Magnetoresistance of insulating amorphous Ni_xSi_{1-x} films exhibiting Mott variable-range hopping laws

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Magnetoresistance (MR) ratios $R(B,T)/R(0,T)$ have been measured in an *insulating* three-dimensional amorphous nickel-silicon film that exhibits the Mott variable-range hopping (VRH) law in its zero-field resistance behavior. Surprisingly, the resistance displayed a *decrease* in *small* fields; only in *moderately strong* magnetic fields did the resistance exhibit a *large increase* over its zero-field value. These results are described by a phenomenological empirical model of two hopping processes acting simultaneously—the orbital magnetoconductance (forward-interference) model yielding negative magnetoresistances and the wave-function shrinkage model contributing positive magnetoresistances. The fits use numerical values for estimating the $R(B,T)/R(0,T)$ ratios, based upon the wave-function shrinkage model. The model includes three fitting parameters, whose magnitudes are extracted from the MR ratio data at $T=10.5$ K. Agreement between the predicted and measured data is acceptable at high temperatures. A crossover of the conductivity to an Efros-Shklovskii (ES) variable-range hopping law is observed around $T=6$ K. At lower temperatures for this ES case, predicted values for the $R(B,T)/R(0,T)$ ratios are fitted to the data. For a second *weakly insulating* film, which also exhibits a Mott VRH law in its resistance, the negative magnetoresistance contribution is greatly depressed.

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I. INTRODUCTION

The resistance of *insulating* films often follows a Mott variable-range hopping (VRH) law below room temperature.[1] The Mott VRH law is based upon a constant density of states around the Fermi energy. In the liquid-helium temperature region, a crossover of the resistance from the Mott VRH law to an Efros-Shklovskii (ES) VRH behavior is commonly observed.[2] The ES VRH law arises from Coulomb repulsion, where the density of states is predicted to vanish quadratically very close to the Fermi energy.[3] Sometimes even a stronger activated temperature dependence is observed in the resistance, arising from the ''soft gap'' in the density of states.[4] At high temperatures, there is sufficient phonon energy to promote hops between occupied states located sufficiently below the Fermi energy to unoccupied states located sufficiently above the Fermi energy. Thus, the hopping electron samples mainly the states described by the ''Mott'' constant density of states and ignores the states in the ''Coulomb gap.'' At much lower temperatures where the phonon energy is very small, the hopping electron is forced to sample states described by the ES quadratic density of states very close to the Fermi energy. Owing to the relatively few available states, the resistance displays a stronger exponential temperature dependence, known as the Efros-Shklovskii variable-range hopping law.

Recently, numerical predictions have been made for the resistance behavior in large magnetic fields, provided that the zero-field resistance can be described by a general VRH law.[5] Experimentally, fabricating an appropriate three-dimensional (3D) film for testing the numerical predictions is challenging. If the film is too insulating and has too large a Mott temperature T_{Mott}, then the crossover to the Efros-Shklovskii hopping law will take place between liquid-nitrogen and liquid-helium temperatures. In the liquid-nitrogen temperature region, magnetoresistance (MR) measurements are generally difficult to perform owing to temperature drifts. If the film is weakly insulating and has too small a Mott temperature, then the theories predict such small magnitudes in the liquid-helium temperature region that the measurements must be performed in the mK temperature region in order to observe reasonable magnitudes of the MR.

There has been published some excellent MR data in the Mott VRH law regime. In some cases, no theoretical fits were included. In other cases, fits were presented employing models that we feel are incorrect or incomplete. Benzaquen, Walsh, and Mazuruk presented clear experimental evidence of negative magnetoresistance $\{R(B,T)/R(0,T)\text{ratios}<1\}$ at small fields and then positive magnetoresistance $\{R(B,T)/R(0,T)\text{ratios}>1\}$ at higher fields in 3D *n*-type GaAs epitaxial layers.[6] Biskupski then reconfirmed the initial negative magnetoresistance behavior at low magnetic fields and the positive behavior in higher fields for a lightly doped sample of $CdAs_2$.[7] Similar results have been reported by Agrinskaya and co-workers on doped CdTe crystals.[8,9] Recently, extensive measurements have been made on *quasicrystalline* samples, some of which exhibit *insulating* behavior. For example, Wang and co-workers observed MR ratios $R(B,T)/R(0,T)<1$ in small fields and ratios >1 in larger fields for two quasicrystalline samples that displayed Mott

VRH laws in their zero-field resistance behavior.[10,11] Again this MR behavior is very similar to that observed in amorphous-insulating and semiconductor-insulating samples. In contrast, Dai, Friedman, and Sarachik observed only positive magnetoresistance behavior and were puzzled by the absence of any negative magnetoresistance in small fields in their doped silicone samples.[12] We now present a phenomenological model that can explain the majority of these data over a broad range of magnetic fields and temperatures.

