Crossed sliding Luttinger liquid phase

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We study a system of crossed spin-gapped and gapless Luttinger liquids. We establish the existence of a stable non-Fermi-liquid state with a finite-temperature, long-wavelength, isotropic electric conductivity that diverges as a power law in temperature T as $T \rightarrow 0$. This two-dimensional system has many properties characteristic of a true isotropic Luttinger liquid, though at zero temperature it becomes anisotropic. This model can easily be extended to three dimensions.

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For over two decades a central theme in the study of correlated electronic systems has been the drive to understand and classify electronic states that do not conform to Landau's Fermi-liquid theory. A clear example of such "non-Fermi liquid" physics occurs in one dimension (1D), where arbitrarily weak interactions destroy the Fermi surface and invalidate the notion of independent quasiparticles at low energy. Away from charge-density-wave instabilities, the interacting 1D electron gas forms a Luttinger liquid in which the discontinuity in occupation at the Fermi energy of a normal Fermi liquid is replaced by a power-law singularity, and the low-lying excitations are bosonic collective modes in which spin and charge decouple.

Following the discovery of high-temperature superconductivity, Anderson suggested that the unusual normal-state properties of the cuprates were the result of similar non-Fermi-liquid physics in two dimensions.² However, the study of non-Fermi liquids in higher dimensions has proven to be quite difficult. Since the Fermi liquid is stable for weak interactions, perturbative methods fail.³ Moreover, generalizations of the bosonization technique to isotropic systems in higher dimensions have indicated that Fermi-liquid theory survives provided the interactions are not pathologically long ranged.4 An alternative approach has been to study anisotropic systems consisting of arrays of parallel weakly coupled 1D wires.⁵ It has recently been proposed^{6,7} that for a range of interwire charge and current interactions, there is a smectic-metal (SM) phase in which Josephson, charge- and spin-density-wave, and single-particle couplings are irrelevant. This phase is an anistropic sliding Luttinger-liquid phase whose transport properties exhibit power-law singularities like those of a 1D Luttinger liquid. It is the quantum analog of the sliding phases of coupled classical XY models found by O'Hern et al. 8,9

We consider a square network of 1D wires formed by coupling two perpendicular smectic metals 10 and show that it exhibits a new *crossed sliding Luttinger Liquid* (CSLL) phase. We establish a range of couplings for which both this phase and the anisotropic two-dimensional smectic-metal phase from which it is constructed are stable with respect to a large class of operators. At finite temperature T, the CSLL phase is an isotropic 2D Luttinger liquid with an isotropic long-wavelength conductivity that diverges as a power-law in T as $T \rightarrow 0$. At T = 0, it is essentially two independent smectic metals. This model could be realized in manmade

structures constructed from quantum wires such as carbon nanotubes. Extension of the model to a three-dimensional stack may be relevant to the stripe phases of the cuprates. Based on neutron and x-ray scattering measurements, it has been suggested that spin-charge stripes in the adjacent CuO₂ plane are orthogonal to each other. ¹¹

The Lagrangian density describing the low-energy behavior of a one-dimensional Luttinger liquid is

$$\mathcal{L}_0 = \frac{1}{2} \kappa \left[v^{-1} (\partial_\tau \phi)^2 + v (\partial_x \phi)^2 \right], \tag{1}$$

where ϕ is a bosonic field¹² and κ and the sound velocity, v, are nonuniversal functions of the coupling constants. For repulsive interactions, $\kappa > 1$. The Lagrangian density in terms of the dual phase variable θ has the same form as Eq. (1), but with v replaced by 1/v. For spin-1/2 fermions, the spin excitations could either be gapped or gapless. In the spin-gap, Luther-Emery regime, the system can be described by a single Luttinger liquid for charge. In the gapless case, both spin and charge are dynamical degrees of freedom, and there are two Luttinger parameters (κ_c , κ_s), and two velocities (v_c , v_s).

