Vortex state in a doped Mott insulator

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We analyze the recent vortex core spectroscopy data on cuprate superconductors and discuss what can be learned from them about the nature of the ground state in these compounds. We argue that the data are inconsistent with the assumption of a simple metallic ground state and exhibit characteristics of a doped Mott insulator. A theory of the vortex core in such a doped Mott insulator is developed based on the $U(1)$ gauge-field slave boson model. In the limit of vanishing gauge-field stiffness such a theory predicts two types of singly quantized vortices: an insulating ''holon'' vortex in the underdoped and a metallic ''spinon'' vortex in the overdoped region of the phase diagram. We argue that the holon vortex exhibits a pseudogap excitation spectrum in its core qualitatively consistent with the existing experimental data on $Bi_2Sr_2CaCu_2O_8$. As a test of this theory we propose that the spinon vortex with a metallic core might be observable in the heavily overdoped samples.

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I. INTRODUCTION

The nature of the ground state as a function of doping remains one of the recurring unresolved issues in the theory of high-T_c cuprate superconductors. The problem is partly due to formidable difficulties related to the theoretical description of doped Mott insulators and partly due to experimental hurdles in accessing the normal-state properties in the $T\rightarrow 0$ limit because of the intervening superconducting order. Probes that suppress superconductivity and reveal the properties of the underlying ground state are therefore of considerable value. So far only pulsed magnetic fields[1] in excess of H_{c2} and impurity doping beyond the critical concentration[2] have been used towards this goal. Here we argue that the vortex core spectroscopy performed using scanning tunneling microscope (STM) can provide insights into the nature of the ground state in cuprates. We analyze the existing experimental data[3–6] and conclude that they imply a strongly correlated ''normal'' ground state, presumably derivable from a doped Mott insulator. We then develop a theoretical framework for the problem of tunneling in the vortex state of such a doped Mott insulator.

In the vortex core the superconducting order parameter is locally suppressed to zero and the region within a coherence length ξ from its center can be to the first approximation thought of as normal. Spectroscopy of the vortex core therefore provides information on the normal-state electronic excitation spectrum in the $T\rightarrow 0$ limit. More accurately, the core spectroscopy reflects the spectrum in the spatially nonuniform situation where the order parameter amplitude rapidly varies in response to the singularity in the phase imposed by the external magnetic field. In order to extract useful information regarding the underlying ground state from such measurements a detailed understanding of the vortex core physics is necessary. So far the problem has been addressed using the weak coupling approach based on the Bogoliubov–de Gennes theory generalized to the *d*-wave symmetry of the order parameter,[7–10] and semiclassical calculations.[11–13] The early theoretical debate focused on the existence or absence of the vortex core bound states.[12,14,15] This debate, now resolved in favor of the absence of any bound states in the pure $d_{x^2-y^2}$ state,[9,10,16] has somewhat eclipsed the possibly more important issues related to the nature of the ground state in cuprates.

The body of work based on mean-field, weak-coupling calculations[8–10,13] yields results for the local density of states in the vortex core which exhibit two generic features: (i) the coherence peaks (occurring at $E=\pm\Delta_0$ in the bulk) are suppressed, with the spectral weight transferred to a (ii) broad featureless peak centered around the zero energy. Here we wish to emphasize the heretofore little appreciated fact that these features are *qualitatively inconsistent* with the existing experimental data on cuprate superconductors. STM spectroscopy on $Bi_2Sr_2CaCu_2O_8$ (BSCCO) at 4.2 K indicates a ''pseudogap'' spectrum in the vortex core with the spectral weight from the coherence peaks at $\pm\Delta_0\simeq 40$ meV transferred to *high energies*, and no peak whatsoever around $E=0$.[4] Recent high-resolution data on the same compound[6] confirmed these findings down to 200 mK and found evidence for weak bound states at ± 7meV. Experiments on $YBa_2Cu_3O_7$ (YBCO) (Ref. 3) also indicate low-energy bound states, but are somewhat more difficult to interpret because of the high zero-bias conductance of unknown origin appearing even in the absence of magnetic field.

The fundamental discrepancy between the theoretical predictions and the experimental findings strongly suggests that models based on a simple weak coupling theory break down in the vortex core. The pseudogap observed in the core hints that the underlying ground state revealed by local suppression of the superconducting order parameter is a doped Mott insulator and not a conventional metal. Taking into account the effects of strong correlations appears to be necessary to consistently describe the physics of the vortex core. Conversely, studying the vortex core physics could provide information essential for understanding the nature of the underlying ground state in cuprates.

One step in this direction was taken by Arovas *et al.*[17] who proposed that within the framework of the SO(5) theory[18] vortex cores could become antiferromag-

netic (AF). They found that such AF cores can be stabilized at low T but only in the close vicinity of the bulk AF phase. In contrast, experimentally the pseudogap in the core is found to persist into the overdoped region.[4] More recently, microscopic calculations within the same model[19] revealed electronic excitations in such AF cores with behavior roughly resembling the experimental data. Quantitatively, however, these spectra exhibit asymmetric shifts in the coherence peaks (related to the fact that spin gap in the AF core is no longer tied to the Fermi level) not observed experimentally. These discrepancies suggest that generically cores will not exhibit the true AF order. Finally, these previous approaches are still of the Hartree-Fock-Bogoliubov type and cannot be expected to properly capture the effects of strong correlations.

Here we consider a model for the vortex core based on a version of the $U(1)$ gauge-field slave boson theory formulated recently by Lee.[20] Originally proposed by Anderson[21] the slave boson theory was formulated to describe strongly correlated electrons in the CuO_2 planes of the high-T_c cuprates. Various versions of this theory have been extensively discussed in the literature.[22–26] Interest in spin-charge separated systems revived recently[27–29,20] due to the realization that they provide a natural description of the pseudogap phenomenon observed in the underdoped cuprates. The common ingredient in these theories is ''splintering'' of the electron into quasiparticles carrying its spin and charge degrees of freedom. Within the theories based on Hubbard and t-J models this splintering is formally implemented by the decomposition of the electron creation operator

$$c_{i\sigma}^{\dagger}=f_{i\sigma}^{\dagger}b_i \tag{1}$$

into a fermionic spinon $f_{i\sigma}$ and bosonic holon b_i. The local constraint of the single occupancy $b_i^{\dagger}b_i+f_{i\sigma}^{\dagger}f_{i\sigma}=1$ is enforced by a fluctuating $U(1)$ gauge field **a**. The mean-field phase diagram is known to contain four phases distinguished by the formation of spinon pairs, $\Delta_{ij}=\langle \epsilon_{\sigma\sigma'}f_{i\sigma}^{\dagger}f_{j\sigma'}^{\dagger}\rangle$, and Bose-Einstein condensation of the individual holons $b=\langle b_i\rangle$,[26] and is illustrated in Fig. 1.

