Possibility of a π Josephson junction and switch in superconductors with spiral magnetic order

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(Received 20 September 2000; revised manuscript received 1 June 2000; published 13 February 2001)

It is shown in tunneling junctions made of two magnetic superconductors with spiral magnetic order that the Josephson current depends on the magnetic phase, $\theta=\theta_L-\theta_R$, as well as the spiral helicity, $\chi=\chi_L\chi_R$. The former describes the relative orientation of magnetizations on the left (L) and right (R) banks of the contact, while the latter characterizes the left (right)-handed spiral with respect to the surface unit vector. The Josephson current $J=(J_c-J_{-\chi}\cos\theta)\sin\varphi$ is calculated as a function of the superconducting order parameters, $\Delta_{L(R)}$, the exchange energies, $h_{L(R)}$, and the spiral wave vectors, $\mathbf{Q}_{L(R)}$. It turns out that the π Josephson junction can be realized in some range of microscopic parameters. A possibility of making switches, which are based on this new phase relation, is also analyzed.

DOI: 10.1103/PhysRevB.63.104503 PACS number(s): 74.50.+r

I. INTRODUCTION

The physics of magnetic superconductors is interesting due to the competition of magnetic order and singlet superconductivity in bulk materials. The problem of their coexistence was first set up theoretically in the pioneering work by V. L. Ginzburg[1] in 1956. The experimental progress in the field begun after the discovery of the ternary rare earth (RE) compounds (RE)Rh_4B_4 and (RE)Mo_6X_8 (X=S,Se) (Ref. 2) with a regular distribution of localized RE magnetic moments. It turned out that in many of these systems superconductivity (with the critical temperature T_c) coexists rather easily with antiferromagnetic order (with the critical temperature T_N). Usually the situation with $T_N<T_c$ is realized.[2] Due to their antagonistic characters singlet superconductivity and ferromagnetic order cannot coexist in bulk samples with realistic physical parameters. However, under certain conditions the ferromagnetic order is transformed, in the presence of superconductivity, into a spiral or domainlike structure—depending on the type and strength of magnetic anisotropy in the system.[3,4] As a result of this competition, these two types of order coexist in a limited temperature interval $T_{c2}<T<T_m$ (the reentrant behavior) in $ErRh_4B_4$ and $HoMo_6S_8$, or even down to $T=0$ K in $HoMo_6Se_8$. In $ErRh_4B_4$ the coexistence region is narrow ($T_c=8.7$ K, $T_m\approx0.8$ K, $T_{c2}\approx0.7$ K), while in $HoMo_6S_8$ it is even narrower with $T_c=1.8$ K, $T_m\approx0.74$ K, $T_{c2}\approx0.7$ K—see Refs. 2, 3, and 4. In most of the new quaternary rare-earth compounds (RE)Ni_2B_2C antiferromagnetic order and superconductivity coexist up to $T=0$ K (Ref. 5), while in $HoNi_2B_2C$ an additional oscillatory magnetic structure is realized in a limited temperature interval. The latter magnetic structure competes strongly with superconductivity causing a reentrant behavior in this compound.[6]

A new and very interesting field of research was opened recently by Pobell's group,[7] which discovered the coexistence of superconductivity and nuclear magnetic order in $AuIn_2$ with $T_c=0,207$ K and $T_m=35$ μK. This result was interpreted in Ref. 8 to be due to the appearance of a spiral or domainlike nuclear magnetic order in the superconducting state.

Recently, it was reported that antiferromagnetic order (which is probably accompanied by a weak ferromagnetism) coexists with superconductivity in the layered perovskite superconductor $RuSr_2GdCu_2O_8$.[9] The magnetic order appears at the temperature $T_N=137$ K with magnetic moments placed in Ru-O planes. The superconductivity sets in at $T_c<45$ K and it occurs in Cu-O planes. We point out that in all of the above cited magnetic superconductors the effect of the exchange interaction (between localized magnetic moments and conduction electrons) on superconductivity is much larger than the electromagnetic interaction. Note that the electromagnetic interaction between localized magnetic moments and superconductivity is due to the dipolar magnetic field $\mathbf{B}_M$. The latter is caused by the localized magnetic moments and it affects the orbital motion of superconducting electrons.

Due to these facts, in the following we study the Josephson effect in magnetic superconductors with spiral magnetic order in the framework of the direct exchange (EX) model in which the exchange interaction dominates. It will be shown below that the Josephson current depends on a new quantity—the relative phase of the magnetic order parameter at the contact surfaces.

II. THE MODEL

The microscopic theory of magnetic superconductors, which is described in detail in Ref. 3, takes into account the interaction between *localized magnetic moments* and *conduction electrons* that goes: (a) via the direct exchange (EX) interaction and (b) via the electromagnetic (EM) interaction.

