

Quasicondensate and superfluid fraction in the two-dimensional charged boson gas at finite temperature

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The Bogoliubov–de Gennes equations are solved for the Coulomb Bose gas describing a fluid of charged bosons at finite temperature. The approach is applicable in the weak-coupling regime and the extent of its quantitative usefulness is tested in the three-dimensional fluid, for which diffusion Monte Carlo data are available on the condensate fraction at zero temperature. The one-body density matrix is then evaluated by the same approach for the two-dimensional fluid with e^2/r interactions, to demonstrate the presence of a quasicondensate from its power-law decay with increasing distance and to evaluate the superfluid fraction as a function of temperature at weak coupling.

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I. INTRODUCTION

The fluid of pointlike spinless charged bosons embedded in a uniform neutralizing background has attracted attention in the literature mainly as a model in quantum statistical mechanics, which is complementary to the physically more relevant fermionic gas of electrons.¹ It was proposed by Schafroth² as a model for superconductors prior to the BCS theory and has received renewed interest after the discovery of ceramic superconductors.³ In some viewpoints a Bose-Einstein condensate of tightly bound pairs of small polarons could be a relevant model for high- T_c superconductivity in the layered cuprates.⁴ The model also has some astrophysical relevance in describing pressure-ionized helium in cold stellar matter^{5–7} and the fusion of α particles inside a dense helium plasma.^{8,9}

A number of theoretical and computational studies have been addressed to the three-dimensional charged-boson fluid (3D CBF) at zero temperature. The properties of main interest for the theory have been the ground-state energy and structure and the static and dynamic dielectric response. The early theoretical work was concerned with evaluating the ground-state energy and the elementary excitations in the weak-coupling (high density) limit.^{10–12} Both variational calculations based on Jastrow-Feenberg wave functions^{13–18} and self-consistent treatments of correlations^{19–21} have subsequently been used to evaluate the intermediate and strong-coupling regime. Quantum Monte Carlo studies of the 3D CBF (Refs. 16 and 22–25) have covered the whole range of coupling strength up to the regime of Wigner crystallization driven by the Coulomb repulsions. Extensive data on the condensate fraction and the momentum distribution in dependence of the coupling strength have become available through the diffusion Monte Carlo (DMC) work of Moroni *et al.*²⁵

The properties of the 2D CBF at zero temperature have also been investigated with both e^2/r and $\ln(r)$ interactions. In the latter case Magro and Ceperley²⁶ have shown, using a sum-rule argument from Pitaevskii and Stringari,²⁷ that the presence of the long-wavelength plasmon in the excitation spectrum rules out the existence of a condensate even at zero

temperature. They further showed from a DMC study that the noncondensed fluid exhibits a power-law decay of the one-body density matrix and that above a threshold density the momentum distribution diverges at low momenta, although no condensate forms.

In the 2D CBF with e^2/r interactions (henceforth referred to as quasi-2D or Q2D CBF) a condensate is present in the ground state, but its density vanishes at finite temperature. This behavior parallels that of the neutral 2D Bose gas.^{28,29} The theory of correlations in the latter system has been developed by Kagan, Svistunov, and Shlyapnikov³⁰ (see also Kagan *et al.*³¹). As its temperature is lowered at constant density (or as its density is increased at constant temperature) across the Kosterlitz-Thouless transition, a weakly interacting gas enters the superfluid regime in which the phase correlation length R_c is much larger than the density correlation length r_c . In this situation the one-body density matrix $\rho(r)$ decays asymptotically to zero with an inverse-power law rather than exponentially.^{32,33} The idea of a “quasicondensate” emerges from the behavior of $\rho(r)$ at intermediate distances $r_c \ll r \ll R_c$. The local properties of the quasicondensate are the same as those of a genuine condensate, so that it turns to the latter as the phase correlation length starts to exceed the dimensions of the sample.

The main purpose of the present work is to study this behavior in the 2D CBF, using the Bogoliubov approach to describe the charged fluid both in the ground state and at finite temperature in the weak-coupling regime (corresponding in this case to high density). The Bogoliubov–de Gennes equations are presented for convenience in Sec. II and are first solved for the 3D CBF in Sec. III, where the approach is quantitatively tested at $T=0$ by comparing its results for the condensate fraction with the available DMC data. Section IV reports our main results regarding the quasicondensate and the superfluid fraction in the Q2D CBF. Finally, Sec. V gives a brief summary and our conclusions.

