Interlayer *c*-axis transport in the normal state of cuprates

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A theoretical model of *c*-axis transport properties in cuprates is proposed. Interplane and in-plane charge fluctuations make hopping between planes incoherent and diffusive (the in-plane momentum is not conserved after tunneling). The non-Drude optical conductivity $\sigma_c(\omega)$ and the power-law temperature dependence of the dc conductivity are generically explained by the strong fluctuations excited in the process of tunneling. Several microscopic models of the charge fluctuation spectrum are considered.

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Despite the strongly two-dimensional layered structure of the high-temperature cuprate superconductors, features associated with the third dimension, perpendicular to the CuO_2 planes, may be an important ingredient in their superconductivity. In fact, it is well accepted that a certain degree of Josephson-type coupling between different planes is necessary to suppress the two-dimensional fluctuations, which will otherwise destroy the superconducting long-range order. However, the systematic dependence of the critical temperature T_c on the number of layers in the unit cell (together with the absence of evidence for strong fluctuations effects above T_c at optimal doping, which suggests that these fluctuations are not the major reason for this systematic dependence) points almost unambiguously to the conclusion that theories formulated for a single plane cannot be the whole story. Either hopping between planes,[1] or Coulomb interaction between them,[2] or both, is an important factor in raising the critical temperature (and, perhaps, in some cases also for lowering it, see Ref. 2). In light of this, the study of the *c*-axis optical and transport properties is more than just a minor diversion from the main issue.

These *c*-axis optical and transport properties are very puzzling and anomalous.[3] Most remarkable is the fact that the temperature dependence of the dc *c*-axis resistivity $\rho_c(T)$, in sharp contrast to the well-known linear T dependence of the in-plane resistivity $\rho_{ab}(T)$, is nonuniversal, being described in most cases by a power law $\rho_c(T)\sim T^{\gamma}$, where, however, the exponent γ can be anything in between approximately $+1$ and -2. The optical conductivity $\sigma_c(\omega)$ is roughly frequency independent from low frequencies up to midinfrared frequencies [except in the case of some overdoped cuprates (Y-123 and overdoped La-214)]; the numerical value is below the Mott-Ioffe-Regel minimum metallic conductivity. This behavior, which is dramatically different from the behavior of the in-plane resistivity, has been christened ''confinement.''[1] Thus, despite the dramatic differences in the raw data between the different cuprate families, one can isolate at least two elements which may be legitimately called ''universal:'' a non-Drude optical conductivity of a magnitude below the Mott limit, and a power-law temperature dependence of the dc resistivity $\rho_c(T)$ (albeit with a material-specific exponent). In this paper we develop a framework for the explanation of these universalities, which will hopefully shed light on the role of the interplane and in-plane Coulomb interaction as well as on the more obvious one of the interlayer hopping.

On the basis of the above experimental observations, we can make the assumption of incoherent transport (diffusive tunneling) in the *c* direction. The interplane (or rather interunit cell) hopping time $\tau_{\rm hop}$ can be estimated from the dc *c*-axis resistivity. Relating the diffusion constant D to the hopping time $\tau_{\rm hop}$ by $D=d^2/\tau_{\rm hop}$, we can derive a model-independent relation between the conductivity σ_c and the hopping time $\sigma_c=e^2\nu_{2D}(d/\tau_{\rm hop})$, where ν_{2D} is a two-dimensional density of states. Using this formula and the experimental values for the *c*-axis conductivity we can estimate the *c*-axis hopping time $\tau_{\rm hop}$. The strong two dimensionality of the electron motion becomes obvious if we compare the hopping time $\tau_{\rm hop}$ with the in-plane scattering time τ_{ab}. As is well known, the in-plane scattering time τ_{ab} is of order of $\hbar/kT$. Direct comparison[4] shows that for most materials the *c*-axis hopping time is *much longer* than the scattering time in the plane. These two times are comparable only for overdoped Y-123 and La-214 suggesting a crossover to a different regime of *c*-axis transport; this is confirmed by the experimental observation of the Drude-like frequency dependence of the conductivity $\sigma_c(\omega)$ for these compounds.

Many approaches have been suggested[1,3] to describe the *c*-axis transport properties. Most of them stem from phenomenologically assumed in-plane Green functions. One remarkable example is a non-Fermi ''Luttinger'' liquid theory which explains naturally the ''confinement'' for *c*-axis motion in the normal state.[1] Many other theories are essentially based on the Fermi liquid theory modified by strong correlations.[3,5]

We take a quite different approach to the problem. Although in our approach the in-plane motion (expressed by in-plane Green functions) is undeniably important, we show that most of the *c*-axis properties can be qualitatively understood on the basis of knowledge of the spectrum of in-plane and interplane charge density fluctuations excited during the process of the interplane tunneling. The spectrum of charge fluctuations can be directly measured experimentally by optical reflectivity and electron-energy-loss spectroscopy (EELS).

