Intrinsic inhomogeneities in superconductors and the pseudogap phenomenon

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(Received 15 August 2000; published 24 January 2001)

High-T_c oxides are intrinsically inhomogeneous materials. The density of states is evaluated for such an inhomogeneous system, and it displays a gap structure above $T_c^{\rm res}$ (pseudogap). Thus, the pseudogap phenomenon can be caused by an inhomogeneity of the metallic phase. As a result, the critical temperature is spatially dependent. Various types of nonuniform structure are described (inhomogeneous carrier distribution, nonuniform distribution of pair breakers). The transition to a dissipationless state ($R=0$) corresponds to the percolation threshold.

DOI: 10.1103/PhysRevB.63.064524 PACS number(s): 74.80.−g, 74.72.−h, 74.25.−q

I. INTRODUCTION

The problem of the pseudogap in the high-T_c cuprates has attracted a lot of interest. Indeed, the existence of a gap structure, which is a precursor to the dissipationless state below $T_c^{\rm res}$ ($T_c^{\rm res}$ is the resistive transition to the superconductive state so that the resistance $R=0$ at $T<T_c^{\rm res}$) is unusual from the point of view of the conventional theory of superconductivity. The pseudogap has been observed for a broad range of temperatures above $T_c^{\rm res}$ (up to the pseudogap temperature T_c^*) using various experimental techniques, such as tunneling (see, e.g., Refs. 1–3), photoemission,[4,5] heat capacity,[6–8] NMR,[9] and spectroscopy.[10–13] The most direct observation is provided by tunneling spectroscopy, which displays a dip in the measured density of states.

Understanding the nature of such a peculiar phenomenon as a gap structure above T_c is of definite interest. Of course, the presence of an energy gap, that is the discrete structure of the energy spectrum, has several causes, some not even related to superconductivity. But we think that the continuous smooth change of the gap structure while crossing $T_c^{\rm res}$ (see, e.g., Refs. 1–3), as well as the recent observation of the isotopic dependence of T_c^*, similar to that for $T_c^{\rm res}$,[14] implies that the gap structure above $T_c^{\rm res}$ is related to the gap parameter in the dissipationless state at $T<T_c^{\rm res}$.

We think that the appearance of the pseudogap is due to a peculiar, intrinsically inhomogeneous structure of the compound. We stated our point of view briefly in Ref. 15. Here we present a more detailed analysis and some new results.

The structure of the paper is as follows: Sec. II describes our theoretical approach. At first, we present a qualitative picture, and then we focus on a detailed theory for two possible scenarios of inhomogeneity. Section III contains an analysis of experimental data and a discussion.

II. THEORY

A. Qualitative picture: Two scenarios

Consider an inhomogeneous superconducting compound. Inhomogeneity leads to a spatial dependence of the critical temperature, so that $T_c \equiv T_c(\mathbf{r})$. As a result, it is realistic to imagine the sample containing regions with different values of local critical temperature (T_c^L). Of course, for some of them $T_c^L > \overline{T_c}$, where $\overline{T_c}$ is the average value of the critical temperature.

Consider the case when these regions form small clusters with local critical temperature T_c^L that is larger than $T_c^{\rm res}$ (Fig. 1). These clusters are embedded in a normal metallic matrix above $T_c^{\rm res}$. If the distance between the clusters L exceeds some value L_0 (the criteria will be introduced below), then the compound is in the normal resistive state. At the same time, in the region $T_c^L > T > T_c^{\rm res}$, pairing still exists in the clusters, which manifests itself in spectroscopy (pseudogap), a diamagnetic moment above $T_c^{\rm res}$, heat capacity (loss of entropy), etc.

One should emphasize an important aspect of this scenario we described. The clusters are not isolated but are embedded in the normal metallic matrix. As a result, the proximity effect between the cluster and the matrix plays a very important role and has to be directly included in the analysis. The proximity effect imposes serious restrictions on the scale of inhomogeneites, and on the distance between clusters.

There are two possible scenarios for the inhomogeneites of interest: (1) an inhomogeneous distribution of pair breakers (as is known, the presence of a pair breaker leads to a local depression in T_c), and (2) an inhomogeneous distribution of carriers leading to a spatial dependence of the coupling constant $\lambda \equiv \lambda(\mathbf{r})$. Below we will consider both scenarios.