The effects of magnetic field on such a spin charge separated system is most conveniently studied in the framework of an effective Ginzburg-Landau (GL) theory for the condensate fields Δ and b. The corresponding effective action can be constructed[30,31] based on the requirements of local gauge invariance with respect to the physical electromagnetic vector potential **A** and the internal gauge field **a**:

$$f_{\mathrm{GL}}=|(\nabla-2i\mathbf{a})\Delta|^2+r_{\Delta}|\Delta|^2+\frac{1}{2}u_{\Delta}|\Delta|^4$$
$$+|(\nabla-i\mathbf{a}-ie\mathbf{A})b|^2+r_b|b|^2+\frac{1}{2}u_b|b|^4+v|\Delta|^2|b|^2$$
$$+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2+f_{\mathrm{gauge}}. \tag{2}$$

The factor of 2 in the spinon gradient term reflects the fact that *pairs* of spinons were assumed to condense. f_{gauge} describes the dynamics of the internal gauge field **a**. We note that unlike the physical electromagnetic field **A** the gauge field **a** has no independent dynamics in the underlying microscopic model since it serves only to enforce a constraint. Sachdev[30] and Nagaosa and Lee[31] assumed that upon integrating out the microscopic degrees of freedom a term

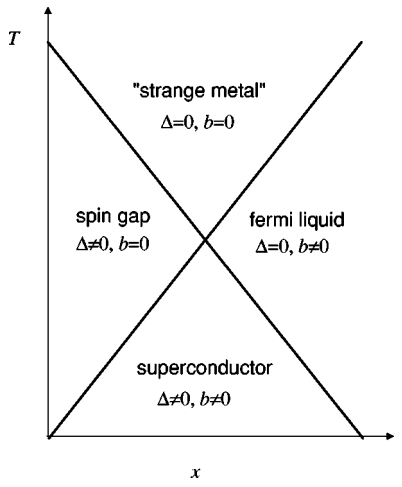

FIG. 1. Schematic phase diagram of the system with spin-charge separation in the doping-temperature plane, as applied to cuprate superconductors.

$$f_{\mathrm{gauge}}=\frac{\sigma}{2}(\nabla\times\mathbf{a})^2 \tag{3}$$

is generated in the free energy. They then analyzed vortex solutions of the free energy (2) and came to the conclusion that two types of vortices are permissible: a ''holon vortex'' with the singularity in the b field and a ''spinon vortex'' with the singularity in the Δ field. Because holons carry electric charge e the holon vortex is threaded by electronic flux quantum hc/e, i.e., twice the conventional superconducting flux quantum $\Phi_0=hc/2e$. Spinons on the other hand condense in pairs, and the spinon vortex therefore carries flux Φ_0. Stability analysis then implies that spinon vortex will be stable over the most of the superconducting phase diagram, while the hc/e holon vortex can be stabilized only in the close vicinity of the phase boundary on the underdoped side.[30,31] This is a direct consequence of the fact that singly quantized vortices are always energetically favorable.[32,33]

As far as the electronic excitations are concerned, the spinon vortex is virtually indistinguishable from the vortex in a conventional weak-coupling mean-field theory: the spin gap Δ, which gives rise to the gap in the electron spectrum, vanishes in the core. Consequently, the vortex state based on the results of Sachdev-Nagaosa-Lee (SNL) theory[30,31] does not exhibit the pseudogap in the core and suffers from the same discrepancy with the experimental data as the weak-coupling theories[7–10] based on the conventional Fermi-liquid description. Moreover, no evidence exists at present for stable doubly quantized holon vortices predicted by SNL. What is needed to account for the experimental data is a *singly quantized holon vortex* stable over the large portion of the superconducting phase in the phase diagram of Fig. 1. In the core of such a holon vortex the spin gap Δ remains finite

and leads naturally to the pseudogap excitation spectrum. In what follows we show that under certain conditions the free energy (2) permits precisely such a solution.

The results of the SNL theory are predicated upon the assumption that the ''stiffness'' σ of the gauge field is relatively large and that singular configurations in which $\nabla\times\mathbf{a}$ contains a full flux quantum through an elementary plaquette are prohibited. Consider now a precisely opposite physical situation, allowing unconstrained fluctuations in **a**. This amounts to the assumption that the $f_{\rm gauge}$ term (3) can be neglected in Eq. (2), i.e., $\sigma\to 0$. Physically this corresponds to the ''extreme type-I'' limit of the GL ''superconductor'' (2) with respect to fluctuations in **a**. Based on Elitzur's theorem,[34] Nayak[35] recently argued that the exact local $U(1)$ symmetry of the model cannot be broken, implying absence of the phase stiffness term (3) at all energy scales. Our assumption therefore appears reasonable and in Sec. III we shall give a more thorough discussion of the significance of the $f_{\rm gauge}$ term for the vortex solutions of interest here. For the time being we shall assume that $f_{\rm gauge}$ can be neglected and explore physical consequences of the resulting theory.

$f_{\rm GL}$ given by Eq. (2) is quadratic in **a** and with the $\nabla\times\mathbf{a}$ term absent the gauge fluctuations can be trivially integrated out. Within the closely related microscopic model this procedure has been recently implemented by Lee.[20] The resulting effective free-energy density reads

$$f=f_{\rm amp}+\frac{\rho_\Delta^2\rho_b^2}{4\rho_\Delta^2+\rho_b^2}(\nabla\phi-2\nabla\theta+2e\mathbf{A})^2+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2, \tag{4}$$

where we have set $\Delta=\rho_\Delta e^{i\phi}$, $b=\rho_b e^{i\theta}$, and

$$f_{\rm amp}=(\nabla\rho_\Delta)^2+r_\Delta\rho_\Delta^2+\frac{1}{2}u_\Delta\rho_\Delta^4+(\nabla\rho_b)^2+r_b\rho_b^2+\frac{1}{2}u_b\rho_b^4+v\rho_\Delta^2\rho_b^2 \tag{5}$$

is the amplitude piece. The most important feature of the effective free energy (4) is that it no longer depends on the individual phases ϕ and θ but only on their particular combination

$$\Omega=\phi-2\theta. \tag{6}$$

Since the physical superconducting order parameter $\Psi=\Delta^* b^2=\rho_\Delta\rho_b^2 e^{-i(\phi-2\theta)}$ it is reasonable to identify Ω with the phase of a *Cooper pair*. Physically, the unconstrained fluctuations of the gauge field in Eq. (2) resulted in partial restoration of the original electronic degrees of freedom in Eq. (4). In the underlying microscopic model this means that on long length scales spinons and holons are always confined, in agreement with Elitzur's theorem.[34,35] On length scales shorter than the confinement length, such as inside the vortex core, spinons and holons can still appear locally decoupled. In the present effective theory this aspect is reflected by two amplitude degrees of freedom present in Eq. (4). More detailed discussion of these issues is given in Refs. 20 and 35.

We have thus arrived at an effective theory of a spin-charge separated system containing one phase degree of freedom Ω and two amplitudes, ρ_Δ and ρ_b. Deep in the superconducting phase, where both amplitudes are finite, the physics of Eq. (4) will be very similar to that of a conventional GL theory. In the situations where the superconducting order parameter Ψ is strongly suppressed, such as in the vortex core, near an impurity or a wall, the theory has an extra degree of richness, associated with the fact that it is sufficient (and generally preferred by the energetics) that only *one* of the two amplitudes is suppressed. Since the two amplitudes play very different roles in the electronic excitation spectrum, the effective theory (4) will lead to a number of nontrivial effects.