The essential physical picture of our approach is that the *c*-axis tunneling is strongly suppressed by charge fluctuations excited in the process of tunneling.[6] This anomaly (the so-called Coulomb blockade) is widely observed in many strongly correlated and mesoscopic systems. Other examples of this class of phenomena are orthogonality catastrophes

and a zero-bias anomaly in diffusive systems. The ubiquity of the Coulomb blockade phenomena (static or dynamic) in correlated systems indicates that the anomalous *c*-axis transport properties may be merely a consequence of strong correlations in the cuprates. The necessary condition for the appearance of the Coulomb blockade phenomenon is the strong effective coupling of the tunneling electron to the collective excitations of the liquid. In fact, we can think about the Coulomb blockade as a ''high-energy'' phenomenon of order of Coulomb energy per electron and independent of the low-energy quasiparticle spectral properties, be they Fermi- or non-Fermi liquid. In other words, the tunneling electron couples to the excitations in the broad range of frequencies from low up to high frequencies. The non-Fermi-liquid property (property not present for a three-dimensional Fermi liquid or for a electron gas in the RPA approximation) which is responsible for the anomalous *c*-axis properties is simply the large density of ''detuning'' charge fluctuations (as observed by optical and Raman spectroscopy) over a broad frequency range up to midinfrared frequencies.

Empirically, an overview of the in-plane conductivity and the *c*-axis conductivity in various families of cuprates does not reveal any obvious correlation between their temperature dependencies. On the other hand, the perturbative diagrammatic expression (assuming the equivalent Green's functions in each plane and uncorrelated impurities) for the in-plane conductivity and out-of-plane conductivity[9] are equivalent up to the vertex functions. Therefore the difference in the temperature and frequency dependence of the in- and out-plane conductivities must come *exclusively* from the interplane tunneling probability. For instance, the notion of the ''two-dimensional Luttinger'' liquid is not sufficient by itself to explain *the difference* between the in-plane and out-of-plane resistivities.[7]

One particular approach to explaining the difference between the resistivities along the different directions is based on the highly anisotropic form of the tunneling matrix element $t_\perp(k_x,k_y)$ as a function of the in-plane momentum. Several authors[5] developed a phenomenological approach (assuming as well a strong anisotropy of quasiparticle lifetimes and the density of states around the fermi surface) which seem to fit successfully the experimental data. Two remarks are in order. First, this approach assumes that the tunneling conserves the in-plane momentum. This may be the case in certain situations (possibly, in the superconducting state and in the overdoped regime), but in general this assumption deserves close scrutiny by experiment and theory. In fact, we will argue that the in-plane momentum is not conserved when $\tau_{\rm hop} \gg \tau_{ab}$. Second, the anisotropy of the tunneling matrix element $t_\perp(k_x,k_y)$ is different for some variations of cuprates,[8] and, in general, it can be doping dependent. Thus in some cases, $t_\perp(k_x,k_y)$ may not vanish along the diagonals of the Brillouin zone making the contribution of the diagonal quasiparticles ($k_x=\pm k_y$) to the *c*-axis conductivity nonvanishing contrary to the assumptions of Ref. 5.

In the rest of the paper, we begin by generalizing the standard tunneling formalism, introducing a nontrivial tunneling probability which accounts for the inelastic and elastic (momentum scattering) processes. Then, we calculate this tunneling probability from assumed spectra of ''the detuning fluctuations.'' After that, we calculate the experimentally measured optical conductivity $\sigma_c(\omega,T)$ and the tunneling conductance $\sigma_c(V)$. Finally, we discuss the complex experimental situation and possible extensions of the proposed theory. In the Appendix we discuss several microscopic models giving the spectrum of the charge fluctuations.

Theory. The tunneling formalism. The tunneling of an electron from one plane to another plane can be considered by using the time-dependent tunneling Hamiltonian formalism. The ''blockade effect'' due to the excitation of the electromagnetic modes is accounted by the modulation of the tunneling matrix element by the Coulomb interaction. Thus the part of the Hamiltonian responsible for the transfer of electrons between planes is

$$H_c=\sum_{r_1,r_2} t_\perp(r_1,r_2;t)[a_1^+(r_1)a_2(r_2)+a_2^+(r_2)a_1(r_1)], \tag{1}$$

where the quasiclassical tunneling matrix element $t_\perp(r_1,r_2;t)$ is equal to $t_\perp(r_1,r_2)\exp[-(ie/c)\int_{r_1}^{r_2}A(z,t)dz]$. Gauge invariance dictates the presence of the phase factor $\varphi(r_1,t)=(e/c)\int_{r_1}^{r_2}A(z,t)dz$, where the integral is taken over a path connecting two points r_1 and r_2 on the different planes. Since the optimal tunneling trajectory is perpendicular to the planes (along the z axis), the tunneling matrix element can be written as $t_\perp(r_1,r_2)=t_\perp\delta(r_1-r_2)\exp[-(ie/c)\int_{z_1}^{z_2}A(z,t)dz]$. It conserves the in-plane momentum. Using the tunneling Hamiltonian formalism,[9,10] we get the following expression for the tunneling current $I(t)$ between two planes:

$$\begin{aligned} I(t)=&-\frac{2e}{\hbar^2}\mathrm{Re}\int\int dr dr'\int_{-\infty}^{+\infty}dt' e^{-i(eVt/\hbar)}|t_\perp|^2 \\ &\times P(r-r',t-t')S(r-r',t-t'), \\ P(r-r',t-t')\equiv&\langle e^{i\varphi(r,t)}e^{-i\varphi(r',t')}\rangle, \\ S(r-r',t-t')\equiv&\Theta(t-t')\langle G_R^<(r-r',t-t') \\ &\times G_L^>(r'-r,t'-t)-G_L^<(r-r',t-t') \\ &\times G_R^>(r'-r,t'-t)\rangle, \end{aligned} \tag{2}$$