As temperature decreases towards $T_c^{\rm res}$, the region occupied by clusters is growing and $T_c^{\rm res}$ corresponds to the per-

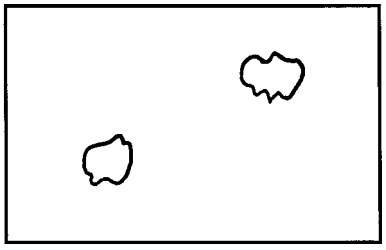

FIG. 1. Inhomogeneous structure. ''Islands'' are characterized by values of T_c's higher than the matrix.

colation threshold, that is, to the formation of an infinite cluster. This cluster forms the channel for a macroscopic superconducting current.

As was noted above, the presence of local superconducting regions within the interval $T_c^L > T \cong T_c^{\rm res}$ leads to a diamagnetic moment above $T_c^{\rm res}$. This problem was considered by us in Ref. 16. In this paper, we focus on the pseudogap phenomenon. Let us turn to a more detailed evaluation.

B. Main equations and density of states

Our goal is to evaluate the density of states, because the energy gap structure should be directly reflected in the shape of the density of states. Let us consider first the case of an inhomogeneous distribution of pair breakers. As we know, magnetic impurities act as pair breakers.[17,18] The pair-breaking effect leads to a depression in T_c, so that the value of T_c depends directly on the impurity concentration $n_{\rm imp}$. As a result, an inhomogeneous distribution of magnetic impurities leads to the dependence $T_c(\mathbf{r})$. Some regions (clusters) with a small concentration of impurities have a higher value of $T_c^L > \overline{T_c}$.

Below we consider the case of *s*-wave pairing. Note, that for the *d*-wave picture, the pair-breaking effect is also quite important, but the scenario looks even stronger, because in addition, nonmagnetic scatters also act as pair breakers.

We employ the method of integrated Green's functions that was developed by Eilenberger,[19] and independently by Larkin and Ovchinnikov,[20] (see review in Ref. 21). This method is very convenient for treating an inhomogeneous system. This method was used by us in Ref. 16.

The main equations have the form:

$$\alpha\Delta - \beta\omega_n + (D/2)[\alpha\partial_r^2\beta - \beta\partial_r^2\alpha] = \alpha\beta\Gamma(\mathbf{r}), \tag{1}$$

$$\alpha^2 + \beta^2 = 1, \tag{1'}$$

$$\Delta = 2\pi T\lambda \sum_{\omega_n > 0} \beta(\omega_n). \tag{1''}$$

Here, α and β are the usual and pairing Green's functions averaged over energy, Δ is the order parameter, $\partial_r = (\partial/\partial\mathbf{r})$, $\omega_n = (2n+1)\pi T$, and $\Gamma = \tau_s^{-1}$, τ_s is a spin-flip relaxation time (for the magnetic impurities).[17] Of course, α, β, and Δ are spatially dependent functions. It is essential that, because of an inhomogeneous distribution of magnetic impurities, Γ is a spatially dependent quantity, so that $\Gamma \equiv \Gamma(\mathbf{r})$. We consider the ''dirty'' case, so that D is the diffusion coefficient. The Green's functions α and β are normalized by the function α_N, that is, by the value of α in the normal state.

Our goal is to calculate the density of states ν_s. It will be shown that, indeed, ν_s displays a pseudogap phenomenon (see below). At $T = T_c^*$, the order parameter Δ becomes equal to zero (then $\nu_s = \nu_n$).

Let us start by calculating the local value of the critical temperature, T_c^L; this will allow us to explicitly separate the impact of the proximity effect. In order to evaluate T_c^L, we use the method employed by us in Ref. 16 for the calculation of the diamagnetic moment for an inhomogeneous system above $T_c^{\rm res}$. According to Ref. 16,

$$\ln(T_c^0/T_c^L) = \Psi[0.5 + (\Gamma_\infty + \lambda_1)/2\pi T_c^L] - \Psi(0.5). \tag{2}$$

Here, T_c^0 is the critical temperature in the absence of pair breakers (this is an ''intrinsic'' value of T_c introduced by us in Ref. 22), Γ_∞ is the value of Γ outside of the inhomogeneous region (grain), and λ_1 is the minimum eigenvalue of the equation

$$-[(D/2)\partial_-^2 - \Gamma(\mathbf{r})]\Delta_0 = (\lambda_1 + \Gamma_\infty)\Delta_0. \tag{3}$$

For concreteness, consider the case when $\Gamma(\mathbf{r})$ is described by the following dependence:

$$\Gamma = \Gamma_\infty(\rho > \rho_0), \quad \Gamma = \Gamma_0(\rho < \rho_0). \tag{4}$$

Here, ρ_0 is the radius of the small region. Assume $\Gamma_0 < \Gamma_\infty$, so that the region ρ_0 is characterized by a higher value of local $T_c \equiv T_c^L$.