To illustrate this consider what will happen in the core of a superconducting vortex. Under the influence of the magnetic field the phase Ω will develop a singularity such that $\nabla\Omega\sim 1/r$ close to the vortex center. For the free energy to remain finite the amplitude prefactor in the second term of Eq. (4) must vanish for $r\to 0$. This is analogous to $|\Psi|$ vanishing in the core of a conventional vortex. In the present case, however, it is sufficient when the product $\rho_\Delta\rho_b$ vanishes. Since suppressing any of the two amplitudes costs condensation energy, in general only one amplitude will be driven to zero. Which of the two is suppressed will be determined by the energetics of the amplitude term (5). On general grounds we expect that the state in the vortex core will be the same as the corresponding bulk ''normal'' state obtained by raising the temperature above T_c. Thus, very crudely, we expect that the holon vortex will be stable in the underdoped while the spinon vortex will be stable in the overdoped region of the phase diagram Fig. 1.

An important point by which our approach differs from the SNL theory is that in the present theory *both* types of vortices carry the *same* superconducting flux quantum Φ_0 and thus compete on equal footing. This is a direct consequence of our assumption of the vanishing phase stiffness σ.

In what follows we study in detail the vortex solutions of the free energy (4). Our main objective is to obtain the precise estimates for the energy of the two types of vortices as a function of temperature and doping and deduce the corresponding phase diagram for the state inside the vortex core. We show that for generic parameters in Eq. (4) the singly quantized holon vortex with a pseudogap spectrum in the core can be stabilized over a large portion of the superconducting phase, as required by the experimental constraints discussed above.

II. SOLUTION FOR A SINGLE VORTEX

A. General considerations

In order to provide a more quantitative discussion we now adopt some assumptions about the coefficients entering the free energy (4). We assume that

$$r_i=\alpha_i(T-T_i),\quad i=b,\Delta, \tag{7}$$

where T_i are corresponding ''bare'' critical temperatures, which we assume depend on doping concentration x in the following way:

$$T_\Delta = T_0(2x_m - x), \quad T_b = T_0 x. \tag{8}$$

Here x_m denotes the optimal doping and T_0 sets the overall temperature scale. We furthermore assume that u_i and v are all positive and independent of doping and temperature. It is easy to see that such choice of parameters qualitatively reproduces the bulk phase diagram of cuprates in the x-T plane shown in Fig. 1. The effect of the v term is to suppress T_c from its bare value away from the optimal doping. In real systems fluctuations will lead to additional suppression of T_c which we do not consider here.

In the absence of perturbations the bulk values of the amplitudes are given by

$$\overline{\rho_\Delta^2} = -(r_\Delta u_b - r_b v)/D,$$

$$\overline{\rho_b^2} = -(r_b u_\Delta - r_\Delta v)/D, \tag{9}$$

with $D = u_b u_\Delta - v^2$. In analogy with conventional GL theories we may define coherence lengths for the two amplitudes[30]

$$\xi_\Delta^{-2} = -(r_\Delta - r_b v/u_b),$$

$$\xi_b^{-2} = -(r_b - r_\Delta v/u_\Delta), \tag{10}$$

one of which always diverges at T_c as $(T - T_c)^{-1/2}$.

Minimization of the free energy (4) with respect to the vector potential $\mathbf{A}$ yields an equation

$$\nabla \times \nabla \times \mathbf{A} = e\rho_s(\nabla\Omega - 2e\mathbf{A}), \tag{11}$$

where

$$\rho_s = \frac{4\rho_\Delta^2 \rho_b^2}{4\rho_\Delta^2 + \rho_b^2} \tag{12}$$

is the effective superfluid density. The term in brackets can be identified as twice the conventional superfluid velocity

$$\mathbf{v}_s = \frac{1}{2}\nabla\Omega - e\mathbf{A}.$$

Making use of the Ampere's law $4\pi\mathbf{j} = \nabla \times \mathbf{B}$, we see that Eq. (11) specifies the supercurrent in terms of superfluid density and velocity: $\mathbf{j} = 2e\rho_s \mathbf{v}_s$. Minimization of Eq. (4) with respect to Ω then implies $\nabla \cdot \mathbf{j} = 0$; the supercurrent is conserved.

Minimizing the free energy (4) with respect to the amplitudes results in the pair of coupled GL equations:

$$-\nabla^2\rho_\Delta + r_\Delta\rho_\Delta + u_\Delta\rho_\Delta^3 + v\rho_b^2\rho_\Delta + \frac{4\rho_\Delta^2\rho_b^2}{(4\rho_\Delta^2 + \rho_b^2)^2}\mathbf{v}_s^2 = 0, \tag{13a}$$

$$-\nabla^2\rho_b + r_b\rho_b + u_b\rho_b^3 + v\rho_\Delta^2\rho_b + \frac{16\rho_\Delta^2\rho_b^2}{(4\rho_\Delta^2 + \rho_b^2)^2}\mathbf{v}_s^2 = 0. \tag{13b}$$

We are interested in the behavior of the amplitudes in the vicinity of the vortex center. In this region, for a strongly type-II superconductor, we may neglect the vector potential $\mathbf{A}$ in the superfluid velocity $\mathbf{v}_s$. In a singly quantized vortex Ω winds by 2π around the origin leading to a singularity of the form $\mathbf{v}_s \simeq \frac{1}{2}\nabla\Omega = \hat{\varphi}/2r$. First, for the *holon* vortex we assume that ρ_b vanishes in the core as some power $\rho_b(r) \sim r^\nu$ and $\rho_\Delta(r) \approx \overline{\rho_\Delta}$ remains approximately constant. Eq. (13b) then becomes

$$\left(\frac{1}{4} - \nu^2\right) r^{\nu-2} + (r_b + v\overline{\rho_\Delta^2})r^\nu + u_b\overline{\rho_b^2}r^{3\nu} = 0, \tag{14}$$

where we have neglected $\rho_b^2(r)$ compared to $4\overline{\rho_\Delta^2}$ in the denominator of the last term in Eq. (13b). The most singular term in Eq. (14) is the first one and we must demand that the coefficient of $r^{\nu-2}$ vanishes. This implies $\nu = \frac{1}{2}$. The asymptotic short distance behavior of the holon amplitude therefore can be written as

$$\rho_b(r) \simeq c_b\overline{\rho_b}\left(\frac{r}{\xi_b}\right)^{1/2}, \tag{15}$$

where c_b is a constant of order unity which may be determined by the full integration of Eqs. (13a) and (13b). Similar analysis of Eq. (13a) in the vicinity of the *spinon* vortex yields

$$\rho_\Delta(r) \simeq c_\Delta\overline{\rho_\Delta}\left(\frac{r}{\xi_d}\right), \tag{16}$$

with ρ_b approximately constant.