Now consider a two-dimensional array of parallel quantum wires. To begin with, we consider the *spin gapped* case, so that the spin fluctuations on each wire are effectively frozen out at low energies. It has been suggested that this case might describe the stripe phases of high-temperature superconductors.¹³ In general, we expect a generalized current-current interaction between the wires, which can be represented by a Lagrangian density of the form

$$\mathcal{L}_{\text{int}} = \frac{1}{2} \sum_{n,n',\mu} j_{\mu,n}(x,\tau) \widetilde{W}_{\mu}(n-n') j_{\mu,n'}(x,\tau), \qquad (2)$$

where $j_{\mu,n} = [\rho_n(x,\tau), J_n(x,\tau)]$ with $\rho_n = \partial_x \phi_n(x,\tau)$ the density and $J_n = \partial_\tau \phi_n(x,\tau)$ the current on the nth wire. This interaction is marginal and should be included in the fixed-point action. It is invariant under the "sliding" transformations $\phi_n \to \phi_n + \alpha_n$ and $\theta_n \to \theta_n + \alpha_n'$. Equations (1) and (2) define the fixed-point action of the smectic-metal phase, which can be written in Fourier space as

$$S = \sum_{Q} \frac{1}{2} \{ W_0(q_\perp) \omega^2 + W_1(q_\perp) q_\parallel^2 \} |\phi(Q)|^2, \tag{3}$$

where $Q = (\omega, q_{\parallel}, q_{\perp})$, with q_{\parallel} the momentum along the chain and q_{\perp} perpendicular to the chains.

We can study perturbatively the relevance of various operators to ascertain the stability of the smectic-metal (SM) phase. Due to the spin gap, single-particle hopping between chains is irrelevant, and the only interchain interactions involving only pairs of chains that could become relevant are the Josephson (SC) and CDW couplings, whose respective Hamiltonian densities are

$$\mathcal{H}_{SC,n} = \sum_{i} \mathcal{J}_{n} \cos[\sqrt{2\pi}(\theta_{i} - \theta_{i+n})],$$

$$\mathcal{H}_{\text{CDW},n} = \sum_{i} \mathcal{V}_{n} \cos[\sqrt{2\pi}(\phi_{i} - \phi_{i+n})], \tag{4}$$

where \mathcal{J}_n are the interchain Josephson couplings and \mathcal{V}_n the interchain particle-hole (CDW) interactions. The scaling dimensions of $\mathcal{H}_{SC,n}$ and $\mathcal{H}_{CDW,n}$ are, respectively,

$$\Delta_{\text{SC},n} = \int_{-\pi}^{\pi} \frac{dq_{\perp}}{2\pi} (1 - \cos nq_{\perp}) \kappa(q_{\perp}),$$

$$\Delta_{\mathrm{CDW},n} = \int_{-\pi}^{\pi} \frac{dq_{\perp}}{2\pi} \frac{(1 - \cos nq_{\perp})}{\kappa(q_{\perp})}, \tag{5}$$

where $\kappa(q_{\perp}) = \sqrt{W_0(q_{\perp})W_1(q_{\perp})}$. For the smectic-metal phase to be stable, these perturbations should be irrelevant, which implies

$$\Delta_{\text{CDW},n} > 2, \quad \Delta_{\text{SC},n'} > 2$$
 (6)

for all n and n'. In addition to the pairwise operators of Eq. (4), there are multiwire operators of the form $\mathcal{H}_{\text{CDW},\{\sigma_n\}} = \sum_i \mathcal{T}(\sigma_n) \cos[\sqrt{2\pi}(\sum_n \sigma_n \theta_{i+n})]$ where the σ'_n s are integers satisfying $\Sigma \sigma_n = 0$. The overall strengths of these interactions measured by $\mathcal{T}(\sigma_n)$ are much smaller than those of $\mathcal{H}_{\text{CDW},n}$, and they become important only at very small temperatures even if they are relevant. We will therefore ignore them in this article, delaying a more complete study of their effects to a future publication.