where $P(r-r',t-t')$ is a phase-phase correlation function between two planes averaged over the equilibrium fluctuations, and V is an applied voltage. The definitions of Green's functions and essential details of the derivation can be found in the Ref. 10. The fact that the hopping time $\tau_{\rm hop}$ is much longer than the in-plane scattering time τ_{ab} allows us to separate the in-plane propagation $S(r-r',t-t')$ and the tunneling probability $P(r-r',t-t')$. It is important to remark at this stage that the long-wavelength fluctuation modes (with the wavelength much longer than the in-plane mean free path) can suppress the tunneling probability without effecting the in-plane motion. The scattering by the

short-wavelength fluctuations is accounted by the spectral properties of the in-plane propagation $S(r-r',t-t')$. The understanding of the properties of the tunneling probability $P(r-r',t-t')$, describing the effect of the ''detuning fluctuations,'' is imperative for any particular problem of the tunneling. The importance of this correlation function was first described in Ref. 11. The novel element here is the discussion of the spatial dependence of the tunneling probability function $P(r-r',t-t')$. The spatial dependence appears to be very important for many questions of the *c*-axis transport properties. As mentioned above, in previous studies of *c*-axis transport in cuprates, specific properties of the tunneling probability (e.g., the tunneling with or without the conservation of the in-plane momentum $k_\parallel$) were assumed. Here we analyze and calculate the tunneling probability from the fluctuation spectrum of the electromagnetic field. We can call the tunneling ''diffusive'' if the in-plane momentum is not conserved (if the momentum is conserved, it can be called specular). In other words, the tunneling is diffusive, if the tunneling probability $P(r-r',t-t')$ is significant only if $|r-r'|/l \lesssim 1$ (where l is a short length scale of order of a lattice constant). It should be noted that, generally speaking, the question of the conservation of in-plane momentum during tunneling is another aspect of tunneling not equivalent to the question of coherence or incoherence of tunneling (that is the question of the dephasing of an electron). Equation (2) can be rewritten in the following form:[10]

$$I(V)=\frac{2eSt_\perp^2}{\hbar}\int dE dE' dk dk' A_1(k,E)A_2(k',E') \times\{f(E)[1-f(E')]P(E+eV-E',k-k') - f(E')[1-f(E)]P(E'-eV-E,k-k')\}, \quad (3)$$

where $A_{1,2}(k,E)$ are the spectral functions, $f(E)$ is a Fermi function, and $P(E,k)$ is a Fourier transform of the function $P(r-r',t-t')$.

The tunneling probability. We need to calculate the correlation function $\langle e^{i\hat{\varphi}(r,t)}e^{-i\hat{\varphi}(r',t')}\rangle$. The averaging can be done if we assume the field $\varphi(r,\tau)$ is Gaussian correlated. Using gauge invariance, the phase $\varphi(r,\tau)$ can be rewritten as $\varphi(r,\tau)=\int_{-\infty}^{\tau}\delta V(r,t)dt$, where $\delta V(r,t)$ is the local voltage difference between two planes. This way we get an expression for the tunneling probability (the calculation is a generalization of the exact calculation from Ref. 9, pp. 273–277):

$$P(\delta r=r-r',\delta t=t-t')\equiv\exp[-R(\delta r,\delta t)],$$

$$R(\delta r,\delta t)\equiv\int\frac{d\omega}{\omega^2}\int d^2q\langle\delta V_{q,\omega}^2\rangle[1-\cos(\omega\delta t+\vec{q}\vec{\delta r})] \times\coth\frac{\omega}{2T}. \quad (4)$$

In this paper, we assume that the most effective ''detuning'' fluctuations are the voltage fluctuations (or related charge fluctuations). It is important to point out that the same method can be used to calculate the ''blocking'' of the tunneling due to any mechanism of in-plane scattering. Since the nature of the ground state of cuprates and therefore the spectrum of the fluctuations is not known, later we examine several general forms of the spectrum. The problem of incoherent tunneling between a couple of two-dimensional planes is a natural generalization of the spin-boson model of quantum dissipation.

In view of the importance of the spatial dependence of the tunneling probability, we give *several different arguments* proving the diffusive nature of the tunneling (if $\tau_{\rm hop}\gg\tau_{ab}$) in the normal state. First of all *a qualitative argument*: if the hopping time is much longer the in-plane scattering time, an electron experiences many inelastic and elastic scattering processes (both not conserving the direction of the in-plane momentum) before the hopping between planes. Thus it is intuitively natural to think that the momentum is not conserved after the hopping. *A straightforward quantitative argument* is given by the analysis of the function $R(\delta r,\delta t)$ in the exponent of the expression for the tunneling probability [Eq. (4)]. To separate the spatial and time dependence of the function $R(\delta r,\delta t)$, we rewrite the multiplier in the integral as $[1-\cos(\omega\delta t+q_x\delta r)]=[1-\cos(\omega\delta t)]+\cos(\omega\delta t)[1-\cos(q_x\delta r)]-\sin(q_x\delta r)\sin(\omega\delta t)$. The integral with the last term vanishes, because this term makes the integral expression antisymmetric with respect to integration over q_x. Thus, we can write