The Hamiltonian of the system has the form

$$\hat{H}=\hat{H}_0+\hat{H}_{BCS}+\int d^3r\left\{\hat{\psi}^{\dagger}(\mathbf{r})\hat{V}_{ex}(\mathbf{r})\hat{\psi}(\mathbf{r})+\frac{[\mathrm{curl}\,\mathbf{A}(\mathbf{r})]^2}{8\pi}\right\}+\sum_i[-\mathbf{B}(\mathbf{r}_i)g\mu_B\hat{\mathbf{J}}_i+\hat{H}_{CF}(\hat{\mathbf{J}}_i)]. \quad (1)$$

The operator $\hat{H}_0\equiv\hat{H}_0(\hat{\mathbf{p}}-(e/c)\mathbf{A})$ describes the quasiparticle motion in the magnetic field $\mathbf{B}(\mathbf{r})=\mathrm{curl}\,\mathbf{A}(\mathbf{r})$, where the

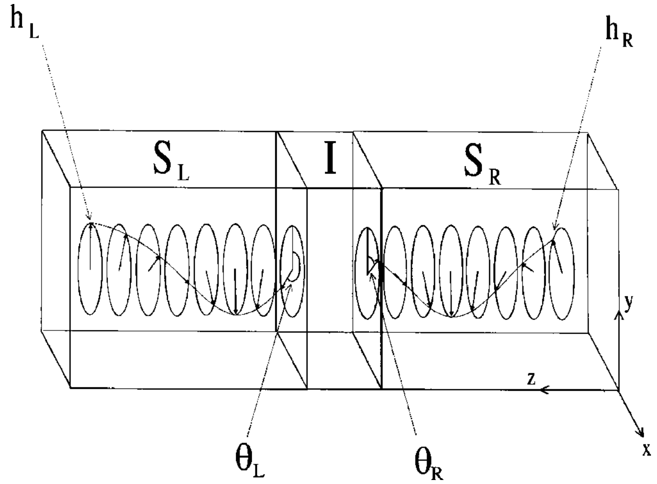

FIG. 1. The Josephson contact I between two magnetic superconductors S_L and S_R with spiral magnetic order. The corresponding exchange fields $\vec{h}_{L,R}$ at the surface make angles $\theta_{L,R}$ with the y axis. The wave vectors, $\mathbf{Q}_{L,R}$, of the spirals are along the z axis.

latter is created by the localized magnetic moments and by the induced Meissner current. The BCS pairing Hamiltonian $\hat{H}_{BCS}\equiv\hat{H}_{BCS}(\Delta(\mathbf{r}))$ describes superconductivity with a singlet order parameter $\Delta(\mathbf{r})$, while $\hat{V}_{ex}(r)=\hat{\mathbf{h}}(\mathbf{r})\boldsymbol{\sigma}$ is the exchange energy operator, where $\hat{\mathbf{h}}(\mathbf{r})=\Sigma_i J_{ex}(\mathbf{r}-\mathbf{r}_i)(g-1)\hat{\mathbf{J}}_i$. Here, $J_{ex}(\mathbf{r})$ is the exchange integral between the conduction electron spin $\boldsymbol{\sigma}=(\sigma_1,\sigma_2,\sigma_3)$ and the localized magnetic moment. The latter is characterized by the total angular momentum operator $\hat{\mathbf{J}}_i$ at the ith site. If the magnetic anisotropy in the system is pronounced then the crystal-field term $\hat{H}_{CF}(\hat{\mathbf{J}}_i)$ has to be added to $\hat{H}$.

In the following we assume that both banks of the junction are made of clean s-wave magnetic superconductors with spiral magnetic order. The extension of the theory to other than spiral oscillating magnetic order is discussed in Sec. IV. The spiral is characterized by the wave vector $\mathbf{Q}$ along the z axis, $\mathbf{Q}=Q_z\hat{\mathbf{z}}=\pm Q\hat{\mathbf{z}}$, and by the helicity $\chi=Q_z/Q=\pm 1$. In what follows we consider magnetic superconductors with an easy-plane magnetic anisotropy in which case the localized magnetic moments lie in the xy plane (see Fig. 1) and the spiral structure is favorable in the superconducting state. The exchange effects are considered in the mean-field approximation in which case the exchange potential $\hat{V}(\mathbf{r})\equiv\langle\hat{V}_{ex}(\mathbf{r})\rangle$ reads

$$\hat{V}(\mathbf{r})=\begin{pmatrix} 0 & he^{-i(\chi Qz+\theta)} \\ he^{i(\chi Qz+\theta)} & 0 \end{pmatrix}. \tag{2}$$

Here, $h[=n_m(g-1)J_{ex}(0)\langle|\hat{\mathbf{J}}|\rangle]$ is the exchange energy and $J_{ex}(0)$ is the $\mathbf{q}=0$ Fourier component of $J_{ex}(\mathbf{r}-\mathbf{r}_i)$, n_m is the concentration of regularly distributed localized magnetic moments—see Ref. 3. For further purposes we define $h_\theta=he^{i\theta}$, where the magnetic phase θ characterizes the orientation of the exchange field (magnetization) at the surface $z=0$ of the junction. As is well known the Josephson current depends on the anomalous Green's functions $F^\dagger_{\sigma_1\sigma_2}(x_1,x_2)=\langle\hat{T}\hat{\psi}^\dagger_{\sigma_1}(x_1)\hat{\psi}^\dagger_{\sigma_2}(x_2)\rangle$, as well as on their conjugates, where $x=(\mathbf{r},\tau)$ and $\sigma=\uparrow,\downarrow$. In the case of magnetic superconductors with spiral magnetic order these functions were calculated in Ref. 4 as a function of Matsubara frequencies, $\omega_n=\pi T(2n+1)$, and are given by