II. BOGOLIUBOV APPROACH TO A CHARGED-BOSON FLUID

The fluid of charged bosons on a uniform neutralizing background is described by the Hamiltonian

$$H = \int d\mathbf{r} \psi^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m} - \mu \right) \psi(\mathbf{r}) + \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}') V(|\mathbf{r}-\mathbf{r}'|) \psi(\mathbf{r}') \psi(\mathbf{r}), \quad (1)$$

where $V(r) = e^2/r$, $\psi(\mathbf{r})$ is the field operator and μ the chemical potential. The role of the background is to set to zero the average potential felt by each particle. The coupling strength is measured by the dimensionless parameter r_s , defined by $r_s a_B = (4\pi n/3)^{-1/3}$ in 3D and by $r_s a_B = (\pi n)^{-1/2}$ in 2D with a_B the Bohr radius and n the mean particle density.

We consider first the fluid at $T=0$. The Bogoliubov transformation³⁴ (for recent presentations, see Refs. 35 and 36) introduces a macroscopic order parameter ψ_0 by writing the field operator as $\psi(\mathbf{r}) = \psi_0 + \tilde{\psi}(\mathbf{r})$. The operator $\tilde{\psi}(\mathbf{r})$ describes the gas of Bose particles promoted out of the condensate. This gas is treated in the Hartree approximation, assuming that the contribution from terms nonlinear in $\tilde{\psi}(\mathbf{r})$ is small.

One finds $\mu=0$ and, with the linear transformation

$$\tilde{\psi}(\mathbf{r}, t) = \sum_\nu [u_\nu(\mathbf{r}, t) a_\nu + v_\nu^*(\mathbf{r}, t) a_\nu^\dagger] \quad (2)$$

for the Heisenberg field operator, one has to solve the coupled linear equations

$$i \frac{\partial u_\nu(\mathbf{r}, t)}{\partial t} = -\frac{1}{2m} \nabla^2 u_\nu(\mathbf{r}, t) + n_0 \int d\mathbf{r}' V(|\mathbf{r}-\mathbf{r}'|) [u_\nu(\mathbf{r}, t) + u_\nu(\mathbf{r}', t) + v_\nu(\mathbf{r}', t)] \quad (3)$$

and

$$-i \frac{\partial v_\nu(\mathbf{r}, t)}{\partial t} = -\frac{1}{2m} \nabla^2 v_\nu(\mathbf{r}, t) + n_0 \int d\mathbf{r}' V(|\mathbf{r}-\mathbf{r}'|) \times [v_\nu(\mathbf{r}, t) + v_\nu(\mathbf{r}', t) + u_\nu(\mathbf{r}', t)]. \quad (4)$$

Here, $n_0 = \psi_0^2$ is the uniform condensate density. The subsidiary condition

$$\sum_\nu [u_\nu(\mathbf{r}, t) u_\nu^*(\mathbf{r}', t) - v_\nu(\mathbf{r}', t) v_\nu^*(\mathbf{r}, t)] = \delta(\mathbf{r}-\mathbf{r}') \quad (5)$$

embodies the commutation rules on the field operators.

In a uniform fluid the state index ν is the wave vector \mathbf{k} . Equations (3)–(5) are solved by taking $u_{\mathbf{k}}(\mathbf{r}, t) = u_{\mathbf{k}} \exp[i(\mathbf{k} \cdot \mathbf{r} - \varepsilon_{\mathbf{k}} t)]$ and $v_{\mathbf{k}}(\mathbf{r}, t) = v_{\mathbf{k}} \exp[i(\mathbf{k} \cdot \mathbf{r} - \varepsilon_{\mathbf{k}} t)]$, with the results

$$u_{\mathbf{k}}^2 = \frac{1}{2} \left\{ 1 + \varepsilon_{\mathbf{k}}^{-1} \left[n_0 V(k) + \frac{k^2}{2m} \right] \right\} \quad (6)$$

and

$$v_{\mathbf{k}}^2 = -\frac{1}{2} \left\{ 1 - \varepsilon_{\mathbf{k}}^{-1} \left[n_0 V(k) + \frac{k^2}{2m} \right] \right\} \quad (7)$$

where

$$\varepsilon_{\mathbf{k}} = \left[\frac{n_0 k^2 V(k)}{m} + \left(\frac{k^2}{2m} \right)^2 \right]^{1/2}. \quad (8)$$