Then, (see Ref. 16)

$$\lambda_1 = \delta\Gamma + \gamma, \tag{5}$$

where

$$-\delta\Gamma = \Gamma_\infty - \Gamma_0 \tag{5'}$$

and

$$\gamma = 0.5D(z_0/\rho_0)^2. \tag{5''}$$

z_0 is the lowest zero of the Bessel function $J_0(z)$. As a result, we obtain

$$\ln(T_c^0/T_c^L) = \Psi[0.5 + (\Gamma_0 + \gamma)/2\pi T_c^L] - \Psi(0.5). \tag{6}$$

Using this equation and the expression[17] $\ln(T_c^0/T_c^{\rm is.}) = \Psi[0.5 + \Gamma_0/T_c^{is\cdot}] - \Psi(0.5)$, where $T_c^{is\cdot}$ is the critical temperature of an isolated grain, we obtain the following equation for T_c^L:

$$\ln(T_c^{is}/T_c^L) = \Psi[0.5 + (\Gamma_0 + \gamma)/2\pi T_c^L] - \Psi(0.5 + \Gamma_0/2\pi T_c^{is}), \tag{7}$$

where γ is defined by Eq. (5). One can see directly from Eq. (7) that the value T_c^L is depressed relative to $T_c^{\rm is}$, and this is due to the proximity effect between the grain and the matrix; the proximity effect is described by the parameter γ.

Let us turn to calculation of the density of states. The order parameter and the Green's function β in the first approximation have the form:[16]

$$\Delta = C\Delta_0; \quad \beta = C\Delta_0(\omega + \lambda_1 + \Gamma_\infty)^{-1}, \tag{8}$$

where Δ_0 is the normalized solution of Eq. (3).
The dependence $C \equiv C(T)$ can be evaluated with the use of higher terms in β and is described by relation

$$C^2(T)=A(T_c^L)[\ln(T_c^L/T)+\Psi(0.5+\lambda/2\pi T_c^L)-\Psi(0.5+\lambda/2\pi T)] \quad (9)$$

$$A^{-1}(T)=(4\pi T)^{-1}[\Psi'(0.5+\lambda/2\pi T)+(\lambda/4\pi T)\times\Psi''(0.5+\lambda/2\pi T)]-(D/2(2\pi T)^2)\times\Psi''(0.5+\lambda/2\pi T)\int d\vec{\rho}\Delta_0^2(\partial\Delta_0/\partial\rho)^2,$$

where $\lambda=\lambda_1+\Gamma_\infty$.

As was mentioned above, our main goal is to evaluate the average value of the density of states ν, that is, $\langle\nu\rangle=\mathrm{Re}\langle\alpha\rangle$. It is natural to assume that near T_c^L the deviation of the Green's function α from its normal value inside the matrix is caused by the proximity effect and is small. As a result, we arrive at the following expression for the average value of α:

$$\langle\alpha\rangle=1-(n_cC^2/2)[\lambda^2-\omega^2+2i\lambda\omega](\lambda^2+\omega^2)^{-2}. \quad (10)$$

Here, $C\equiv C(T)$ and λ_1 are determined by Eqs. (9) and (5), n_c is the concentration of superconducting clusters (''spots''). The density of states is defined $\langle\nu\rangle=\mathrm{Re}\langle\alpha\rangle$ and can be determined directly from Eq. (10). The density of states is peaked near the value $\omega\cong\tilde{\Gamma}\equiv\omega_p$ [see Fig. 2(a)].

One can see directly from Eq. (10) and Fig. 2(a) that there is a ''softening'' of the low-energy part of the density of states, and this is a manifestation of the pseudogap phenomenon.

If the temperature is above T_c^{res} and increases towards T_c^*, $(T_c^*\cong\max\{T_c^L\})$ then $C^2\to0$ [see Eq. (10)], and the difference $\Delta\nu=\nu_{\max}-\nu_{\min}\to0$. At the same time the position of the peak is independent of T. This feature is very specific for a pseudogap phenomenon caused by an inhomogeneous distribution of pair breakers.

