

## Ultrasonic studies of the spin-triplet order parameter and the collective mode in $\text{Sr}_2\text{RuO}_4$

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The elastic constant  $C_T$  and ultrasonic attenuation  $\alpha_s$  for  $(C_{11} - C_{12})/2$  have been measured on  $\text{Sr}_2\text{RuO}_4$  with  $T_c^* = 1.42$  K. In contrast to the  $A_{1g}$  and  $E_g$  strains, the  $B_{1g}$  strain ( $\epsilon_{xx} - \epsilon_{yy}$ ) strongly couples to the superconducting state, which dominantly originate from hybridized Ru- $4d_{xy}$  electrons on the  $\gamma$  Fermi surface. Taken into account impurity induced in-gap states, the  $T^3$  dependence of  $\alpha_s$  is found to be consistent with an existence of line nodes. The large reduction in  $C_T$  below  $T_c^*$  evidences the two-component order parameter and chiral  $p$ -wave state. Anomalous  $C_T$  and  $\alpha_s$  around  $T_a = 1.34$  K suggest a collective mode.

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Layered perovskite superconductor  $\text{Sr}_2\text{RuO}_4$  has attracted much attention,<sup>1</sup> because the pairing symmetry could be of  $p$  wave. Two-dimensional electronic states are formed on the alternating  $\text{RuO}_2$  planes. There exist three cylindrical Fermi surfaces named  $\alpha$ ,  $\beta$ , and  $\gamma$  running along  $[001]$  in a tetragonal structure.<sup>2-4</sup> The nuclear relaxation rate  $T_1$  measured by  $^{101}\text{Ru}$  nuclear quadrupole resonance (NQR) never exhibits a Hebel-Slichter coherence peak, and the Knight shift of  $^{17}\text{O}$  nuclear magnetic resonance (NMR)<sup>5</sup> is temperature independent below and above  $T_c$ . The muon spin relaxation indicates a broken time-reversal symmetry in the superconducting state.<sup>6</sup> These experimental results were interpreted with  $\mathbf{d}$  vector  $z\Delta_0(k_x + ik_y)$  for an isotropic gap.<sup>7</sup> Here  $z$  and  $k_x + ik_y$  represent spin and orbital parts, and  $\Delta_0$  is a constant.

Recent specific heat<sup>8</sup> and  $^{101}\text{Ru}$  NQR measurements,<sup>9</sup> however, show power-law behaviors at low temperatures, suggesting an existence of a line-node gap. An anisotropic gap model<sup>10</sup> shows the  $p$ -wave superconductivity induced by the short-range ferromagnetic spin fluctuation. Various  $f$ -wave pairing states are obtained by the product of two irreducible representations in a  $D_{4h}$  point group.<sup>11</sup>

In  $p$ -wave superfluid  $^3\text{He}$ , many collective modes were extensively studied by ultrasonic measurements.<sup>12</sup> However, collective modes caused by multicomponent order parameters in an unconventional superconductor<sup>13</sup> have been little known experimentally at least.

We have already reported a small superconducting anomaly below  $T_c = 1.2$  K in the longitudinal mode  $C_{33}$  inducing a volume strain with  $A_{1g}$  representation of  $D_{4h}$ .<sup>4</sup> On the other hand, a large temperature dependence is found in the longitudinal mode  $C_{11}$ , which is reducible to  $A_{1g}$  and  $B_{1g}$ . With this indication that the  $B_{1g}$  strain  $\epsilon_{xx} - \epsilon_{yy}$  might strongly couple to the superconductivity, we have measured the ultrasound velocity and attenuation of  $(C_{11} - C_{12})/2$  mode to clarify the superconducting order parameter and the collective mode.