$$F^\dagger_{\downarrow\uparrow}(\mathbf{k},\mathbf{k}';\omega_n)=\delta(\mathbf{k}-\mathbf{k}')\Delta\frac{\omega_n^2+\Delta^2-h^2+(\varepsilon_{\mathbf{p}}+\rho_{\mathbf{k}})^2}{[\omega_n^2+E_{1\mathbf{k}}^2][\omega_n^2+E_{2\mathbf{k}}^2]}. \tag{3}$$

$$F^\dagger_{\uparrow\uparrow}(\mathbf{k}+\mathbf{Q}/2,\mathbf{k}'-\mathbf{Q}/2;\omega_n)=-\delta(\mathbf{k}-\mathbf{k}')2\Delta h^*_\theta\frac{[i\omega_n-\rho_{\mathbf{k}}]}{[\omega_n^2+E_{1\mathbf{k}}^2][\omega_n^2+E_{2\mathbf{k}}^2]}. \tag{4}$$

The quasiparticle excitation spectrum in the coexistence phase, $E_{1,2\mathbf{k}}$, is given by

$$E_{1,2\mathbf{k}}^2=\varepsilon_{\mathbf{k}}^2+\rho_{\mathbf{k}}^2+h^2+|\Delta|^2\mp 2\sqrt{\varepsilon_{\mathbf{k}}^2(\rho_{\mathbf{k}}^2+h^2)+|\Delta|^2h^2}, \tag{5}$$

where

$$\varepsilon_{\mathbf{k}}=[\xi(\mathbf{k}+\mathbf{Q}/2)+\xi(\mathbf{k}-\mathbf{Q}/2)]/2$$

$$\rho_{\mathbf{k}}=[\xi(\mathbf{k}+\mathbf{Q}/2)-\xi(\mathbf{k}-\mathbf{Q}/2)]/2. \tag{6}$$

For calculational simplicity we consider in the following a parabolic quasiparticle spectrum, $\xi(\mathbf{k})=\mathbf{k}^2/2m-E_F$, in which case $\varepsilon_{\mathbf{k}}=\xi(\mathbf{k})$ and $\rho_{\mathbf{k}}=\mathbf{v}_F\cdot\mathbf{Q}$, where $\mathbf{v}_F$ is the Fermi velocity.

The pair function $F^\dagger_{\downarrow\downarrow}$ is obtained from $F^\dagger_{\uparrow\uparrow}$ by replacing $h^*_\theta\to h_\theta$, i.e., $\theta\to-\theta$ and $\mathbf{Q}\to-\mathbf{Q}$. It is worth mentioning here, that in the magnetic superconductor with singlet pairing $(\Delta_{\uparrow\downarrow}=\Delta)$ and with spiral magnetic order triplet pairs with amplitudes, $F^\dagger_{\uparrow\uparrow}$ and $F^\dagger_{\downarrow\downarrow}$, have a finite lifetime although there is no triplet pairing in the system $(\Delta_{\uparrow\uparrow}=0)$. The finite lifetime of the triplet pairs, which is of the order $\hbar/\Delta$, leads to nontrivial effects in the Josephson current studied below.

The above theory[3,4] predicts that if in a magnetic superconductor the exchange interaction dominates then superconductivity modifies the ferromagnetic order into a spiral, or domainlike, one. In that case the spiral wave vector, $\vec{Q}$, fulfills the condition $\xi_0^{-1}\ll Q\ll k_F$. The latter means, that the exchange field, $\langle\mathbf{h}\rangle_{\xi_0}$, which is averaged over the superconducting coherence length ξ_0, is practically zero, thus reducing the depairing effect of the exchange interaction. As already mentioned the two types of order coexist in a narrow temperature interval in $ErRh_4B_4$, $HoMo_6S_8$, or down to $T=0$ K in $HoMo_6Se_8$, while in $AuIn_2$ superconductivity coexists with the nuclear (spiral) magnetic order down to $T=0$ K.[7,8]

III. JOSEPHSON CURRENT

We consider the tunneling Josephson junction between two magnetic superconductors, where in both of them the spiral magnetic order is realized—see Fig. 1. The latter is characterized by the wave vector $\mathbf{Q}_{L,R}$ and exchange fields

$h_{\theta_{L(R)}}=h_{L(R)}e^{i\theta_{L(R)}}$, respectively, while the superconductivity in the banks is described by the order parameter $\Delta_{L,R}=|\Delta_{L,R}|e^{i\varphi_{L,R}}$. To simplify the calculation we assume that: (i) $|\Delta_L|=|\Delta_R|=\Delta$, $h_L=h_R=h$, while $\varphi=\varphi_L-\varphi_R\neq 0$ and $\theta=\theta_L-\theta_R\neq 0$; (ii) $|\mathbf{Q}_L|=|\mathbf{Q}_R|=Q$ where $\mathbf{Q}_{L,R}=\chi_{L,R}Q\hat{\mathbf{z}}$ are orthogonal to the tunneling barrier. The spiral helicity $\chi_{L(R)}$ takes values ± 1 for $\mathbf{Q}_{L,R}$ along or opposite to the z axis. In that case one has $\rho_{\mathbf{k}_{L(R)}}=\mathbf{v}_F\cdot\mathbf{Q}_{L(R)}=\chi_{L(R)}\rho=\chi_{L(R)}\rho_0\cos\beta$, where $\rho_0=Qv_F$ and β is the angle between the Fermi velocity $\mathbf{v}_F$ and the z axis.

