Pinning force and peak effect in superconductor/normal-metal multilayers

V. N. Kushnir and S. L. Prischepa
State University of Informatics and RadioElectronics, P. Brovka Street 6, Minsk 220600, Belarus

C. Attanasio and L. Maritato
Departimento di Fisica and INFM, Università degli Studi di Salerno, Baronissi (Sa) I-84081, Italy
(Received 30 June 2000; published 29 January 2001)

We analyzed the temperature, the magnetic field, and the normal layer thickness dependencies of the pinning force F_p in superconductor/normal metal multilayers in the framework of the Ginzburg-Landau (GL) theory. In particular, we studied the temperature dependence of the magnetic field H_p at which are observed both the peak of the critical current density J_c curves versus the external magnetic field H_0 and the maximum pinning force $F_{p\,\max}$ in the $F_p(H_0)$ dependencies. We show that, well below the critical temperature T_c, the magnetic field H_p corresponds to the magnetic field H_D of the periodic solution of the GL equations and the H_D versus temperature dependence well describes the experimental data.

DOI: 10.1103/PhysRevB.63.092503 PACS number(s): 74.60.Ge, 74.60.Jg, 74.20.De, 74.80.Dm

Superconducting multilayered structures have been intensively studied during the last years.[1] In particular, superconductor/normal-metal (S/N) multilayers have attracted great attention because they allow to investigate the influence of the reduced dimensionality on different physical properties and to obtain more information on analogous effects observed in high-temperature superconductors. The properties of S/N multilayers in the region of the superconducting transition temperature T_c were firstly described by de Gennes.[2] Later an exhaustive interpretation of the temperature dependence of the upper critical field was given by Takahashi and Tachiki using a microscopic theory.[3–5] In spite of the classical work of Eilenberger[6] the application of a microscopic theory far away the critical region is still an open question, and many experimental evidences are waiting for a better understanding. For example, one of the most interesting effects observed far away from the critical region in S/N layered samples—the appearance of a peak in the external parallel magnetic field H_0 dependence of the critical current density J_c—did not receive a full theoretical description.

The properties of S/N multilayers could also be studied by using the Ginzburg-Landau (GL) theory. In this case due to the conditions of applicability of the GL theory (see, e.g., Ref. 7), only systems, in which the normal layer plays a role of a perturbation of the superconducting state, can be analyzed.[8–11]

Ami and Maki[8] directly solved the problem of the weak action of the layered structure on the vortex lattice, calculating the $J_c(H_0)$ dependencies by using the perturbation theory[12] in conjunction with the variation principle. The idea of commensurability (originally contributed by Pippard for explaining the peak effect in type II superconductors[13]) between the period of the multilayer structure and the vortex lattice spacing was put forward to account for the peak effect in the $J_c(H_0)$ dependencies experimentally observed at the beginning of the seventies by Raffy *et al.*[14,15] However the Ami-Maki solution is valid close to the upper critical magnetic field H_{c2} and does not explain the observed temperature dependence of the magnetic field H_p at which the peak effect occurs.[15,16]

Kulić and Rys[9] calculated the $J_c(H_0)$ dependence in the case of twinning planes considered as perturbations in the range of the magnetic fields $H_{c1} \ll H_0 \ll H_{c2}$, where H_{c1} is the lower critical magnetic field. Again, their model did not describe the temperature dependence of the H_p field.

Kugel *et al.*[11] calculated H_p considering the elementary pinning force maximum on the S/N boundary for samples in which the thickness of the superconducting layers are essentially larger than the correlation length in the superconductor and the thickness of the *N* layer are finite. The total pinning force was obtained by multiplying the elementary pinning forces. From this and from the boundary conditions for the GL wave function follows the *D*-periodic nature of the solution (*D* is the period of the multilayered structure). This model is valid for magnetic fields well below H_{c2} and gives a temperature dependence of H_p related to that of the superconducting parameters in the normal zones. However, the above physical picture is indeed more appropriate for high-temperature superconductors rather than for artificially layered structures.

