Crystal and magnetic structure of orthorhombic HoMnO₃

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o-HoMnO₃ crystallizes in the GdFeO₃-type structure (*Pbnm*) with no crystal structure anomalies below 295 K and with the Mn-O-Mn angle as low as 144.1°. The Mn³⁺ and Ho³⁺ magnetic moments are ordered below $T_N = 38$ K and $T_{Ho} = 11$ K, respectively. The ordering is incommensurate with a propagation vector $\mathbf{k} = k_y \mathbf{b}^*$, where k_y is between 0.395 and 0.431 depending on temperature. Symmetry analyses have been carried out for both magnetic sites. The ordering was determined to be A_y for the Mn³⁺ moments, modulated in a longitudinal sinusoidal wave along the propagation vector. The Ho³⁺ moments order with the same propagation vector as Mn³⁺, but the site is split in two orbits of two ions, each ordered in an $a_x f_y$ pattern with a phase factor between them. At the lowest temperatures an antiferromagnetic (commensurate) magnetic phase appears in competition with the main (incommensurate) magnetic phase. The arrangement of the moments is discussed in terms of the magnetic exchange interactions and compared with the closely related magnetic structures of LaMnO₃, TbMnO₃, and o-YMnO₃. k_y is found to increase with decreasing size of the rare-earth component.

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I. INTRODUCTION

Perovskite phases with the formula $R_{1-x}A_xBO_3$ (R = La, Pr, Nd; A = Ca, Sr, Ba, Pb) and B = Mn have recently been subject to intense studies owing to the findings of colossal magnetoresistance.^{1,2} Phases with different transition metals on *B* site have also been studied and their magnetoresistive properties have proved to be rather dependent on the chemical nature of B.² However, there are still many problems to be solved for the nonsubstituted manganites of the heavy *R*'s. Here we shall address problems related to the crystal and magnetic structure of one particular manganite: metastable, orthorhombic (o)HoMnO₃.

The stoichiometric manganites $RMnO_3$ (R=La-Lu) can structurally be divided into two different groups. The larger sized R^{3+} 's ($r_{R^{3+}} \ge r_{Dy^{3+}}$) adopt an orthorhombically deformed perovskite-type structure (Pbnm), while the smaller R^{3+} 's take a hexagonal structure ($P6_3cm$) which is not related to the atomic arrangements in the perovskite family.³ However, a perovskite form of the smaller R's can also be prepared by means of special synthesis methods.

The unit-cell dimensions are known for the entire orthorhombic RMnO₃ series and indicate an increasing deformation from the ideal perovskite-type arrangement, as R becomes smaller.⁴ The perovskite atomic arrangement is characterized by RO_3 close-packed layers, and readjustment of the oxygen atoms is necessary to maintain connections between R^{3+} and O^{2-} when R^{3+} becomes increasingly smaller than O^{2-} . The rearrangement does mainly manifest itself as increasing tilts of the MnO₆ octahedra, which increase the orthorhombic splitting of the unit-cell dimensions compared with the cubic perovskite-type cell.

Positional parameters are available for R = La, Pr, Nd, Tb, Dy, Ho, Y, and Er^{5-10} through Rietveld refinements of neutron powder diffraction (NPD) data.

o-HoMnO₃ has been synthesized under high pressure¹¹

and through decomposition of citrate-based precursors.^{4,12} Large-scale samples of *o*-HoMnO₃ synthesized by the latter procedure are used in this work to examine its crystal and magnetic structure by means of NPD.

The magnetic arrangement of stoichiometric RMnO₃ has been reported to be of the antiferromagnetic A type (R = La, Pr, Nd; for nomenclature for magnetic modes see Ref. 13) or the phase is said to be metamagnetic (R = Gd, Tb, Dy, Ho).^{5,11,14–17} In addition, a weak ferromagnetic component may be present.¹⁴

The Néel temperature (T_N) for the ordering of the manganese moments decreases from 141 K for La to 40 K for Tb.^{5,15–18} This is in accordance with the trend of decreasing T_N with decreasing *R* size for the rare earth titanites, vanadites, chromites, and orthoferrites.¹⁵ Ordering of the *R* moments occurs at a low temperature; below 1.5 K for Pr and Nd,⁵ and at 7 K for Tb.¹⁶

In *o*-YMnO₃ the antiferromagnetic arrangement has been reported to be helical, derived from an *A* mode with $\mathbf{k} = 0.458\mathbf{b}^*$ and $T_N = 42 \text{ K}$.¹⁹ TbMnO₃ has a sinusoidal-wavetype ordering with $\mathbf{k} \approx 0.28\mathbf{b}^*$ below $T_N = 40 \text{ K}$; above 20 K in an *A* mode and below 20 K with *A* and *G* modes.¹⁶ The ordering of the Tb moments has not yet been completely established, but the arrangement is reported to take a complex short-range ordering with $\mathbf{k} = 0.415\mathbf{b}^*$ and participation of *A*, *C*, and *G* modes.¹⁶

The magnetic susceptibility of TbMnO₃ shows a broad maximum around $T_{\rm Tb}$ =7 K,¹⁶ (Ref. 16 uses the nomenclature T_{N2} for $T_{\rm Tb}$) and a similar maximum in magnetic susceptibility is seen for *o*-HoMnO₃ at 6.5 K.¹²

It is important to understand the physical reasons underlying the complex magnetic behavior of these materials. We have attempted to go a further step in this mission by solving the magnetic structure of o-HoMnO₃, for which it will be shown that the low-temperature magnetic behavior is far from simple. The remaining part of the paper is organized as follows. Section II describes the experiments and Sec. III the results and discussion. The crystal structure is considered in Sec. III A and the magnetic ordering temperatures in Sec. III B. Symmetry analyses are reported in Sec. III C and the findings are used to determine the orderings of the magnetic moments of Mn^{3+} and Ho^{3+} described in Secs. III D and *F*, respectively. The manganese ordering is discussed in terms of the exchange interactions (Sec. III E) and a comparison with other rare-earth manganites is given in Sec. III G. Conclusions are given in Sec. IV.

II. EXPERIMENT

 Ho_2O_3 (Strem Chem., 99.9%; pre-annealed) and Mn(CH₃COO)₂·4H₂O (Fluka, 99%; with thermogravimetric determined water content) were dissolved in melted citric acid monohydrate (Riedel de Häen, 99.5%) and water was added. Stepwise heating of the mixture produced in this way were performed up to a final treatment temperature of 900 °C, where crystallization of *o*-HoMnO₃ took place in oxygen. For details see Ref. 12.