To explore the regions of stability of the SM phase, we follow Refs. 7 and 9 and take

$$\kappa(q_{\perp}) = K[1 + \lambda_1 \cos(q_{\perp}) + \lambda_2 \cos(2q_{\perp})]. \tag{7}$$

We define $\Delta_{\text{SC},n} = a_n K$ and $\Delta_{\text{CDW},n} = b_n / K$, where $a_1 = (1 - \lambda_1 / 2)$, $a_2 = (1 - \lambda_2 / 2)$ and $a_n = 1$ for $n \neq 1,2$. The SM phase becomes unstable to interchain Josephson couplings for K less than $K_{\text{SC}} = \max_n (2/a_n)$ and unstable to interchain CDW interactions for K greater than $K_{\text{CDW}} = \min_n (b_n / 2)$. Thus the smectic metal phase is stable with respect to pairwise interactions over a window of K, $K_{\text{SC}} < K < K_{\text{CDW}}$, provided

$$\beta \equiv \frac{K_{\text{CDW}}}{K_{\text{SC}}} = \frac{a_n b_m}{4} \bigg|_{\substack{\text{min wrt } m \& n}} > 1.$$
 (8)

If β <1, the system goes directly from a 2D superconducting (SC) phase to a CDW crystal as K is increased, without

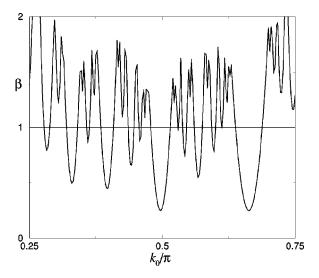


FIG. 1. Plot of $\beta \equiv K_{\rm CDW}/K_{\rm SC}$ as a function of k_0/π . For $\beta > 1$ there exists a region of K over which the non-Fermi-liquid phase is stable to all pairwise interactions.

passing through the SM phase. $\kappa(q_\perp)$ reaches a minimum, $\kappa(k_0) \equiv \Delta K$ at some $q_\perp = k_0$. The SM phase is stabilized at small Δ , that is, when the system is close to a CDW instability in which there is a periodic modulation of charge on different wires. Setting $\Delta = 10^{-5}$, we plot β as a function of k_0 . We note that there are regions of stability of the smectic phase with respect to \mathcal{H}_{SC} and \mathcal{H}_{CDW} , for positive as well as negative values of λ_1 . (See Fig. 1).

We consider next a square grid of wires, again for the spin-gapped case. There are two arrays of quantum wires, the *X* and *Y* arrays running, respectively, parallel to the *x* and *y* directions. Each wire now sees a periodic one-electron potential from the array of wires crossing it that leads to a new band structure with new band gaps. We assume that the Fermi surface is between gaps so that the wires would be conductors in the absence of further interactions. By removing degrees of freedom with wavelengths smaller than the inverse wire separation, we obtain an effective theory whose form is identical to the theory before the periodic potential was introduced. Thus, in the absence of two-particle interactions between crossed arrays, the system could be in a phase consisting of two crossed, noninteracting smectic-metal states.

We will now demonstrate that the sliding phase in a crossed grid is stable if the sliding phase in the component planar arrays is stable. In addition to the interwire couplings within each array, we need to consider Coulomb interactions between wires on the X array and wires on the Y array. These interarray couplings are marginal and should be included in the fixed point. They do not, however, change the dimensions of the operators, except by renormalizing $\kappa(q_\perp)$. For a stable sliding phase, additional interactions between the two arrays, such as the Josephson and CDW couplings, have to be irrelevant. We will show that it is possible to tune $\kappa(q_\perp)$ such that this is indeed the case.