We have used a single crystal of  $\text{Sr}_2\text{RuO}_4$  grown by a floating zone technique with the size of  $3.02([110]) \times 2.10([1\bar{1}0]) \times 2.51([001])$  mm<sup>3</sup>. Ultrasonic experiments

based on a phase comparison method with high sensitivity of  $\Delta v/v = 10^{-5} \sim 10^{-6}$  have been carried out down to 0.17 K with a  $^3\text{He}$ - $^4\text{He}$  dilution refrigerator. The attenuation coefficient has been measured by the amplitude variation of the 2nd-echo signal with the input power of  $\sim 30$   $\mu\text{W}$ . The 5th-higher overtone  $\sim 43.4$  MHz has been employed to increase the resolution. The transverse ultrasound propagating and polarizing along  $[110]$  and  $[1\bar{1}0]$  is excited and detected by the  $\text{LiNbO}_3$  transducers stuck to both the parallel sample surfaces by RTV silicone.

The superconducting transition for the present crystal is determined by the specific heat using a quasiadiabatic heat pulse method. As shown in Fig. 1(a), the mean-field superconducting transition temperature is obtained as  $T_c^* = 1.42$  K by the mid point of the specific heat jump. The jump between  $T_c^{\text{offset}} = 1.37$  K and  $T_c^{\text{onset}} = 1.50$  K is estimated to be  $\Delta C_p = 4.5 \times 10^3$  erg/cm<sup>3</sup> K. The transition width is attributed to the sample inhomogeneity due to the spatial distribution of  $T_c$ . In a spin-triplet superconductor, the non-magnetic impurities act as a pair breaker. According to the analysis with the Abrikosov-Gor'kov theory,<sup>14</sup>  $T_c^0$  for an ideal case has been estimated to be 1.5 K,<sup>15</sup> and the quasi-particle life time due to the impurity scattering is obtained as  $\tau_s = 7.5 \times 10^{-11}$  sec for the present crystal.

The temperature dependence of the elastic constant  $C_T = (C_{11} - C_{12})/2$  is depicted in Fig. 1(b).  $\Delta C_T = C_T(T, H = 0) - C_T(T, H = 0.1 \text{ T})$  is the difference in the elastic constant between superconducting and normal states, where  $H = 0.1$  T applied along  $[001]$  is higher than the upper critical field at  $T = 0$ . The experimental data deviate from  $\Delta C_T = 0$  below  $T_c^{\text{onset}}$ . The difference  $\Delta C_T$  is theoretically calculated by the second derivative of a Helmholtz free energy based on a two-fluid model  $\Delta F = -\phi(\epsilon_F)[1 - T^2/T_c^{*2}(\epsilon_F)]^2$  with respect to the strain  $\epsilon_F$ .<sup>16</sup> Here the condensation energy  $\phi$  at 0 K and  $T_c^*$  depends on the strain. In general, the longitudinal elastic constant exhibits a jump at  $T_c^*$  due to the first-order derivative term  $dT_c^*/d\epsilon_F$ .

For the transverse strain  $\epsilon_F$ ,

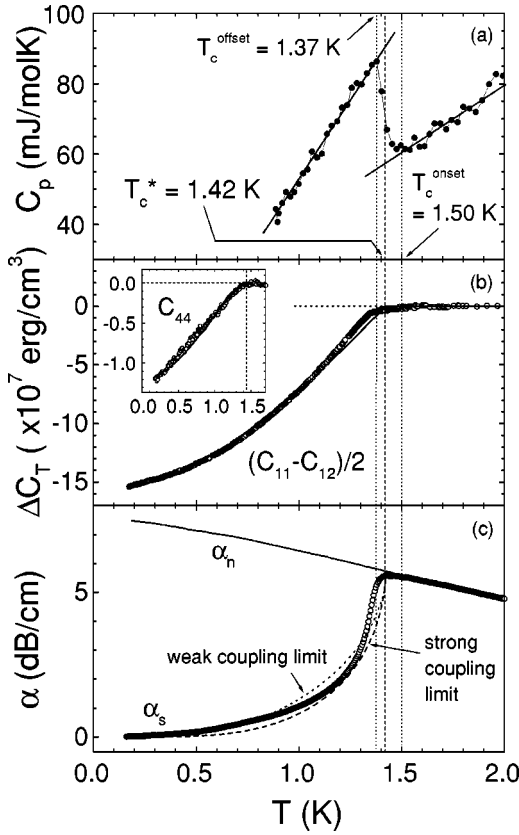


FIG. 1. Temperature dependence of the specific heat (a), the elastic constant (b), and the attenuation coefficient (c) for  $(C_{11} - C_{12})/2$ . The inset in (b) shows  $\Delta C_T$  in  $C_{44}$ . See the text for details.