Two different samples were synthesized. The first (denoted I) consisted of three different specimens (each ~ 0.6 g), which each were proved to be single-phase *o*-HoMnO₃ by x-ray powder diffraction (XPD; Siemens D5000) and then mixed together. The unit-cell dimensions thus obtained were in good agreement both mutually and with earlier literature values.

The second sample (denoted II) was a mixture of six different batches, which XPD (Guinier-Hägg camera) confirmed to be single phase with fully consistent unit-cell dimensions.

The fraction of Mn^{4+} was determined to be 2(1)% by cerimetric titration. This gives $HoMnO_{3.009(5)}$ or more precisely $Ho_{0.997(2)}Mn_{0.997(2)}O_3$.

Magnetic-susceptibility measurements were performed in the temperature range 5–300 K in magnetic fields up to 5.5 T with a superconducting quantum interference device instrument (Magnetic Property Measurement System; Quantum Design); the sample size was approximately 11 mg.

XPD data for sample I was obtained with a Siemens D5000 diffractometer with $CuK\alpha_1$ radiation, primary monochromator, position sensitive detector (PSD) and Si as internal standard. For sample II a camera with Guinier-Hägg geometry, $CuK\alpha_1$ radiation, primary monochromator, and Si internal standard was used.

NPD data at 8 and 295 K for sample I were collected with the PUS two-axis diffractometer at the JEEP II reactor, Kjeller. Monochromatized neutrons of wavelength 1.5527 Å were obtained by reflection from Ge (511) and detected by two 20° (in 2 θ) PSD banks. Lower temperatures were obtained by means of a Displex system. An instrumental impurity peak, which appears at $2\theta \approx 32^\circ$, was excluded from the data refinement.

NPD data at 2.8 K were collected at the high-resolution powder diffractometer D2B at ILL, Grenoble, France for sample I. Monochromatic neutrons of wavelength 1.5938 Å were obtained by reflection from Ge (335) and detected by 64 ³He counters.

NPD data for sample II were collected in the diffractometer G4.1 (at the reactor Orphée, Saclay, France), that has a position sensitive detector of 800 cells, spanning an angular range of 80° (2θ from 10 to 90°). A wavelength of 2.426 Å was obtained by reflection (002) from pyrolitic graphite. Diffraction patterns were collected every 3 K from 1.4 to 58.0 K. Each data point is separated by 0.1° (in 2θ).

The crystal and magnetic structures were refined by using the Rietveld refinement program FULLPROF98.²⁰ A pseudo-Voigt profile function was used. Isotropic temperature factors were refined for the different types of atoms, with typical values (in Å²): B_{Ho} =0.50, B_{Mn} =0.60, B_{O} =0.70.

III. RESULTS AND DISCUSSION

A. Crystal structure of o-HoMnO₃

There is no evidence of magnetic order at 295 K (*vide infra*), and the diffraction pattern (Fig. 1) was found to be in full accordance with the assumed $GdFeO_3$ -type structure (Fig. 2, here described in the space group *Pbnm*). Rietveld refinement of the crystal structure gave the unit-cell dimensions and positional parameters listed in Table I.

The octahedral coordination of O around Mn is (2+2)+2 (Table II) as a result of Jahn-Teller deformation. Ho is surrounded by 8 O, arranged in a deformed square antiprism. The calculated bond valence for Mn and Ho is 3.17 and 2.96, respectively, and the average Mn-O-Mn angle is as low as 144.1°.

Up to quite recently, there have not been enough consistent structural data to establish precisely the Jahn-Teller distortion as a function of the R component in the $RMnO_{300}$ series. The data for LaMnO₃₀₀ (Refs. 8 and 9) and o-HoMnO₃ as well as relatively old data for PrMnO₃, NdMnO₃, and TbMnO₃ (Ref. 5) gave a hint about the development in the size of the distortion. The structure data for the three latter phases are mutually consistent and indicate an increasing difference between the two longest and the average of the four shortest Mn-O distances. However, this is not compatible with the findings for R = La and Ho. In this situation it was therefore a relief to see the results of the recent room-temperature NPD study¹⁰ of the series with R = La, Pr, Nd, Tb, Dy, Ho, Y, and Er (which are reasonably compatible with the mentioned findings for R = La, Nd, and Ho, but certainly not with those for R = Pr and Tb). Thus it seems established that the Jahn-Teller deformation increases with decreasing R^{3+} size and the consequent increasing tilts of the MnO₆ octahedra. However, the size of the deformation appears to go through a broad maximum around R = Dy.

Apart from the thermal expansion in the unit-cell dimensions (Fig. 3 and Table I), there is no evidence of distinct alterations in the crystal structure of o-HoMnO₃ between 2.8 and 295 K. It is, in particular, worth noting that the passages of the magnetic ordering temperatures for the Mn³⁺ and Ho³⁺ sublattices (*vide infra*) have virtually no impact on the variables of the crystal structure.



B. Néel temperature

At low temperature several extra reflections, originating from cooperative magnetism, are revealed in the NPD patterns. The strongest magnetic reflection (which later will be recognized as 001^{\pm}) and its weaker neighbor (011^{-}) is



FIG. 2. The GdFeO₃-type crystal structure of o-HoMnO₃ (see Table I); Ho³⁺ (open circles), Mn³⁺ (black circles), and O²⁻ (shaded circles). The numbering of the Ho³⁺ and Mn³⁺ atoms refers to the magnetic arrangement.

FIG. 1. Rietveld refinements (upper line) of NPD data collected with PUS (circles) for o-HoMnO₃ at 295 K. Positions of Bragg reflections (nuclear contributions) are shown with bars. The difference between observed and calculated intensity is shown with the bottom line.

shown as a function of temperature in Fig. 4. Two different magnetic ordering schemes are immediately recognized; one between ca. 36 and 13 K and the other below ca. 13 K, the latter with higher intensity. From the trend in the magnetic properties of the $RMnO_3$ perovskites it is reasonable to conclude that the magnetic moments of manganese order (approaching saturation near 13 K) in the highest temperature interval, while the holmium moments order at lower tem-

TABLE I. Crystal and magnetic structure parameters for o-HoMnO₃ according to Rietveld refinements of NPD data. This description is based on space group *Pbnm* with Ho in 4c, Mn in 4b; O1 in 4c, and O2 in 8d.