The tunneling process of a left-side electron with momentum and spin $(\mathbf{k}_L,\sigma)$ into a right-side electron $(\mathbf{k}_R,\sigma)$ is described by the standard tunneling Hamiltonian.[10] For the calculational simplicity the standard assumption is made[10] that the tunneling amplitude $T_{\mathbf{k}_L,\mathbf{k}_R}$ is weakly dependent on energy and momentum, i.e., $T_{\mathbf{k}_L,\mathbf{k}_R}\approx T_0\Theta(k_{L,z}k_{R,z})$. The Heaviside function $\Theta(k_{L,z}k_{R,z})$ takes into account that before tunneling the left electron moves toward the barrier, while after tunneling it moves as a right electron away from the barrier, and vice versa. We point out that the assumed form for $T_{\mathbf{k}_L,\mathbf{k}_R}$ is more suitable for a diffusive tunneling barrier, i.e., for an incoherent tunneling process.

The standard theory of the first-order Josephson effect[10] (in which the current is proportional to $|T_{\mathbf{k}_L,\mathbf{k}_R}|^2$) applied to the above system gives the Josephson current $J(\varphi,\theta)$ and the junction energy $E_J(\varphi,\theta)$ in the following form:

$$J(\varphi,\theta)=(J_c-J_{-\chi}\cos\theta)\sin\varphi, \tag{7}$$

$$E_J(\varphi,\theta)=-\frac{\Phi_0 J_c}{2\pi c}(1-R_{-\chi}\cos\theta)\cos\varphi+\text{const}. \tag{8}$$

Here, $\varphi=\varphi_L-\varphi_R$, $\theta=\theta_L-\theta_R$, Φ_0 is the flux quantum and $R_{-\chi}=J_{-\chi}/J_c$. The first term in the bracket of Eq. (7), which is proportional to J_c, is the standard Josephson term due to the tunneling of singlet pairs, i.e., $J_c\sim T\Sigma_{\mathbf{k}_L,\mathbf{k}_R,\omega_n}|T_{\mathbf{k}_L,\mathbf{k}_R}|^2F^{\dagger}_{\uparrow\downarrow}(\mathbf{k}_L,\omega_n)F_{\uparrow\downarrow}(\mathbf{k}_R,-\omega_n)$. In what follows a finite temperature is assumed. In the calculation of $J(\varphi,\theta)$ the summation over $\mathbf{k}$ is replaced by the integration over ξ and $\rho(\equiv\rho_0\cos\beta)$, i.e.,

$$\sum_{\mathbf{k}}(\cdots)=\frac{N(0)}{2\rho_0}\int_0^{\rho_0}d\rho\int_{-\infty}^{\infty}d\xi(\cdots), \tag{9}$$

where $N(0)$ is the density of states on the Fermi level, $\rho_0=Qv_F$. By integrating over ξ a straightforward calculation leads to the expression for J_c:

$$J_c=4e\pi^2N^2(0)|T_0|^2\Delta^2T\sum_{n=1}^{\infty}\left[\int_0^1 I(\omega_n,y)dy\right]^2,$$

$$I(\omega_n,y)=\frac{a_n+\sqrt{a_n^2-4\Delta^2h^2}-2h^2}{\sqrt{a_n^2-4\Delta^2h^2}\sqrt{a_n-2\rho_0^2y^2-2h^2+\sqrt{a_n^2-4\Delta^2h^2}}}, \tag{10}$$

where $y=\cos\beta$, $a_n=\omega_n^2+\rho_0^2y^2+h^2+\Delta^2$ and $\rho_0=Qv_F$.

The second term in Eq. (7), which is proportional to $J_{-\chi}$, is absent in standard Josephson junctions based on nonmagnetic superconductors. This term depends on the relative magnetic phase of the exchange fields at the barrier surfaces as well as on the helicity $\chi=\chi_L\chi_R$. It is due to the tunneling of Cooper pairs that are for a short time in the triplet state; i.e.,

$$J_{-\chi}\sim -T\sum_{\mathbf{k}_L,\mathbf{k}_R,\omega_n}|T_{\mathbf{k}_L,\mathbf{k}_R}|^2\{F^{\dagger}_{\uparrow\uparrow}(\mathbf{k}_L,\omega_n)[F^{\dagger}_{\uparrow\uparrow}(\mathbf{k}_R,-\omega_n)]^*+F^{\dagger}_{\downarrow\downarrow}(\mathbf{k}_L,\omega_n)[F^{\dagger}_{\downarrow\downarrow}(\mathbf{k}_R,-\omega_n)]^*\}.$$

Since $J_{-\chi}\sim h_Lh_R$ it follows that $J_{-\chi}=0$ in the absence of magnetic order in at least one bank of the junction, i.e., $J_{-\chi}=0$ for $h_L=0$ or $h_R=0$.