II. THE WAVE-FUNCTION SHRINKAGE MODEL

At moderately high temperatures, many highly insulating 3D samples exhibit resistances that follow the Mott VRH law in zero magnetic field[1]

$$R(0,T)=R_{\mathrm{Mott},0}\exp(T_{\mathrm{Mott}}/T)^{1/4}, \quad (1)$$

where $R_{\mathrm{Mott},0}$ is the prefactor. T_{Mott} is the characteristic Mott temperature that can be determined from the zero-field resistance data using the $w=-d\ln R/d\ln T$ method of Zabrodskii and Zinov'eva;[13] this method also yields a value for the hopping exponent s. According to theory, $T_{\mathrm{Mott}}=18.1/(k_B g_0 a_0^3)$, where g_0 is the constant Mott density of states and a_0 is the Bohr radius or localization length.[1,2] Note that this localization length is expected to diverge to infinity as the metal-insulator transition (MIT) is approached from below. Thus, $T_{\mathrm{Mott}}\rightarrow 0$ K just below the MIT. For the Mott model to be valid, the optimum hopping distance r_{opt} must be greater than the localization length a_0 (or Bohr radius). This implies that the measurement temperatures must satisfy this relation: $T<T_{\mathrm{Mott}}$ since

$$r_{\mathrm{opt}}=(3/8)a_0(T_{\mathrm{Mott}}/T)^{1/4}.$$

Strong positive increases of the resistance with application of a magnetic field had been predicted originally by Tokumoto, Mansfield, and Lea,[14] and by Shklovskii[15,16] and elaborated by Shklovskii and Efros[17] using the wave-function shrinkage model. Numerical calculations for predicting the $R(B,T)/R(0,T)$ ratios for small and intermediate magnetic fields for the case of the Efros-Shklovskii VRH law had been made by Schoepe.[18] The application of a magnetic field decreases the overlap probability between two sites, thus resulting in an increase of the resistance with field.

We now summarize the positive MR ratio predictions of the wave-function shrinkage model for the case when the resistance exhibits a Mott VRH law. (a) For very small magnetic fields, Shklovskii and Efros found this expression for the MR ratio for the case of the Mott 3D VRH:[17]

$$R(B,T)/R(0,T)\approx\exp[t_1(e^2a_0^4/\hbar^2)(T_{\mathrm{Mott}}/T)^{3/4}B^2]. \quad (2)$$

Here, t_1 is predicted to be $t_1\approx 5/2016=0.00248$; and a_0 is the Bohr radius, approximately equal to the localization length. $R(0,T)$ is the resistance in zero field at temperature T, given by Eq. (1). The applied field B is ''small'' compared to a characteristic field B_c if $B\ll B_c$.[18] Here $B_c=6\hbar/[ea_0^2(T_0/T)^s]$, where T_0 is the characteristic temperature given by the Mott or ES temperatures and s is the hopping exponent, equal to $\frac{1}{4}$ for the Mott VRH case and to $\frac{1}{2}$ for the ES VRH case. Typical magnitudes for the characteristic field B_c range from 1 to 20 T, and thus the small-field range lies much below 1 T. Recalling that the typical ''magnetic field length'' is given by $\lambda=(\hbar/eB)^{1/2}$, this characteristic field B_c corresponds to a characteristic length that is of the same order as the localization or Bohr radius length. (b) For high fields, Shklovskii and Efros suggest the following expression, again for the Mott VRH case:[17]

$$R(B,T)\approx R_{\mathrm{Mott},B}\exp[(ea_0^2/6\hbar)^{1/3}(T_{\mathrm{Mott}}/T)^{1/3}B^{1/3}]. \quad (3)$$

Here $R_{\mathrm{Mott},B}$ is the prefactor, different from the zero-field prefactor. The high-field range is defined as $B\gg B_c$ and thus is on the order of hundreds of Tesla. (c) For the interval of intermediate and large fields where $B\approx B_c$, there are no analytical predictions for the $R(B,T)/R(0,T)$ ratio. Here we strongly rely on a procedure described by Schoepe in Ref. 18, where values for the percolation parameter (or optimum hopping probability parameter) in moderately strong fields are calculated for the ES VRH case. The difficult problem is to estimate the corresponding hopping volume V_ξ around a donor site, which gradually changes from an isotropic sphere at small fields to a double paraboloid in large fields. Schoepe uses the volume expression suggested by Ioselevich.[19,18] Recently, numerical calculations have been summarized in tables for the Mott, ES, and ''soft-gap'' VRH cases.[5] For the Mott VRH case, values for the normalized percolation parameter $\xi_c(B)/\xi_c(0)$ can be read off from Fig. 1 as a function of the normalized magnetic field B/B_c and inserted into Eq. (4) below to estimate values for $R(B,T)/R(0,T)$.