| <i>T</i> (K) | 295 (PUS) | 8 (PUS) | 2.8 (D2B) |
|-------------------------------|------------|------------|---------------|
| a (Å) | 5.2549(2) | 5.2481(2) | 5.24765(10) |
| <i>b</i> (Å) | 5.8263(2) | 5.8188(2) | 5.81744(11) |
| <i>c</i> (Å) | 7.3648(3) | 7.3431(3) | 7.3447(2) |
| $x_{\rm Ho}$ | 0.9832(5) | 0.9833(5) | 0.9830(4) |
| ${\cal Y}_{ m Ho}$ | 0.0848(3) | 0.0850(3) | 0.0836(3) |
| <i>x</i> ₀₁ | 0.3902(5) | 0.3895(5) | 0.3894(5) |
| <i>Y</i> 01 | -0.0368(5) | -0.0355(5) | -0.0369(4) |
| <i>x</i> _{O2} | 0.7014(4) | 0.7007(4) | 0.6999(3) |
| <i>Y</i> _{O2} | 0.3261(4) | 0.3264(4) | 0.3275(3) |
| Z _{O2} | 0.0529(3) | 0.0533(3) | 0.0533(2) |
| $\mu_{AF,Mn}(\mu_B)$ | | 3.96 | 3.96 |
| $\mu_{AF,\mathrm{Ho}}(\mu_B)$ | | 4.85(10) | $7.65(9)^{a}$ |
| k/b* | | 0.4289(11) | 0.4263(5) |
| R_{nucl} (%) | 3.59 | 4.02 | 3.64 |
| $R_{\rm mag}$ (%) | | 12.1 | 7.26 |
| χ^2 | 1.22 | 1.76 | 1.75 |
| | | | |

^aThe G4.1 data gave 7.69(5) μ_B at 1.4 K.

TABLE II. Selected distances (in Å) and angles (degrees) from Rietveld refinements of NPD data of o-HoMnO₃. Estimated standard deviation of the distances is 5–8 in the last digit.

| T (K) | 295 | 8 | 2.8 |
|--------------------------------|-------|-------|-------|
| Mn-O1(×2) | 1.907 | 1.907 | 1.908 |
| Mn-O2(\times 2) | 1.940 | 1.935 | 1.937 |
| Mn-O2(\times 2) | 2.206 | 2.206 | 2.210 |
| $\overline{d}_{\mathrm{Mn-O}}$ | 2.018 | 2.018 | 2.018 |
| $Ho-O1(\times 1)$ | 2.252 | 2.243 | 2.246 |
| $Ho-O1(\times 1)$ | 2.299 | 2.306 | 2.306 |
| $Ho-O1(\times 1)$ | 3.193 | 3.139 | 3.192 |
| $Ho-O1(\times 1)$ | 3.681 | 3.670 | 3.671 |
| Ho-O2(\times 2) | 2.306 | 2.297 | 2.287 |
| Ho-O2(\times 2) | 2.501 | 2.501 | 2.513 |
| Ho-O2(\times 2) | 2.559 | 2.554 | 2.552 |
| Ho-O2(\times 2) | 3.664 | 3.665 | 3.669 |
| $\overline{d}_{	ext{Ho-O}}$ | 2.410 | 2.407 | 2.407 |
| $Mn-O2-Mn(\times 4)$ | 144.6 | 144.3 | 144.0 |
| $Mn-O1-Mn(\times 2)$ | 142.9 | 142.9 | 142.9 |

perature. By plotting the peak intensities against temperature, T_N could be determined to be close to 40 K and T_{Ho} near 15 K (left inset to Fig. 5). A more accurate approach for determination of T_N is to plot the reduced magnetic moment (obtained by Rietveld refinements) against T/T_N for different T_N and make comparison with the Brillouin function for J=S = 2, corresponding to Mn³⁺. The best fit is obtained by using $T_N=38$ K (Fig. 5). T_{Ho} could not be determined accurately by this method, but the best fit to the Brillouin function for J=4 (corresponding to Ho³⁺) was obtained with T_{Ho} = 11 K (right inset to Fig. 5). This is supported by the fact

that the deviation from the Curie-Weiss law (dominated by the Ho³⁺ moments) starts at 11.0 ± 0.5 K (Fig. 6).

The magnetic ordering of manganese is not revealed in the susceptibility curve of o-HoMnO₃. The same appears to be the case for several other members of the RMnO₃ series where R^{3+} carries a large magnetic moment.^{11,14,15} Nevertheless, the trend of decreasing T_N with decreasing size of R^{3+} is well established from NPD data.^{5,15-18} The explanation for this size dependence is related to the structural change in the $e_g^1 - 2p_\sigma - e_g^1$ and $t_{2g}^3 - 2p_\pi - t_{2g}^3$ superexchange paths.²¹ As R^{3+} becomes smaller, the MnO₆ octahedra turn more in the tilting process and the Mn-O-Mn angle decreases [average angle for o-HoMnO₃ is 144° compared to 155° for LaMnO₃ (Refs. 8-10)], thus weakening the superexchange interaction and decreasing T_N . The splitting of the unit-cell parameters of an adopted pseudocubic $2 \times 2 \times 2$ perovskite cell (an indicator of the Mn-O-Mn angle; Fig. 7) levels out with the decreasing size of R^{3+} when the stability limit of the GdFeO₃-type arrangement is approached,⁴ and this could explain the minor variation in T_N for the larger R^{3+} 's.

 T_R seems to have the opposite dependence on the size of R^{3+} , but the actual ordering temperatures are generally very low and the values more scattered. The role of the crystal-field effects is very important in the magnetic anisotropy of R^{3+} and this may influence the ordering temperature for the R sublattices. Hence, the safest is to maintain that T_R is independent of the size of R^{3+} .

For a TbMnO₃ single-crystal, neutron diffraction proved $T_{\rm Tb}$ to be 7 K, where the magnetic susceptibility is found to exhibit a maximum.¹⁶ For *o*-HoMnO₃, $T_{\rm Ho}$ is slightly higher than the temperature of the maximum in magnetic susceptibility. In fact, the behavior of the magnetic susceptibility for a perovskite with magnetic ions on both *A* and *B* sites and a large number of different exchange interactions is very difficult to predict.



FIG. 3. Thermal-expansion data for o-HoMnO₃ between 2.8 and 295 K. Numerical values for the linear expansion coefficients are given on the illustration.



FIG. 4. NPD data collected with G4.1 for *o*-HoMnO₃. The strongest magnetic peak 001^{\pm} and its neighbor 011^{-} are shown below (open circles) and above (filled circles) $T_{\rm Ho}$. The temperature difference between each of these recordings is approximately 3 K.