The condensate density is given by

$$n_0 = n - \langle \tilde{\psi}^\dagger(\mathbf{r}) \tilde{\psi}(\mathbf{r}) \rangle = n - \sum_{\mathbf{k} \neq 0} v_{\mathbf{k}}^2 \quad (9)$$

and the one-body density matrix is

$$\rho(r) = n_0 + \langle \tilde{\psi}^\dagger(\mathbf{r}) \tilde{\psi}(0) \rangle = n_0 + \sum_{\mathbf{k} \neq 0} v_{\mathbf{k}}^2 \exp(i\mathbf{k} \cdot \mathbf{r}). \quad (10)$$

We have assumed unitary volume in writing Eqs. (9) and (10).

After these transformations and approximations the Hamiltonian (1) has been reduced to that of a system of independent Bosonic excitations described by the operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$. The extension of the theory to finite temperature is then effected by means of the Bose-Einstein distribution function

$$f_{\mathbf{k}} = [\exp(\beta \varepsilon_{\mathbf{k}}) - 1]^{-1}. \quad (11)$$

In particular, Eqs. (9) and (10) become

$$n_0 = n - \sum_{\mathbf{k} \neq 0} [v_{\mathbf{k}}^2 + f_{\mathbf{k}}(u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2)], \quad (12)$$

and

$$\rho(r) = n_0 + \sum_{\mathbf{k} \neq 0} [v_{\mathbf{k}}^2 + f_{\mathbf{k}}(u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2)] \exp(i\mathbf{k} \cdot \mathbf{r}), \quad (13)$$

respectively.

III. THREE-DIMENSIONAL CHARGED-BOSON FLUID

We introduce the parameter $A = (3n_0/nr_s^3)^{1/2} e^2/(a_B k_B T)$ and the reduced distance $R = (12n_0/nr_s^3)^{1/4} (r/a_B)$. Then Eqs. (9), (12), and (13) can be rewritten as follows:

$$\left[\left(1 - \frac{n_0}{n} \right) \left(\frac{n_0}{n} \right)^{-3/4} \right]_{T=0} = \frac{2^{1/2} r_s^{3/4}}{3^{1/4} \pi} \int_0^\infty dx \left[\frac{f(x)}{g(x)} - 2x^2 \right], \quad (14)$$

$$\left[\left(1 - \frac{n_0}{n} \right) \left(\frac{n_0}{n} \right)^{-3/4} \right]_{T \neq 0} = \frac{2^{1/2} r_s^{3/4}}{3^{1/4} \pi} \int_0^\infty dx \left\{ \frac{f(x)}{g(x)} \left[1 + \frac{2}{\exp[Ag(x)] - 1} \right] - 2x^2 \right\} \quad (15)$$

and

$$\frac{\rho(r)}{n} = 1 - \frac{2^{1/2}}{3^{1/4} \pi} \left(\frac{n_0 r_s}{n} \right)^{3/4} \int_0^\infty dx \left[1 - \frac{\sin(Rx)}{Rx} \right] \times \left\{ \frac{f(x)}{g(x)} \left[1 + \frac{2}{\exp[Ag(x)] - 1} \right] - 2x^2 \right\}, \quad (16)$$

TABLE I. Condensate fraction in the 3D CBF at zero temperature from the Bogoliubov approach (B), compared with the diffusion Monte Carlo data (DMC, from Moroni *et al.*, Ref. 25).

r_s	1	2	5	10	20	50	100	160
$(n_0/n)_B$	0.818	0.722	0.549	0.401	0.264	0.132	0.072	0.047
$(n_0/n)_{\text{DMC}}$	0.827	0.722	0.542	0.359	0.206	0.053	0.0104	0.004

where we have defined $f(x) = 1 + 2x^4$ and $g(x) = (1 + x^4)^{1/2}$. Of course, Eq. (14) can also be obtained from Eq. (15) in the limit $T \rightarrow 0$. In the following we use the energy e^2/a_B as the unit of the thermal energy $k_B T$.