$$R(\delta r,\delta t)=R_0(\delta r,\delta t)+R_1(\delta r,\delta t),$$

$$R_0(\delta t)=\int\frac{d\omega}{\omega^2}\int dq_x q_y\langle\delta V_{q,\omega}^2\rangle\coth\frac{\omega}{2T}[1-\cos(\omega\delta t)],$$

$$R_1(\delta r,\delta t)=2\int\frac{d\omega}{\omega^2}\int dq_x q_y\langle\delta V_{q,\omega}^2\rangle\coth\frac{\omega}{2T}\cos(\omega\delta t) \times\sin^2[(q_x\delta r)/2]. \quad (5)$$

The space-independent part $R_0(\delta t)$ is calculated later in the paper [see Eq. (9)]. Below we calculate the function $R_1(\delta r,\delta t)$ which describes the spatial dependence of the tunneling probability. For this calculation we assume that the fluctuations are uncorrelated in two planes (see below for a more general discussion), and the interplane noise is just twice the in-plane Johnson-Nyquist (JN) noise (voltage fluctuations).[12] The Johnson-Nyquist noise can be calculated from the spectral density of Coulomb noise in the two-dimensional plane valid in the hydrodynamic approximation[13]

$$\langle\delta V_{q,\omega}^2\rangle\simeq 4\pi\sigma_Q\frac{\sigma_2\omega}{\omega^2+4\pi^2\sigma_2^2q^2}, \quad (6)$$

where σ_2 is a two-dimensional conductance, and $\sigma_Q\equiv e^2/\hbar$. We substitute Eq. (6) into the expression (5) for $R_1(\delta r,\delta t)$ and impose an upper cutoff q_c on the q integration to take into account of the fact that the expression (5) is strictly valid only in the long-wavelength limit; thus we take $q\ll q_F$ (but still of the general order of q_F). We also impose on the ω integration a lower cutoff ω_l, the choice of which

will be discussed below. Then, taking into account the fact that we are interested in values of δt which are of the general order of magnitude $\tau_{\rm hop}$ and thus several orders of magnitude larger than $(\sigma_2 q_c)^{-1}$, we see that to a good approximation $R_1(\delta r,\delta t)$ factorizes into a product of a function of δr and a function of δt:

$$R_1(\delta r,\delta t)\simeq F(\delta r)G(\delta t),$$

$$F(\delta r)\equiv\int_0^{2\pi}d\theta\int_0^{q_c\delta r}dx\frac{1-\cos(x\cos\theta)}{x},$$

$$G(\delta t)\equiv\frac{2\sigma_Q}{\pi\sigma_2}\int_{\omega_l}^{\infty}\frac{d\omega}{\omega}\coth\left(\frac{\omega}{2kT}\right)\cos(\omega\delta t). \tag{7}$$

The expression $F(\delta r)$ is approximately $(\pi/8)(q_c\delta r)^2$ for $q_c\delta r\ll 1$ and $2\pi\ln(q_c\delta r)$ for $q_c\delta r\gg 1$. As for the function $G(\delta t)$, we will see below that the ''interesting'' values of δt [i.e., those for which the function $R_0(\delta t)$ does not suppress $P(\delta r,\delta t)$ too badly] are less or of order of $\hbar/2\pi\alpha kT$, where the dimensionless quantity α is typically less or of order of 1. Under these conditions, provided $\hbar\omega_l\ll kT$ which will be satisfied by our choice of ω_l (see below), the integral defining $G(\delta t)$ is dominated by its lower limit and approximately given by the δt-independent expression

$$G(\delta t)\simeq\frac{2\sigma_Q}{\pi\sigma_2}\frac{kT}{\hbar\omega_l}. \tag{8}$$

We will make the choice $\omega_l\sim 1/\tau_{\rm hop}$, on the grounds that once we need to allow for appreciable interplane hopping, Eq. (6) for the noise is no longer applicable [and we expect the expression for $\langle\delta V^2_{q,\omega}\rangle$ to decrease as a higher power of ω for $\omega\to 0$, thereby effectively cutting off the integral (7)]. We thus have for $R_1(\delta r,\delta t)$ the approximate expression

$$R_1(\delta r,\delta t)\simeq(kT\tau_{\rm hop}/\hbar)(\sigma_Q/\sigma_2)(2/\pi)F(q_c\delta r).$$

The salient point, now, is that the quantity $kT\tau_{\rm hop}/\hbar$, which is essentially the ratio of the ab plane and c-axis conductivities, is of order 10^2-10^4 for most of the cuprates, while the ratio σ_2/σ_Q is never greater than about 10. Thus, the quantity R_1 has a value large compared to unity for values of δr small compared to $1/q_c$, and we can approximate the expression $F(\delta r)$ by its limiting form $(\pi/8)(q_c\delta r)^2$. Thus, the ''effective area'' $S_{\rm eff}\equiv(\delta r_{\rm eff})^2$ for which R_1 is appreciable is defined by

$$S_{\rm eff}\sim\frac{4(\sigma_2/\sigma_Q)}{q_c^2(kT\tau_{\rm hop}/\hbar)}$$

and by the above argument this is much smaller than $1/q_c^2$ and thus at most of the order of $1/q_F^2$. At distances of this order formulas such as Eq. (6) should no longer be taken seriously, but the crucial upshot of the argument is that coherence between tunneling events separated in space by more than $\sim 1/q_F$ can be simply neglected. To put it differently, the tunneling is effectively local (diffusive); the effective rms change in momentum in the course of a tunneling event is of order of q_F (see below). It is noteworthy that this is so even if the momentum cutoff q_c on the voltage fluctuations is only a small fraction of q_F; consideration of shorter-wavelength fluctuations can only strengthen this conclusion.