$$R(B,T)/R(0,T)=\exp\{(T_{\mathrm{Mott}}/T)^{1/4}[\xi_c(B)/\xi_c(0)-1]\}, \quad (4)$$

where $R(0,T)$, the resistance in zero field, is given by Eq. (1). $\xi_c(0)$ is defined as $\xi_c(0)=(T_0/T)^s=(T_{\mathrm{Mott}}/T)^{1/4}$ for the Mott case. There is one free fitting parameter $B_c(T)$, the normalizing characteristic field that must be extracted from one set of MR ratio data points. But since $B_c(T)$ has the following temperature dependence:[18]

$$B_c(T)=6\hbar/[ea_0^2\xi_c(0)]=(6\hbar)(T/T_{\mathrm{Mott}})^{1/4}/(ea_0^2), \quad (5)$$

$B_c(T)$ can also be estimated at all other temperature measurement points once it is determined at one known temperature.

In the limit of small fields, the numerical calculations yield a quadratic B^2 dependence for the $R(B,T)/R(0,T)$ ratio, and in the very high-field limit, the logarithm of the ratio tends to a $B^{1/3}$ dependence.

III. THE ORBITAL MAGNETOCONDUCTIVITY THEORY

The orbital magnetoconductivity (MC) theory or forward-interference theory predicts small *negative* magnetoresistances. This model takes into account the *forward* interference among random paths in the hopping process. Nguyen, Spivak, and Shklovskii (NSS)[20,21] considered the effect of interference among the various paths associated with the hopping between two sites spaced at a distance equal to the optimum hopping distance r_{opt}. NSS found that the interfer-

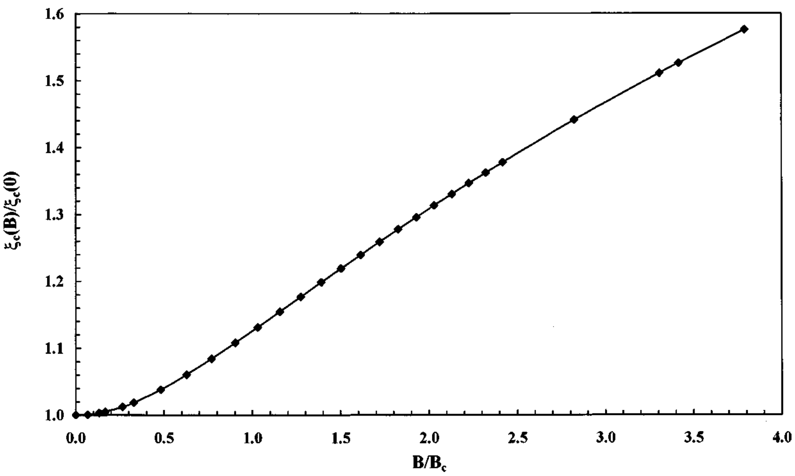

FIG. 1. The normalized percolation parameter (optimum hopping probability parameter) $\xi_c(B)/\xi_c(0)$ as a function of normalized magnetic field for the case when the resistance follows a Mott VRH law. Values for the magnetoresistance ratio $R(B,T)/R(0,T)$ can be estimated using this curve and Eq. (4).

ence between all possible paths within a cigar-shaped area S of length r_{opt} and width $(a_0 r_{\text{opt}})^{1/2}$ will change the hopping probability between two sites. Averaging numerically the logarithm of the conductivity over many random impurity realizations, they obtained under most conditions a negative MR (positive MC that is linear in magnetic field).

Sivan, Entin-Wohlman, and Imry (SE-WI) expanded the NSS model by using a critical percolating resistor method rather than the logarithmic averaging method.[22] Their calculated MC is always *positive* for strong fields and is predicted to *saturate* at sufficiently large fields. The field at which saturation starts to occur, B_{sat} is given by this approximate formula:

$$B_{\text{sat}} \approx 0.7(h/e)(8/3)^{3/2}(1/a_0^2)(T/T_{\text{Mott}})^{3/8}. \tag{6}$$

For this case, the saturation field $B_{\text{sat}} \propto T^{3/8}$, and thus the MC saturates at smaller fields as the temperature is lowered.