A numerical solution of Eqs. (14) and (15) for n_0/n in the physical range $0 \leq n_0/n \leq 1$ can be found for all values of the system parameters. We should bear in mind, however, that we are using a weak-coupling theory so that the results can be significant only when the depletion of the condensate is small (i.e., for n_0/n close to unity, in line of principle). This statement is quantitatively tested in Table I, where we compare our results for n_0/n at $T=0$ with the DMC data of Moroni *et al.*²⁵ over the whole fluid range up to Wigner crystallization. It is clear from Table I that, rather surprisingly, the Bogoliubov approach is almost fully quantitative up to $r_s \approx 5$, i.e., for values of n_0/n down to almost 0.5. Very similar results are obtained in the same range of r_s by the integrodifferential equations approach of Cherny and Shanenko.³⁷

Table II reports our results for n_0/n as a function of temperature for $r_s=1$ and $r_s=2$. In the lack of data for a quantitative test, one may hope from the test shown in Table I that the Bogoliubov approach could again be reasonably accurate for values of n_0/n larger than 0.5. According to this crude criterion, it appears from Table II that at such weak-couplings the theory could perhaps be useful up to a fairly sizable value of the reduced temperature $k_B T a_B / e^2$ —perhaps as large as unity for $r_s \approx 1$.

Finally, the one-body density matrix $\rho(r)$ (in units of the particle density n) is shown in Fig. 1 for $r_s=1$ (left panel) and $r_s=2$ (right panel), at various values of the reduced temperature. The asymptotic value of $\rho(r)/n$ in the limit $r \rightarrow \infty$ is the condensate fraction n_0/n , that we have already presented in Table II.

 TABLE II. Condensate fraction in the 3D CBF as a function of the reduced temperature from the Bogoliubov approach, for two values of the coupling strength r_s .

T	$(n_0/n)_{r_s=1}$	T	$(n_0/n)_{r_s=2}$
0	0.818	0	0.722
0.3	0.816	0.1	0.718
0.5	0.793	0.2	0.652
0.75	0.720	0.3	0.466
1.0	0.593	0.32	0.410
1.25	0.407	0.34	0.345
1.4	0.247	0.36	0.266
1.482	0.092	0.38	0.153

IV. QUASI-TWO-DIMENSIONAL CHARGED-BOSON FLUID

Taking $V(k) = 2\pi e^2/k$, Eq. (9) for the condensate fraction at $T=0$ yields

$$\left[\left(1 - \frac{n_0}{n} \right) \left(\frac{n_0}{n} \right)^{-2/3} \right]_{T=0} = r_s^{2/3} \int_0^\infty dx \left[\frac{f(x)}{g(x)} - 2x^3 \right], \quad (17)$$

where we have defined $f(x) = 1 + 2x^6$ and $g(x) = (1 + x^6)^{1/2}$. The numerical solution of Eq. (17) yields the values of the condensate fraction which are reported in Table III over a range of values for the coupling strength well below the Wigner phase transition³⁸ at $r_s \approx 35$. We should expect that the role of correlations becomes more important in lowered dimensionality, and indeed the crude criterion introduced in Sec. III suggests that in the Q2D fluid at $T=0$ the Bogoliubov approach may be useful only up to $r_s \approx 1$.

It is easily seen that the condensate fraction must instead vanish at $T \neq 0$. Indeed, if we assume $n_0/n \neq 0$ then Eq. (12) yields

$$\left[\left(1 - \frac{n_0}{n} \right) \left(\frac{n_0}{n} \right)^{-2/3} \right]_{T \neq 0} = r_s^{2/3} \int_0^\infty dx \left\{ \frac{f(x)}{g(x)} \left[1 + \frac{2}{\exp[Axg(x)] - 1} \right] - 2x^3 \right\}, \quad (18)$$