Another argument estimates directly the change of momentum in the process of tunneling. The change of the momentum due to a fluctuation of the electromagnetic potential is $\delta p\sim(e\delta A)/c$, therefore $\langle\delta p^2\rangle=(e/c)^2\langle\delta A^2\rangle$. We can relate the correlation function of the vector potential with the correlation function of the scalar potential by gauge transformation (assuming only longitudinal fluctuations) $\langle A^2_{q,\omega}\rangle=(c^2q^2/\omega^2)\langle\delta V^2_{q,\omega}\rangle$. Using Eq. (6) at low frequencies $\omega<\sigma_2 q$, the correlation function can be approximated as $\langle A^2_{q,\omega}\rangle=(4\pi\sigma_2c^2/\omega^2)[q^2/(\omega^2+\sigma_2^2q^2)]\simeq(4\pi c^2/\omega^2\sigma_2)$. Thus the variance of momentum $\langle\delta p^2\rangle$ is $\langle\delta p^2\rangle=(e^2/\hbar c^2)\int dq q\int d\omega(c^2/\omega^2)[\omega\coth(\omega/2T)/\sigma_2]$. We see that the integral over frequency is diverging for $\omega<T$ as $\int(d\omega/\omega^2)$. This integral can be cut off again on $1/\tau_{\rm hop}$, therefore $\langle\delta p^2\rangle\sim q_c^2(\sigma_Q/\sigma_2)(\tau_{\rm hop}kT)$ (at any finite temperature). This estimate gives a result equivalent to the earlier calculation. This indicates again that the in-plane momentum is completely randomized after the process of tunneling. All these arguments validate theories of c-axis transport in the normal state assuming nonconservation of in-plane momentum during tunneling. The fact of the nonconservation of momentum $k_\parallel$ in the normal state (if $\tau_{\rm hop}\gg\tau_{ab}$) is quite general, a sufficient condition as can be seen from the above discussion is the ohmic density of the noise $\langle\delta V_r(\omega)^2\rangle\sim\omega$ for $\omega\to 0$. It is important to stress that the ''diffusivity'' of the tunneling is due to specific form of the spectrum of the voltage fluctuations (and not due to short links or impurities), for instance, it may not be true in the superconducting state. It is important to point out that the question of the in-plane momentum conservation for c-axis tunneling can be examined experimentally[14] supporting or disproving the above arguments.

When the tunneling is diffusive, the ''detuning fluctuations'' are simply the local voltage fluctuations

$$\langle\delta V^2_\omega\rangle=\int d^2q\langle\delta V^2_{q,\omega}\rangle\equiv\alpha\omega,$$

where α is the microscopic parameter describing the ohmic density of the noise. As can be seen below this microscopic parameter α is sufficient to describe all dc and ac dependencies of the c-axis conductivity. When the two planes are widely separated and isolated (the situation possibly realized in Bi-2201), the noise spectra in each plane are uncorrelated, so that the interplane noise spectrum is just the sum of the noise spectra in each plane. In such a case, the coefficient α should be determined only by the properties of the copper-oxygen plane. If the planes are moved closer together, so that the interplane Coulomb interaction become relevant, the intensity of the interplane noise (assuming no interplane hopping) increases due to the presence of the acoustic (out-of-phase) plasmon in this bi-layer structure. Unfortunately, realistically the noise between planes can be suppressed and correlated at low frequencies because of interplane hopping and become dependent on the interlayer structure thus imply-

ing different values of α for different cuprate materials. It is known experimentally that the detailed temperature dependence of the *c*-axis resistivity is very sensitive to several factors (sample preparation, interlayer structure and doping). The role of the interlayer structure (different intercalating atoms, chains and additional layers present in some compounds) and the structure of the tunneling matrix element $t_\perp(k_x,k_y)$ (e.g., in-plane anisotropy) is not clear, it makes the question of the interlayer noise and the *c*-axis transport properties even more complex.