After the ξ integration $J_{-\chi}$ reads

$$J_{-\chi}=16e\pi^2N^2(0)|T_0|^2\Delta^2h^2T\times\sum_{n=0}^{\infty}\left\{\left(\omega_n\int_0^1K(\omega_n,y)dy\right)^2-\chi\left(\rho_0\int_0^1 yK(\omega_n,y)dy\right)^2\right\},$$

where $K(\omega_n,y)$ is given by

$$K(\omega_n,y)=\frac{I(\omega_n,y)}{a_n+\sqrt{a_n^2-4\Delta^2h^2}-2h^2}. \tag{11}$$

In order to calculate J_c and $J_{-\chi}$ we have to know the equilibrium values of Δ, h, and Q, which minimize the free energy $F(\Delta,h,Q)$ of the system. This problem has been discussed extensively in the past—see Ref. 3, and references therein, where it was shown that the equilibrium values of Δ, h, and Q depend on microscopic parameters $k_F,v_F,n_m,J_{ex},\Delta_0$. This problem is not the subject here but we stress, that in systems that would be ferromagnetic in the absence of superconductivity the equilibrium value of Q is strongly affected by superconductivity. It turns out that $Q\sim\xi_0^{-1/3}$ for the spiral structure, while in systems with an easy-axis magnetic anisotropy, the domain structure with $Q\sim\xi_0^{-1/2}$ is realized.[3] We point out that a situation is possible when the oscillating magnetic order is due to Fermi surface properties in the normal state in which case Q is independent on superconductivity. As a typical example for such a behavior serves $TmNi_2B_2C$ with $T_c=11$ K, where at $T_m=1.5$ K an oscillating magnetic order appears with the wave vector $Q=0.24$ Å^{-1}, i.e., $Q^{-1}\ll\xi_0\approx 250$ Å.[5,11] This discussion demonstrates that in singlet magnetic superconductors an oscillating magnetic structure can be realized with the equlibrium wave vector Q laying in the range $\xi_0^{-1}<Q<k_F$.

The current ratio $R_{-\chi}(m,p)=J_{-\chi}/J_c$ is calculated numerically, for the temperature $T=0.1$ K, as a function of the parameters $m=\Delta/h$ and $p=(\rho_0/h)(=Qv_F/h)$, which are supposed to be equilibrium values. The obtained results are shown in Fig. 2, where $R_{-\chi}(m,p=\text{const})$ is plotted, and in Fig. 3, where the case $R_{-\chi}(m=\text{const},p)$ is presented.

