Ballistic versus diffusive magnetoresistance of a magnetic point contact

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The quasiclassical theory of a nanosize point contacts (PC's) between two ferromagnets is developed. The maximum available magnetoresistance values in PC's are calculated for ballistic versus diffusive transport through the area of a contact. In the ballistic regime the magnetoresistance in excess of a few hundred percent is obtained for the iron-group ferromagnets. The necessary conditions for realization of so large a magnetoresistance in PC's, and the experimental results by García *et al.* are discussed.

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In recent experiments studying Ni-Ni and Co-Co point contacts (PC's), a surprisingly high negative magnetoresistance exceeding 200% has been discovered.[1,2] The setup of the experiment was typical for the observation of giant magnetoresistance (GMR), the effect observed earlier in hybrid systems involving ferromagnetic and normal multilayer metals.[3,4] However, for multilayer structures the typical change of the resistance reached 10%–30%, which is considerably lower than the corresponding values of Refs. 1 and 2. So one can come easily to the conclusion that the main contribution to the MR comes from the region of the PC's.

A negative magnetoresistance can be due to scattering on domain walls (DW's) and this effect has been considered in a number of works[5–8] giving typical values of MR in the range of a few percent. Such considerably low values of the MR were obtained assuming that realistic widths of the DW's were large, which resulted in low scattering amplitudes. Considering sharp DW's in the ballistic regime one comes to values $\sim 70\%$.[8]

The fact that a sharp DW may give large MR was used in Ref. 2 to explain the anomalously large values of MR in the experiments on point contacts.[1,2] However, the theory in Ref. 2 is perturbation theory, and it cannot be applied to a explanation of the 300% effect. A diminishing of the width of DW's, when decreasing the size of the constriction, was demonstrated by Bruno.[9] The DW width becomes comparable to the PC length, and magnetization rotates almost abruptly inside the constriction. This conclusion holds until the diameter of the PC is smaller than its actual length. With a further increase of constriction size (diameter) the wall will bend outside of the PC, and simple energy considerations show that the DW width will be of the order of the PC size.[10]

The regime of conductance quantization in magnetic PC's has been considered by Imamura *et al.*[11] They obtained that, if the spin of the conduction electron cannot rotate in DW's pinned to the constriction, then magnetoresistance acquires oscillations as a function of PC size with the amplitude exceeding 1000%.

In this paper we develop a quasiclassical theory of electric transport through magnetic PC's taking into account scattering by impurities, thus covering the ballistic ($l>a$) and diffusive ($l<a$) regimes (l is the mean free path and a is the radius of the contact). The typical PC size, which is beyond the quantization regime, $2a \geqslant 8$ Å, may be well described within the quasiclassical (QC) approximation ($2a \gg \lambda_F = 2\pi/p_F \sim 6$ Å; λ_F and p_F are the Fermi wavelength and momentum).

We believe that extremely large magnetoresistance can be obtained if the strong reflection of spin-polarized current carriers on the PC area is achieved at *antiparallel* (AP) alignment of magnetizations in contacting ferromagnets. This is realized if there is mismatch in the spin-subband Fermi momenta of contacting magnets. For AP alignment, $p_{F1\uparrow} = p_{F2\downarrow}$ and $p_{F1\downarrow} = p_{F2\uparrow}$. Let us assume that $p_{F1\uparrow} \gg p_{F1\downarrow}$. Then a subband with a smaller value of the Fermi momenta, which is the minority subband, cannot accept momenta transferred from the opposite side of the PC, which is a majority subband with the same spin projection. As a result, only a narrow incidence angle cone around the normal to the interface is responsible for the charge transport across the PC. Electrons with more inclined trajectories are completely reflected. Thus, the partial transmission at the steep incidence and the total reflection at slanting incidence provide the high boundary resistance of PC's.

The necessary condition for realization of the above scenario is the conservation of electron spin orientation when crossing the domain wall. The orientation conserves if the DW width d_w is shorter than the length d_s, at which the electron spin quantization axis adjusts the varying direction of the local exchange field. For ballistic transmission through PC's, $d_s = v_F T_1$, where T_1 is the longitudinal relaxation time of the conduction electron magnetization—the Overhauser time.[12] At this condition the transmission process looks like transmission through abrupt DW's, and the description of the electron transport through PC's with boundary conditions at the PC interface is valid.