C. Inhomogeneous distribution of coupling constants

In this section, we consider the case when the coupling constant is spatially dependent: $\lambda\equiv\lambda(\mathbf{r})$. Here, the set of equations for the integrated Green's functions has a form [cf. Eq. (1)]

$$\alpha\Delta-\beta\omega+(D/2)[\alpha(\partial^2\beta/\partial\mathbf{r}^2)-\beta(\partial^2\alpha/\partial\mathbf{r}^2)]=0,$$

$$\alpha^2+\beta^2=1;\quad \Delta=2\pi T|\lambda(\mathbf{r})|\sum_{\omega_n>0}\beta(\omega). \quad (11)$$

Equations (11), similar to Eq. (1), determine the spatially dependent functions α, β, and Δ. As was mentioned above, we focus on a layered conductor and, in a first approximation, we neglect the interlayer hopping and consider a two-dimensional system. We also assume that the spatial dependence of λ is such that in some small region it has a large value, so that

$$\lambda=\lambda_0(\rho>\rho_0);\quad \lambda=\lambda_1(\rho<\rho_0);\quad \lambda_1>\lambda_0.$$

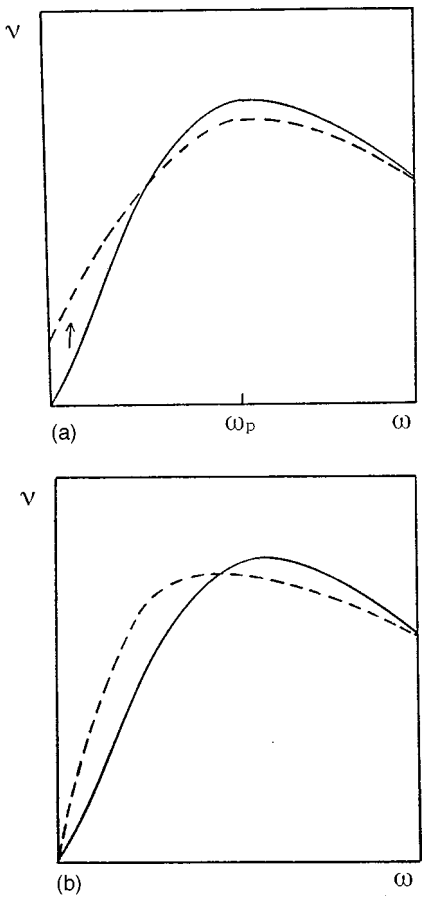

FIG. 2. The behavior of the density of states for different types of inhomogeneites. The dashed line corresponds to higher temperature. (a) Density of states for an inhomogeneous distribution of pair breakers. (b) Density of states for an inhomogeneous distribution of carriers and, correspondingly, the coupling constants.

ρ_0 is the radius of the spot. In order to determine the transition temperature of the cluster (''spot''), we consider the linearized Eq. (11):

$$\Delta-\beta\omega+\frac{D}{2}[\rho^{-1}\partial/\partial\rho(\rho\partial\beta/\partial\rho)]=0. \quad (12)$$

Based on Eq. (12), we can write for the region inside the spot

$$\beta=B-(B_1/2)\rho^2;\quad \Delta_0=DB_1+\omega B, \quad (13)$$

where B,B_1 are some constants, and Δ_0 is the value of the order parameter at $\rho=0$.

For the region $\rho>\rho_0$ (matrix) where $\lambda=\lambda_0$, one can write

$$\beta=AK_0\left(\rho\sqrt{\frac{2\omega}{D}}\right), \quad (14)$$

where K_0 is the Bessel function and A is a constant.

With the use of continuity conditions, we obtain the following values of the constants A, B, and B_1:

$$A=(\Delta_0/\omega)f(a); \quad B=(\Delta_0/\omega)g(a), \tag{15}$$

$$f(a)=[K_0(a)+\tilde{a}K_1(a)]^{-1}, \tag{15'}$$

$$g(a)=[K_0(a)+(a/2)K_1(a)]f(a). \tag{15''}$$

Here,

$$a=a(\omega_n)=\rho_{\mathbf{0}}(2\omega_n/D)^{1/2}; \quad \tilde{a}=(a/2)+(2/a). \tag{16}$$

As a result, we arrive at the following expression for the transition temperature of the cluster T_c^L:

$$\ln(T_c^{\mathbf{0}}/T_c^L)=2\pi T\sum_{\omega_n>0}\frac{1}{\omega_n}[1-g(a)], \tag{17}$$

where $g(a)$ is defined by Eq. (5''). Here T_c^0 is the critical temperature of the bulk superconductor with coupling constant λ_1. One can see directly from Eq. (17) that, indeed $T_c^L<T_t^0$, that is, the proximity effect between the cluster ($\rho<\rho_0$) and the matrix leads to decrease in T_c; T_c^0 corresponds to the absence of the proximity effect. Note that the measured quantity is T_c^L.