We notice the different power laws in the holon and spinon results. Operationally this difference arises from different numerical prefactors of the respective superfluid velocity terms in Eqs. (13). Physically, the unusual r dependence of the holon amplitude in the core reflects the fact that the field b describes a condensate of single holons, each carrying charge e. Superconducting vortex with the flux quantum Φ_0 represents a magnetic ''half flux'' for the holon field which results in nonanalytic behavior of $\rho_b(r)$ at the origin. Singly quantized holon vortex is therefore a peculiar object and we shall discuss it more fully in Sec. III. Here we note that the physical superconducting order parameter amplitude $|\Psi| = \rho_\Delta\rho_b^2$ remains analytic in the core of both the spinon and the holon vortex.

B. Holon vs spinon vortex: the phase diagram

We are now in the position to estimate the energies of the two types of vortices and deduce the phase diagram for the ''normal'' state in the vortex core. To this end we consider a single isolated vortex centered at the origin. The total vortex line energy can be divided into electromagnetic and core contributions.[33] The electromagnetic contribution consists of the energy of the supercurrents and the magnetic field outside the core region. It may be estimated by assuming that the amplitudes ρ_Δ and ρ_b have reached their bulk values $\overline{\rho_\Delta}$ and $\overline{\rho_b}$, respectively. Taking curl of Eq. (11) and noting that $\nabla \times \nabla\Omega = 2\pi\delta(\mathbf{r})$ for a singly quantized vortex we obtain the London equation for the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ of the form

$$B-\lambda^2\nabla^2 B=\Phi_0\delta(\mathbf{r}), \tag{17}$$

where

$$\lambda^{-2}=8\pi e^2\frac{4\overline{\rho_\Delta^2}\,\overline{\rho_b^2}}{4\overline{\rho_\Delta^2}+\overline{\rho_b^2}} \tag{18}$$

has the meaning of the London penetration depth for the effective GL theory (4). Aside from the unusual form of λ, Eq. (17) is identical to the conventional London equation. The corresponding electromagnetic energy is therefore the same for both types of vortices and can be calculated in the usual manner[32,33,30] obtaining

$$E_{\mathrm{EM}}\simeq\left(\frac{\Phi_0}{4\pi\lambda}\right)^2\ln\kappa, \tag{19}$$

with $\kappa=\lambda/\max(\xi_\Delta,\xi_b)$ being the generalized GL ratio.

To estimate the core contribution to the vortex line energy we assume that one of the amplitudes is suppressed to zero in the core

$$\rho_i(r)=0,\ r<\xi_i, \tag{20}$$

while the other one stays constant and equal to its bulk value. This is a very crude approximation which we justify below by an exact numerical computation. With these assumptions, the core energy is

$$E_{\mathrm{core}}^{(i)}\simeq\left(\frac{\Phi_0}{4\pi\lambda_i}\right)^2, \tag{21}$$

where $i=\Delta,b$ for spinon and holon vortex, respectively, and

$$\lambda_i^{-2}=8\pi e^2\overline{\rho_i^2}\,. \tag{22}$$

Such a crude approximation overestimates the core energy. A more accurate analysis,[32,33] which we do not pursue here, allows for a more realistic variation of $\rho_i(r)$ in the core and indicates that the value of $E_{\mathrm{core}}^{(i)}$ has the same form as Eq. (21) multiplied by a numerical factor $c_1\approx 0.5$.[36,37] Thus, the total energy of the vortex line can be written as

$$E^{(i)}=\left(\frac{\Phi_0}{4\pi\lambda}\right)^2\ln\kappa+c_1\left(\frac{\Phi_0}{4\pi\lambda_i}\right)^2, \tag{23}$$

where again $i=\Delta,b$ for spinon and holon vortex, respectively. Equation (23) parallels the Abrikosov expression for the vortex line energy in a conventional GL theory[32] where λ and λ_i are identical and equal to the ordinary London penetration depth.

In the vortex state described by the free energy (4) the vortex with lower energy $E^{(i)}$ will be stabilized. Equation (23) implies that the difference in energy between the two types of vortices comes primarily from the core contribution, as expected on the basis of the physical argument presented above. Condition $\lambda_\Delta=\lambda_b$ marks the transition point between the two solutions. For fixed GL parameters T_0, x_m, α_i, u_i, and v this defines a transition line in the x-T plane. According to Eq. (22) the equation for this line is

$$\overline{\rho_\Delta}(x,T)=\overline{\rho_b}(x,T). \tag{24}$$

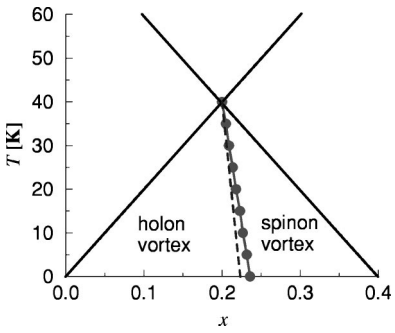

FIG. 2. Vortex core phase diagram for GL parameters chosen as follows: $\alpha_\Delta=0.13$, $\alpha_b=0.10$, $T_0=200$ K, $x_m=0.2$, $u_\Delta=u_b=1.0$, and $v=0.5$. Dashed line marks the phase boundary $T_g(x)$ obtained from Eq. (25) while the solid circles correspond to the numerical calculation with the same parameters.

Using Eqs. (7)–(9) one can obtain an explicit expression for the transition temperature T_g between two types of vortices as a function of doping

$$T_g(x)=T_0\left[\frac{2x_m-x}{1-\beta}+\frac{x}{1-\beta^{-1}}\right], \tag{25}$$

with

$$\beta=\frac{\alpha_b(u_\Delta+v)}{\alpha_\Delta(u_b+v)}. \tag{26}$$

Equation (25) describes a straight line in the x-T plane, originating at $[x_m,T_0x_m]$, i.e., maximal T_c at optimum doping, and terminating at $[2x_m/(1+\beta),0]$. Generically, we expect that parameters α_i and u_i will be comparable in magnitude for the holon and spinon channels. Parameter β defined in Eq. (26) will therefore be of order unity. The typical situation for $\beta=0.77$ is illustrated in Fig. 2. More generally the quartic coefficients u_i and v could exhibit weak doping and temperature dependences leading to a curvature in the phase boundary.

The appealing feature of the present theory is that parameter β may vary from compound to compound. Thus, the experimental fact that in BSCCO the pseudogap in the core persists into the overdoped region is easily accounted for in the present theory. It would be interesting to see if the transition from holon to spinon vortex as a function of doping could be experimentally observed. A good candidate for such observation would be LSCO, where the transport measurements in pulsed magnetic fields[1] established a metal-insulator transition around optimal doping, i.e., $\beta\approx 1$. The current theory predicts a holon vortex with the pseudogap spectrum in the underdoped (insulating) region and spinon vortex with conventional metallic spectrum on the overdoped side.