Usually the $J_c(H_0)$ peak effect is observed in samples in which the influence of the normal layers cannot be considered as a perturbation (the thickness of the *N* layer d_N is of the same order of magnitude of the thickness of the *S* layer d_S and d_S is of the order of the perpendicular coherence length $\xi_\perp$). Moreover, analysis show that in the region of the $J_c(H_0)$ peak effect the fields of the maximum values of the J_c and the $F_{p\,\max}$ coincide within the accuracy of a few percents and their temperature dependencies are the same.[16]

In this article we apply to the $J_c(H_0)$ peak effect problem a rather simple interpolation procedure based on the GL theory for S/N multilayers in the case of $d_N \approx d_S$. Dediu *et al.*[17] proposed a version of such a model restoring the temperature dependencies of the upper critical fields, which were obtained in microscopic theories and were in agreement with the experimental results. In the present work we propose another version of the GL model. We explain the $H_p(T)$ dependence in the limit of high values of the GL parameter κ. We argue that the H_p values correspond to one of the *D*-periodic solutions of the GL equations at least for H_p sufficiently lower than the parallel critical magnetic field

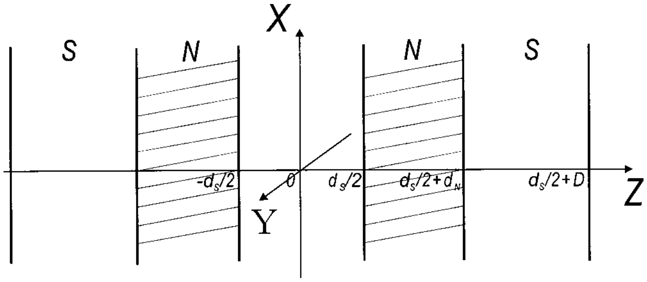

FIG. 1. The coordinate system of the proposed model.

$H_{c2\parallel}$. In our analysis the surface superconductivity[18] is ignored as well as the possibility of kinks in the vortex structure.[1,19] The agreement of our theoretical results with the $H_p(T)$ behavior observed in Nb/Pd multilayers[16] is quite good.

We use the coordinate system sketched in Fig. 1. The XOY plane is parallel to the layers and corresponds to on of the symmetry surface of the system. The plane XOZ is perpendicular to the layers and corresponds to the other symmetry surface of the sample. The parallel external magnetic field H_0 is oriented along the OY axis and the vortices move along the OZ axis.

The GL free energy G for a layered superconductor, neglecting a possible Josephson coupling between the superconducting planes, can be written[20] according to standard scaling procedure[7] as

$$G=\int d^3r\left\{\left|\left(\frac{1}{\kappa_S}\nabla-i\mathbf{A}\right)\Psi(\mathbf{r})\right|^2-\eta(z)|\Psi(\mathbf{r})|^2+\frac{1}{2}|\Psi(\mathbf{r})|^4+\mathbf{B}^2(\mathbf{r})\right\}. \quad (1)$$

Here $\mathbf{B}$ is the induction, $\mathbf{A}$ is the vector potential, $\mathbf{r}$ is the coordinate, $\eta(z)=1$ inside the superconductor, $\eta(z)=-\xi_S^2(T)/\xi_N^2(T)$ inside the normal layers, ξ_S is the coherence length in the superconducting layers, ξ_N is the coherence length in the normal layers, and $\kappa_S=\lambda_S(T)/\xi_S(T)$ is the GL parameter, where λ_S is the superconducting penetration depth.

The pinning force $F_p(Z)$ on the vortex lattice due to the inhomogeneities of the sample can be determined from the GL free energy G taking into account that the function $\eta(z)$ from Eq. (1) depends on the translation variable Z, the generalized coordinate of the vortex lattice, as $\eta(z)\rightarrow\eta(z-Z)$. Finding the collective variable Z isn't a trivial task, as it depends on the variables $\psi(\mathbf{r})$, $\psi^*(\mathbf{r})$ and $B(\mathbf{r})$, but in the static case such complexity does not appear. Then

$$F_p(Z)=-\frac{dG}{dZ}=\left(1+\frac{\xi_S^2(T)}{\xi_N^2(T)}\right)\sum_n^{N_L}\int dx\left\{\left|\Psi\left(x,Z-\frac{d_S}{2}+nD\right)\right|^2-\left|\Psi\left(x,Z+\frac{d_S}{2}+nD\right)\right|^2\right\}, \quad (2)$$

where N_L is the number of bilayers.