The Coulomb interactions between electrons on intersecting wires gives rise to a term in the Hamiltonian of the form $V_{m,n}^c(x,y)\rho_{x,m}(x)\rho_{y,n}(y)$, where $\rho_{x,m}(x)[\rho_{y,m}(y)]$ is the

electron density on the mth wire on the X(Y) array at position x(y). We expect $V^c_{m,n}(x,y)$ to have the form $V^c(x-na,y-ma)$, where a is the distance between parallel wires. If all parameters for the X and Y arrays are the same, the crossed-grid action as a functional of the θ and ϕ variables can be written as

$$S = \frac{1}{2} \int \frac{d\omega dq_{x} dq_{y}}{(2\pi)^{3}} [V^{\theta}(q_{y})q_{x}^{2}|\theta_{x}|^{2} + V^{\theta}(q_{x})q_{y}^{2}|\theta_{y}|^{2} + V^{\phi}(q_{y})q_{x}^{2}|\phi_{x}|^{2} + V^{\phi}(q_{x})q_{y}^{2}|\phi_{y}|^{2} + V^{C}(q_{x}, q_{y})q_{x}q_{y}\{\phi_{x}\phi_{y}^{*} + \text{c.c.}\} - i\omega q_{x}\{\theta_{x}^{*}\phi_{x} + \text{c.c.}\} - i\omega q_{y}\{\theta_{y}^{*}\phi_{y} + \text{c.c.}\}]$$

$$(9)$$

with obvious definitions for $\phi_x = \phi_x(\omega, q_x, q_y), \phi_y, \theta_x$, and θ_y . It should be noted that this is an effective theory with $-\pi/a < q_x, q_y < \pi/a$. Integrating out the ϕ variables, we are left with an effective action

$$S_{\theta} = \frac{1}{2} \int \frac{d\omega dq_{x}dq_{y}}{(2\pi)^{3}} \left[\frac{1}{\kappa_{x}(\mathbf{q})} \left(v_{x}(\mathbf{q}) q_{x}^{2} + \frac{\omega^{2}}{v_{x}(\mathbf{q})} \right) |\theta_{x}|^{2} + \frac{1}{\kappa_{y}(\mathbf{q})} \left(v_{y}(\mathbf{q}) q_{y}^{2} + \frac{\omega^{2}}{v_{y}(\mathbf{q})} \right) |\theta_{y}|^{2} + V_{R}^{c}(\mathbf{q}) \omega^{2} \{\theta_{x}\theta_{y}^{*} + \text{c.c.}\} \right],$$

$$(10)$$

where

$$\kappa_{x}(\mathbf{q}) = \sqrt{\frac{\gamma(\mathbf{q})}{V^{\theta}(q_{x})V^{\theta}(q_{y})}}, \quad v_{x}(\mathbf{q}) = \sqrt{\frac{V^{\theta}(q_{y})\gamma(\mathbf{q})}{V^{\phi}(q_{x})}}$$

with $\gamma(\mathbf{q}) = V^{\phi}(q_x)V^{\phi}(q_y) - (V^c(\mathbf{q}))^2$, and $\kappa_y(\mathbf{q}) = \kappa_x(P\mathbf{K})$; $v_y(\mathbf{q}) = v_x(P\mathbf{q})$ where $P\mathbf{q} = P(q_x, q_y) = (q_y, q_x)$. Correlation functions for θ_x and θ_y can be calculated directly from Eq. (10). $\theta_x - \theta_y$ cross correlations are nonsingular, whereas, $\theta_x - \theta_x$ and $\theta_y - \theta_y$ correlations have singular parts with exactly the same functional forms as they have in the absence of coupling between layers, but with the $\kappa(q)$ function in expressions for the scaling exponents replaced by

$$\kappa(q_{\perp}) = \kappa_{\nu}(0, q_{\perp}) = \kappa_{\nu}(q_{\perp}, 0). \tag{11}$$

Thus, other than renormalizing $\kappa(q)$, the coupling $V_{m,n}^c$ between the two arrays leaves the dimensions of all operators *unchanged*. Equations (10) and (11) define a 2D non-Fermi liquid with scaling properties to be discussed below.