For small magnetic fields, the SE-WI model predicts a quadratic magnetic field dependence of the MC, but the magnitudes are extremely small and difficult to observe experimentally in most cases.[22]

There is no simple ''hand-waving'' argument to explain the *positive* sign of the MC. Numerical calculations by the various theoretical groups show that the MC is *positive* in almost all cases. This *forward-interference* model considers a hopping path either between an occupied donor to an empty donor or an alternative second hopping path that includes a scattering event off a third donor. The two hopping paths generate a triangle of area S. The location of this third donor is random, and Raikh and co-workers[23,24] describe its location by a distribution function. When an external field is applied, an additional phase factor results from the magnetic flux cutting this area. By averaging over all different areas resulting from the random location of the third donor, Raikh presents a convincing mathematical argument that the MC is positive.[23] Alternatively, Schirmacher and co-workers argue that there is a particular area or spatial region called the ''interference hole.''[25,26,27] Inside this hole no destructive interference can occur. This hole produces Aharonov-Bohm-like oscillations upon application of a magnetic field, resulting in an initial positive MC. And lastly, Shklovskii and Spivak[28] argue that the resistance is dependent upon averaged logarithmic terms that contain overlap integrals. The magnitudes of these two overlap integrals determine the sign of the MC, which is predicted to be positive. Again, we are not aware of an intuitive simple picture that can predict the sign of the MC to be positive.

We approximate the orbital MC contribution by the following expression:

$$\sigma(B,T)/\sigma(0,T) \approx 1 + c_{\text{sat}}[B/B_{\text{sat}}(T)]/[1+B/B_{\text{sat}}(T)]. \tag{7}$$

Eq. (7) saturates at high fields to a value of $(1+c_{\text{sat}})$ and yields a linear dependence upon B at intermediate fields. Here c_{sat} is a temperature-independent fitting parameter. Inverting Eq. (7), we obtain for the orbital contribution to $R(B,T)/R(0,T)$

$$R(B,T)/R(0,T) = 1/\{1 + c_{\text{sat}}[B/B_{\text{sat}}(T)]/[1+B/B_{\text{sat}}(T)]\}. \tag{8}$$

For the case of a small B/B_{sat} ratio and a small prefactor c_{sat},

$$R(B,T)/R(0,T) \approx 1 - c_{\text{sat}} B/B_{\text{sat}}. \tag{9}$$

We now make the assumption that the resistive contributions from both the wave-function shrinkage theory and the orbital magnetoconductance theory can be added, based upon the behavior of our MR data. Many experimental groups have used this assumption, since ''acceptable'' fits to the low-field data can be obtained.[6,7] A more rigorous theory is certainly needed, which would probably consider a hopping probability that is composed of these two processes acting simultaneously.[27] Thus, in our phenomenological empirical model, the final expression for the magnetoresistance ratio $R(B,T)/R(0,T)$ takes this form:

$$R(B,T)/R(0,T) \approx \exp\{\xi_c(0)[\xi_c(B)/\xi_c(0) - 1]\} + 1/\{1 + c_{\text{sat}}[B/B_{\text{sat}}(T)]/[1+B/B_{\text{sat}}(T)]\} - 1, \tag{10}$$

where c_{sat}, $B_{\text{sat}}(T)$, and $B_c(T)$ are three fitting parameters. The last term, -1, is needed to assure that the ratio has the

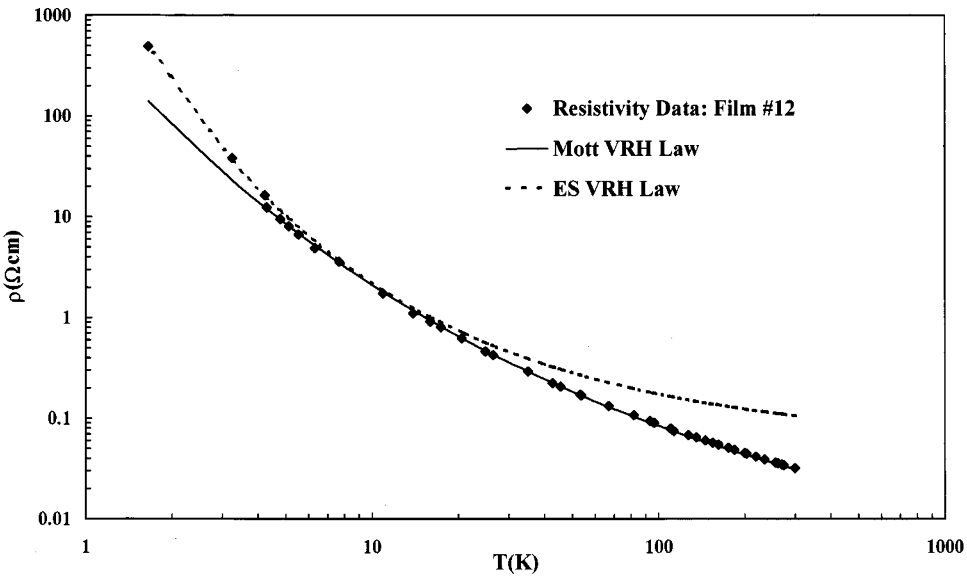

FIG. 2. A Mott VRH law fitted to the zero-field resistivity of an amorphous Ni_xSi_{1-x} film No. 12 of series No. 540. There is a crossover to an Efros-Shklovskii hopping law below 6 K. The Mott temperature is sizable, $T_{Mott} = 27\,610$ K, making this film strongly insulating; also, $T_{ES} = 138$ K.

correct limit when $B \to 0$, namely that $R(B,T)/R(0,T) \to 1$. Recall that $B_{sat}(T) \propto T^{3/8}$ and $B_c(T) \propto T^{1/4}$, and c_{sat} should be independent of temperature.