where we have set $A = 2(n_0/nr_s^2)^{2/3}e^2/(a_B k_B T)$. The second term in the square brackets on the right-hand side of Eq. (18) yields a contribution of order x^{-1} to the integrand for $x \rightarrow 0$, so that the integral diverges. On the other hand, the solution $n_0/n=0$ at $T \neq 0$ is consistent with the Bogoliubov–de Gennes equations: in this case they yield $v_{\mathbf{k}}=0$ and $u_{\mathbf{k}}=1$, and the particle density is related to the (now finite) chemical potential by

$$n = \sum_{\mathbf{k} \neq 0} \left\{ \exp \left[\beta \left(\frac{k^2}{2m} - \mu \right) \right] - 1 \right\}^{-1}. \quad (19)$$

We can at this point introduce the quasicondensate for the Q2D CBF at finite temperature. The essential point is the power-law decay of the one-body density matrix, which becomes slow at sufficiently low temperature and weak coupling.^{30–33} We first show this by an heuristic procedure that we shall justify later below. We isolate the singular term in $\rho(r)$ and resum it to infinite order to obtain

$$\rho(r) = \tilde{\rho}(r) \exp[-\Lambda(r)], \quad (20)$$

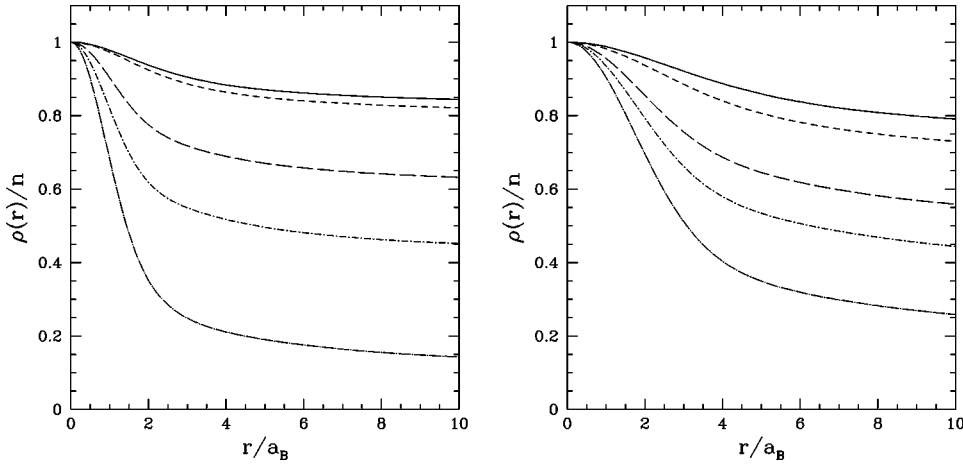


FIG. 1. The one-body density matrix $\rho(r)/n$ as a function of r/a_B in the 3D CBF. Left: for $r_s = 1$ at values of the reduced temperature T equal to 0, 0.5, 1.0, 1.25, and 1.482 (from top to bottom). Right: for $r_s = 2$ at $T = 0, 0.2, 0.3, 0.34,$ and 0.38 .

where with the help of Eq. (13) we have set

$$\begin{aligned} \Lambda(r) &= \int \frac{d^2k}{(2\pi)^2} [1 - \cos(\mathbf{k} \cdot \mathbf{r})] \frac{V(k)}{\tilde{\epsilon}_{\mathbf{k}}} f(\tilde{\epsilon}_{\mathbf{k}}) \\ &= 2 \left(\frac{nr_s^2}{\tilde{n}_0} \right)^{1/3} \int_0^\infty dx \frac{1 - J_0(x^2 R)}{g(x) \{ \exp[\tilde{A}xg(x)] - 1 \}} \end{aligned} \quad (21)$$

and

$$\begin{aligned} \frac{\tilde{\rho}(r)}{n} &= 1 - \frac{1}{n} \int \frac{d^2k}{(2\pi)^2} [1 - \cos(\mathbf{k} \cdot \mathbf{r})] \\ &\quad \times \left[\frac{1}{2} \left(\frac{\tilde{n}_0 V(k) + k^2/2m}{\tilde{\epsilon}_{\mathbf{k}}} - 1 \right) + \frac{k^2/2m}{\tilde{\epsilon}_{\mathbf{k}}} f(\tilde{\epsilon}_{\mathbf{k}}) \right] \\ &= 1 - \left(\frac{\tilde{n}_0 r_s}{n} \right)^{2/3} \int_0^\infty dx [1 - J_0(x^2 R)] \\ &\quad \times \left\{ \frac{f(x)}{g(x)} + \frac{4x^6}{g(x)(\exp[\tilde{A}xg(x)] - 1)} - 2x^3 \right\}, \end{aligned} \quad (22)$$