First, however, we must verify that it is possible to choose potentials so that this 2D non-Fermi liquid is stable with respect to perturbations. All pairwise couplings within a given array, i.e., $\mathcal{H}_{\text{SC},n}^{X}$, $\mathcal{H}_{\text{CDW},n}^{X}$, $\mathcal{H}_{\text{SC},n}^{Y}$ and $\mathcal{H}_{\text{CDW},n}^{Y}$ defined as obvious generalizations of Eq. (4), can be rendered irrelevant by choosing $\kappa(q_{\perp})$ as in the case of an individual array. We must also consider Josephson and CDW couplings between the two arrays, which operate at the points of crossing (x,y)=(na,ma) of wire m in the X array and wire n of the Y array and, respectively, take the form

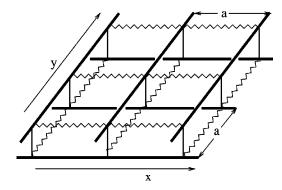


FIG. 2. A schematic depiction of the 2D non-Fermi liquid as a resistor network, with two parallel arrays of wire running along the *x* and *y* axes, with nodes in the *z* direction

$$\mathcal{H}_{\mathrm{SC}}^{XY} = \mathcal{J}^{XY} \cos[\sqrt{2\pi}(\theta_{x,m}(na) - \theta_{y,n}(ma))]$$

$$H_{\text{CDW}}^{XY} = \mathcal{V}^{XY} \cos[\sqrt{2\pi}(\phi_{x,m}(na) - \phi_{y,n}(ma))].$$
 (12)

The dimensions of these operators are, respectively,

$$\Delta_{SC,\infty} \equiv \int_{-\pi}^{\pi} \frac{dq}{2\pi} \kappa(q) = K,$$

$$\Delta_{\text{CDW},\infty} \equiv \int_{-\pi}^{\pi} \frac{dq}{2\pi} \frac{1}{\kappa(q)} \approx \frac{1}{K} \frac{1}{\sqrt{C\Delta}}, \quad (13)$$

where we assume that $\kappa(q)$ has the form given by Eq. (7), Δ is defined as before, and $C \equiv \kappa''(k_0)/2K$. If κ is chosen such that Eq. (7) is satisfied for each array, then \mathcal{H}^{XY}_{SC} and \mathcal{H}^{XY}_{CDW} are automatically irrelevant. We do not need any further fine tuning of κ to get this 2D sliding Luttinger liquid phase, and there is a stable CSLL phase.

We now investigate the transport properties of the CSLL phase. The conductivities of an array of parallel wires has been considered by Emery $et~al.^6$ In the presence of impurities, the resistivity along the wires, ρ_{\parallel} , vanishes as $T^{\alpha_{\parallel}}$, with $\alpha_{\parallel} = \Delta_{\text{CDW},\infty} - 2$. The perpendicular conductivity, σ_{\perp} , goes as $T^{\alpha_{\perp}}$ with $\alpha_{\perp} = 2\Delta_{\text{SC}} - 3$, where Δ_{SC} is the minimum of $\Delta_{\text{SC},1}$ and $\Delta_{\text{SC},2}$. The conductance, σ_c , arising from the Josephson coupling at the contact between the crossed wires satisfies $\sigma_c \sim T^{\alpha_c}$, where $\alpha_c = 2\Delta_{\text{SC},\infty} - 3$.

Thus we can model our 2D non-Fermi liquid as the resistor network depicted in Fig. 2 with nodes at the vertical Josephson junctions between the arrays at r_{mn} . The nodes of the X(Y) array are connected by nearest neighbor resistors with conductances $\sigma_{\parallel} = \rho_{\parallel}^{-1}$ if they are parallel to the x(y) axis and σ_{\perp} if they are perpendicular to the x axis(y axis). Nearest-neighbor nodes of the X and Y arrays are connected by resistors of conductance σ_c . In the continuum limit, the 2D current densities in the plane of the α grids ($\alpha = X, Y$) is $J_i^{\alpha} = \sigma_{ij}^{\alpha} E_j$ where $\sigma_{ij}^{X} = \sigma_{\parallel} e_{xi} e_{xj} + \sigma_{\perp} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\parallel} e_{yi} e_{yj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\perp} e_{xj} e_{xj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\perp} e_{xj} e_{xj}$ and $\sigma_{ij}^{Y} = \sigma_{\perp} e_{xi} e_{xj} + \sigma_{\perp} e_{xj} e_{xj}$ and $\sigma_{\perp} e_{xj} e_{xj} e_{xj}$ and $\sigma_{\perp} e_{xj} e_{xj} e_{xj} e_{xj} e_{xj}$ and $\sigma_{\perp} e_{xj} e_{xj} e_{xj} e_{xj} e_{xj} e_{xj}$ and $\sigma_{\perp} e_{xj} e_$