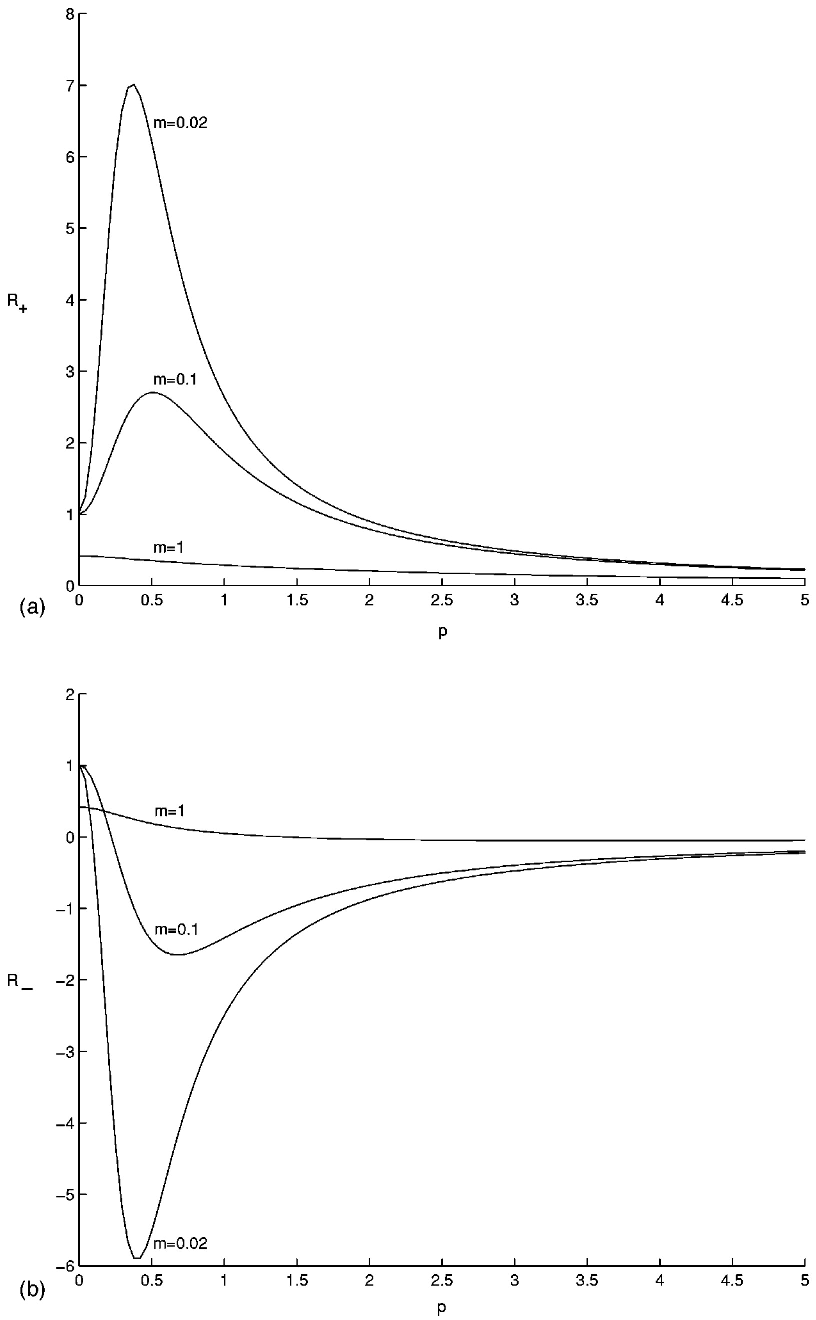

FIG. 2. The ratio $R_{-\chi}(m=\text{const},p)=J_{-\chi}/J_c$ for $m=1;0.1;0.02$. (a) $\chi=-1$ and (b) $\chi=1$. For $|R_{-\chi}|>1$ the π contact is realized. Here, $m=\Delta/h$ and $p=Qv_F/h$.

IV. DISCUSSION AND CONCLUSION

The θ dependence of the Josephson current gives rise to interesting physical effects in this junction—we call it the MSJ junction. In the following we discuss some of them, like, for instance, the tuning from 0- to π junctions, i.e., switching ability of the MSJ junction. The extension of the theory to other than spiral magnetic order is discussed at the end of this section.

(i) The π Josephson junction

The theory for the MSJ junction predicts that both cases, $|R_{-\chi}|<1$ and $|R_{-\chi}|>1$, can be realized in some region of parameters: $m(=\Delta/h)$ and $p(=Qv_F/h)$—see Figs. 2 and 3. Equation (8) tells us, that if $|R_{-\chi}|>1$ the π junction is realized, in which case the MSJ-junction energy [given by Eq. (8)] has a minimum for $\varphi=\pi$. For $\chi=1$ and for $R_{-}<-1$, the π contact is favorable if the magnetic phase θ takes values $\cos\theta<-1/|R_{-}|$, while in the case when $\chi=-1$ and $R_{+}>1$, the π contact is realized for $\cos\theta>1/R_{+}$. It is seen in Figs. 2 and 3 that the π junction is preferable in the parameter region $m<1$, $p<2$, where $|R_{-\chi}|>1$. We point out that the parameter $m(\equiv\Delta/h_{ex})$ is a measure of the relative strength of the superconducting order parameter with respect to the exchange energy, while $p(=Qv_F/h_{ex}\equiv L_{ex}/L_{spiral})$ measures the relative ratio between the ''exchange'' length, $L_{ex}=2\pi v_F/h_{ex}$, and the spiral period, $L_{spiral}=2\pi/Q$. Hence the condition $p<2$ means that the π contact is favorable in magnetic superconductors with a longer period of the spiral structure. However, theory[3] shows that the latter property is less favorable for the coexistence of superconductivity and spiral magnetic order. In existing