The PC model we consider is the circular hole of the radius a made in a membrane, which divides the space on two half-spaces, occupied by single-domain ferromagnetic metals. The membrane is impenetrable for quasiparticles carrying a current; however, the thickness of the membrane in the model is assumed to be vanishing. The z axis of the coordinate system is chosen perpendicular to the membrane plane. The electron motion on both sides of the contact can be described by the equations for QC Green functions (GF's) derived by Zaitsev.[13] They are in fact the Boltzmann equations in the τ approximation:

$$v_z \frac{\partial g_a}{\partial z} + \mathbf{v}_\parallel \frac{\partial g_s}{\partial \vec{\rho}} + \frac{1}{\tau}(g_s - \overline{g_s}) = 0,$$

$$v_z \frac{\partial g_s}{\partial z} + \mathbf{v}_\parallel \frac{\partial g_a}{\partial \vec{\rho}} + \frac{g_a}{\tau} = 0. \tag{1}$$

g_s and g_a are symmetric and antisymmetric with respect to z projection of the quasiparticle momentum QC GF (Green functions integrated over the energy variable), $\mathbf{v}$ is the vector of the Fermi velocity, $v_z = v_F \cos\theta$, $v_\parallel^2 = v_F^2 - v_z^2$, the angle θ is measured from the z axis, v_F is the modulus of $\mathbf{v}$, the overbar over $\overline{g_c}$ means averaging over the solid angle. We assume that the spin-mixing process is weak; therefore we consider spin channels as independent and omit the spin-channel indices in Eqs. (1) and expressions below.

The boundary conditions to Eqs. (1) for the specular scattering ($p_{F1\alpha}\sin\theta_1 = p_{F2\alpha}\sin\theta_2 \equiv p_\parallel$) at the interface $z=0$ are[13]

$$g_{a1}(0) = g_{a2}(0) = \begin{cases} g_a(0), & p_\parallel < p_{F1}, p_{F2}, \\ 0, & \min(p_{F1}, p_{F2}) < p_\parallel, \end{cases}$$

$$2Rg_a(0) = -D(g_{s2} - g_{s1}), \tag{2}$$

where a subscript 1 or 2 labels the left- or right-hand side of the contact, respectively, p_{Fi} is the Fermi momentum of the ith side, and $p_\parallel$ is the projection of the Fermi momentum vector on the PC plane. D and $R = 1 - D$ are the exact quantum mechanical transmission and reflection coefficients that can be considered either as phenomenological parameters or calculated for the models of interest. The second line in the first boundary condition in Eqs. (2) explicitly quantifies the total reflection for inclined trajectories, described qualitatively above.

The density of a current through the contact may be written as

$$j^z(z,\vec{\rho},t) = -\frac{e p_{F\,\min}^2}{2\pi} \int_0^{\pi/2} d\Omega_\theta \cos\theta\, g_a(z,\vec{\rho},t). \tag{3}$$

The total current through the area of the contact is

$$I^z(z \to 0,t) = a \int_0^\infty dk J_1(ka) j^z(0,k,t). \tag{4}$$

In the above equations $p_{F\,\min} = \min(p_{F1}, p_{F2})$, $J_1(x)$ are the Bessel functions, and $j^z(0,k,t)$ is the Fourier transform of the current density, Eq. (3), over the in-plane coordinate ρ. The cylindrical symmetry of the problem has been used upon derivation of Eq. (4).

We search a solution for g_s in the form ($k_B = \hbar = 1$):

$$g_s(\varepsilon) = \tanh\frac{\varepsilon}{2T} + f_s(\varepsilon), \tag{5}$$

where the first term is the equilibrium value of g_s in the leads far away from of PC's. Substitution of Eq. (5) into Eqs. (1) and Fourier transformation over the variable ρ leads to equations, the exact solution of which reads

$$f_s(z) = g_a(z)\,\mathrm{sgn}(z) + \frac{1}{l_z} \int_{-\infty}^{\infty} d\xi e^{-\varkappa|\xi - z|} \overline{f_s}(\xi,k), \tag{6}$$

where

$$\varkappa = \frac{1 - i\mathbf{k}\mathbf{l}_\parallel}{l_z}, \tag{7}$$

$l = \tau v_F$ is the mean free path, $l_z = l\cos\theta$, and $l_\parallel^2 = l^2 - l_z^2$. Integrating Eq. (6) over the solid angle we obtain

$$\overline{f_s}(z>0) = \overline{g_a} + \int_z^\infty d\xi K(\xi - z) \overline{f_s}(\xi,k), \tag{8}$$

where the kernel $K(\eta)$ is ($x = \cos\theta$)

$$K(\eta) = \frac{1}{l} \int_0^1 dx \frac{e^{-\eta/lx}}{x} J_0\left(k\eta \frac{\sqrt{1-x^2}}{x}\right). \tag{9}$$