C. Symmetry analysis

In order to determine the magnetic structure of the Mn³⁺ and R^{3+} sites in RMnO₃ perovskites it is very important to classify the possible spin configurations according to the irreducible representations (ireps) of the space group for the propagation vector. For *o*-HoMnO₃, all the magnetic reflections can be indexed using a propagation vector $\mathbf{k}=k_y\mathbf{b}^*$ with $k_y \leq \frac{1}{2}$. Symmetry analysis for space group *Pbnm* with this kind of propagation vector has earlier been performed for the 4*b* position, obtaining the different basis functions of the ireps describing the magnetic ordering at this site.²² We have extended this analysis for all magnetic sites and the final results are summarized in Table III.

The decomposition of the full reducible magnetic representation Γ (of dimension 3 times the number of atoms in the primitive cell) for 4*b* is: $\Gamma_{4b} = \Sigma_{\oplus} n_{\nu} \Gamma_{\nu} = 3\Gamma_1 \oplus 3\Gamma_2 \oplus 3\Gamma_3 \oplus 3\Gamma_4$, the corresponding decomposition for the two 4*c* sites is: $\Gamma_{4c1,4c2} = \Gamma_1 \oplus 2\Gamma_2 \oplus \Gamma_3 \oplus 2\Gamma_4$. The number of free parameters for a magnetic structure corresponding to an irep is equal to the product $(n_{\text{free}} = n_{\nu} d_{\nu})$ of the number of times (n_{ν}) the irep Γ_{ν} is contained in Γ and the dimension (d_{ν}) of the irep. This is also the total number of independent basis function vectors.



FIG. 5. Reduced magnetic moment of Mn^{3+} as a function of reduced temperature compared to the Brillouin function for J=S= 2 and $T_N=38$ K. Right inset: Reduced magnetic moment of Ho³⁺ as a function of reduced temperature compared to the Brillouin function for J=4 and T_{Ho} = 11 K. Left inset: Reduced intensity of the magnetic reflections 001[±] and 011⁻ as a function of temperature (see Fig. 4).



FIG. 6. Inverse susceptibility vs temperature; onset of deviations from Curie-Weiss law at 11.0 ± 0.5 K. Left inset: The entire inverse susceptibility characteristic below room temperature. Right inset: Magnetic moment as a function of magnetic field at 5, 10, 15, 30, and 50 K.

The magnetic moment of the atom "j", in the cell with origin at the lattice point \mathbf{R}_n , in a magnetic structure characterized by the set of propagation vectors {k}, is given by the Fourier series: $\mathbf{m}_{nj} = \sum_{\{\mathbf{k}\}} \mathbf{S}_{\mathbf{k}j} \cdot e^{-2\pi i \mathbf{k} \cdot \mathbf{R}_n}$, where the sum is extended to the whole set of propagation vectors. For **k** at the interior of the Brillouin zone the vector $-\mathbf{k}$ should also be present and $\mathbf{S}_{\mathbf{k}j} = \mathbf{S}_{-\mathbf{k}j}^*$. The physical meaning of the basis functions of the irep Γ_{ν} in describing a magnetic structure is that the Fourier coefficients $\mathbf{S}_{\mathbf{k}j}$ are linear combinations of the constant vectors $\mathbf{S}_{\alpha}(\mathbf{k}, \nu, \lambda | j)$, so that

Eu Gd

$$\mathbf{S}_{\mathbf{k}i} = \boldsymbol{\Sigma}_{\alpha\lambda} C_{\alpha\lambda} \mathbf{S}_{\alpha}(\mathbf{k}, \nu, \lambda | j)$$

6

Т

1.00

1.02

0.98

0.96

0.94

 $r_{R^{3+}}$ (Å)

Pr Nd

56^L

140

120

100

80

60

40

²⁰ 1.04

 $T_{\rm N}({\rm K})$

For a given propagation vector **k** and a given irep Γ_{ν} , the index α runs between 1 and n_{ν} and λ between 1 and d_{ν} (for details, see Ref. 23). The coefficients $C_{\alpha\lambda}$ can be real or purely imaginary. By varying these coefficients we obtain the whole class of magnetic structures satisfying the symmetry of the propagation vector. Let us consider two examples for the manganese sublattices constructed with the representation Γ_3 . The irep is of dimension 1 (so $\lambda = 1$ and the summation index over λ is therefore removed) and it is contained 3 times in Γ_{4b} , so the number of coefficients is 3 ($\alpha = 1, 2,$ 3). The possible magnetic structures described by this irep are given by

> 0.52 0.48

0.36

0.32

0.28

0.5

0.4

0.3 **र**

0.2

0.1

0.0

0.86

b*

I I Yb Lu





0.90

0.88

HoY Er Tm

INCOMMEN

SURATE

0.92

Tb Dy

TABLE III. Irreducible representations (ireps) for the group of the propagation vector $\mathbf{G}_{\mathbf{k}} \mathbf{k} = k_y \mathbf{b}^*$ $(k_y < \frac{1}{2})$ in the space group *Phnm*. For $\mathbf{G}_{\mathbf{k}} = Pb2_1m$ the center of symmetry is lost. The coset representatives of $\mathbf{G}_{\mathbf{k}}$ are $\{1, m_z, 2_{1y}, b_x\} = \{(x, y, z), (x, y, -z + \frac{1}{2}), (-x + \frac{1}{2}, y + \frac{1}{2}, -z + \frac{1}{2}), (-x + \frac{1}{2}, y + \frac{1}{2}, z)\}$. There are four one-dimensional irreducible representations. The basis functions of positions 4b and 4c are also given. We have used an extension to complex basis functions with the Bertaut (Ref. 13) notations for the modes *F*, *A*, *C*, *G*. The value of ω is $e^{i\pi k_y}$. The numbering of atoms in the 4b site is: $\mathrm{Mn}_1(\frac{1}{2},0,0)$, $\mathrm{Mn}_2(\frac{1}{2},0,\frac{1}{2})$, $\mathrm{Mn}_3(0,\frac{1}{2},\frac{1}{2})$, $\mathrm{Mn}_4(0,\frac{1}{2},0)$. The atoms in the 4c site split in two orbits (previously related by the center of symmetry) having the same basis functions. The coupling between these two orbits is usually positive or negative, but in the general case there is a phase factor between them. The holmium atoms of the first orbit are: $\mathrm{Ho}_1(x, y, \frac{1}{4})$, $\mathrm{Ho}_4(1\frac{1}{2}-x, y+\frac{1}{2}, \frac{1}{4})$, and those of the second orbit: $\mathrm{Ho}_2(1-x,1-y,\frac{3}{4})$, $\mathrm{Ho}_3(x-\frac{1}{2}, \frac{1}{2}-y,\frac{3}{4})$, with $x \approx 0.985$ and $y \approx 0.085$. The symbols (*f*,*a*) used to describe the basis functions for the Ho orbits correspond to *F* and *A* for a two-atoms orbit.