If N_L goes to infinity the function $F_p(Z)$ is periodic, i.e., $F_p(Z)=F_p(Z+D)$. Due to the symmetry of the system we also have that the function is odd and that the function zeroes are given by $F_p(lD)=F_p(lD+D/2)=0$ with $l=0,\pm1,\pm2,\ldots$.

Equation (2) allows us to give some remarks about the dependence of the pinning force on H_0, d_N, and T. For $N_L\rightarrow\infty$ the main contribution to the sum in Eq. (2) will give only the function modes with the wave numbers $(2\pi/D)\times m$ where m is integer. Then for H_0 values, at which the solution of the GL equation represents a D-periodic function, the expression for the pinning force is written as

$$F_p=N_L\cdot\left(1+\frac{\xi_S^2}{\xi_N^2}\right)\cdot\int dx\left\{\left|\Psi\left(x,Z-\frac{d_S}{2}\right)\right|^2-\left|\Psi\left(x,Z+\frac{d_S}{2}\right)\right|^2\right\}\equiv N_L\cdot\left(1+\frac{\xi_S^2}{\xi_N^2}\right)\cdot\left(f^2\left(Z-\frac{d_S}{2}\right)-f^2\left(Z+\frac{d_S}{2}\right)\right), \quad (3)$$

where the pinning force depends directly on the large value N_L and $\int dx|\Psi(x,z)|^2\equiv\overline{|\Psi|^2}\equiv f^2(z)$.

The meaning of the D-periodic solution is the following: one of the vortex lattice constants is oriented parallel to the layers, and the z component of the other vortex lattice vector is equal to D. From Eqs. (2) and (3) formally follows that only D-periodic solution can exist in an infinite S/N layered structure because all other solutions will be unstable with respect to small perturbations if one neglects pinning besides that due to N layers. Consequently, the peak effect in $F_p(H_0)$ curve will be associated to the most stable of the periodic solutions. The possibility of the existence of this peak follows from Eq. (3). In fact, while the magnetic field increases, the modulus of the wave function obviously decreases. But, on the other hand, the convergence of the maxima and minima of the wave function at constant distance ($\sim D/2$), see Fig. 2, (vortices slip, increasing their density in N layers with increasing H_0) can lead to the increasing of the difference $\overline{|\Psi|^2}_{\max}-\overline{|\Psi|^2}_{\min}\propto F_p$ in some region of the magnetic field.

When the temperature increases, the pinning force decreases both due to the decreasing of the modulus of the wave function and to the decreasing of the difference between moduli in Eq. (2). The last fact is related to the higher inhomogeneity of the wave function due to the increasing of the coherence length with temperature. When d_N decreases and becomes much smaller than ξ_S, F_p has to decrease too. In this case, supposing the continuity of the GL wave functions on the S/N boundary, one gets

$$F_p\approx d_n\cdot\left(1+\frac{\xi_S^2(T)}{\xi_N^2(T)}\right)\sum_n^{N_L}\int dx\frac{d}{dZ}\left|\Psi\left(x,Z+\frac{D}{2}+nD\right)\right|^2.$$

To perform a quantitative analysis we take into account that from Eq. (3) it follows that $F_{p\,\max}\propto f^2(0)-f^2(D/2)$, that the measured values of the H_p field are of the order of $\Phi_0/2\pi D^2\gg H_{c1\parallel}(T)$ and that $\lambda_\perp\gg D$.[21] The change of magnetic field inside the S layer is then

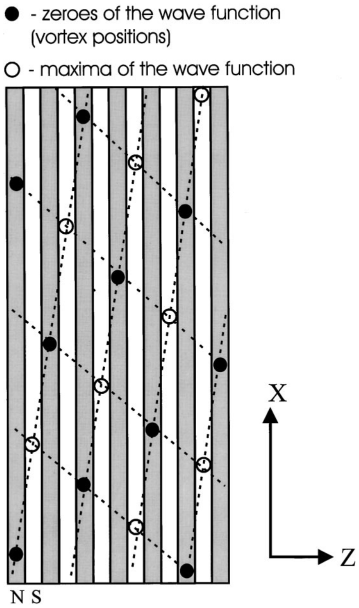

FIG. 2. Example of a possible stable D-periodic solution for a GL wave function in a SNS multilayer.