C. Numerical results

In order to put the above analytical estimates on firmer ground we now pursue numerical computation of the vortex line energy. For simplicity we consider the strongly type-II situation ($\kappa\gg 1$) where the vector potential term in $\mathbf{v}_s$ can be

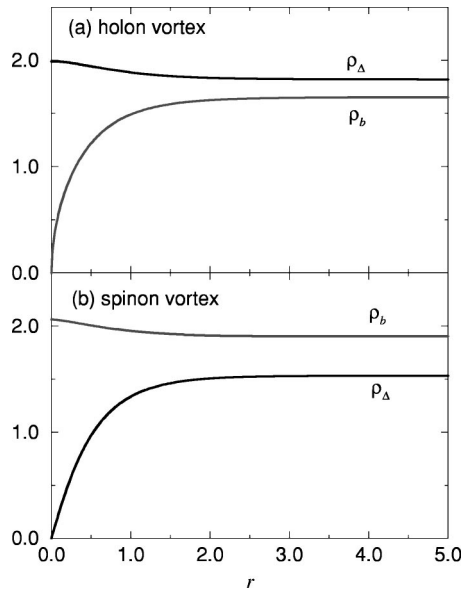

FIG. 3. Order parameter amplitudes near a single isolated vortex for GL parameters specified in Fig. 2. The holon vortex is plotted for $T=0$ and $x=0.22$ (implying coherence lengths $\xi_\Delta=0.63$ and $\xi_b=0.70$), while the spinon vortex is plotted for $T=0$ and $x=0.24$ (implying $\xi_\Delta=0.75$ and $\xi_b=0.60$).

neglected to an excellent approximation, as long as we focus on the behavior close to the core. We are then faced with the task of numerically minimizing the free energy (4) with respect to the two cylindrically symmetric amplitudes $\rho_\Delta(r)$ and $\rho_b(r)$. As noted by Sachdev[30] direct numerical minimization of the free energy (4) provides a more robust solution than the numerical integration of the coupled differential Eq. (13).

We discretize the free-energy functional (4) on a disk of a radius $R \gg \xi_i$ in the radial coordinate r with up to $N=2000$ spatial points. We then employ the Polak-Ribiere variant of the conjugate gradient method[38] to minimize this discretized functional with respect to $\rho_\Delta(r_j)$ and $\rho_b(r_j)$, initialized to suitable single vortex trial functions. The procedure converges very rapidly and the results are insensitive to the detailed shape of the trial functions as long as they saturate to the correct bulk values outside the vortex core.

Typical results of our numerical computations are displayed in Fig. 3 and are in complete agreement with the analytical considerations of the preceding subsections. Note in particular that $\rho_b(r)$ in the holon vortex vanishes with infinite slope, consistent with Eq. (15). Plotting $\rho_b^2(r)$ confirms that the exponent is indeed 1/2. In the spinon vortex $\rho_\Delta(r)$ is seen to vanish linearly as expected on the basis of Eq. (16). The nonvanishing order parameter is slightly elevated in the core reflecting the effective ''repulsion'' between the two amplitudes contained in the v term of the free energy. The results for the spinon vortex are consistent with those of Ref. 30.

We explored a number of other parameter configurations and obtained similar results. We find that Eq. (24) is a good predictor of the transition line between the holon and spinon vortex, although the precise numerical value of the transition temperature T_g for given x tends to deviate slightly from the value predicted by Eq. (25). This is illustrated in Fig. 2 where we compare the vortex core phase diagrams obtained numerically and from Eq. (25). Interestingly, the deviation always tends to enlarge the holon vortex sector of the phase diagram at the expense of the spinon vortex sector. This is presumably because the sharper $\sim\sqrt{r}$ suppression of the holon order parameter in the core costs less condensation energy.

III. GAUGE FLUCTUATIONS AND THE SPECTRAL PROPERTIES IN THE CORE

The theory of the vortex core based on the effective action (4) appears to yield results consistent with the STM data on cuprates[4,6] in that it implies a stable holon vortex solution over the large portion of the superconducting phase diagram. The state inside the core of such a holon vortex is characterized by vanishing amplitude of the holon condensate field, $|b|=0$, and a finite-spin gap $|\Delta|\approx\Delta_{\rm bulk}$. This is the same state as in the pseudogap region above T_c. One would thus expect the electronic spectrum in the core to be similar to that found in the normal state of the underdoped cuprates, in agreement with the data.[4,6] The holon vortex with this property carries conventional superconducting flux quantum Φ_0, in accord with experiment. This general agreement between theory and experiment would suggest that the effective action (4) provides the sought for phenomenological description of the vortex core physics in cuprates. In what follows we amplify our argumentation that it is also tenable in a broader theoretical context in that it naturally follows from the $U(1)$ slave boson models extensively studied in the classic and more recent high-T_c literature. We then provide a more detailed discussion of the vortex core spectra and propose an explanation for the experimentally observed core bound states.

A. Significance of the $f_{\rm gauge}$ term

Derivation of the effective action (4) from the more general $U(1)$ action (2) hinges on our assumption that the stiffness σ of the gauge field **a** is low and that the $f_{\rm gauge}$ term (3) can be neglected. Assumption of large σ by SNL leads to very different vortex solutions[30,31] which appear inconsistent with the recent experimental data. We first expand on our discussion as to why the $f_{\rm gauge}$ term is important and then we argue why it may be permissible to neglect it in the realistic models of cuprates.

To facilitate the discussion let us rewrite Eq. (2) by resolving the complex matter fields into amplitude and phase components:

$$f_{\rm GL}=f_{\rm amp}+\rho_\Delta^2(\nabla\phi-2\mathbf{a})^2+\rho_b^2(\nabla\theta-\mathbf{a}-e\mathbf{A})^2 + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2+\frac{\sigma}{2}(\nabla\times\mathbf{a})^2, \tag{27}$$

with $f_{\rm amp}$ specified by Eq. (5). Now consider a situation in which the sample is subjected to uniform magnetic field $\mathbf{B}=\nabla\times\mathbf{A}$. Two scenarios (discussed previously by SNL) ap-

pear possible. In the first, the internal gauge field develops no net flux, $\langle \nabla \times \mathbf{a} \rangle = 0$, and the holon phase θ develops singularities in response to $\mathbf{A}$ such that

$$\nabla \times \nabla \theta = 2\pi \sum_j \delta(\mathbf{r} - \mathbf{r}_j),$$

where $\mathbf{r}_j$ denotes the vortex positions. The holon amplitude ρ_b is driven to zero at $\mathbf{r}_j$, essentially to prevent the free energy from diverging due to the singularity in the phase gradient. Since holons carry charge e, each vortex is threaded by flux hc/e, i.e. twice the superconducting flux quantum $\Phi_0 = hc/2e$. This solution represents the doubly quantized holon vortex lattice, considered by SNL.