As was noted above, our goal is to evaluate the density of states. It will allow us to conclude whether or not there is a gap structure (''pseudogap''), that is, a softening in the density of states at low energies. We focus on the region $\overline{T_c}<T<T_c^L$. If the region occupied by the clusters is small, then $\overline{T_c}\cong T_c^{\mathrm{vol}}$, where T_c^{vol} is determined by the coupling constant λ_0. The density of states $\nu\propto \mathrm{Re}\,\alpha$. The function α for the matrix slightly deviates from the normal value and, in a first approximation, can be written in the form:

$$\alpha\cong 1-\beta^2/2=1-(\Delta(0)^2/2\omega^2)K_0(a)\times[K_0(a)+\tilde{a}K_1(a)]^{-2}, \tag{18}$$

where a and $\tilde{a}$ are defined by Eq. (16); recall that the function α is normalized to its normal value α_N. As was noted above, $\Delta_0\equiv\Delta(0,T)$ is the value of the order parameter at $\rho=0$. The temperature dependence $\Delta(0,T)$ can be obtained from Eqs. (11) and (18), and is described by the relation

$$\Delta^2(0,T)=\pi^2\left\{(T_c^L)^2\ln^2\left(\frac{\tilde{\omega}}{T_c^L}\right)^{1/2}-T^2\ln^2\left(\frac{\tilde{\omega}}{T}\right)^{1/2}\right\}\Big/ I. \tag{19}$$

Here, $\tilde{\omega}=D/(\pi\rho_0^2)$, and $I=\int_1^\infty dy\, y\ln^5 y(1+\ln y)^{-4}=1.055$. One can see that, indeed, $\Delta(0,T)\to 0$ as $T\to T_c^L$.

The average density of states $\langle\nu\rangle\propto\langle\mathrm{Re}\,\alpha\rangle$ can be calculated with the use of Eqs. (18) and (19), and the corresponding analytical continuation. The result of the calculations is presented in Fig. 2. The density of states is peaked in the region $\omega_p<\omega<D/\rho_0^2$, where

$$\omega_p\cong\pi n_c\rho_0^4\Delta^2(0,T)/4D. \tag{20}$$

In this region $\langle\alpha\rangle$ has a form

$$\langle\alpha\rangle=1+\pi n_c\rho_0^6\Delta^2(0,T)\ln(2/a(\omega))/4D^2 \tag{21}$$

Here, n_c is a concentration of the clusters with $\lambda=\lambda_1$, and $a(\omega)$ is defined by Eq. (18). The density of states is suppressed in the region $\omega<\omega_p$, so that the total number of states is conserved. This suppression is a direct manifestation of the pseudogap.

Note that an increase in temperature $T\to T_c^*$ ($T_c^*\cong\max T_c^L$) is accompanied by decrease in $\Delta(0,T)$ [see Eq. (19)] and, consequently, by the shift (towards to $\omega=0$) of the peak position in the density of states [Fig. 2(b)].

Herein lies a major difference between two scenarios considered in the present paper. In both cases, we are dealing with the appearance of the pseudogap. However, if it is due to an inhomogeneous distribution of pair breakers (see Sec. II B), the peak position does not depend on temperature as $T\to T_c^*$ (the depth of the pseudogap is getting smaller), whereas the present case [$\lambda\equiv\lambda(\mathbf{r})$] is characterized by a continuous shift of the peak position and corresponding decrease of the pseudogap region. Real superconductors can combine both factors.

III. EXPERIMENTAL DATA: DISCUSSION

The phenomenon of the pseudogap has been observed in many studies, e.g., Refs. 1–14. The most direct technique to observe the pseudogap is tunneling spectroscopy, which measures explicitly the density of states. Such experiments were described in Refs. 1–3. There are also interesting photoemission data,[4,5] the observation of a loss of entropy in heat capacity measurements,[6–8] infrared spectroscopy data,[12] and a decrease in nuclear relaxation in the normal state.[9]

According to our approach, the appearance of the pseudogap is due to intrinsic inhomogeneites that lead to the existence of regions (clusters) with values of local $T_c=T_c^L$ higher than $\overline{T_c}=T_c^{\mathrm{res}}$. We described above two possible scenarios of inhomogeneity: (1) an inhomogeneous distribution of pair breakers, and (2) a spatial dependence of the coupling constant: $\lambda\equiv\lambda(\mathbf{r})$. Both scenarios lead to a pseudogap, but they each have distinctive behaviors of the peak positions (see above).

The density of states and its temperature dependence were directly measured by tunneling spectroscopy in Ref. 1. One can see directly from the data (see Fig. 1 in Ref. 1), that the gap structure (pseudogap) persists above T_c, but the peak position does not depend on temperature. This is in a direct correspondence with the first scenario discussed above (see Sec. II B) and, therefore, indeed, for the $Bi_2Sr_2CaCu_2O_{8+\delta}$ sample studied in Ref. 1 the pseudogap phenomenon is caused by an inhomogeneous distribution of the pair breakers.

