2 . This can be accounted for by the intervention of the $\nu = \frac{5}{3}$ and $\frac{4}{3}$ FQHE's. B_{eff} 's for ± 1 enter (or are about to enter) the region of deep downward slope for the FQHE's, whose amplitudes are orders of magnitude larger than the present CO. The FQHE becomes more pronounced with decreasing temperature, making ± 1 minima more difficult to recognize. Qualitatively the same things happen for the Weiss oscillation of electrons. At $B \geq 0.45$ T (at 4.2 K), oscillations with lower indices are modified both in their amplitude and phase, and sometimes totally hidden, by the SdH effect for our small a and V_0 systems (see Fig. 1 and Ref. 15). Another factor that affects the oscillation amplitude is one peculiar to $\nu = \frac{3}{2}$ CF's. From Eqs. (1) and (2), $k_F = \sqrt{4\pi n_{CF}} = \sqrt{4\pi(n_e - 2B_{\text{eff}}/\Phi_0)/3}$, hence $L_{CF} = \hbar k_F \mu / e$ decrease with B_{eff} . This will make the dips in the $B_{\text{eff}} > 0$ side less pronounced than the corresponding dips in the $B_{\text{eff}} < 0$ side, which can actually be observed in Fig. 2.

We have also investigated the effect of in-plane field B_{\parallel} by tilting the sample (Fig. 4). Quite unexpectedly, the PMR becomes asymmetric with a tilt angle θ : the PMR on the $B_{\text{eff}} > 0$ side grows larger, extending up to larger B_{eff} , while it shrinks on the $B_{\text{eff}} < 0$ side. On the other hand, the positions of the CO minima marked by vertical lines remain practically unchanged at least for $B_{\text{eff}} < 0$,²⁵ although their amplitudes show a nonmonotonic change with θ . For $B_{\text{eff}} > 0$, the CO becomes less visible by the intervention of the enhanced PMR. A plausible source of the asymmetry is an asymmetric L_{CF} : L_{CF} becomes smaller (larger) with $|B_{\text{eff}}|$ for $B_{\text{eff}} > 0$ ($B_{\text{eff}} < 0$). However, no simple reason can be readily found for this asymmetry to be enhanced by B_{\parallel} . For the $\nu = \frac{1}{2}$ CF's,³ the PMR is seen to become asymmetric with an increase of the period. Also, our $a = 115$ nm sample shows an

asymmetric dent around $\nu = \frac{3}{2}$, as mentioned above. A similar trend can occur if L_{CF} is decreased with a fixed period. However, ρ_{xx} at $\nu = \frac{3}{2}$ for our sample does not change by tilting, suggesting that L_{CF} remains unaffected by B_{\parallel} . A recent theory²⁰ demonstrated that the presence of both potential and B_{eff} modulations leads to an asymmetric PMR. But again, it is difficult to connect this to B_{\parallel} . The tilt-induced asymmetry remains unexplained at the moment. It would be quite interesting to know whether or not similar asymmetries also occur for the $\nu = \frac{1}{2}$ CF's. Unfortunately tilt experiments for the $\nu = \frac{1}{2}$ CF's generally require much higher fields than are available in superconducting magnets.

In summary, we have successfully observed PMR and CO of $\nu = \frac{3}{2}$ CF's under unidirectional B_{eff} modulation, by employing a short-period modulation. Most of the CO minima

are found to be consistent with the fully spin-polarized k_F , which varies with the field. We have also observed a single minimum whose position is apparently consistent with the fully unpolarized k_F . Together with the unexpected evolution of asymmetry in the PMR with B_{\parallel} , this calls for further study.

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¹⁷Higher-temperature traces in Fig. 2 show wiggling structures around $\nu = \frac{4}{3}$. The origin of those is not known at present, but may be related to the use of relatively high excitation current. Since they are also observed for the unmodulated part, they are apparently not caused by the potential modulation. Their temperature dependence is qualitatively different from that of the features of our present interest around $\nu = \frac{3}{2}$.

¹⁸Explicitly, the positions are $B_{\text{eff}}(n) = [-A(n) \pm \sqrt{A(n)^2 + 4\bar{B}_2 A(n)}] / 2$, with $A(n) \equiv [(8/3\pi)(\Phi_0/a^2)f] / (n + \frac{1}{4})^2$, where $f = 1$ ($f = 0.5$) for the spin-polarized (unpolarized) case.

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