In the second scenario, $\mathbf{a}$ develops a net flux such that $\mathbf{a} \approx -e\mathbf{A}$, which screens out the $\mathbf{A}$ field in the holon term but produces a net flux $-2e\mathbf{A}$ in the spinon term. In response to this flux, spinon phase ϕ develops singularities such that

$$\nabla \times \nabla \phi = 2\pi \sum_j \delta(\mathbf{r} - \tilde{\mathbf{r}}_j),$$

corresponding to the spinon vortex lattice. $\tilde{\mathbf{r}}_j$ denotes vortex positions which will be different from $\mathbf{r}_j$ since at the fixed field B there will be twice as many spinon vortices as holon vortices. (Spinon vortices carry conventional superconducting quantum of flux Φ_0.) In this case, ρ_Δ is driven to zero at the vortex centers. In this scenario one pays a penalty for nucleating the net flux in $\nabla \times \mathbf{a}$ due to last term in Eq. (27). This energy cost can be estimated as

$$E_\sigma \simeq 8\pi\sigma e^2 \left(\frac{\Phi_0}{4\pi\lambda} \right)^2 \tag{28}$$

per vortex. Stiffness σ must be small enough so that E_σ is small compared to the vortex energy (23). Taking the dominant E_{EM} term and neglecting $\ln \kappa$, this implies that

$$\sigma \ll \frac{1}{8\pi e^2}, \tag{29}$$

which is the same condition as considered in Ref. 30.

Now consider a *third* scenario in which a *singly quantized* holon vortex emerges. As a starting point consider the spinon vortex solution just described. In the underdoped regime the amplitude piece f_{amp} would favor suppressing the holon amplitude in the core instead of the spinon amplitude but according to our previous considerations this would ordinarily require formation of a doubly quantized vortex whose magnetic energy is too large. However, if the gauge-field-stiffness σ is sufficiently small, the system could lower its free energy by setting up singularities in $\mathbf{a}$ which would precisely cancel the singularities in $\nabla\phi$ and shift them to the holon term. To arrive at this situation imagine contracting the initially uniform flux $\nabla \times \mathbf{a}$ so that it becomes localized in the individual vortex core regions. Taking this procedure to the extreme, i.e., taking the limit $\sigma \to 0$, the gauge field will form ''flux spikes'' of the form

$$2(\nabla \times \mathbf{a}) = -\nabla \times \nabla \phi = -2\pi \sum_j \delta(\mathbf{r} - \tilde{\mathbf{r}}_j), \tag{30}$$

completely localized at the vortex centers. The gauge field of this form indeed completely cancels the singularities in the spinon phase gradient in Eq. (27) and ρ_Δ is no longer forced to vanish in the core. The singularities now appear in the holon term, but they stem from $\mathbf{a}$ rather than $\nabla\theta$ which remains nonsingular. Consequently, ρ_b is forced to vanish in the vortex cores. By construction, the vortices are located at $\tilde{\mathbf{r}}_j$ and are therefore singly quantized. This is the singly quantized holon vortex solution identical to the one discussed above on the basis of the free energy (4). Based on the above discussion the singly quantized holon vortex can be thought of as a composite object formed by attaching half quantum of fictitious gauge flux $\nabla \times \mathbf{a}$ to the spinon vortex.

In the framework of the free energy (27) one pays a penalty for such a singular solution due to the gauge stiffness term. In the present continuum model this penalty per single vortex is actually infinite, since according to Eq. (30) it involves a spatial integral over $[\delta(\mathbf{r} - \tilde{\mathbf{r}}_j)]^2$. Thus, in the continuum model the singular solutions of this type are prohibited. In reality, however, we have to recall that our effective action (2) descended from a microscopic lattice model for spinons and holons in which the gauge field $\mathbf{a}$ lives on the nearest-neighbor bonds of the ionic lattice. The ionic lattice constant d therefore provides a natural short-distance cutoff and the δ function in Eq. (30) should be interpreted as a flux quantum Φ_0 piercing an elementary plaquette of the lattice. The energy cost per vortex thus becomes finite and is given by

$$E'_\sigma \simeq \frac{\sigma e^2}{2} \left(\frac{\Phi_0}{d} \right)^2. \tag{31}$$

Again, for the solution to be stable, E'_σ must be negligible compared to the vortex energy (23). This implies

$$\sigma \ll \frac{1}{8\pi^2 e^2} \left(\frac{d}{\lambda} \right)^2, \tag{32}$$

which is a much more stringent condition than Eq. (29) since in cuprates $d \ll \lambda$.

When condition (32) is satisfied it is permissible to neglect the f_{gauge} term in the effective action (2) and it becomes fully equivalent to Eq. (4) as far as the vortex solutions are concerned. Equation (32) gives the precise meaning to the requirement of the weak stiffness of the gauge field loosely stated when deriving the effective action (4).

B. Microscopic considerations

As mentioned in the introduction, the gauge field $\mathbf{a}$ has no dynamics in the original $U(1)$ microscopic model, as it only serves to enforce a constraint on spinons and holons. The stiffness term (3) in the effective theory was assumed to arise in the process of integrating out the microscopic degrees of freedom.[30,31] While such a term is certainly permitted by symmetry, assessing its strength σ is a nontrivial issue since even deep in the superconducting phase neither holons nor

spinons are truly gapped. Thus, in general, integrating out these degrees of freedom may lead to singular and nonlocal interactions between the condensate and the gauge fields. To our knowledge, the procedure has not been explicitly performed for the $U(1)$ model and the precise form or magnitude of the gauge stiffness term is unknown. General considerations[35] suggest that the gauge stiffness term is negligible in the class of models with exact local $U(1)$ symmetry connecting the phases of holons and spinons.

Consider now an intermediate representation of the problem where only high-energy microscopic degrees of freedom have been integrated out. In the presence of a cutoff this is a well defined procedure even for gapless excitations, as explicitly shown by Kwon and Dorsey[39] for a simple BCS model. The corresponding effective Lagrangian density of the present $U(1)$ model can be written as

$$\begin{aligned}\mathcal{L}_{\rm eff}=&\frac{\kappa_\Delta^\mu}{2}(\partial_\mu\phi-2a_\mu)^2+\frac{\kappa_b^\mu}{2}(\partial_\mu\theta-a_\mu-eA_\mu)^2-f_{\rm amp}\\&+(\partial_\mu\phi-2a_\mu)J_{\rm sp}^\mu+(\partial_\mu\theta-a_\mu-eA_\mu)J_h^\mu\\&+\mathcal{L}_{\rm sp}[\psi_{\rm sp},\psi_{\rm sp}^\dagger;\rho_\Delta]+\mathcal{L}_h[\psi_h,\psi_h^\dagger;\rho_b]+\mathcal{L}_{\rm EM}[A_\mu].\end{aligned}\tag{33}$$

The Greek index μ runs over time and two spatial dimensions, κ_i^0 are compressibilities of the holon and spinon condensates, while

$$\kappa_i^j=-2(\rho_i)^2,\quad i=\Delta,b,\quad j=1,2\tag{34}$$

are the respective phase stiffnesses. $J_{\rm sp}^\mu$ and J_h^μ are spinon and holon three currents, respectively, and $\mathcal{L}_{\rm sp}$ and $\mathcal{L}_h$ are the low-energy effective Lagrangians for the fermionic spinon field $\psi_{\rm sp}$ and bosonic holon field ψ_h. $\mathcal{L}_{\rm EM}$ is the Maxwell Lagrangian for the physical electromagnetic field. Thus, $\mathcal{L}_{\rm eff}$ describes an effective low-energy theory of spinons and holons coupled to their respective collective modes and a fluctuating $U(1)$ gauge field. A similar theory has been recently considered by Lee.[20]