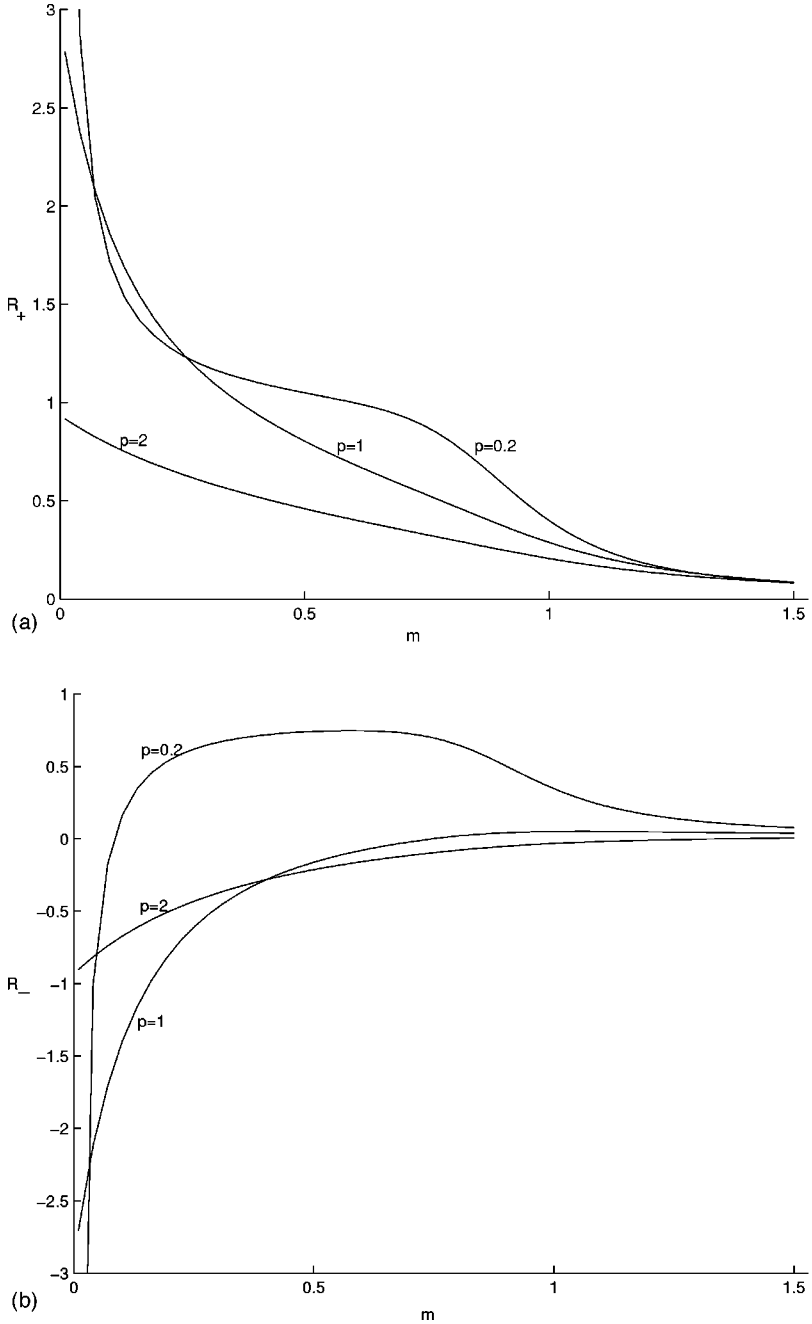

FIG. 3. The ratio $R_{-\chi}(m,p=\text{const})=J_{-\chi}/J_c$ for $p=2;1;0.2$. (a) $\chi=-1$ and (b) $\chi=1$. For $|R_{-\chi}|>1$ the π contact is realized. Here, $m=\Delta/h$ and $p=Qv_F/h$.

(ferro)magnetic superconductors $ErRh_4B_4$, $HoMo_6S_8$, $HoMo_6Se_8$, and $AuIn_2$, the parameter m decreases from $m \gg 1$ to $m<1$ by lowering T from T_m, while the value $p>10$ is realized. The latter fact means that the current ratio $|R_{-\chi}|$ is small in these systems[3] and the π junction is unfavorable in MSJ junctions based on these compounds.

(ii) Tuning of the magnetic phase θ in the MSJ-junction

If $\theta(x,y,t)$ can be tuned in space and time in a controllable way it offers possibilities for applications of the MSJ junctions. The elaboration of coupled equations for $\varphi(x,y,t)$ and $\theta(x,y,t)$ is a matter for future investigation and at present this problem can only be analyzed qualitatively. In the MSJ junction the Josephson phase φ can be tuned by external currents, like in standard junctions, while the magnetic phase θ can in principle be tuned by external magnetic fields. For instance, in the case of magnetic superconductors with an easy-plane magnetic anisotropy (the x-y plane in the geometry of Fig. 1) θ can be tuned in the range $(0,\pi)$ by applying a magnetic field on lateral surfaces (which are far away from the contact plane) of the left and/or right superconducting bank. By applying magnetic field directly to the contact one can change both φ and θ along the contact.

(iii) Switching of MSJ junctions

The controllable change of θ in a MSJ junction might be useful in switching and similar operations. For instance, if the system is characterized by $|R_{-\chi}|>1$ one can tune the MSJ junction continuously from the 0- to π junction by rotating the magnetic field (which is applied far away from the contact). If such a π junction is part of a superconducting ring circuit with large inductance L then a spontaneous current flows.[12] Moreover, it is imaginable that one can realize different logic circuits by combining more such junctions.

Note that the tuning property of the MSJ junction, from a 0- to π state in the same setup, is unique. Such tuning in the same setup is hard to realize in junctions with high-T_c superconductors, where due to *d*-wave pairing one can have either a 0- or π junction. It should also be stressed that even those MSJ junctions with $|R_{-\chi}|<1$ (when only 0 junctions are possible) might be useful in applications because spatial and time variations of θ would modulate the Josephson current.