An interesting interlayer tunneling spectroscopy for overdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ compound was described in Ref. 2. The authors[2] also observed the occurrence of the pseudogap below 150 K, that is at much higher temperatures than the resistive $T_c\cong 87$ K. The data,[2] like Ref. 1, is also consistent with the picture of an inhomogeneous distribution of pair breakers. The break junction technique was employed in Ref. 3 for $T>T_c^{\mathrm{res}}$ to study the $(Bi,Pb)_2Sr_2Ca_2Cu_3O_{10+\delta}$ compound. A slight shift in the peak position as $T\to T_c^*$ reflects the presence of some inhomogeneity of λ.

The proximity effect leads to restrictions on the scale of

the regions, namely, on the size of the superconducting clusters $\rho \geq \xi_s = h v_F / 2\pi T_c^L$; otherwise, the proximity effect will destroy the pairing. On the other hand, in the pseudogap region ($T_c^* > T > T_c^{\rm res}$) the distance between the clusters $L \geq \xi_N = h v_F / 2\pi T$; then we are dealing with normal transport ($R \neq 0$).

The pseudogap region is characterized by a normal resistance ($R \neq 0$), because the distance between the superconducting clusters is large and there is no macroscopic superconducting path (see below). Nevertheless, one can expect a frequency-dependent superconducting response of the clusters. As we know, the conductivity $\sigma \equiv \sigma(\omega)$ in the superconducting state has the form: $\sigma = \sigma_1 + i\sigma_2$, $\sigma_2 \cong n_s$, where n_s is the ''superfluid'' density. Recent microwave measurements[23] show that in the pseudogap region ($T_c^{\rm res} < T < T_c^*$) $\sigma_2 \neq 0$ and this is consistent with our picture. We will analyze the microwave response in detail elsewhere.

The presence of superconducting clusters can also contribute to the diamagnetic response of the sample above $T_c^{\rm res}$. Such response, indeed, has been observed above T_c for the overdoped Tl-based cuprate.[24] Based on the described model, we calculated the diamagnetic moment and its temperature dependence,[16] in good agreement with the data.[24]

The pseudogap is most evident in underdoped cuprates, and inhomogeneities in this region of the phase diagram have been observed in a number of experiments. Scanning tunneling microscopy (STM) at low temperatures into cleaved single crystals of BiSrCaCuO (Ref. 25) has revealed the presence of large variations in the tunneling conductance (reflecting large variations in the superconducting gap) depending on the position of the STM tip. This variation was more pronounced in samples with lower resistive transition temperatures than those with T_c's closer to the optimally doped 93 K.[26] Of course, it will be necessary to map out in detail this variation of the density of states above the resistive transition in the pseudogap temperature region to determine if experimental evidence for *local* superconductivity persists in underdoped BiSrCaCuO above the resistive transition. Recently, anomalous diamagnetism above the resistive T_c has been observed in underdoped YBaCuO from magnetization and Cu NMR-nuclear quadrupole resonance relaxation measurements.[27] This diamagnetism has been observed well into the pseudogap regime but did not seem to persist above the maximum known T_c for YBaCuO, at least within the sensitivity of the reported measurements.

We have recently learned about another interesting series of measurements[28] in which an electrically discontinuous (insulating) Pb film is covered with increasing thicknesses of Ag. The Ag act to couple the superconducting Pb grains via the proximity effect. The resistive transition, as well as tunneling spectra, has been taken on a series of these films. The most insulating film has no resistive transition but a full Pb gap as revealed by the tunneling spectra. This gap is reduced as silver is added reflecting the decrease in the mean-field T_c of the Pb grains. At some point, the composite film becomes continuous and superconducting with a low resistive transition temperature. The evolution of the mean-field transition temperature and the resistive transition temperature with increasing Ag thickness mimics the phase diagram of the cuprates with doping. The mean-field transition temperature resembles the pseudogap onset temperature and the resistive T_c resembles the superconducting transition temperature, with the mean-field transition temperature lying above the resistive transition.

We believe that the results on the Pb/Ag artificial inhomogeneous superconductor model the behavior of the cuprates. The cuprates are doped substitutionally and inhomogenously. At some concentration of doping there are regions with a high enough concentration of carriers to locally superconduct and therefore reduce the low energy density of states. The evolution of these islands into a percolating resistanceless state would resemble the percolating proximity coupling described above. It is not then surprising that the phase diagrams would be nearly identical.