the quasicondensate density \tilde{n}_0 being defined by

$$\tilde{n}_0 = \lim_{r \rightarrow \infty} \tilde{\rho}(r). \quad (23)$$

In these equations $\tilde{\epsilon}_{\mathbf{k}} = [\tilde{n}_0 V(k)k^2/m + k^4/4m^2]^{1/2}$, $\tilde{A} = 2(\tilde{n}_0/nr_s^2)^{2/3} e^2/(a_B k_B T)$, $R = 2(\tilde{n}_0/nr_s^2)^{1/3}(r/a_B)$ and $J_0(y)$ is the Bessel function of zeroth order.

Figure 2 (upper panels) reports our numerical results for $\rho(r)$ at two values of the coupling strength and at various values of the reduced temperature. It is evident that at low

temperature the decay of the density matrix in space is slow and the notion of a quasicondensate thereby acquires physical significance.

A power-law decay of the one-body density matrix at low temperature and coupling strength can be demonstrated directly from Eqs. (20)–(23). The function $J_0(x^2 R)$ in the integrand in Eq. (21) provides a lower limit of integration going as $r^{-1/2}$, while the upper limit is set by a cutoff wave vector $k_0 = 1/L$ associated with the quasicondensate region (see Popov³²). Equation (21) thus yields

$$\begin{aligned} \Lambda(r) &\rightarrow 2\tilde{A}^{-1} (nr_s^2/\tilde{n}_0)^{1/3} \int_{r^{-1/2}}^{L^{-1/2}} dx \frac{1}{x} \\ &= 2\tilde{A}^{-1} (nr_s^2/\tilde{n}_0)^{1/3} \ln(r/L)^{1/2}. \end{aligned} \quad (24)$$

Hence since $\tilde{\rho}(r) \rightarrow \tilde{n}_0$ we find from Eq. (20)

$$\rho(r) \rightarrow \tilde{n}_0 (r/L)^{-\alpha} \quad (25)$$

with the value of the exponent given by

$$\alpha = \frac{n}{2\tilde{n}_0} r_s^2 \frac{k_B T}{e^2/a_B}. \quad (26)$$

Figure 2 (bottom panels) evidentiates the power-law decay of the density matrix by plotting the curves in log-log scale, as compared with the asymptotic behavior predicted by Eqs. (25) and (26) (dots in the figure).

A. Asymptotic behavior of the single-particle Green's function

We can actually show that the power-law decay of $\rho(r)$ that we have obtained just above derives from the correlations between phase fluctuations in the Q2D CBF. We follow the method proposed for the neutral 2D gas in the work of Popov³² (see also Fisher and Hohenberg³³). The fluctuations

TABLE III. Condensate fraction in the Q2D CBF at zero temperature from the Bogoliubov approach.

r_s	0.01	0.1	0.2	0.4	0.6	0.8	1	2	5
n_0/n	0.968	0.863	0.794	0.700	0.633	0.580	0.537	0.398	0.230