| | | | | | Basis functions vectors $\mathbf{S}(\mathbf{k}, \nu, \lambda j)$ for 4 <i>b</i> site | | | Basis functions vectors $\mathbf{S}(\mathbf{k}, \nu, \lambda j)$ for two 4 <i>c</i> orbits | | | | | |
|----------------|---|-------|-----------|-----------|--|-----------------|-----------------|--|---------------|-----------------|---|-----------------|---|
| Γ_{ν} | 1 | m_z | 2_{1y} | b_x | Mn ₁ | Mn ₂ | Mn ₃ | Mn ₄ | | Ho ₁ | Ho_4 | Ho ₂ | Ho ₃ |
| Γ_1 | 1 | 1 | ω | ω | (100) | (-100) | $(-\omega 00)$ | (<i>w</i> 00) | A_x | | | | |
| | | | | | (010) | (0 - 10) | $(0\omega 0)$ | $(0-\omega 0)$ | G_{v} | | | | |
| | | | | | (001) | (001) | $(00-\omega)$ | $(00-\omega)$ | C_{z} | (001) | $(00-\omega) a_{z}$ | (001) | $(00-\omega) a_{z}$ |
| Γ_2 | 1 | -1 | ω | $-\omega$ | (100) | (100) | $(-\omega 00)$ | $(-\omega 00)$ | $\tilde{C_x}$ | (100) | $(-\omega 00) a_x$ | (100) | $(-\omega 00) a_x$ |
| | | | | | (010) | (010) | $(0\omega 0)$ | $(0\omega 0)$ | F_{v} | (010) | $(0\omega 0) f_{y}$ | (010) | $(0\omega 0) f_{y}$ |
| | | | | | (001) | (00 - 1) | $(00-\omega)$ | (00ω) | A_z | | 2 | | 2 |
| Γ_3 | 1 | 1 | $-\omega$ | $-\omega$ | (100) | (-100) | $(\omega 00)$ | $(-\omega 00)$ | G_x | | | | |
| | | | | | (010) | (0 - 10) | $(0-\omega 0)$ | $(0\omega 0)$ | A_{v} | | | | |
| | | | | | (001) | (001) | (00ω) | (00ω) | F_z | (001) | $(00\omega) f_z$ | (001) | $(00\omega) f_z$ |
| Γ_4 | 1 | -1 | $-\omega$ | ω | (100) | (100) | $(\omega 00)$ | $(\omega 00)$ | F_{x} | (100) | $(\omega 00) f_x$ | (100) | $(\omega 00) f_x$ |
| | | | | | (010) | (010) | $(0-\omega 0)$ | $(0-\omega 0)$ | C_{y} | (010) | $(0-\omega 0) a_y$ | (010) | $(0-\omega 0) a_y$ |
| | | | | | (001) | (00 - 1) | (00ω) | $(00-\omega)$ | G_z | | , i i i i i i i i i i i i i i i i i i i | | , i i i i i i i i i i i i i i i i i i i |

$$\mathbf{S}_{\mathbf{k}1} = \boldsymbol{\Sigma}_{\alpha} C_{\alpha} \mathbf{S}_{\alpha}(\mathbf{k}, \nu, 1 | 1) = C_1(1, 0, 0) + C_2(0, 1, 0) + C_3(0, 0, 1)$$

$$=(C_1, C_2, C_3)$$

$$\begin{split} \mathbf{S}_{\mathbf{k}2} &= \mathbf{\Sigma}_{\alpha} C_{\alpha} \mathbf{S}_{\alpha}(\mathbf{k}, \nu, 1 | 2) \\ &= C_1(-1, 0, 0) + C_2(0, -1, 0) + C_3(0, 0, 1) \\ &= (-C_1, -C_2, C_3) \\ \mathbf{S}_{\mathbf{k}3} &= \mathbf{\Sigma}_{\alpha} C_{\alpha} \mathbf{S}_{\alpha}(\mathbf{k}, \nu, 1 | 3) \\ &= C_1(\omega, 0, 0) + C_2(0, -\omega, 0) + C_3(0, 0, \omega) \\ &= \omega(C_1, -C_2, C_3) \end{split}$$

$$\mathbf{S}_{\mathbf{k}4} = \mathbf{\Sigma}_{\alpha} C_{\alpha} \mathbf{S}_{\alpha}(\mathbf{k}, \nu, 1 | 4)$$

= $C_1(-\omega, 0, 0) + C_2(0, \omega, 0) + C_3(0, 0, \omega)$
= $\omega(-C_1, C_2, C_3).$

If, e.g., $C_1 = C_3 = 0$, the structure corresponds to a collinear longitudinal sinusoidal structure. If $C_2 = 0$, and $C_1 = iC_3$ with C_3 purely imaginary, the structure corresponds to a helix with a cylindrical envelope. If $C_1 = 0$ and $C_2 = iC_3$ the structure is a cycloid, etc.

To solve the magnetic structure one has to look for the set of coefficients that give rise to the best agreement between observed and calculated magnetic intensities. This part of the analysis may be undertaken just by trial and error by hand or using Monte Carlo techniques.

D. Magnetic ordering of Mn³⁺

Just below T_N the magnetic reflections can be indexed with a propagation vector nearly equal to the propagation vector for o-YMnO₃. Depending on temperature, **k** varies in the region 0.395**b*** to 0.431**b*** (inset to Fig. 8). Susceptibility measurements (Fig. 6) show no indication of a ferromagnetic component. This is consistent with the observed **k** vector in the interior, $\mathbf{k} \neq (0,0,0)$, of the Brillouin zone. The fact that manganese is in a special position, allows a simple de-



FIG. 8. Magnetic moment amplitudes for *o*-HoMnO₃ refined from NPD data obtained with G4.1 as a function of temperature for Ho³⁺ (filled circles) and Mn³⁺ (open circles) compared with Brillouin functions (lines). Inset: Magnitude of propagation vector k_y as a function of temperature.