If the mean free path l is short ($l \ll a$), the second term in Eqs. (6) and (8) dominates and the integrand of Eq. (8) is the product of a rapidly decreasing on the distance l kernel $K(\eta)$ and slowly decreasing function $\overline{f_s}$. That is why we may take out $\overline{f_c}(k,\xi)$ from the integral (8) at the point $\xi = z$. Within this approximation we obtain

$$\overline{f_s}(z,k) = \overline{g_a}(z,k)[1 - \lambda(k)]^{-1}, \tag{10}$$

where

$$\lambda(k) = \int_0^\infty d\xi K(\xi - z) = \frac{1}{kl}\arctan kl. \tag{11}$$

Substituting Eq. (10) into Eq. (6), and using the boundary conditions (2), we obtain the equation for the antisymmetric combination g_a:

$$g_a(0,k) = -\frac{1}{2} D\left(\tanh\frac{\varepsilon}{2T} - \tanh\frac{\varepsilon - eV}{2T}\right)\gamma_k - \frac{D}{1-\lambda_1}\frac{1}{2l_{z1}} \int_{-\infty}^0 d\xi e^{\varkappa_1 \xi} \overline{g_{a1}}(\xi) - \frac{D}{1-\lambda_2}\frac{1}{2l_{z2}} \int_0^\infty d\xi e^{-\varkappa_2 \xi} \overline{g_{a2}}(\xi), \tag{12}$$

where

$$\gamma_k = \int_0^a \rho d\rho \int_0^{2\pi} e^{i\mathbf{k}\vec{\rho}} d\varphi = \frac{2\pi a}{k} J_1(ka), \tag{13}$$

and V is the bias voltage.

To find $g_a(0,k)$ we average Eq. (12) over the solid angle, exploit the continuity of g_a at the interface, Eq. (2), and again use the fact that in the limit $l \ll a$ the kernel in the integral over x in the second and third terms of the averaged equation (12) is a function, rapidly decreasing at distance l. Of course, in the ballistic regime ($l > a$) this approximation is no longer valid, but in this regime the first (exact) term in Eq. (12) dominates the approximate terms with integrals. So in the ballistic limit the approximation does not bring a large error either. Although the approximation may not be valid in the intermediate regime, the suggested scheme can be used as an interpolation.

Now we find easily $\overline{g_a}(0,k)$, make consecutive substitutions into Eqs. (12), (3), and (4), and, finally, obtain the general expression for the current through PC's:

$$I^z = \frac{e^2 p_{F\,\min}^2 a^2 V}{2\pi} \int_0^\infty \frac{dk}{k} J_1^2(ka) \langle D\, F(k,\theta) \cos\theta \rangle, \quad (14)$$

where

$$F(k,\theta) = 1 - \left[\frac{1}{2(1-\lambda_1)\varkappa_1 l_{z1}} + \frac{1}{2(1-\lambda_2)\varkappa_2 l_{z2}} \right] \times \frac{\overline{D}}{1 + \dfrac{\tilde{\lambda}_1}{2(1-\lambda_1)} + \dfrac{\tilde{\lambda}_2}{2(1-\lambda_2)}}, \quad (15)$$

$$\tilde{\lambda}_i = \left\langle \frac{D}{\varkappa_i l_{zi}} \right\rangle = \int_0^1 dx \frac{D(x)}{\sqrt{1 + k^2 l_i^2 (1-x^2)}}, \quad (16)$$

$\langle \cdots \rangle$ means averaging over the solid angle. Equations (14) and (15) are the basic analytical result of the paper, which expresses the current in terms of the parameters D, l, a, and p_F characterizing the system.