The precise form of the microscopic Lagrangians $\mathcal{L}_{\rm sp}$ and $\mathcal{L}_h$ is not important for our discussion. The salient feature which we exploit here is that only the amplitude of the respective condensate field enters into $\mathcal{L}_{\rm sp}$ and $\mathcal{L}_h$. Coupling to the phases and the gauge field is contained entirely in the Doppler shift terms [second line of Eq. (33)]. Such form of the coupling is largely dictated by the requirements of the gauge invariance and the particular form of Eq. (33) can be explicitly derived by gauging away the respective phase factors from the ψ fields.[29,39]

The gauge field a_μ enters the effective Lagrangian (33) only via two gauge invariant terms: $(\partial_\mu\phi-2a_\mu)$ and $(\partial_\mu\theta-a_\mu-eA_\mu)$, which may be interpreted as the three velocities of the spinon and holon condensates, respectively. Furthermore, the only coupling between holons and spinons arises from a_μ. Therefore, if we now proceed to integrate out the remaining microscopic degrees of freedom from $\mathcal{L}_{\rm eff}$, the two velocity terms will not mix. This consideration suggests that upon integrating out all of the microscopic degrees of freedom, the resulting gauge stiffness term will be of the form

$$f'_{\rm gauge}=\frac{\sigma_\Delta}{2}[\nabla\times(2\mathbf{a}-\nabla\phi)]^2+\frac{\sigma_b}{2}[\nabla\times(\mathbf{a}+e\mathbf{A}-\nabla\theta)]^2.\tag{35}$$

Clearly, such a term is permitted by the gauge symmetry. Furthermore, we note that for smooth (i.e., vortex-free) configurations of phases the gradient terms will contribute nothing and we recover the gauge term considered in Ref. 31.

In the presence of a vortex in ϕ or θ the $f'_{\rm gauge}$ term will contribute formally divergent energy. Regularizing this on the lattice, as discussed above Eq. (31), this energy will become finite and can be interpreted simply as the energy of the spinon or holon vortex core states, which have been integrated out. In the microscopic theory (33) such energy would arise upon solving the relevant fermionic or bosonic vortex problem.

We stress that, as concluded in the preceding subsection, the main theoretical obstacle to the formation of a singly quantized holon vortex in the original SNL theory was the appearance of a formally divergent contribution in the $f_{\rm gauge}$ term (3). The argument above suggests that $f_{\rm gauge}$ in Eq. (2) should be replaced by Eq. (35), in which such formally divergent contribution appears for *arbitrary* vortex configuration and upon regularization has a simple physical interpretation in terms of the energy of the vortex core states. Usage of the physically motivated term (35) in place of Eq. (3) therefore removes the bias against the singly quantized holon vortex solution, which appears to be realized in real materials. With Eq. (35) any bias between the holon and spinon vortex solutions can result only from the difference between the two stiffness constants σ_Δ and σ_b. It is reasonable on physical grounds to assume that constants σ_Δ and σ_b are of the similar magnitudes. Furthermore, on the basis of Ref. 35 we expect these constants to be negligibly small in the physically relevant models. Consequently we expect that neglecting the $f_{\rm gauge}$ term as in our derivation of effective action (4) will result in accurate determination of the phase diagram for the state in the vortex core.

C. Vortex core states

The phenomenological theory based on the effective action (4) does not allow us to address the interesting question of the nature of the fermionic states in the vortex core. To do this we need to consider the microscopic Lagrangian density (33). While the fully self-consistent calculation is likely to be prohibitively difficult, one can obtain qualitative insights by first solving the GL theory (4) as described in Sec. II, and then using the order parameters ρ_Δ and ρ_b as an input to the fermionic and bosonic sectors of the theory specified by Eq. (33). The work on a detailed solution of this type is in progress. Here we wish to point out some interesting features of such a theory and argue that it may indeed exhibit structure in the low-energy spectral density similar to that found experimentally.[3,6]

It is instructive to integrate out the gauge fluctuations from the Lagrangian (33) as first discussed by Lee.[20] Since $\mathcal{L}_{\rm eff}$ is quadratic in a_μ the integration can be explicitly performed resulting in the Lagrangian of the form

$$\mathcal{L}'_{\rm eff}=\frac{1}{2}K_\mu(v_s^\mu)^2-f_{\rm amp}+\mathcal{L}_{\rm EM}$$
$$-\frac{2\kappa_b^\mu}{4\kappa_\Delta^\mu+\kappa_b^\mu}(v_s^\mu J_{\rm sp}^\mu)+\frac{4\kappa_\Delta^\mu}{4\kappa_\Delta^\mu+\kappa_b^\mu}(v_s^\mu J_h^\mu)$$
$$+\mathcal{L}_{\rm sp}+\mathcal{L}_h-\frac{1}{2}\frac{1}{4\kappa_\Delta^\mu+\kappa_b^\mu}(2J_{\rm sp}^\mu+J_h^\mu)^2, \qquad (36)$$

where $K_\mu=4\kappa_\Delta^\mu\kappa_b^\mu/(4\kappa_\Delta^\mu+\kappa_b^\mu)$ and

$$v_s^\mu=\left(\partial_\mu\theta-\frac{1}{2}\partial_\mu\phi-eA_\mu\right) \qquad (37)$$

is the physical superfluid velocity. The first line reproduces the GL effective action (4) for the condensate fields, the second line describes the Doppler shift coupling of the superfluid velocity to the microscopic currents, and the third line contains spinon and holon pieces with additional current-current interactions generated by the gauge fluctuations.[20]

We now discuss the physical implications of Eq. (36) for the two types of vortices. We focus on the static solutions (i.e., we ignore the time dependences of various quantities, e.g., taking $v_s^0=0$) of $\mathcal{L}'_{\rm eff}$ in the presence of a single isolated vortex. We are interested in the local spectral function of a physical electron. This is given by a convolution in the energy variable of the spinon and holon spectral functions. According to the analysis presented in Ref. 28, at low temperatures the electron spectral function will be essentially equal to the spinon spectral function. Convolution with the holon spectral function which is dominated by the sharp coherent peak due to the condensate merely leads to a small broadening of the order T. In the following we therefore focus on the behavior of spinons in the vicinity of the two types of vortices.

By inspecting Eq. (36) it is easy to see that the excitations inside the *spinon vortex* will be qualitatively very similar to those found in the conventional vortex described by the weak coupling d-wave BCS theory.[8–10] In particular, according to Eq. (16) we have $\kappa_\Delta\sim r^2$, and $\kappa_b\sim$ const in the core. Recalling furthermore that $|\mathbf{v}_s|\sim 1/r$ we observe that the spinon current $\mathbf{J}_{\rm sp}$ is coupled to a term that diverges as $1/r$ in the core (just as in a conventional vortex), while the holon current $\mathbf{J}_h$ is coupled to a nonsingular term. Thus, one may conclude that holons remain essentially unperturbed by the phase singularity in the spinon vortex while the spinons obey the essentially conventional Bogoliubov–de Gennes equations for a d-wave vortex.