$$-\sigma_{ij}^X \partial_i \partial_j V^X + \frac{\sigma_c}{a^2} (V^X - V^Y) = \mathcal{T}^X$$

$$-\sigma_{ij}^{Y}\partial_{i}\partial_{j}V^{Y} - \frac{\sigma_{c}}{a^{2}}(V^{X} - V^{Y}) = \mathcal{T}^{Y}, \tag{14}$$

where \mathcal{T}^X and \mathcal{T}^Y are current densities (current/area) injected, respectively, into the X and Y grids. If no currents are injected, then this equation is solved by $V^X = V^Y = -\mathbf{E} \cdot \mathbf{x}$ to produce a total in-planar current density

$$J_{i} \equiv J_{i}^{X} + J_{i}^{Y} = (\sigma_{ij}^{X} + \sigma_{ij}^{Y}) E_{i} = (\sigma_{\parallel} + \sigma_{\perp}) E_{i}.$$
 (15)

Thus under a uniform electric field, the double layer behaves like an isotropic 2D material with in-plane conductivity $\sigma = \sigma_{\parallel} + \sigma_{\perp} \simeq \sigma_{\parallel}$, or equivalently with an isotropic resistivity that vanishes as $\rho_{\parallel} \sim T^{\alpha_{\parallel}}$. If currents are spatially nonuniform, as they are, for example, when current is inserted at one point and extracted from another, there is a crossover from isotropic to anisotropic behavior at length scales less than $l = a \sqrt{(\sigma_{\parallel} + \sigma_{\perp})/\sigma_c} \sim T^{-(\alpha_{\parallel} + \alpha_c)/2}$ that diverges as $T \to 0$.

This two-layer CSLL model can, quite simply, be extended to three dimensions by stacking alternate arrays in the third direction. It is still possible, although more difficult, for a stable CSLL phase to exist. This phase is characterized by an isotropic in-plane conductivity $\sigma_{\parallel} \sim T^{-\alpha_{\parallel}}$ and a conductivity $\sigma_{c}/a \sim T^{\alpha_{c}}$ in the direction perpendicular to the planes

of the wires. Thus the conductivity along the planes is much larger than the perpendicular conductivity.

The extension of the above analysis to a system of coupled Luttinger liquids where both charge and spin excitations are *gapless* is straightforward. On each wire, there is a Luttinger liquid for charge and for spin. To maintain gapless Luttinger liquids and SU(2) symmetry, we require that the spin degrees on each wire be represented by a Lagrangian of the form Eq. (2) with κ_s =1 and that at the fixed point there be no spin coupling between the wires. The fixed point for the charge degrees of freedom has the same form as for the gapped case. However, now, single-particle tunneling as well as Josephson and CDW couplings may be relevant. The phase diagram is quite complicated and will be discussed in a future publication. There *is*, however, a small but finite region of phase space where the sliding phase is stable.

In conclusion, we have demonstrated the existence of a non-Fermi metallic phase in two dimensions that maintains spin-charge separation and is stable to all pairwise potentials. This is a remarkable phase, which could be identified as a two-dimensional Luttinger liquid. We delay to a future publication the investigation of the stability of this phase with respect to all multiwire interactions.

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¹²There has been some inconsistency in the literature regarding the definitions of θ and ϕ . In this paper, we follow the notation of Ref. 6, and define θ as the phase variable, and ϕ as the conjugate density variable.

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