Now we calculate the magnetoresistance of PC's between two identical ferromagnets. It can be expressed via the conductances $\sigma = I/V$ as follows:

$$MR = \frac{R^{AP} - R^P}{R^P} = \frac{\sigma^P - \sigma^{AP}}{\sigma^{AP}}, \quad (17)$$

where R^P (σ^P) stands for the resistance (conductance) at *parallel* alignment of magnetizations of contacting ferromagnets, and R^{AP} (σ^{AP}) is for the *antiparallel* alignment of magnetizations. For the *parallel* alignment the net current is the sum of currents for both (independent) spin channels, $D=1$, $\tilde{\lambda}_i = \lambda_i$. Labeling the quantities by arrow-up and down notation we write

$$\sigma^P = \sigma^z_{\uparrow\uparrow} + \sigma^z_{\downarrow\downarrow} = \frac{e^2 (p_{F\uparrow}^2 + p_{F\downarrow}^2)(\pi a^2)}{4\pi^2} \int_0^\infty \frac{dk}{k} J_1^2(ka) \times \left\{ \frac{p_{F\uparrow}^2}{p_{F\uparrow}^2 + p_{F\downarrow}^2} \frac{k^2 l_\uparrow^2}{(1 + \sqrt{1 + k^2 l_\uparrow^2})^2} + (\uparrow \rightleftarrows \downarrow) \right\}. \quad (18)$$

The prefactor in Eq. (18) is nothing but the sum of Sharvin[14] conductances for the spin channels. For the AP alignment of magnetizations the conductance is

$$\sigma^{AP} = \frac{e^2 p_{F\downarrow}^2 (\pi a^2)}{\pi^2} \int_0^\infty \frac{dk}{k} J_1^2(ka) \int_0^1 dx x (D(x))_{\uparrow\downarrow} \times \left\{ 1 - \left[\frac{1-\lambda^\uparrow}{\sqrt{1 + k^2 l_\uparrow^2 (1-x^2)}} + \frac{1-\lambda^\downarrow}{\sqrt{1 + k^2 l_\downarrow^2 (1-x^2)}} \right] \times \frac{(\overline{D})_{\uparrow\downarrow}}{2(1-\lambda^\uparrow)(1-\lambda^\downarrow) + \tilde{\lambda}^\uparrow_{\uparrow\downarrow}(1-\lambda^\downarrow) + \tilde{\lambda}^\downarrow_{\uparrow\downarrow}(1-\lambda^\uparrow)} \right\}, \quad (19)$$

where $(D(x))_{\uparrow\downarrow}$ stands for the transmission coefficient of the interface at AP alignment. For the mechanism of magnetoresistance discussed above, $(D(x))_{\uparrow\downarrow}$ can be found from the solution of the Schrödinger equation for the particle moving in the steplike potential landscape:[15]

$$(D(x))_{\uparrow\downarrow} = \frac{4 (v_{z1}^\uparrow)_\uparrow (v_{z2}^\uparrow)_\downarrow}{[(v_{z1}^\uparrow)_\uparrow + (v_{z2}^\uparrow)_\downarrow]^2} = (D(x))_{\downarrow\uparrow}, \quad (20)$$

with $v_{z2}^\uparrow = v_{z1}^\downarrow$ for the *antiparallel* alignment. The transmission coefficient (20) gives maximum available magnetoresistance values for a particular parameters choice. Neglecting the difference of the effective masses in the spin-subbands we may write down

$$(D(x))_{\uparrow\downarrow} \simeq \frac{4x\sqrt{b^2 + x^2}}{(x + \sqrt{b^2 + x^2})^2}, \quad (21)$$

where

$$b^2 = \frac{1-\delta^2}{\delta^2}, \qquad \delta = \frac{p_{F\downarrow}}{p_{F\uparrow}} = \frac{v_{F\downarrow}}{v_{F\uparrow}} \leqslant 1. \quad (22)$$

For purely ballistic transport [$a/l_\uparrow \to 0$, where $l_\uparrow$ ($l_\downarrow$) is the majority (minority) electrons mean free path] all integrals in Eqs. (18) and (19) are evaluated analytically, and the magnetoresistance reads

$$\mathrm{MR} = \frac{(1-\delta)\{5\delta^3 + 15\delta^2 + 9\delta + 3\}}{8\delta^3(\delta + 2)}. \quad (23)$$

If $\delta = 1$, then $\mathrm{MR} = 0$; i.e., the magnetoresistance vanishes. For the set of δ values we obtain, from Eq. (23), $\delta = 0.5$, $\mathrm{MR} = 238\%$; $\delta = 0.4$, $\mathrm{MR} = 455\%$; $\delta = 0.33$, $\mathrm{MR} = 780\%$; amd $\delta = 0.3$, $\mathrm{MR} = 1012\%$.