FIG. 9. The magnetic arrangement in o-HoMnO₃. Top: Ordering of the magnetic moments of Mn in the *ab* plane for three crystallographic unit cells. The sinusoidal modulation of the magnetic moments is also shown. Bottom: Ordering of the magnetic moments of Ho³⁺ in the *ab* plane with corresponding sinusoidal modulation of the moments for the two different orbits. Note that at a given temperature, there is only one common propagation vector, but there may be phase differences between the three magnetic sites.

termination of the magnetic structure from the extinction rules of the satellites as deduced from the expression of the magnetic structure factor. The observed satellites verify that the parent reflections (*hkl*) satisfy h+k=2n and l=2n+1. Hence only pure A modes are compatible with these observations in the temperature range 13-36 K. Other possible modes present in the magnetic structure are below the detection limit of the present experiments. Then, there are three possibilities left: $\Gamma_1(A_x 00)$, $\Gamma_2(00A_z)$, and $\Gamma_3(0A_v 0)$ if the magnetic structure is described by a single irep. The mixing of ireps is also possible, so modes of the form Γ_1 - $\Gamma_1 \oplus \Gamma_2 \oplus \Gamma_3(A_x A_y A_z)$ $\oplus \Gamma_2(A_x 0 A_z)$ or even are conceivable.²³ The first three alternatives correspond to sinusoidal structures (transverse for Γ_1 and Γ_2 and longitudinal for Γ_3) and the mixing of ireps offers the possibility of having helical, cycloidal, or sinusoidal orderings. Rietveld refinements of $\Gamma_1(A_r,00)$ and $\Gamma_2(00A_r)$ with a magnetic moment perpendicular to the propagation vector inevitably resulted in enhanced calculated intensity for several reflections, while $\Gamma_3(0A_v0)$, e.g., $C_1 = C_3 = 0$ and $C_2 \neq 0$, gave a fully satisfactory fit. Hence the moments of Mn³⁺ order with



FIG. 10. The magnetic ordering of Mn^{3+} in $R\text{MnO}_3$ [R = La, Pr, Nd, Tb, Ho (and Y)] in two adjacent *ab* slabs. The numerical values refer to the periodicity just below T_N .

the magnetic moment modulated in a longitudinal sinusoidal wave along the propagation vector. This ordering is visualized in Fig. 9 and compared to the magnetic arrangement in $LaMnO_3$ and $TbMnO_3$ in Fig. 10.

The amplitude of the magnetic moment was refined for the G4.1 data at all available temperatures. The nuclear structure and isotropic temperature factors were kept constant and equal to the refined D2B data at 2.8 K. Special care was taken to select the correct background position around the largest magnetic peaks because the determination of the moment is fairly sensitive to this. Below $T_{\rm Ho}$ the moment of Mn³⁺ was kept constant in order to stabilize the refinements (see Sec. III F). The thus derived magnetic structure parameters are listed in Table I and illustrated in Figs. 8–10.

E. Exchange interactions

The occurrence of a sinusoidal magnetic structure as the first magnetic ordered state for manganese in o-HoMnO₃ is the consequence of the competition between magnetic interactions which gives rise to a certain degree of frustration. An analysis of the exchange integrals has been partially performed in Ref. 19. The first magnetic ordered state corresponds to the maximum eigenvalue for the Fourier transform Hermitian matrix of the exchange integrals (see Ref. 24, but note that we use a different sign convention: here a negative exchange integral corresponds to antiferromagnetic coupling, the magnetic energy of a pair is given by $E_{ij} = -J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$):

$$\xi_{ii}(\mathbf{k}) = \boldsymbol{\Sigma}_n J_{ii}(\mathbf{R}_n) \cdot e^{-2\pi i \mathbf{k} \cdot \mathbf{R}_n},$$

where the index pair *ij* refers to a particular numbering of atoms within a primitive cell so that $J_{ij}(\mathbf{R}_n)$ is the exchange integral between an atom *i* in the reference (origin) cell and an atom *j* in the cell with origin at \mathbf{R}_n . If only isotropic exchange interactions are considered, the magnetic structure is degenerated with respect to a complete rotation of all the spins in the crystal. The eigenvectors of $\xi_{ij}(\mathbf{k})$ are the modes that may be combined to construct general helical structures. A list of exchange interactions that may be considered as contributing to determine the magnetic structure of o-HoMnO₃ is given in Table IV.

The problem of the ground state is too complicated to be treated here, however, we can make some simplifications in order to get some insight into the conditions which must be satisfied by the exchange integrals in order to arrive at the first ordered magnetic state of Mn. The 4×4 matrix $\xi_{ii}(\mathbf{k})$

TABLE IV. Exchange paths possibly contributing to determine the magnetic ground state of o-HoMnO₃ (*Pbnm* setting). The numbering of atoms is as in Table III. A lattice translation has been added to appropriate second atoms in order to identify representative pairs for each exchange integral. The notations for the Mn-Mn (MM) exchange integrals are the same as in Ref. 19; MH is used for Mn-Ho interactions, and HH for Ho-Ho interactions.

| Exchange integral | Representative atom pair | Number of equivalents | Separation (in Å) at 2.8 K | Exchange path via oxygen |
|-------------------|--|-----------------------|-------------------------------|--------------------------|
| J ₁₂ | Mn ₁ -Mn ₂ | 2 | 3.672 | MM superexchange |
| J_{14} | Mn ₁ -Mn ₄ | 4 | 3.917 | MM superexchange |
| J_{11a} | $Mn_1 - Mn_1(100)$ | 2 | 5.248 | MM super-superexchange |
| J_{11b} | $Mn_1 - Mn_1(010)$ | 2 | 5.818 | MM super-superexchange |
| J_{13} | Mn ₁ -Mn ₃ | 8 | 5.370 | MM super-superexchange |
| J_{MR1} | $Mn_1-Ho_2(0\overline{1}0);Ho_4(00\overline{1})$ | 2 | 3.036 | MH superexchange |
| $J_{\rm MR2}$ | $Mn_1-Ho_1;Ho_3(011)$ | 2 | 3.178 | MH superexchange |
| J _{MR3} | $Mn_1-Ho_1(\overline{1}00);Ho_3(1\overline{11})$ | 2 | 3.303 | MH superexchange |
| $J_{\rm MR4}$ | Mn_1 -Ho ₂ ;Ho ₄ (011) | 2 | 3.865 | MH superexchange |
| J_{RR1} | $Ho_1-Ho_3(1\overline{1}0)$ | 2 | 3.805 | HH superexchange |
| J_{RR2} | Ho ₁ -Ho ₂ | 2 | 3.816 | HH superexchange |
| J _{RR3} | Ho ₁ -Ho ₂ (100) | 2 | 4.022 | HH superexchange |

for the Mn sublattices (see Figs. 2, 9, and 10) can be easily constructed for the exchange integrals of Table IV. This matrix has been discussed in detail by Bertaut¹³ and applied to the case of o-YMnO₃.¹⁹ As discussed by these authors the eigenvalue corresponding to the *A* mode, along the direction $\mathbf{k} = k_y \mathbf{b}^*$, can be written as

$$\lambda_A(0,k_y,0) = 2(J_{11a} - J_{12}) + 2J_{11b}\cos(2\pi k_y) -4(2J_{13} - J_{14})\cos(\pi k_y).$$

The derivative of λ_A with respect to k_v

$$\partial \lambda_A / \partial k_y = 4 \pi \sin(\pi k_y) [-2J_{11b} \cos(\pi k_y) + 2J_{13} - J_{14}] = 0$$

should be zero for a maximum.