$$\Delta C_T = \frac{d^2 T_c^*}{d\epsilon_T^2} \left[ -\frac{T^2}{2T_c^{*2}} \left( 1 - \frac{T^2}{T_c^{*2}} \right) \right] \Delta C_p - \frac{d^2 \phi}{d\epsilon_T^2} \left( 1 - \frac{T^2}{T_c^{*2}} \right)^2. \quad (1)$$

Equation (1) only includes the second-order derivative term  $d^2 T_c^*/d\epsilon_T^2$ , so that the discontinuity in the slope  $d\Delta C_T/dT$  occurs at  $T_c^*$ . As shown by a solid line in Fig. 1(b), Eq. (1) with parameters  $d^2 T_c^*/d\epsilon_{B1g}^2 = 7 \times 10^4$  K and  $\phi^{-1}(d^2 \phi/d\epsilon_{B1g}^2) = 2 \times 10^5$ , where  $\epsilon_{B1g} = \epsilon_{xx} - \epsilon_{yy}$ , can be fit well to the data except just below  $T_c^*$ .

For the transverse mode  $C_{44}$  with  $E_g$  inducing  $\epsilon_{Eg} = \epsilon_{zx}$  in the inset of Fig. 1(b), we obtain  $d^2 T_c^*/d\epsilon_{Eg}^2 = 3 \times 10^3$  K and  $\phi^{-1}(d^2 \phi/d\epsilon_{Eg}^2) = 1 \times 10^4$ . Equation (1) reproduces the data quite well just below  $T_c^*$ . In case of  $C_{33}$ ,<sup>4</sup>  $d^2 T_c^*/d\epsilon_{A1g}^2 = 2 \times 10^4$  K, while the first-order derivative term is extremely small because the longitudinal elastic constant does not exhibit a remarkable jump at  $T_c$ . Therefore the superconducting electrons couple strongly to the transverse  $B_{1g}$  strain, which induces the in-plane perturbation for the two-dimensional electronic states.

Here we consider the electron-strain coupling which is measurable via an area coefficient  $\Lambda_\Gamma$  derived by the acoustic de Haas–van Alphen (dHvA) effect. The  $\gamma$  oscillation has not been detected by this effect owing to the large effective mass. The coefficient for  $C_{33}$  with the  $\alpha$  Fermi surface,<sup>4</sup>

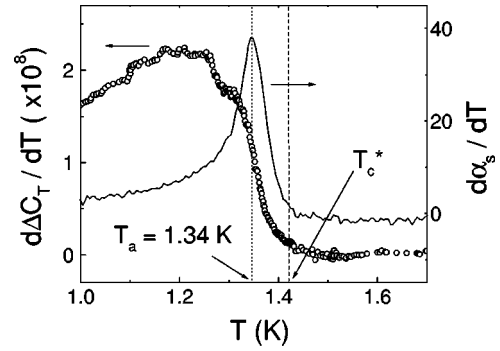


FIG. 2. Temperature derivative of  $\Delta C_T$  (open circles) and  $\alpha_s$  (solid line).

$\Lambda_{A1g}^\alpha$ , is quite large, about 20, while those of  $(C_{11} - C_{12})/2$  with  $\alpha$  and  $\beta$  Fermi surfaces are quite small,<sup>17</sup> less than 1. Therefore, the  $\alpha$  and  $\beta$  Fermi surfaces constructed by the hybridized  $\text{Ru-}4d_{yz(zx)}$  orbitals as predicted from the band-structure calculations<sup>2</sup> couple strongly with the  $A_{1g}$  strain and quite weakly with the  $B_{1g}$  strain. These results are totally consistent with the orbital-strain symmetry. Since the superconducting state exclusively couples to the  $B_{1g}$  strain as mentioned above, the hybridized  $\text{Ru-}4d_{xy}$  orbital electron on the  $\gamma$  Fermi surface is concluded to be dominantly responsible for the superconductivity.