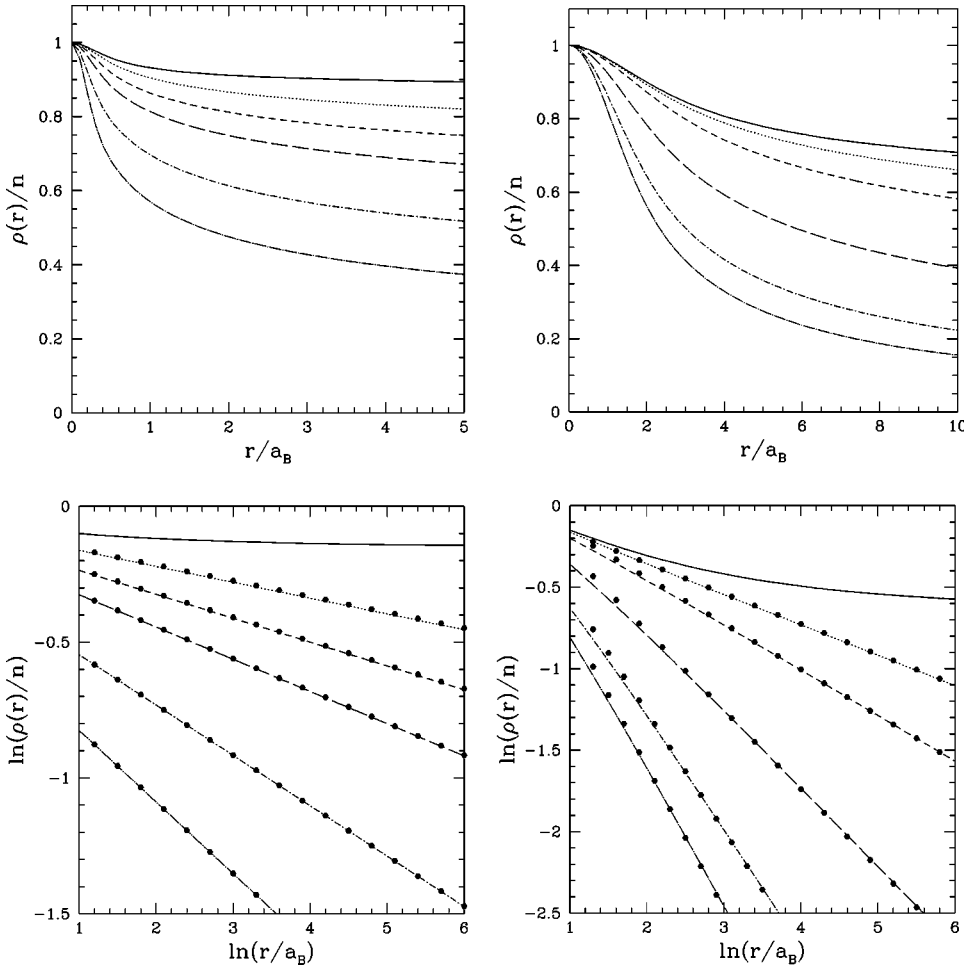


FIG. 2. The one-body density matrix $\rho(r)/n$ as a function of r/a_B in the Q2D CBF, both in linear scale (upper panels) and in logarithmic scale (lower panels). Left panels: for $r_s=0.1$ at values of the reduced temperature T equal to 0, 10, 15, 20, 30, and 40 (from top to bottom). Right panels: for $r_s=1$ at $T=0, 0.2, 0.3, 0.5, 0.7,$ and 0.8 . The dots in the lower panels give the asymptotic behavior of $\rho(r)/n$ at large r as evaluated analytically in Eqs. (25) and (26). The length L in Eq. (25), which is not determined by the asymptotic calculation, appears in logarithmic scale as an additive constant and is here fixed by requiring that the analytic result overlaps the numerical one as r increases.

of the phase $\phi(\mathbf{x}, t)$ determine the single-particle Green's function in the low-momentum regime (below the cutoff momentum k_0) according to

$$G(\mathbf{x}, \tau; \mathbf{x}_1, \tau_1) \approx \tilde{n}_0 \exp\left\{-\frac{1}{2}\langle[\phi(\mathbf{x}, \tau) - \phi(\mathbf{x}_1, \tau_1)]^2\rangle\right\}. \quad (27)$$

From Eq. (19.16) in Chap. 6 of Popov's book³² we find

$$\langle\phi(\mathbf{k}, \omega)\phi(-\mathbf{k}, -\omega)\rangle \rightarrow \frac{V(k)}{\omega^2 + \tilde{n}_0 V(k)k^2/m} \quad (28)$$

for the Q2D CBF at long wavelengths and frequencies, so that

$$\begin{aligned} & \frac{1}{2}\langle[\phi(\mathbf{x}, \tau) - \phi(\mathbf{x}_1, \tau_1)]^2\rangle \\ & \rightarrow \frac{1}{2\beta} \sum_{k < k_0} \sum_{\omega} \frac{V(k)}{\omega^2 + \tilde{n}_0 V(k)k^2/m} \\ & \quad \times |\exp[i(\mathbf{k} \cdot \mathbf{x} - \omega \tau)] - \exp[i(\mathbf{k} \cdot \mathbf{x}_1 - \omega \tau_1)]|^2. \end{aligned} \quad (29)$$