In the general case the angular integrals in Eqs. (16) and (19) can be still evaluated analytically, whereas the integrations over k can be done only numerically. The results for the magnetoresistance (17) as a function of the contact radius are shown in Fig. 1. The curves show the maximum available MR, that could be realized in PC's with physical parameters displayed in the figure. MR exponentially drops when the size of the contact approaches the mean free path of a material. Then it shows a smooth crossover from ballistic to diffusive regimes of conduction.

Let us discuss the experimental data on the magnetoresistance of magnetic PC's by García *et al.* Ni-Ni PC's showed maximal $\mathrm{MR} \simeq 280\%$,[1] and Co-Co PC's showed maximal

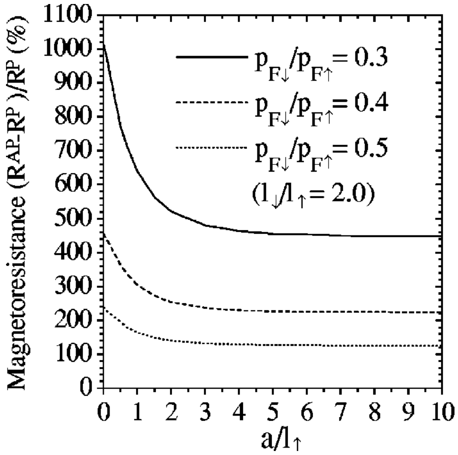

FIG. 1. The dependence of the magnetoresistance on the PC radius.

MR$\simeq$230%.[2] In a recent paper[16] they quote maximal MR $\simeq$33% for Fe-Fe PC's. To obtain the MR values 280% (Ni) and 230% (Co) we have to use the values $\delta(\mathrm{Ni})\simeq 0.47$ and $\delta(\mathrm{Co})\simeq 0.5$. These numbers are in the range of values obtained experimentally from the single-photon threshold photoemission, $\delta(\mathrm{Co})\simeq 0.4$,[19] and from ferromagnet and superconductor point contact spectroscopy: $\delta(\mathrm{Ni})\simeq 0.59$–0.65,[17] $\delta(\mathrm{Ni})\simeq 0.71$;[18] $\delta(\mathrm{Co})\simeq 0.62$–0.65,[17] $\delta(\mathrm{Co})\simeq 0.68$.[18]

If we use the experimental data of Ref. 17 for iron, $\delta(\mathrm{Fe})\simeq 0.59$–0.65, then in our theory we obtain MR(Fe) $=(100$–$140)\%$, which is larger than the experimentally measured 33%.[16] The justification of our model suggests that the observed MR is not solely confined to a value of the polarization δ. We believe that the basic condition for the observation of an upper MR limit $d_w \ll d_s$ is not fulfilled in the Fe-Fe PC experiment.[16] T_1 is proportional to the squared magnetic moment and the integral of exchange between conduction electrons and localized moments, and proportional to conduction electrons density of states at the Fermi level. All these physical parameters for iron are larger than for cobalt, and especially than for nickel. Therefore we expect that $T_1(\mathrm{Fe})$ is at least one order of magnitude shorter than T_1 for Co and Ni. When $d_s(\mathrm{Fe})\sim d_w(\mathrm{Fe})$, the electron spin almost tracks the local exchange field in the domain wall. As a result the reflection of the electrons from DW's decreases, and the observed MR does not reach its maximal value.

Let us discuss now the magnetoresistance in the diffusive regime of transport, when the radius of the nanohole is much larger than the mean free path $(a \gg l_\uparrow, l_\downarrow)$. The giant MR values can be obtained if the condition of validity of our model, $d_w \ll d_s$, will be realized in an experiment. In the opposite limit $d_w > d_s$, when PC size is so large that the DW becomes smooth and wide, the electron spin will track the local exchange field in the domain wall, and MR will level off at the Levy-Zhang[7] impurity scattering enhancement mechanism, which can give 2–11 % magnetoresistance. The requirement of abrupt DW's with constant width, irrespective of the PC size, can be technologically controlled if a very thin (two to four monolayers of thickness $\sim\lambda_F$) nonmagnetic interlayer is deposited on the PC plane before depositing the second electrode. Then, just like in CPP transport in multilayers,[3,4] the contacting domains will be exchange decoupled, so the magnetization will acquire a sudden reversal within spacer thickness $\sim\lambda_F$. In this case our analysis is valid for an *arbitrary* size of PC.

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