IV. COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS FOR AN INSULATING MOTT FILM

We now compare the numerical calculations to the MR ratio data taken on an insulating 2000 Å amorphous Ni_xSi_{1-x} film, No. 12 of series No. 540. Values for the hopping exponent $s = 0.252$ and for the Mott characteristic temperature $T_{Mott} = 27\,610$ K were obtained using the method described by Zabrodskii and Zinov'eva.[13] The zero-field resistivity data and the Mott VRH fit, where $\rho(0,T) = 0.00139\exp(27\,610/T)^{0.252}$ in Ω cm, are shown in Fig. 2 and agreement is excellent down to 7 K. Moreover, we note that for the Mott VRH model to be valid, the optimum hopping distance $r_{opt}(T)$ must satisfy the criterion that $r_{opt}(T)/a_0 \approx 0.375(T_{Mott}/T)^{1/4} > 1$. This criterion is well satisfied in the temperature region of interest, owing to the large value of the Mott characteristic temperature $T_{Mott} = 27\,610$ K.

Below 7 K there is a clear crossover to the ES VRH law, and the ES VRH law is well behaved below 3 K, as observed in Fig. 2. Below 3 K the zero-field resistivity can be described by the ES VRH law, where $\rho(0,T) = 0.0537\exp(138/T)^{0.50}$ in Ω cm; here $T_{ES} = 138$ K.

In Fig. 3, the MR ratio data taken in the Mott VRH region are shown with the fits of Eq. (10) using the empirical model. Values for the fitting parameters are $c_{sat} = 0.075$, $B_{sat}(10.5\text{ K}) = 4$ T and $B_c(10.5\text{ K}) = 66.6$ T, $T_{Mott} = 27\,610$ K and $s = 0.25$. At $T = 8.4$ K, B_{sat} was scaled down to $B_{sat} = 3.7$ T according to its theoretical $T^{3/8}$ dependence, c_{sat} was kept at this temperature-independent value of 0.075, and B_c was scaled down to $B_c(8.4\text{ K}) = 63.0$ T according to its theoretical $T^{1/4}$ dependence. The agreement is acceptable. A value of 29 Å for the Bohr radius (localization length) was obtained using Eq. (5). Unfortunately, there was insufficient user's time at the National High Magnetic Field Laboratory to extend measurements above 10.5 K.

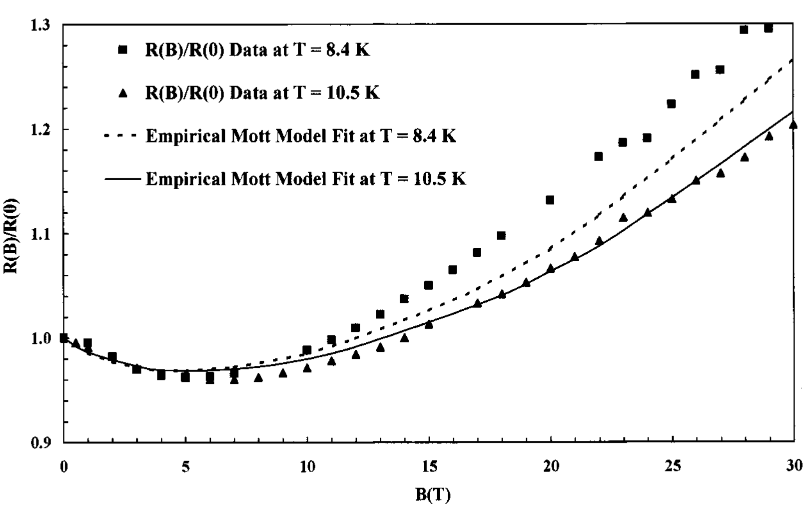

FIG. 3. The magnetoresistance ratios $R(B,T)/R(0,T)$ measured at temperatures where the Mott VRH law describes the resistivity behavior of film No. 12. Notice the *negative* magnetoresistance behavior (ratios <1) for fields less than 12 T. The lines are fits using the empirical model consisting of contributions from the orbital magnetoconductivity model and from the wave-function shrinkage model. Both models assume Mott VRH. See text for fitting details.