In Eq. (29) we carry out the summation over the Matsubara frequencies for $\tau_1 = \tau^+$ and keep the most diverging contribution in the integral over the momenta to obtain

$$\frac{1}{2}\langle[\phi(\mathbf{x}, \tau) - \phi(\mathbf{x}_1, \tau^+)]^2\rangle \rightarrow \alpha \ln \frac{r}{L} + \text{const}, \quad (30)$$

where $r = |\mathbf{x} - \mathbf{x}_1|$ and the quantity α is given by Eq. (26). Using this result in Eq. (27) we see that the Green's function decays to zero with the law $r^{-\alpha}$.

B. Superfluid fraction and quasicondensate fraction

As already noted in Sec. I, the notion of a quasicondensate becomes meaningful at temperatures below the Kosterlitz-Thouless transition. A superfluid component is therefore present at these temperatures and its density n_s is given by³³

$$\frac{n_s}{n} = 1 - \frac{\beta}{2nm} \sum_{\mathbf{k} \neq 0} k^2 \frac{\exp(\beta \epsilon_{\mathbf{k}})}{[\exp(\beta \epsilon_{\mathbf{k}}) - 1]^2}. \quad (31)$$

From Eq. (31) we have

$$\frac{n_s}{n} = 1 - 2\tilde{A} \left(\frac{r_s \tilde{n}_0}{n}\right)^{2/3} \int_0^\infty dx \frac{\exp[\tilde{A}(x+x^4)^{1/2}]}{\{\exp[\tilde{A}(x+x^4)^{1/2}] - 1\}^2}. \quad (32)$$

TABLE IV. Superfluid and quasicondensate fraction in the Q2D CBF as a function of reduced temperature from the Bogoliubov approach, for two values of the coupling strength r_s .

T	$(n_s/n)_{r_s=0.1}$	$(\tilde{n}_0/n)_{r_s=0.1}$	T	$(n_s/n)_{r_s=1}$	$(\tilde{n}_0/n)_{r_s=1}$
0	1	0.863	0	1	0.537
10	0.991	0.861	0.2	0.995	0.536
15	0.972	0.854	0.3	0.979	0.533
20	0.944	0.843	0.5	0.905	0.517
30	0.865	0.809	0.7	0.779	0.482
40	0.763	0.758	0.8	0.696	0.457

The value of the superfluid fraction n_s/n that we obtain from Eq. (32) are reported in Table IV for two values of the coupling strength and at various values of the reduced temperature.

In Table IV we also show the values taken by the quasicondensate fraction \tilde{n}_0/n at the same values of the coupling strength. The quasicondensate density is similar to the superfluid density at very low coupling, but rapidly decreases as the coupling strength is increased.

V. SUMMARY AND OUTLOOK

Summarizing, we have studied quasicondensation and superfluidity in a weakly interacting 2D fluid of charged

bosons with e^2/r interactions at finite temperature. By comparison of results on the 3D fluid with diffusion Monte Carlo data²⁵ we have found that the Bogoliubov approach may yield quantitatively useful predictions over a surprisingly wide range of coupling strengths and of deviations of the condensate fraction from unity. At finite temperature the behavior of the charged 2D gas is qualitatively wholly similar to that of its better known neutral analog: well below the Kosterlitz-Thouless transition a slow power-law decay is seen in the one-body density matrix, heralding extended-range correlations in phase fluctuations and the formation of a condensate over regions of finite size.

From the diffusion Monte Carlo results of Magro and Ceperley²⁶ it appears that the charged 2D fluid with logarithmic interactions may also show quasicondensation, even in the absence of macroscopic condensation in the ground state. Other interesting questions in this area regard how the transition from 3D to 2D behavior is effected and how one may develop a sound theoretical description of the momentum distribution with increasing coupling strength.

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