We have examined several models describing the voltage noise to estimate the microscopic parameter α (see the Appendix for the discussion of this question, also Ref. 15). The goal of such exercise is to verify a crude consistency between the estimate of α from microscopic noise and the parameter α required to describe the *c*-axis transport dependencies. The difficulty (not surprising since these calculations assume Fermi-liquid or diffusive spectra of density fluctuations) common to all of the calculations (see the Appendix) is that the ''microscopic'' value of α is significantly smaller than the value of order of 1 necessary to explain the *c*-axis transport properties. For our approach to be valid a large ($\alpha\sim1$) density of ''detuning fluctuations'' (not present in RPA or Fermi-liquid pictures) is vital. *An alternative approach* is to extract the charge fluctuation noise directly from the optical reflectivity measurements. The most dramatic difference between good metals and cuprates seen in Raman and optical measurements of the in-plane dielectric constant $\epsilon_{ab}(q,\omega)$ is that the low-frequency noise for cuprates is ohmic (linear) even for $q\to0$. If we write $\mathrm{Im}[-1/\epsilon_{ab}(q,\omega)]=\gamma\omega$ for $q\simeq0$, then from the experimental data[16] $\gamma\simeq0.2$ eV^{-1}. Since $\langle\delta V_\omega^2\rangle=\int[dqq/(2\pi)^2]V_2(q)\mathrm{Im}(-1/\epsilon_{k,\omega})\simeq(e^2q_c/2\pi)\gamma\omega$, we get $\alpha\simeq(e^2q_c/2\pi)\gamma$. If we take the upper cutoff wave vector $q_c\sim2\pi/a$ (a is a in-plane lattice constant), we get $\alpha\simeq(e^2/a)\gamma\simeq1.2$. It shows that α may be of order of 1, exactly what is required to explain the *c*-axis transport properties. We now use the ''local'' approximation justified above to calculate the dc and ac *c*-axis conductivity.

First of all, we calculate the local tunneling probability $P(r=0,t)$ [or rather its Fourier transform $P(\epsilon,k-k')$]. The quantity $P(\epsilon,k-k')$ can be interpreted as a probability to exchange an energy ϵ and momentum $(k-k')$ with fluctuating fields. Since, as argued above, the tunneling probability is strongly peaked at $r=0$ (if $\tau_{\mathrm{hop}}\gg\tau_{ab}$), the Fourier transform in momentum space $P(\epsilon,k-k')$ is essentially independent of $(k-k')$, therefore we omit the index $(k-k')$ below. For the small values of the coefficient α ($\alpha\ll1$) describing the spectral density of the local voltage noise $\langle\delta V_\omega^2\rangle=\alpha\omega$, the tunneling probability can be calculated analytically. The function $R_0(\delta t)$ in the exponent of the $P(\delta r=0,\delta t)$ is

$$R_0(\delta t)=\int\frac{d\omega}{\omega^2}\langle\delta V_\omega^2\rangle[1-\cos(\omega\delta t)]\coth\left(\frac{\omega}{2kT}\right)$$
$$=\int_0^\infty\frac{d\omega}{\omega^2}\alpha\omega[1-\cos(\omega\delta t)]\coth\left(\frac{\omega}{2kT}\right)$$
$$\simeq\frac{\delta t}{\tau_\phi}+2\alpha\ln(\omega_c/kT), \tag{9}$$

where $\tau_\phi=1/2\pi\alpha kT$ and ω_c is a high-frequency cutoff which we associate with the inverse of the underbarrier traversal time $\hbar/\tau_{tr}\sim1$ eV. After a Fourier transform, it gives the probability of tunneling $P(\epsilon)=S_{\mathrm{eff}}(kT/\omega_c)^{2\alpha}\{2\pi\alpha kT/[\epsilon^2+(2\pi\alpha kT)^2]\}$, where S_{eff} is the effective area of tunneling discussed above. For α of order of 1, the tunneling probability is strongly suppressed and weakly depends on ϵ (for $\epsilon\ll kT$). It cannot be calculated explicitly analytically, but at small ϵ can be approximated as a function independent of ϵ: $P(\epsilon)\simeq S_{\mathrm{eff}}(kT/\omega_c)^{2\alpha}(1/2\pi\alpha kT)$.

The optical conductivity $\sigma_c(\omega,kT)$. *The tunneling conductance* $\sigma_c(V)$. Equation (3) can be further transformed assuming the diffusive tunneling probability. In this case, we can integrate over the in-plane momenta (k,k') separately. The next transformation is due to the detailed balance condition (see Ref. 11), eventually we can write the expression for the dc conductivity $\sigma_c(V)$ as a function of the applied voltage V:

$$\sigma_c(V)=\frac{et_\perp^2d\nu_{2\mathrm{D}}^2}{\hbar}\frac{1-e^{-\beta eV}}{V}\int_{-\infty}^{+\infty}d\epsilon\frac{\epsilon P(eV-\epsilon)}{1-e^{-\beta\epsilon}}. \tag{10}$$

From Eq. (10) in the limit of small voltage ($V\ll kT$) (for all α as long as the tunneling is diffusive) appropriate for the dc measurements, we calculate the temperature dependence of the *c*-axis conductivity

$$\sigma_c(T)=\frac{e^2}{\hbar}t_\perp^2d\nu_{2\mathrm{D}}^2S_{\mathrm{eff}}\left(\frac{kT}{\omega_c}\right)^{2\alpha}\frac{1}{2\pi\alpha}. \tag{11}$$

This result describes the *c*-axis resistivity $\rho_c(T)$ either constant or diverging at low temperatures found experimentally in several compounds (Bi-family, Hg-1201, Tl-2212, Tl-1212, and slightly underdoped La-214).