The pseudogap region is characterized by the coexistence of normal resistance and the gap structure, and it is due to an intrinsic inhomogeneity of the system. The question arises about the dynamics of transition to the dissipationless state with $R=0$ as T decreases towards to $T_c^{\rm res}$. According to our approach, the system contains regions with various T_c's. The decrease in T towards $T_c^{\rm res}$ is accompanied by the formation of new superconducting ''clusters'' and by their growth. This is a percolation picture and $T_c^{\rm res}$ corresponds to the percolation threshold. According to the percolation theory (see, e.g., Ref. 29), this threshold is related to formation of the infinite superconducting cluster. Formation of such a cluster allows us to observe a macroscopic superconducting current with $R=0$ and phase coherence within such cluster.

A similar percolative scenario was introduced by Gor'kov and one of the authors[30] to describe the properties of manganites. Note that a similar approach can be used in order to describe the evolution of the system as a function of the doping level at fixed low temperature.

One should stress that usual inhomogeneous picture (see, e.g., Ref. 31) implies the coexistence of the metallic and insulating phases (phase separation). In this paper, we focus on an additional and different aspect, namely, on the inhomogeneity of the metallic state. This leads to the coexistence of superconducting and normal metallic regions similar to proximity systems.

According to Ref. 32, the Na-doped WO_3 compound is characterized by a diamagnetic moment and by a decrease in resistance. Afterwards, interesting studies were described in Ref. 33. Moreover, recent STM spectroscopy[34] has revealed the dip in the density of states, that is, the gap structure. Probably, this system is inhomogeneous and contains surface superconducting ''islands.'' This corresponds to an inhomogeneous picture described above.

The inhomogeneous structure of the cuprates has been observed with the use of neutron diffraction technique.[35,36] Recently, we have learned about one such interesting study.[36] According to Ref. 36, the underdoped region is very inhomogeneous and the compound becomes more homogeneous if we move towards optimum doping and above. Such a picture is totally consistent with our scenario for the pseudogap, because, as we know, this phenomenon is particularly strong in the underdoped region.

As was mentioned above, recently we learned about the STM measurements[25,37] performed at different locations of the BiSrCaCuo compound (at $T \cong 4.2$ K). The energy gap defined as a distance between the peaks of the density of states displays a strong spatial dependence. The observations[25,37] provide a strong experimental support for the concept of inhomogeneity of the metallic phase. There is, therefore, rather compelling evidence for inhomogeneities playing a very important role in the evolution of the superconducting phase diagram, including the nature and temperature dependence of the pseudogap.

ACKNOWLEDGMENTS

The authors are grateful to S. J. L. Billinge, J. Davis, R. Dynes, T. Egami, S. Sridhar, and S. Reich for sending their papers and fruitful discussions. The research of V.Z.K. was supported by the U.S. Office of Naval Research under Contract No. N00014-99-F0006.