$$\frac{\Delta\overline{H}(z)}{H_0}=\frac{\overline{H}(d_S/2)-\overline{H}_{\min}}{H_0}<\frac{d_S}{2H_0}\cdot\partial_z H_0\left(\frac{d_S}{2}\right)$$

$$\propto\frac{1}{\kappa_S^2}\cdot\frac{d_S^2}{4\xi_S^2}\cdot f^2\left(\frac{d_S}{2}\right)\ll 1,$$

and inside N layer is

$$\frac{\Delta\overline{H}(z)}{H_0}=\frac{\overline{H}_{\max}-\overline{H}(d_S/2)}{H_0}<\frac{d_N}{2H_0}\cdot\partial_z H_0\left(\frac{d_N}{2}\right)$$

$$\propto\frac{1}{\kappa_S^2}\cdot\frac{d_N d_S}{4\xi_S^2}\cdot f^2\left(\frac{d_S}{2}\right)\ll 1.$$

For these reasons, to estimate the F_p value on one period, we can assume the case of the homogeneous magnetic field $\overline{H(x,z)}\equiv\overline{H}(z)\approx H_0$.

Consequently, considering the GL equations on the interval $z\in[-d_S/2,d_S/2+d_N]$ and taking into consideration the symmetry properties for f and $\overline{H}$

$$f\left(-\frac{d_S}{2}\right)=f\left(\frac{d_S}{2}\right)=f\left(\frac{d_S}{2}+d_N\right),\qquad \partial_z\overline{H}(0)=\partial_z\overline{H}(D/2)=0,$$

we may assume $(A_x-\partial_x\chi)^2\propto H_0^2z^2$ inside S layer and $(A_x-\partial_x\chi)^2\propto H_0^2(d_S/d_N)^2(z-D/2)^2$ inside N layer (χ is the phase of the wave function). By expanding on z around the point $z=0$ and on $(z-D/2)$ around point $z=D/2$ and taking account the symmetry of the solution[22] it is possible to show that we may neglect the component A_z in the GL equations. So we may also assume $[A_z(x,z)-\partial_z\chi]^2\propto H_0^2z_0^2$, where z_0 is an unknown parameter.

Let us now consider the nonlinear quartic term in Eq. (1). As we showed in Ref. 21, it is relatively small for zero magnetic fields for the case $d_N\leqslant d_S$. The external magnetic field suppresses even more the wave function amplitude. In the limit of very small $\overline{|\Psi(x,z)|^4}$ term value it is possible to substitute it with the amplitude additive term $\beta\cdot\overline{|\Psi|^2}$.[23]

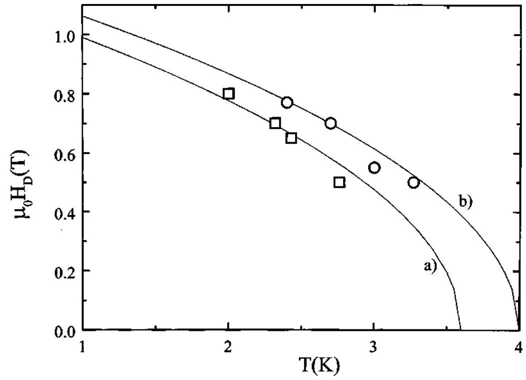

FIG. 3. The temperature dependence of the field of the D-periodic solution for Nb/Pd multilayers (solid curves) together with the H_p values (symbols). (a) $d_N=170$ Å, $d_S=187$ Å, $N_L=10$, and $T_c=3.6$ K; (b) $d_N=132$ Å, $d_S=187$ Å, $N_L=10$, and $T_c=4.0$ K.

So for the case $d_N\approx d_S$ we find the following model equations

$$f''(z)+[1-\varepsilon(H_0)-H_0^2z^2]f(z)=0,\quad z\in\left[-\frac{d_S}{2},\frac{d_S}{2}\right]$$

$$f''(z)-\left[\frac{\xi_S^2(T)}{\xi_N^2(T)}+\varepsilon(H_0)+H_0^2\cdot\frac{d_S^2}{d_N^2}\left(z-\frac{D}{2}\right)^2\right]f(z)=0,$$

$$z\in\left[\frac{d_S}{2},\frac{d_S}{2}+d_N\right],\qquad(4)$$

where $\varepsilon(H_0)=H_0^2z_0^2+H_0z_1$ with the unknown parameters z_0 and z_1. Using the S/N boundary condition

$$\frac{f'\left(\frac{d_S}{2}-0\right)}{f\left(\frac{d_S}{2}-0\right)}=P\cdot\frac{f'\left(\frac{d_S}{2}+0\right)}{f\left(\frac{d_S}{2}+0\right)}\qquad(5)$$