The temperature dependence of the attenuation coefficient is shown in Fig. 1(c) for both the superconducting state ( $\alpha_s$ ) at  $H=0$  and the normal one ( $\alpha_n$ ) under  $H=0.1$  T. With decreasing temperature,  $\alpha_n$  monotonically increases as expected for a pure metal. At  $T_c^{\text{onset}}$ ,  $\alpha_s$  starts to deviate from  $\alpha_n$  as well as the elastic constant mentioned above, and then drops sharply well below  $T_c^*$ .

The attenuation coefficient for a conventional superconductor is written as:<sup>18</sup>

$$\frac{\alpha_s}{\alpha_n} = 2f_0(\Delta(T)) = \frac{2}{1 + \exp(\Delta/kT)}. \quad (2)$$

Here  $\Delta$  is a temperature-dependent energy gap isotropically opened on a Fermi surface. The dotted and broken curves in Fig. 1(c) show the calculated results with Eq. (2) for a weak and strong coupling limit [ $\Delta(0)/k_B T_c^* = 3.52$  and  $3.88$ ]. Both curves become exponentially zero, while the data are still temperature dependent down to 0.17 K.

Figure 2 shows the temperature-derivative of  $\Delta C_T$  and  $\alpha_s$ . There appears a sharp peak in  $d\alpha_s/dT$  at  $T_a = 1.34$  K, around which the discontinuous change in  $d\Delta C_T/dT$  takes place. These anomalies corresponding to the anomalous rounding in  $\Delta C_T$  and  $\alpha_s$  do occur not at  $T_c^*$  but at  $T_a$  well below  $T_c^*$ .

In Fig. 3, the ratio  $\alpha_s/\alpha_n$  near  $T_c^*$  is shown by the solid circles as a function of  $(T - T_c^*)/T_c^*$ . The broken curve is a BCS result with Eq. (2) for a strong coupling limit. To take account of the inhomogeneity effect, a Gaussian distribution function for  $T_c$  is applied to Eq. (2):  $\Psi(T_c) = (2\pi\sigma)^{-1} \exp(-(T_c - T_c^*)^2/2\sigma^2)$ , the sum of which for all the  $T_c$ 's are normalized to unity. The most probable standard

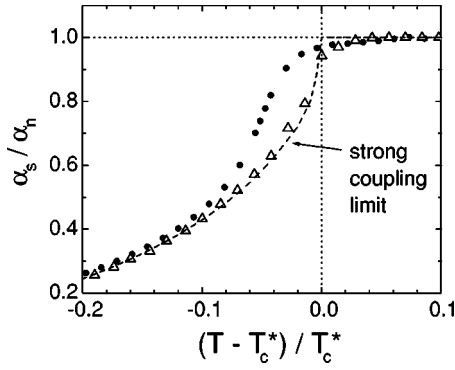


FIG. 3. The ratio  $\alpha_s/\alpha_n$  close to  $T_c^*$  as a function of  $(T - T_c^*)/T_c^*$ .

deviation is obtained as  $\sigma=0.025$ . For the sake of simplicity, we divide the temperature region with 0.02 K step. The open triangles in Fig. 3 are obtained by summing up Eq. (2) weighted by  $\Psi(T_c)$ . The data above  $T_c^*$  are well reproduced by the inhomogeneity effect. However this effect below  $T_c^*$ , which is so small due to the overwhelming temperature dependence [Eq. (2)], cannot explain the data at all. Therefore some additional attenuation exists at temperatures  $\sim 1.2$  K  $\leq T \leq T_c^*$ .

Next we discuss the nonexponential dependence of the attenuation as shown in Fig. 1(c). At high temperatures, say above 0.8 K, the attenuation no longer follows a power-law behavior because of the temperature-dependent gap. The low-temperature data are well fitted to  $T^2$  for  $0.17$  K  $\leq T \leq 0.4$  K and  $T^3$  for  $0.3$  K  $\leq T \leq 0.8$  K, as shown in Fig. 4. The crossover from  $T^3$  to  $T^2$  dependence occurs around 0.4 K with decreasing temperatures. Since the absolute attenuation coefficient due to the electrons is hardly obtained in general, the extrapolation of a  $T^2$  line to  $T=0$  is set to zero.