[1] Ch. Renner, B. Revaz, J.-Y. Genoud, K. Kadowaki, and O. Fischer, Phys. Rev. Lett. **80**, 149 (1998).
[2] M. Suzuki, T. Watanabe, and A. Matsuda, Phys. Rev. Lett. **82**, 5361 (1999).
[3] T. Ekino, Y. Seraki, S. Mashimoto, and H. Fujii, J. Low Temp. Phys. **117**, 359 (1999).
[4] H. Ding, T. Yokoya, J. C. Campuzano, T. Takahashi, M. Randeria, M. Norman, T. Mochiku, K. Kadowaki, and J. Giapintzakis, Nature (London) **382**, 51 (1996).
[5] A. Loeser, Z.-X. Shen, D. S. Dessau, D. S. Marshall, C. H. Park, P. Fournier, and A. Kapitulnik, Science **273**, 325 (1996).
[6] J. Loram, K. Mirza, J. Wade, J. Cooper, and W. Liang, Physica C **235**, 134 (1994).
[7] J. Tallon, J. Cooper, P. de Silva, G. Williams, and J. Loram, Phys. Rev. Lett. **75**, 4114 (1995); J. Tallon and J. Loram (unpublished).
[8] N. Momono, T. Matsuzaki, T. Nagata, M. Oda, and M. Ido, J. Low Temp. Phys. **117**, 353 (1999).
[9] W. Warren, Jr., R. E. Walstedt, G. F. Brennert, R. J. Cava, R. Tycko, F. Bell, and G. Dabbagh, Phys. Rev. Lett. **62**, 1193 (1989).
[10] J. Orenstein, G. A. Thomas, A. J. Millis, S. L. Cooper, D. H. Rapkine, T. Timusk, L. F. Schneemeyer, and J. V. Waszczak, Phys. Rev. B **42**, 6342 (1990).
[11] J. Mason, R. Mallozzi, J. Orenstein, J. Eckstein, and I. Bozovic, Nature (London) **398**, 221 (1999).
[12] R. Kaindl, M. Woerner, T. Elsaesser, D. Smith, J. Ruan, G. Farnan, M. McCurry, and D. Walmsley, Science **287**, 470 (2000); T. Timusk and B. Statt, Rep. Prog. Phys. **62**, 61 (1999).
[13] M. Opel, M. Götzinger, C. Hoffmann, R. Nemetschek, R. Philipp, F. Venturini, R. Hackl, A. Erb, and E. Walker, J. Low Temp. Phys. **117**, 347 (1999).
[14] D. R. Temprano, J. Mesot, S. Janssen, K. Conder, A. Furrer, H. Mutka, and K. Mueller, Phys. Rev. Lett. **84**, 1990 (2000).
[15] V. Kresin, Yu. Ovchinnikov, and S. Wolf, in *Materials and Mechanisms of Superconductivity and High Temperature Superconductivity*, Proceedings of International Conference, Houston [Physica C (to be published)]; *Major Trends in Superconductivity*, Proceedings of International Conference, Klosters, Switzerland [J. Supercond. **13**, 777 (2000)].
[16] Yu. Ovchinnikov, S. Wolf, and V. Kresin, Phys. Rev. B **60**, 4329 (1999).
[17] A. Abrikosov and L. Gor'kov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1961) [Sov. Phys. JETP **12**, 1243 (1961)].
[18] P. de Gennes, J. Phys.: Condens. Matter **3**, 79 (1964).
[19] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[20] A. Larkin and Yu. Ovchinnikov, Zh. Eksp. Teor. Fiz. **55**, 2262 (1968) [Sov. Phys. JETP **28**, 1200 (1969)].
[21] A. Larkin and Yu. Ovchinnikov, in *Non-equilibrium Superconductivity*, edited by D. Landenberg and A. Larkin (Elsevier, Amsterdam, 1986), p. 530.
[22] V. Kresin, S. Wolf, and Yu. Ovchinnikov, Phys. Rev. B **53**, 11 831 (1996); Phys. Rep. **288**, 347 (1997).
[23] S. Sridhar (private communication).
[24] C. Bergemann, A. Tyler, A. Mackenzie, J. Cooper, S. Julian, and D. Farrell, Phys. Rev. B **57**, 14 387 (1998).
[25] A. Truscott, Ph.D. thesis, University of California at San Diego, 2000.
[26] R. Dynes (private communication).
[27] P. Caretta, A. Lascialfari, A. Rigamonti, A. Rosso, and A. Varlamov, Phys. Rev. B **61**, 12 420 (2000).
[28] R. Dynes (private communication).
[29] B. Sklovskii and A. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlag, Berlin, 1984); D. Stuffer and A. Aharony, *Introduction to Percolation Theory* (Taylor and Francis, London, 1992).
[30] L. Gor'kov and V. Kresin, Pis'ma Zh. Eksp. Teor. Fiz. **67**, 934 (1998) [JETP Lett. **67**, 985 (1998)]; J. Supercond. Novel Magnetism **13**, 239 (2000); M. Dzero, L. Gor'kov, and V. Kresin, Eur. Phys. J. B **14**, 459 (2000).
[31] L. Gor'kov and A. Sokol, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 333 (1987) [JETP Lett. **46**, 420 (1987)]; *Phase Separation in Cuprate Superconductors*, edited by E. Sigmund and K. A. Mueller (Springer-Verlag, Berlin, 1994).
[32] S. Reich and Y. Tsabba, Eur. Phys. J. B **9**, 1 (1999).
[33] A. Shengelaya, S. Reich, Y. Tsabba, and K. Mueller, Eur. Phys. J. B **12**, 13 (1999); A. Aird and E. Salje, *ibid.* **15**, 205 (2000).
[34] S. Reich, G. Leitus, Y. Tssaba, Y. levi, A. Sharoni, and O. Millo (unpublished).
[35] T. Egami and S. J. L. Billinge, in *Physical Properties of High Temperature Superconductors*, edited by V. D. Ginzburg (World Scientific, Singapore, 1996); T. Egami, Y. Petrov, D. Louca (unpublished); S. J. L. Billinge, E. Bozin, M. Gutman, H. Takagi (unpublished); M. Gutmann, S. J. L. Billinge, E. L. Brosha, and G. H. Kwei, Phys. Rev. B **61**, 11 762 (2000).
[36] E. Bozin, G. Kwei, H. Takagi, and S. Billinge, Phys. Rev. Lett. **84**, 5856 (2000).
[37] V. Madharan, K. Lang, E. Hudson, S. Pan, H. Eisaki, S. Uchida, and J. Davis, Bull. Am. Phys. Soc. **45**, 416 (2000); J. Davis (private communication).