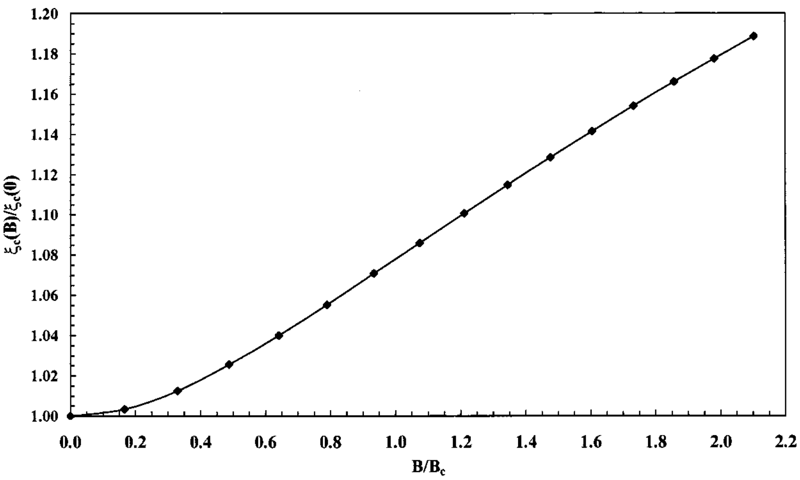

FIG. 4. The normalized percolation parameter (optimum hopping probability parameter) $\xi_c(B)/\xi_c(0)$ as a function of normalized magnetic field for the case when the resistance follows a ES VRH law. Values for the magnetoresistance ratio $R(B,T)/R(0,T)$ can be estimated using this curve and Eq. (4), in which $T_{\rm Mott}$ is replaced by $T_{\rm ES}$ and the Mott exponent of 1/4 is replaced by the ES exponent of 1/2.

At liquid-helium temperatures, one must recall that there is a crossover to the ES VRH hopping law. At $T=1.65\,\mathrm{K}$, this film is already in the ES hopping regime. The Mott fit is completely unacceptable as illustrated in Fig. 6 by the dotted line, a result that is not surprising.

However, if one uses the numerical calculations for the wave shrinkage model based upon the Coulomb gap (or ES) quadratic density of states, then the fit is much better. Again, we use Eq. (10) with $T_{\rm Mott}$ replaced by $T_{\rm ES}$, and we set the hopping exponent s to the value of $\frac{1}{2}$ rather than $\frac{1}{4}$. Values for the normalized percolation parameter $\xi_c(B)/\xi_c(0)$ are taken from Fig. 4 or from the ES table appearing in Ref. 5. The characteristic field B_c now takes on the $(T/T_{\rm ES})^{1/2}$ dependence rather than the $(T/T_{\rm Mott})^{1/4}$ dependence, and $B_{\rm sat}$ now follows an $(T/T_{\rm ES})^{3/4}$ temperature dependence rather than the $(T/T_{\rm Mott})^{3/8}$ dependence. As seen in Fig. 5, the solid line does agree with the low-field MR ratio data at $T=3.25\,\mathrm{K}$ below 12 T. At high fields of 30 T, the ES calculations predict ratios on the order of 3.8, as compared to the experimental values of 2.7. The characteristic field B_c was fixed at $B_c=13\,\mathrm{T}$ and scaled at $T=1.65\,\mathrm{K}$ according to $T^{1/2}$. The calculations are shown by the solid line in Fig. 6 and agreement is fair.

The ratio behavior at $T=1.65\,\mathrm{K}$ with high fields in Fig. 6 is very anomalous and surprising. A saturation of $r=R(B,T)/R(0,T)$ is observed around 25 T followed by a ''turnover'' to smaller values. This behavior is not expected theoretically, since Shklovskii and Efros predict a weak increasing behavior r, where $\ln(r)\propto B^{1/5}$ for the ES case, and where $\ln(r)\propto B^{1/3}$ for the Mott case.[17] The numerical calculations also confirm this behavior.[5] One other group has observed a similar behavior in an insulating quasicrystalline sample.[29] This ''saturation'' behavior strongly suggests that a new conduction process starts to dominate at high fields. We speculate that the ''modified'' density-of-states model proposed by Raikh and co-workers for two-dimensional electron systems might also apply to these three-dimensional systems.[23,24] These authors refer to their model as the ''incoherent mechanism.'' Their model predicts an *increase* of the density of states at the Fermi level with application of a magnetic field. For the Mott VRH case, the characteristic Mott temperature scales inversely with the density of states; hence $T_{\rm Mott}(B)$ *should* decrease. Since the Mott temperature appears in the argument of the exponential function of Eq. (10), this decreasing dependence of $T_{\rm Mott}$ might be responsible for the saturation and turnover of the MR ratio data. We have not tried to include this correction, since four additional fitting parameters are involved. For the ES case, $T_{\rm ES}=7.27e^2/(k_B\kappa a_0)$ does not depend upon the density of states and it is not clear if $T_{\rm ES}$ has a magnetic field dependence.[2]