(P is the transparent coefficient of the S/N boundary) for the solution of Eq. (4) we obtain the following expression for the external magnetic field corresponding to the D-periodic solution of the GL equation

$$1+P=4\cdot P\cdot\overline{\alpha_n}\frac{\Phi(\overline{\alpha_n}+1,3/2;H_0d_Sd_N/4)}{\Phi(\overline{\alpha_n},1,2;H_0d_Sd_N/4)}$$

$$+4\cdot\overline{\alpha_s}\frac{\Phi(\overline{\alpha_s}+1,3/2;H_0d_S^2/4)}{\Phi(\overline{\alpha_s},1/2;H_0d_S^2/4)},\qquad(6)$$

where $\Phi(\alpha,\gamma;z)$ is the confluent hypergeometric function and

$$\overline{\alpha_s}=\frac{1}{4}-\frac{1}{4H_0}+\frac{H_0z_0^2+z_1}{4}$$

$$\overline{\alpha_n}=\frac{1}{4}+\frac{\xi_S^2}{4\xi_N^2H_0}\cdot\frac{d_N}{d_S}+\frac{H_0z_0^2+z_1}{4}\cdot\frac{d_S}{d_N}.$$

The calculated according to Eq. (6) $H_D(T)$ dependencies for two Nb/Pd samples with $d_N = 170$ Å and $d_N = 132$ Å (the d_S value was always equal to 187 Å) are shown in Fig. 3 together with the experimental $H_p(T)$ dependencies.[16] During the numerical procedure it was assumed that

$$\xi_S(T) = \xi_{s0}\left(1 - \frac{T}{T_{cS}}\right)^{-1/2} \quad \text{and} \quad \xi_N(T) = \xi_{Nc}(T/T_c)^{-1/2}$$

(Ref. 7), where ξ_{S0} is the coherence length in the superconducting layer at zero temperature, ξ_{Nc}, is the coherence length in the normal layer at the critical temperature T_c of the multilayer and T_{cS} is the transition temperature for the superconducting material, which was obtained from the de Gennes-Werthammer fit for S/N multilayers ($T_{cS} = 7.8$ K). For the sample with $d_N = 170$ Å we took $\xi_{S0} = 111$ Å, $\xi_{Nc} = 126$ Å, and for the sample with $d_N = 132$ Å we took $\xi_{S0} = 112$ Å, $\xi_{Nc} = 120$ Å as it was obtained in Ref. 21. Theoretical curves were calculated for the strong proximity effect ($P = 1$) with the only two fit parameters, namely, z_0 and z_1. The good agreement between theory and experiment for both the absolute H_p values and their temperature dependencies is quite clear in Fig. 3. Note that the same result could be obtained using a simpler relation extracted from Eq. (6). Indeed, expanding the hypergeometric functions of Eq. (6) into a Bessel series in the first-order approximation we get

$$H_0 = \frac{P}{1+P}\,\frac{2\xi_S}{d_S \cdot \xi_N} \cdot \frac{\nu_2}{\nu_1^{1/2}}\,\frac{Sh\left(\frac{\xi_S d_N}{2\xi_N}\,\nu_1^{1/2}\right)}{Ch\left(\frac{\xi_S d_N}{2\xi_N}\,\nu_3^{1/2}\right)} - \frac{1}{1+P}\,\frac{2}{d_S} \cdot \frac{\sigma_2}{\sigma_1^{1/2}}\,\frac{\mathrm{Sin}\left(\frac{d_S}{2}\,\sigma_1^{1/2}\right)}{\mathrm{Cos}\left(\frac{d_S}{2}\,\sigma_3^{1/2}\right)}, \tag{7}$$

where

$$\sigma_1 = 1 - \varepsilon(H_0) - 2H_0, \quad \sigma_2 = 1 - \varepsilon(H_0) - H_0,$$