(iv) Other types of magnetic order

The proposed theory holds qualitatively for superconductors with antiferromagnetic magnetic order that has an easy-plane magnetic anisotropy. Note that the antiferromagnetic order can be considered as a limiting case of the spiral order. For instance, in some antiferromagnetic heavy-fermion superconductors, like URu_2Si_2 with $T_N \approx 17$ K and $T_c \approx 1.5$ K, one has $p \approx 1-2$, $m \approx 0.01-0.03$, where the small value of p is due to the small Fermi velocity.[13] If one assumes that (anisotropic) *s*-wave pairing takes place in this system in the absence of antiferromagnetic order, then the magnetic order affects superconductivity in the way described above.[14] Since in URu_2Si_2 one has $p \approx 1-2$, $m \approx (0.01-0.03)$ and $|R_\chi| \sim 1$ this means that this system might be favorable for making π junctions. Other heavy fermions, like UPd_2Al_3 ($T_N \approx 14$ K, $T_c \approx 2$ K) and UNi_2Al_3 ($T_N \approx 4.6$ K, $T_c \approx 1$ K), might also belong to the class described by the theory above.[14] The antiferromagnetic superconductor UPt_3, with $T_N \approx 5$ K, $T_c \approx 0.5$ K, $p \approx 1-2$, $m \approx (0.01-0.03)$ is also a candidate for such a system if *s*-wave pairing is realized in it. However, a number of experiments in UPt_3 are better described by assuming unconventional superconductivity[13] with a still unknown structure of the superconducting order parameter.

We would like to point out that the above theory holds qualitatively for magnetic superconductors with an easy-axis magnetic anisotropy. In that case the orientation of the oscillating (sinusoidal or domainlike) magnetic order is fixed and the tuning of corresponding MSJ junctions by a magnetic field is more difficult. This possibility depends strongly on the strength of the magnetic anisotropy.

In conclusion, it is shown that in Josephson junctions made of magnetic superconductors with a spiral magnetic order the Josephson current depends on a new degree of freedom—the relative magnetic phase of the magnetizations at the barrier. In some ranges of the parameters Δ, h and Qv_F the π junction can be realized by tuning θ in a magnetic field. Even in the case when only the 0 junction is realizable the physics of such a junction is interesting due to the sensitivity of the Josephson current to small spatial and time variations of θ. Such systems could be of potential interest for small-scale applications, of course, if the MSJ junction is realizable.

ACKNOWLEDGMENTS

We are thankful to Martin Endres for reading the manuscript. M.L.K. acknowledges the support of the Deutsche Forschungsgemeinschaft through the Forschergruppe ''Transportphänomene in Supraleitern und Suprafluiden.''

[1] V.L. Ginzburg, Zh. Éksp. Teor. Fiz. **31**, 202 (1957)[Sov. Phys. JETP **4**, 153 (1957)].
[2] *Superconductivity and Magnetism*, edited by M. B. Maple and Ø. Fischer (Springer-Verlag, Berlin, 1982).
[3] A.I. Buzdin, L.N. Bulaevskii, M.L. Kulić, and S.V. Panyukov, Adv. Phys. **34**, 176 (1985).
[4] L.N. Bulaevskii, M.L. Kulić, and A.I. Rusinov, Solid State Commun. **30**, 59 (1979); J. Low Temp. Phys. **39**, 255 (1980); A.I. Buzdin, L.N. Bulaevskii, M.L. Kulić, and S.V. Panyukov, Phys. Rev. B **28**, 1370 (1983).
[5] L.J. Chang, C.V. Tommy, D.M. Paul, and C. Ritter, Phys. Rev. B **54**, 9031 (1996).
[6] A. Dertinger, A. Kreyssig, C. Ritter, M. Loewenhaupt, and H.F. Braun, Physica B **284-288**, 485 (2000)
[7] S. Rehmann, T. Herrmannsdörfer, and F. Pobell, Phys. Rev. Lett. **78**, 1122 (1997).
[8] M.L. Kulić, L.N. Bulaevskii, and A.I. Buzdin, Phys. Rev. B **56**, R11 415 (1997).
[9] L. Bauernfeind, W. Widder, and H.F. Braun, J. Low Temp. Phys. **105**, 1605 (1996); C. Bernhard, J.L. Tallon, Ch. Niedermayer, Th. Blasius, R.K. Kremer, E. Brücher, D.R. Noakes, C.E. Stronach, and E.J. Ansaldo, Phys. Rev. B **59**, 14 099 (1999).
[10] B.D. Josephson, Phys. Lett. **1**, 251 (1962); V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963); *ibid.* **11**, 104(E) (1963); I.O. Kulik and I.K. Yanson, *The Josephson Effect in Superconductive Tunneling Structures* (Nauka, Moscow, 1970) (in Russian), Israel Program for Scientific Translations, 1972.
[11] M.L. Kulić, A.I. Buzdin, and L.N. Bulaevskii, Phys. Lett. A **235**, 285 (1997).
[12] L.N. Bulaevskii, V.V. Kuzii, and A.A. Sobyanin, Pis'ma Zh. Éksp. Teor. Fiz. **25**, 240 (1977) [JETP Lett. **25**, 221 (1977)]; Solid State Commun. **25**, 1053 (1978).
[13] G.R. Stewart, Rev. Mod. Phys. **56**, 755 (1984); J.A. Sauls, Adv. Phys. **43**, 113 (1994).
[14] M.L. Kulić, J. Keller, and K.-D. Schotte, Solid State Commun. **80**, 345 (1991).