In the *holon vortex* the situation is quite different. According to Eq. (15) we have $\kappa_b\sim r$ and $\kappa_\Delta\sim$ const in the core. The spinon current $\mathbf{J}_{\rm sp}$ is now coupled to a nonsingular term ($1/r$ divergence in v_s is canceled by $\kappa_b\sim r$). Therefore, there will be no topological perturbation in the spinon sector and we expect the spinon wave functions to be essentially unperturbed by the diverging superfluid velocity. Spinon spectral density in the core should be qualitatively similar to that far outside the core. This is our basis for expecting a pseudogaplike spectrum in the core of a holon vortex.

We now address the possible origin of the experimentally observed vortex core states[3,6] within the present scenario for a holon vortex. To this end consider the effect of the last term in Eq. (36) which we ignored so far. Upon expanding the binomial the temporal component is seen to contain a density-density interaction of the form $J_{\rm sp}^0 J_h^0$, where J_h^0 is the local density of *uncondensed* holons. Since the holon order parameter vanishes in the core and the electric neutrality dictates that the total density of holons must be approximately constant in space, we expect that uncondensed holon density will behave roughly as

$$J_h^0(r)=\overline{\rho_b}-\rho_b(r);$$

$J_h^0(r)$ will have a spike in the core of a holon vortex. Insofar as $J_h^0(r)$ can be viewed as a static potential acting on spinons, the uncondensed holons in the vortex core can be thought of as creating a scattering potential, akin to an impurity embedded in a d-wave superconductor. In fact, formally the spinon problem is identical to the problem of a fermionic quasiparticle in a d-wave superconductor in zero field in the presence of a localized impurity potential. It is known that such a problem exhibits a pair of marginally bound impurity states[40] at low energies which result in sharp resonances in the spectral density inside the gap. Such states have been extensively studied theoretically[41–44] and their existence was recently confirmed experimentally by Pan *et al.*[5] We propose here that, within the formalism of Eq. (36), the same mechanism could give rise to the low-energy quasiparticle states in the core of a holon vortex. Such structure, if indeed confirmed by a microscopic calculation, could explain the spectral features observed experimentally in the vortex cores of cuprate superconductors.[3,6]

IV. CONCLUSIONS

Scanning tunneling spectroscopy of the vortex cores affords a unique opportunity for probing the underlying ''normal'' ground state in cuprate superconductors. The existing experimental data on YBCO and BSCCO strongly suggest that conventional mean-field weak-coupling theories[7–13] fail to describe the physics of the vortex core. Our main objective was to develop a theoretical framework for understanding these spectra and the nature of the strongly correlated electronic system which emerges once the superconducting order is suppressed. We have shown that phenomenological model (2) based on a modern variant of the $U(1)$ gauge-field slave boson theory[20] contains the right physics, provided that the gauge-field stiffness is vanishingly small. The latter assumption is consistent with the general arguments involving local gauge symmetry.[35] In such a theory the gauge field can be explicitly integrated out, resulting in the effective action (4) which contains one phase degree of freedom representing the phase of a Cooper pair and two amplitude degrees of

freedom representing the holon and spinon condensates.

Analysis of the effective theory (4) in the presence of a magnetic field establishes existence of two types of vortices, spinon and holon, with contrasting spectral properties in their core regions. Our holon vortex is singly quantized and therefore differs in a profound way from the doubly quantized holon vortex discussed by SNL.[30,31] As indicated in Fig. 2 such a singly quantized holon vortex is expected to be stable over the large portion of the phase diagram on the underdoped side. Quasiparticle spectrum in the core of a holon vortex is predicted to exhibit a ''pseudogap,'' similar to that found in the underdoped normal region above T_c. This is consistent with the data of Renner *et al.*[4] who pointed out a remarkable similarity between the vortex core and the normal state spectra in BSCCO. Spinon vortex, on the other hand, should be virtually indistinguishable from the conventional *d*-wave BCS vortex and is expected to occur on the overdoped side of the phase diagram. Transition from the insulating holon vortex to the metallic spinon vortex as a function of doping is a concrete testable prediction of the present theory.

Phenomenological theory based on the effective action (4) does not permit explicit evaluation of the electronic spectral function. To this end we have considered the corresponding microscopic theory (36) and concluded that a holon vortex will indeed exhibit a pseudogaplike spectrum. Such qualitative analysis furthermore suggests a plausible mechanism for the sharp vortex core states observed in YBCO (Ref. 3) and BSCCO.[6] We stress that conventional mean-field weak-coupling theories yield neither pseudogap nor the core states. In the core of a holon vortex such states will arise as a result of spinons scattering off of the locally uncondensed holons, in a manner analogous to the quasiparticle resonant states in the vicinity of an impurity in a *d*-wave superconductor.[40–44] The latter conclusion is somewhat speculative and must be confirmed by explicitly solving the fermionic sector of the microscopic theory (36).

On a broader theoretical front the importance of the vortex core spectroscopy as a window to the normal state in the $T\rightarrow 0$ limit lies in its potential to discriminate between various microscopic theories of cuprates. It is reasonable to assume that the observed pseudogap in the vortex core reflects the same physics as the pseudogap observed in the normal state. This means that the mechanism responsible for the pseudogap must be operative on extremely short length scales, of order of several lattice spacings. The $U(1)$ slave boson theory considered in this work apparently satisfies this requirement. Obtaining the correct vortex core spectral functions could serve as an interesting test for other theoretical approaches describing the physics of the underdoped cuprates.[28,45,46]

It will be of interest to explore the implications of the effective theories (4) and (36) in other physical situations. Of special interest are situations where the holon condensate amplitude is suppressed, locally or globally, giving rise to ''normal'' transport properties (vanishing superfluid density) but quasiparticle excitations that are characteristic of a superconducting state. These include the spectra in the vicinity of an impurity, twin boundary, or a sample edge. In the latter case one might hope to observe a signature of the zero bias tunneling peak anomaly (normally seen for certain geometries deep in the superconducting phase in the optimally doped cuprates) even above T_c in the underdoped samples.

Note added in proof. After submission of this manuscript we learned about complementary microscopic treatments of the spin-charge separated state in the vortex core within $U(1)$ (Ref. 47) and $SU(2)$ (Ref. 48) slave boson theories. The former agrees qualitatively with our phenomenological theory. Reference 38 proposes a new type of vortex which takes advantage of the larger symmetry group $SU(2)$. In a related development Senthil and Fisher[49] discussed a Z_2 vortex (which is essentially equivalent to our singly quantized holon vortex) and proposed a ''vision detection'' experiment based on trapping such a vortex in the hole fabricated in a strongly underdoped superconductor. Here we wish to point out that the experiment will produce the same general outcome in a system described by the $U(1)$ theory where the role of a vision will be played by a flux quantum of the fictitious gauge field **a**.

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