$$\sigma_3 = 1 - \varepsilon(H_0),$$

and

$$\nu_1 = 1 + \frac{\xi_N^2 d_S^2}{\xi_S^2 d_N^2}\left(\varepsilon(H_0) + 2H_0\frac{d_N}{d_S}\right),$$

$$\nu_2 = 1 + \frac{\xi_N^2 d_S^2}{\xi_S^2 d_N^2}\left(\varepsilon(H_0) + H_0\frac{d_N}{d_S}\right), \quad \nu_3 = 1 + \frac{\xi_N^2 d_S^2}{\xi_S^2 d_N^2}\,\varepsilon(H_0).$$

Calculations reveal that the fitting procedures according to Eqs. (6) and (7) give the same values for the fitting parameters z_0 and z_1 for both the samples within the accuracy of 5%.

In summary, the GL theory has been used to analyze the pinning forces of S/N multilayers with $d_N \approx d_S$. It was shown that the position of the maximum values of the pinning force in the $F_p(H_0)$ dependence can be explained in the framework of the periodic solution of the GL equations. Introducing a temperature dependent parameter, the field of the periodic solution H_D, we were able to fit the experimental data for S/N multilayers. The model is valid for large values of the GL parameter κ and has an interpolation character. In spite of this, the success in describing the experimental data in the case of Nb/Pd multilayers is very promising for further developments.

[1] A. N. Lykov, Adv. Phys. **42**, 263 (1993).
[2] P. G. de Gennes, Rev. Mod. Phys. **36**, 225 (1964).
[3] S. Takahashi and M. Tachiki, Phys. Rev. B **33**, 4620 (1986).
[4] R. P. W. Koperdraad and A. Lodder, Phys. Rev. B **51**, 9026 (1995).
[5] J. Hara, M. Ashida, and K. Nagai, J. Phys. Soc. Jpn. **68**, 221 (1999).
[6] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[7] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North Holland, Amsterdam, 1988).
[8] S. K. Ami and K. Maki, Prog. Theor. Phys. **53**, 1 (1975).
[9] M. Kulić and F. S. Rys, J. Low Temp. Phys. **76**, 167 (1989).
[10] V. Prokić, D. Davidović, and L. Dobrosavljević-Grujić, Phys. Rev. B **51**, 6013 (1995).
[11] K. I. Kugel, T. Matsushita, E. Z. Melikhov, and A. L. Rakhmanov, Physica C **228**, 373 (1994).
[12] G. Eilenberger, Phys. Rev. **164**, 628 (1967).
[13] A. B. Pippard, Philos. Mag. **19**, 217 (1969).
[14] H. Raffy, J. C. Renard, and E. Guyon, Solid State Commun. **11**, 1679 (1972).
[15] H. Raffy, J. C. Renard, and E. Guyon, Solid State Commun. **14**, 427 (1974).
[16] C. Coccorese, C. Attanasio, L. V. Mercaldo, M. Salvato, L. Maritato, J. M. Slaughter, C. M. Falco, S. L. Prischepa, and B. I. Ivlev, Phys. Rev. B **57**, 7922 (1998).
[17] V. I. Dediu, V. V. Kabanov, and A. A. Sidorenko, Phys. Rev. B **49**, 4027 (1994).
[18] D. Saint-James and P. G. de Gennes, Phys. Lett. **7**, 306 (1963).
[19] B. Roas, L. Schultz, and G. Saemann-Ischenko, Phys. Rev. Lett. **64**, 479 (1990).
[20] B. Y. Jin and J. B. Ketterson, Adv. Phys. **38**, 189 (1989).
[21] S. L. Prischepa, V. N. Kushnir, A. Y. Petrov, C. Attanasio, and L. Maritato, in *Superlattices II: Native and Artificial*, edited by I. Bozovic and D. Pavuna, Proc. SPIE **3480**, 140 (1998).
[22] V. N. Kushnir, A. Y. Petrov, and S. L. Prischepa, Fiz. Nizk. Temp. **25**, 1265 (1999) [Low Temp. Phys. **25**, 948 (1999)].
[23] N. N. Bogolyubov and Y. A. Mitropol'skii, *Asymthotical Methods in the Theory of Nonlinear Oscillations* (Nauka, Moscow, 1974).