In order to calculate the optical conductivity $\sigma_c(\omega)$, we make the following observation. If we apply the external dc voltage, the tunneling probability acquires an additional phase factor $P(t-t')\sim\exp[(i/\hbar)\int_{t'}^t Vd\tau]$ [in Eq. (3) it gives a shift in the energy difference (after a Fourier transform) $P(E+eV-E')$]. Thus schematically, the conductance is $\sigma_c(V)\sim(1/V)\int dt'e^{(i/\hbar)V(t-t')}(\cdots)$. For the ac voltage,

$$P(t-t')\sim\exp\left(\frac{i}{\hbar}\int_{t'}^t Ve^{i\omega\tau}d\tau\right)$$
$$=\exp\left(\frac{i}{\hbar}V\frac{e^{i\omega t}-e^{i\omega t'}}{i\omega}\right)$$
$$\approx1+\frac{V}{\hbar}\frac{e^{i\omega t}}{\omega}(1-e^{i\omega(t-t')}). \tag{12}$$

In the linear response (the optical conductivity is a linear response) and separating a corresponding harmonic of the current proportional to $e^{i\omega t}$, we conclude that the dependence of the conductivity $\sigma_c(\omega)$ on the frequency ω is equivalent to the dependence on the voltage V, such that $\sigma_c(\omega)=\sigma_c(V\to\omega)$ (if the tunneling is incoherent and diffusive). Namely,

$$\sigma_c(\omega)=\frac{et_\perp^2 d\nu_{2D}^2}{\hbar}\frac{1-e^{-\beta\omega}}{\omega}\int_{-\infty}^{+\infty}d\epsilon\frac{\epsilon P(\omega-\epsilon)}{1-e^{-\beta\epsilon}}. \quad (13)$$

A ubiquitous nearly flat optical response for $\sigma_c(\omega)$ is observed experimentally (if measurements exist) in these compounds. Indeed, a qualitative and numerical analysis of the frequency dependence $\sigma_c(\omega)$ of Eq. (13) (or equivalently dependence on the voltage) indicates a very weak dependence on frequency.

It is interesting that under the conditions of incoherent tunneling Ohmic I-V curves [constant conductance $\sigma_c(V)$ as a function of voltage] correspond to a flat optical conductivity $\sigma_c(\omega)$. The correspondence $\sigma_c(\omega)=\sigma_c(V\rightarrow\omega)$ can be directly checked experimentally by comparing the tunneling conductance $\sigma_c(V)=I(V)/V$ and the frequency dependence of $\sigma_c(\omega)$ from the optical reflectivity measurements. The graphs of $\sigma_c(V)$ and $\sigma_c(\omega)$ can be taken from experimental papers.[17] It appears to be roughly true for optimally doped and underdoped compounds in the normal state.

Discussion of experiments. The above results are insufficient to describe all experimental data for different dopings. In optimally doped La-214 and Y-123 and some other overdoped cuprates the c-axis resistivity has a linear temperature dependence with a large ''residual'' value (intercept at $T=0$ K).[18] In these compounds the hopping time becomes comparable to the in-plane scattering time. Thus we can assume that specular tunneling becomes possible. In this crossover situation between two limiting pictures of the tunneling between planes and the anisotropic band along c-axis, we can think about two channels of conduction. One channel is diffusive, while another one is specular. The tunneling probability is the sum of probabilities to tunnel without and with the conservation of the in-plane momentum. In this case, the total c-axis conductivity is the sum of conductivities in each channel. If the Fermi surface has very anisotropic properties, electrons from one part of the Fermi surface can tunnel specularly, while electrons from other parts tunnel diffusively. In this scenario, the second channel of conduction (conserving $k_\parallel$) can be due to the diagonal parts of the Fermi surface in the normal state. It appears from photoemission experiments that the quasiparticles along diagonals of the Brillouin zone ($k_x=\pm k_y$) have longer lifetimes $\tau_{ab,\mathrm{diag}}$ (which should be compared with the hopping time). These quasiparticles can tunnel then with conserved momentum. It is natural to suggest that for overdoped cuprates the c-axis transport is dominated by incoherent, but specular channel, while for underdoped cuprates the diffusive channel is only present.

We can calculate the conductivity of a specular channel, if we assume that the time dependence of the specular tunneling probability is $P(\delta t)\simeq\exp(-\delta t/\tau_\phi)$ with τ_ϕ calculated for a weak detuning, that is $\tau_\phi=1/2\pi\alpha kT$. Thus we substitute the tunneling probability of the form $P_2(\epsilon-\epsilon',k-k')=\delta(k-k')\{2\pi\alpha kT/[(\epsilon-\epsilon')^2+(2\pi\alpha kT)^2]\}$ (or any form $P_2(\epsilon-\epsilon',k-k')\simeq\delta(k-k')1/kTf[(\epsilon-\epsilon')/kT]$) to the Eq. (3), we get the contribution to the c-axis conductivity

$$\sigma_{2,c}(T)=\frac{e^2}{\hbar}t_\perp^2 d\nu_{2D}\frac{A}{kT}, \quad (14)$$

where A is a numerical coefficient. This result is a well-known result for incoherent tunneling with conservation of the in-plane momentum.[19] It is suggested to explain the linear temperature dependence of c-axis resistivity observed in some compounds.[20] Another important consideration is that the band calculations predict a significant angular dependence of $t_\perp$ in some families of cuprates.[8] Due to this reason, the contribution from specular channel (from diagonal parts of the Fermi surface) can be reduced.