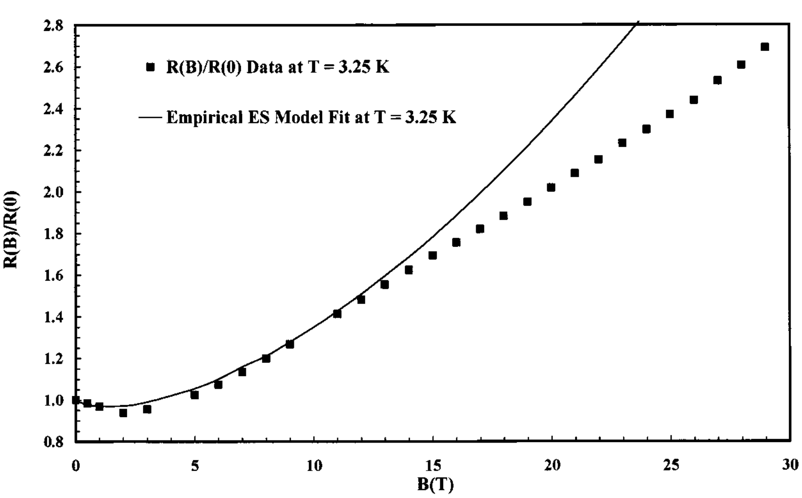

FIG. 5. The magnetoresistance ratios $R(B,T)/R(0,T)$ measured at $T=3.25\,\mathrm{K}$, where the ES VRH law is observed in the resistance of film No. 12. The fit uses the ES VRH expressions and parameters extracted from the low-temperature data of Fig. 2. The agreement below $B=12\,\mathrm{T}$ is quite acceptable.

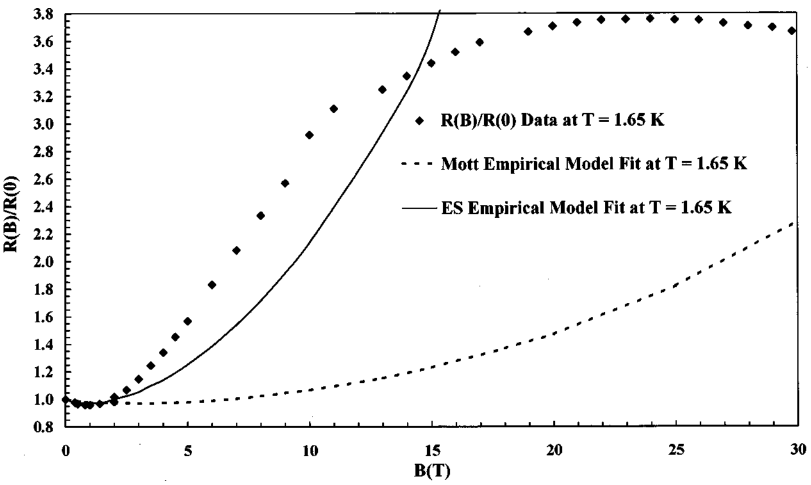

FIG. 6. Magnetoresistance ratios $R(B,T)/R(0,T)$ taken at $T=1.65$ K, where the resistance of film No. 12 exhibits an Efros-Shklovskii VRH law in its resistance. The Mott expressions give an unacceptable fit to the data. The ES parameters give a much improved fit, but there are still bad deviations between theory and data above $B=15$ T that are not currently understood. A possible explanation for the saturation and turnover behavior above $B=20$ T is given in the text.

V. COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS FOR A *BARELY INSULATING* MOTT FILM

We have recently measured two *quasicrystalline* films that exhibit *insulating* Mott VRH behavior in their resistances. Very surprisingly, these two *insulating* samples *did not* exhibit ratios smaller than 1 (that is, no positive MC was present); yet the Mott VRH law described the resistance behavior nicely. This behavior is similar to the positive magnetoresistance results of Dai, Friedman, and Sarachik.[12] For this special situation, $c_{\mathrm{sat}}=0$; then Eq. (10) reduces to the simple form of Eq. (4), where the wave-function shrinkage theory dominates at all fields. It appears that the positive magnetoconductivity is *absent* or *highly depressed* in *barely insulating* quasicrystalline samples.[10,11] For samples having *very small values* of the Mott characteristic temperature T_{Mott} or equivalently, weakly insulating samples having a small $R(4.2\,\mathrm{K})/R(300\,\mathrm{K})$ ratio of 10 or less, the negative magnetoresistance contribution is not observed experimentally. We refer to the beautiful MR data of Wang and co-workers taken on four *insulating quasicrystalline* (QC) samples located below the metal-insulator transition, which demonstrate the absence of the *positive* magnetoconductivity as the QC samples approach the metal-insulator transition from below.[10,11]