The condition for the mode *A* being more stable than the *G* mode is: $2J_{13}-J_{14}<0.^{19}$ Provided this condition is fulfilled, λ_A is at maximum for $k_y=0$ or for $\cos(\pi k_y)=(2J_{13}-J_{14})/(2J_{11b})$. The first solution corresponds to the antiferromagnetic magnetic structure observed for LaMnO₃, whereas the second solution gives rise to either a sinusoidal or an helical structure (the magnetic isotropic energy is degenerate), depending on whether we consider constant magnetic moments or not. Considering **k** at the interior of the Brillouin zone, the second solution gives as a strong condition for the exchange interactions the relation: $0 < (2J_{13} - J_{14})/(2J_{11b}) < 1$.

There are two exchange integrals, J_{13} and J_{14} , which determine the mode (*A* or *G*) and three (J_{13} , J_{14} and J_{11b}) which determine k_y (see Ref. 19). The nearest-neighbor interaction between Mn₁ and Mn₂ does not have any influence on this. From the equations above it follow that J_{11b} is negative. The strong Jahn-Teller effect in *o*-HoMnO₃ implies that J_{14} is positive and certainly greater than the super-super-exchange interactions J_{13} and J_{11b} that should be of the same order of magnitude. The fact that k_y increases from TbMnO₃

to *o*-HoMnO₃ means that $(2J_{13}-J_{14})/(2J_{11b})$ decreases. This could be due to decreasing J_{14} caused by decreasing Mn-O-Mn angle (see also Ref. 25).

In order to go further into the origin of the incommensurate magnetic structure of o-HoMnO₃ it is necessary to get more insight into the mutual signs and magnitudes of J_{13} , J_{14} , and J_{11b} . In the anticipation of theoretical electronic band-structure based calculations, it has been attempted to extract interaction information from the available crystal structure data. An obvious approach is to try to relate the tilt characteristics of GdFeO₃-type manganites to the magnetic interactions concerned. The relationship between the adapted deformation indicator $(a_p - b_p)$ and $\mathbf{k}/\mathbf{b}^* = k_y$ in Fig. 7 suggests that interesting information may be hidden here. However, the problem is that there are two different tilt angles, α and β , involved,

$$a_p - b_p = 2a_0 \cos \alpha (\cos \alpha - \cos \beta),$$

where a_0 refers to perovskite aristotype cell. In addition comes the deformation of MnO₆ octahedra themselves. Hence the relevant angles and distances for the exchange paths become complicated expressions of the actual deformations. Furthermore, it appears to be no simple relations between angles and distances on the one hand and exchange interactions on the other.

In this situation we limit our concluding considerations on this aspect to the use of the qualitative predictions of the Kanamori-Goodenough superexchange rules.²⁶ This predicts ferromagnetism in the *ab* plane for 180° Mn-O-Mn interaction, due to the Jahn-Teller effect and the antiferrodistortional orbital ordering, and antiferromagnetism along the *c* axis for nearest-neighbors interactions (as observed for LaMnO₃).²⁶ For an intermediate range of angles a competition between ferromagnetic $(d_{z^2}^1 - 2p_{\sigma} - d_{x^2 - y^2}^0)$ and antiferromagnetic $(t_{2g}^3 - 2p_{\pi} - t_{2g}^3)$ interactions in the *ab* plane may re-



FIG. 11. Rietveld refinements (upper line) of NPD data collected with D2B (circles) for o-HoMnO₃ at 2.8 K (λ =1.5938 Å). Simulated nuclear contribution is shaded. Positions of Bragg reflections are shown with bars for the nuclear (upper) and magnetic (incommensurate in middle; commensurate at bottom) contribution. The difference between observed and calculated intensity is shown with the bottom line.

sult in weakening of the effective in-plane Mn-O-Mn nearest-neighbors interactions. Next-nearest-neighbor interactions become competitive and a delicate balance may produce a complex magnetic behavior. In the *o*-HoMnO₃ case the overall frustration obviously results in a sinusoidal wave as the first-ordered state for Mn³⁺. For the particular value of $k_y = 0.395$ obtained just below T_N , the condition $\cos(\pi k_y) = (2J_{13} - J_{14})/(2J_{11b})$ gives: $J_{14} \approx 2(J_{13} - 0.324J_{11b})$, which is positive. Hence with the assumption of negative supersuperexchange interactions J_{13} and J_{11b} , the (strong) condition becomes: $0.324|J_{11b}| > |J_{13}|$.

F. Magnetic ordering of Ho³⁺

The different possible magnetic modes for the holmium site did not give intensity contribution to easily recognizable groups of reflections. Hence in order to solve the Ho³⁺ magnetic structure and establish a good starting model for the Rietveld refinements a Monte Carlo search by means of the integrated intensities was necessary. The Mn³⁺ moment was kept constant and the different representations for Ho³⁺ examined. This clearly showed that a Γ_2 description for Ho³⁺ represents the best solution. Monte Carlo searches with Mn³⁺ in other representations instead did not improve the fit.

With both Ho³⁺ orbits ordering in $\Gamma_2(a_x, f_y, 0)$ and Mn³⁺ in $\Gamma_3(0, A_y, 0)$ the complete magnetic structure could in principle be similar [i.e., $(G_x, A_y, 0)$] for both kinds of magnetic atoms. The obtained magnetic arrangement of Ho³⁺ is illustrated in Fig. 9 and the plot of the Rietveld refinements to the complete nuclear and magnetic structure of *o*-HoMnO₃ at 2.8 K in Fig. 11. There is a phase difference between the two Ho³⁺ orbits of ~34°. A phase difference of 26° would have made the spin amplitude of neighboring Ho³⁺ equal, resulting in a global $(G_x, A_y, 0) = (a_x, f_y, 0)_1 + (a_x, -f_y, 0)_2$ ordering (see the projection on the *ab* plane in Fig. 9). The slightly deviating phase difference makes the present description become an approximation. Hence to a very good approximation (Fig. 9) Ho^{3+} is ordered according to $(G_x, A_y, 0)$, whereas a G_x component is definitely absent for Mn^{3+} . The present NPD data were, according to our simulations, rather insensitive to phase differences between the Mn and Ho sites.