Yet another complexity of cuprates with multiple planes per unit cell is the question of intraunit cell and intercell conduction. In this case, the resistance associated with hopping between planes of the unit cell (intracell resistance) and the resistance associated with hopping between different unit cells (intercell resistance) should be discussed. The total resistance is certainly a sum of intraunit and interunit cell resistances. It may not be correct to assume (as is frequently done in the literature) that the intracell resistance is negligible; a systematic experimental investigation of this question is necessary. In the discussion of this paper we assumed the voltage noise to be uncorrelated between different planes and neglected the interplane Coulomb interaction; these factors must be taken into account in the consideration of the intracell hopping.

In conclusion, a picture of c-axis interlayer (and intercell) tunneling strongly suppressed by voltage fluctuations is proposed. This approach can provide a consistent understanding of observed temperature and frequency dependencies of c-axis conductivity in the normal state.

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APPENDIX

In this appendix, we demonstrate an example of a calculation of the parameter α describing the low frequency voltage fluctuations based on a particular microscopic model of the density fluctuation spectrum. At the end of the Appendix, we list several results calculated for various other microscopic models.

If the two planes are widely separated, then the voltage fluctuations can be treated independently in each plane. In this case, the interplane noise is just twice the in-plane local Johnson-Nyquist noise in each plane. For a high-density electron gas in a hydrodynamic approximation ($\omega\tau\ll1$ and $ql\ll1$), the charge density-density susceptibility of the two-dimensional electron gas can be written as[21]

$$\chi(k,\omega)=\frac{s^2k_{\mathrm{TF}}k^2}{2\pi e^2}\frac{1}{\omega(\omega+i/\tau)-s^2k^2-s^2k_{\mathrm{TF}}k},$$

where $s^2=v_F^2/2$, $k_{\mathrm{TF}}=2\pi n_0 e^2/ms^2$ is the Thomas-Fermi wave number, and τ is a phenomenological relaxation time. Using the fluctuation-dissipation theorem, the spectral density of the voltage fluctuations is

$$\langle V_{k,\omega}^2\rangle = \frac{2\hbar}{\pi} V_k^2 \,\mathrm{Im}\,\chi(k,\omega),$$

where $V_k=2\pi e^2/k$ is the two-dimensional Coulomb interaction, and a factor of 2 in the above expression is due to the doubled Johnson-Nyquist noise. Eventually, we need to calculate the partial frequency-dependent spectral density of the voltage noise $\langle V_\omega^2\rangle=(1/\hbar^2)\int(dkk/2\pi)\langle V_{k,\omega}^2\rangle$. The calculation gives

$$\langle V_\omega^2\rangle = \frac{2}{\pi\hbar}\int \frac{dkk}{2\pi}\frac{(2\pi e^2)^2}{k^2}\frac{s^2 k_{\mathrm{TF}} k^2}{2\pi e^2} \times \frac{\omega(1/\tau)}{(\omega/\tau)^2+(s^2k^2+s^2k_{\mathrm{TF}}k)^2} \simeq \frac{2e^2}{\pi\hbar s^2 k_{\mathrm{TF}}\tau}\omega \ln\left(\frac{s^2k_{\mathrm{TF}}^2\tau}{\omega_c}\right),$$

where ω_c is an infrared cutoff frequency. It implies to the accuracy of the value of the logarithm that the parameter α is

$$\alpha \equiv \frac{e^2}{s^2 k_{\mathrm{TF}}\tau}\frac{2}{\pi\hbar} = \frac{\hbar}{\pi\epsilon_F\tau}.$$

We can rewrite this expression in the following form:

$$\alpha = \frac{2}{\pi^2}\frac{\sigma_Q}{\sigma_{2\mathrm{D}}},$$

where $\sigma_Q=e^2/\hbar$, $\sigma_{2\mathrm{D}}=e^2\nu_{2\mathrm{D}}D=(2/\pi)[e^2(\epsilon_F\tau)/\hbar^2]$ is the two-dimensional conductivity (it can be shown to have a Drude frequency dependence). The above results may be used to estimate the value of the parameter α in the cuprates. Two difficulties can be seen from such literal application of the above model. First of all, the value of α is significantly smaller than one. Second, and more importantly, the parameter α appears to be temperature dependent. It should be realized that the spectrum of charge fluctuations in the cuprates is much more complex and not represented correctly by the above simple model. We investigated several other simple microscopic models in order to get further insight into this question. At this moment, it seems more reasonable to extract the charge fluctuation spectrum directly from experiment as shown in the text of this paper.

The results for several other microscopic models are summarized below. If the spectrum of charge fluctuations is given by the weakly damped acoustic two-dimensional plasmon, then the parameter α is $\alpha\sim\sigma_Q^2/s^2(1+k_{\mathrm{TF}}d)$, where d is the interplane distance. Another calculation taking the voltage noise due to electron-hole pairs of the two-dimensional Fermi liquids gives $\alpha\simeq 1/4\pi$. If the interplane Coulomb interaction is taken into account in the RPA approximation for the same calculation (el-hole pairs), $\alpha=1/(2\pi k_F d)$. We hope to present the details of calculations and expanded arguments elsewhere.

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