The transport mechanisms in the quasicrystalline materials are currently not well understood, and one might question whether the absence of the negative MR behavior in low fields is a special property of quasicrystals. Our viewpoint is the transport properties of quasicrystals are very similar to those of amorphous samples. To check this point, we selected a *weakly insulating* amorphous Ni_xSi_{1-x} film whose resistivity closely followed a Mott VRH law, $\rho(0,T)=0.00889\exp(1140/T)^{0.243}$ in Ω cm. This small Mott temperature of $T_{\mathrm{Mott}}=1140$ K should be contrasted with the much larger value of 27 610 K associated with film No. 12 studied earlier. The MR ratios appearing in Fig. 7 clearly show a minutely small negative MR contribution. The dominating contribution from the wave-function shrinkage process is present over the entire magnetic-field range. The fitting parameters at $T=4.22$ K took on these values: $c_{\mathrm{sat}}=0.0025$, $B_{\mathrm{sat}}=1$ T, $B_c=22$ T, and $T_{\mathrm{Mott}}=1140$ K. Note the extremely small magnitude of c_{sat}.

VI. DISCUSSION

Why is there the absence of the *negative* MR as the metal-insulator transition is approached from below? There are two possibilities: (a) either the orbital magnetoconductivity process is depressed as the MIT is approached, or (b) a second

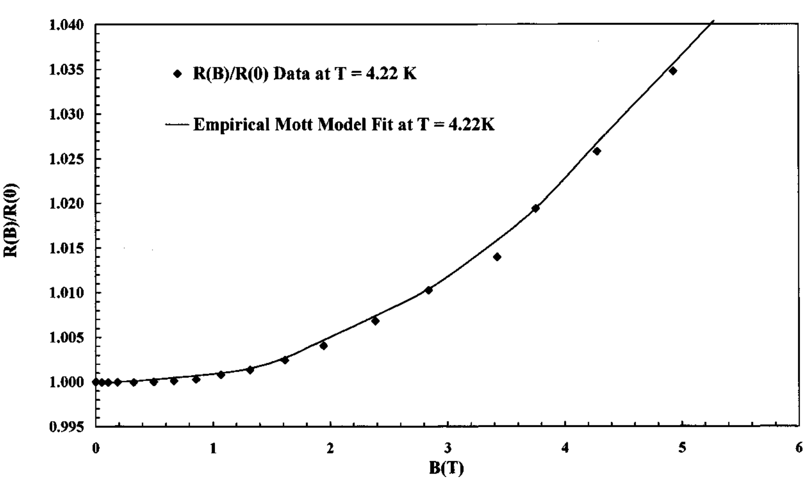

FIG. 7. Magnetoresistance ratios $R(B,T)/R(0,T)$ for a weakly insulating Mott amorphous Ni_xSi_{1-x} film No. 18. This film has a small Mott temperature of $T_{\mathrm{Mott}}=1140$ K, locating this film slightly below the metal-insulator transition (MIT). Notice the *almost complete absence* of the *negative* MR contribution at small fields, using the enlarged scale of the vertical axis. A possible explanation for the absence of the negative MR is presented in the text.

process becomes activated as the MIT is approached, which partially or totally cancels the orbital magnetoconductivity contribution.

The orbital magnetoconductivity process is valid only for *strongly insulating* films, and its validity is not known for *weakly insulating* films located just below the MIT.[30]

We speculate that the orbital magnetoconductivity process is still operational just below the metal-insulator transition, but that a second process starts to become operational. Owing to the close proximity of these films to the metal-insulator transition, the localization length a_0 is large and is expected to diverge to infinity as the MIT is approached from below. This implies that the area S of the scattering sites sampled by the hopping electron also diverges to infinity since $S=r_{opt}(a_0 r_{opt})^{1/2}$, where $r_{opt}=a_0(3/8)\times(T_{Mott}/T)^{1/4}$; note that $S\propto a_0^{7/8}$. Since this area becomes *very large* owing to the diverging localization length, then the weak localization process (the Bergmann backscattering interference process) should also become operative.[31] Because of the strong spin-orbit scattering arising from the heavy nuclei of the Ni, a *negative* magnetoconductance contribution should arise from this back-scattering process, thus partially or totally canceling the *positive* magnetoconductance contribution from the forward-interference process. This cancellation would then explain the absence of the positive magnetoconductance in *weakly insulating* samples located just below the MIT. Recall that the weak localization or back-scattering interference process is present in *metallic* films and involves the constructive interference of two plane waves at the initial site. These wave functions undergo many thousands of elastic-scattering events before returning to the initial site and interfering.[31] But this process should *not* be operative in *strongly insulating* films, where the wave functions are localized and decay exponentially away the site, thus inhibiting elastic-scattering events. We thus speculate that the weak localization process starts to become important just below the metal-insulator transition and dominates above the MIT.

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