In addition to the already considered magnetic contributions from Mn^{3+} and Ho^{3+} , the D2B data contained a small and broad intensity contribution which could be indexed as $(0\frac{1}{2}1)$ for the nuclear unit cell. This could be explained as a secondary magnetic phase with a doubled *b* axis. Although the diffraction pattern did not contain enough information to unequivocally solve this structure, a plausible model which is in accordance with NPD would be a modulated commensurate magnetic structure based on an *A* mode. This gives a ++-- ordering for Mn^{3+} and -0+0 ordering for Ho^{3+} in the *y* direction. Similar findings is reported for the *R*NiO₃ family where such an ordering scheme is observed in two directions.²⁷ This situation appears to comply with what could be imagined to happen if k_y for the incommensurate structure is let free to approach and reach $\frac{1}{2}$.

In the G4.1 data from 1.4 K there is a broad peak [full width at half maximum (FWHM) $\approx 3^{\circ}$] situated between the first two magnetic reflections (001[±] and 110⁻). Profile fitting of these three peaks shows that FWHM of the broad peak increases with increasing temperature. This is of no importance for the refinements above 10 K. The better resolution of the G4.1 data compared with the D2B data at low scattering angles allows us to see in more detail what is seen as a commensurate additional phase in the D2B data. This additional magnetic intensity is of short-range nature or corresponds to a more or less well-defined magnetic structure with short correlation length. The absence of a satisfactory fit using just peak positions generated for a propagation vector of the form $\mathbf{k}_{com} = (0, \frac{1}{2}, 0)$ may be related to the presence of different propagation vectors in the sample. With the present

data we have not attempted to go more deeply into the search for a model which explains the whole magnetic profile and which may be dominated by both a continuous set of propagation vectors between $k_y=0.4$ and $k_y=\frac{1}{2}$ and anisotropic peak broadening owing to differences in correlation lengths along different crystallographic directions.

It is appropriate to emphasize that the present value for the holmium moment (some 7.7 μ_B at 1.4–2.8 K, Table I and Fig. 8) is considerable smaller than the free-ion value of 10 μ_B . This deficiency is clearly rooted in the unaccounted amount of moment hidden in the diffuse scattering.

G. Comparison with other rare-earth manganites

According to Quezel et al.¹⁹ the magnetic ordering of Mn^{3+} in *o*-YMnO₃ is of an incommensurate nature with **k** $=0.458b^*$. Quezel *et al.* reported that the modulation is of the helical type, but our simulations of the magnetic structure in comparison with the published diffraction diagram¹⁹ contradict this. In order to obtain a helical arrangement a C-type mode (see Table III) perpendicular to both the A mode and the propagation vector is necessary, and this demands intensity contribution in the form of extra reflections which are not observed. From the actually observed magnetic contributions a longitudinal sinusoidal wave seems probable. Hence the magnetic arrangement of the Mn³⁺ moments in o-HoMnO₃ and o-YMnO₃ are similar. This is as expected, owing to the close correspondence in size of Ho^{3+} and Y^{3+} , on the assumption that the Ho^{3+} moments do not have an appreciable influence on the exchange interaction between the Mn³⁺ moments above $T_{\rm Ho}$.

The magnetic structure of *o*-HoMnO₃ is also closely related to that of TbMnO₃, which orders as a sinusoidal longitudinal wave with \mathbf{k} =0.295 \mathbf{b} * below T_N =40 K (A_y above 20 K and both G_x and A_y below 20 K). Tb is short-range ordered below T_{Tb} =7 K with a different wave vector, \mathbf{k} = 0.415 \mathbf{b} *. Hence there is less interaction between the Mn³⁺ and R^{3+} moments in TbMnO₃ than in *o*-HoMnO₃. However, an account for this in terms of exchange integrals is beyond the scope of this paper. Furthermore, it should be noted that there is a different temperature dependence of k_y in TbMnO₃ (increasing with the temperature close to T_N) and *o*-HoMnO₃. The origin of this distinction is not known.

The magnetic arrangement of the Mn^{3+} moments of $RMnO_3$ with R=La, Pr, Nd, Tb, Ho, and Y are shown in Fig. 10 and the variation in k_y for the same manganites is

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depicted in Fig. 7. These illustrations brings out a clear correlation between the propagation vector and the size of R^{3+} . In the region of the intermediate sized R^{3+} 's it would be interesting to establish the extension of the commensurate and incommensurate domains. It would indeed also be of considerable interest to follow this trend to the smaller sized R^{3+} 's to see whether k_y continues to increase or get locked in at a commensurate value of, say, $\frac{1}{2}$. Preliminary results for *o*-ErMnO₃ indicate that k_y increases to 0.461 (at 7.7 K).

IV. CONCLUSIONS

Finally we should stress that the incommensurate nature of the magnetic structures observed in $RMnO_3$ comes from an increase of the frustration effects due to exchange competitions: the diminution of the Mn-O-Mn superexchange angle owing to the smaller size of the heavy rare earths, which weakens the most significant exchange interaction in the perovskite structure, and consequently the supersuperexchange interactions become more important than for the lighter rare earths.

An important point is to explain why the magnetic structures are of the sinusoidal type (fluctuating average magnetic moments) instead of the helical type (constant average magnetic moments). Mn³⁺ has four unpaired electrons, so it is expected that at very low temperature all ions have magnetic moments of $4\mu_B$ and the helical type is to be preferred. The failure to explain the observed diffraction patterns by a helical-type magnetic structure indicates clearly that we have to interpret the sinusoidal structures as the presence of an average structure where only one (or two in the case of Ho) component(s) of the magnetic moments is(are) long-range ordered. The other components are disordered. This may also explain why we have a complicated diffraction pattern at low temperature where the correlation between the disordered components start to increase. Hence diffuse features appear together with well-defined magnetic Bragg reflections. In future experiments we will study the behavior of the magnetic structure below 1.4 K in order to find out if the diffuse commensurate component appearing when Ho starts to order is the real ground state of o-HoMnO₃.

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