When we determine the power-law in an anisotropic superconductor, the impurity effect inducing a low-lying in-gap states should be taken into account. The rate of an impurity scattering is represented by the pair-breaking parameter:<sup>10,19</sup>  $\eta_c = \hbar[2\tau_s\Delta(0)]^{-1}$ , where  $\Delta(0)$  is a maximum gap at  $T=0$ . When we assume  $\Delta(0) = (1.75-2.5)k_B T_c^*$ ,  $\eta_c$  is calculated to be 0.014–0.021 for the present crystal. The residual density of states (DOS) is estimated to be about  $0.3N_F$ , where  $N_F$  is a normal-state DOS. Thus the impurity effect plays a significant role below  $T_{\text{imp}} = \eta_c^{1/2} T_c^* \sim 0.2$  K, which locates in the  $T^2$  regime. Therefore, the crossover could be attributed to the impurity effect, which is observed in  $T_1^{-1}$  of the recent <sup>101</sup>Ru NQR<sup>9</sup> deviating from  $T^3$  dependence below 0.15 K.

The  $T^3$  dependence seen at temperatures higher than  $T_{\text{imp}}$  is compared with the theory. According to the analysis for the transverse attenuation in an anisotropic superconductor,<sup>19</sup> the  $T^3$  dependence is characteristic for an existence of line nodes. The  $\mathbf{d}$  vector might be expressed by  $\Delta_0 z(k_x^2 - k_y^2)(k_x + ik_y)$ ,  $\Delta_0 z(k_x k_y)(k_x + ik_y)$ ,<sup>11</sup> or  $\Delta_0 z(\sin k_x + i \sin k_y)$ ,<sup>10</sup> all of which have the same symmetry with  $\Delta_0 z(k_x + ik_y)$ .<sup>7</sup> It is noted that the  $\mathbf{d}$  vector does not necessarily contain  $B_{1g}$ , although the superconducting state strongly couples to  $\epsilon_{xx} - \epsilon_{yy}$ . Because the order parameter

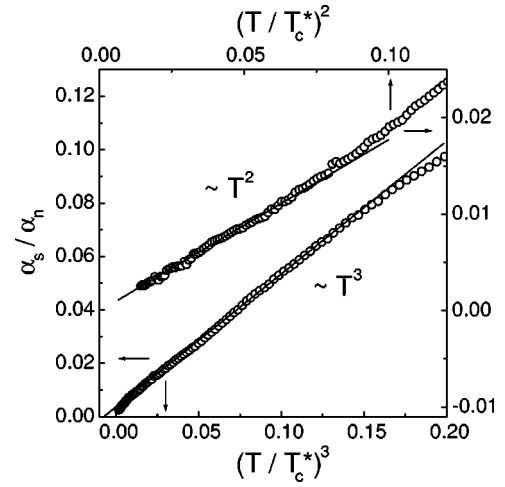


FIG. 4. The ratio  $\alpha_s/\alpha_n$  as a function of  $(T/T_c^*)^n$  ( $n=2$  and  $3$ ).

mentioned later [Eq. (3)] always appears in square in the strain-order parameter coupling.<sup>20</sup>

The elastic anomalies are discussed from the viewpoint of the strain-order parameter coupling.<sup>13,20,21</sup> The complex order parameter is defined as  $(\eta_1, \eta_2) = |\eta|(\cos \theta, e^{i\phi} \sin \theta)$  belonging to the two-dimensional  $E_u$ . The strain couples linearly to the bilinear products of the order parameter.<sup>13</sup> The free energy for the strain-order parameter coupling of the lowest order is expressed as

$$F_{\text{so}} = g_{A1g} \epsilon_{zz} (|\eta_1|^2 + |\eta_2|^2) + g_{B1g} (\epsilon_{xx} - \epsilon_{yy}) \times (|\eta_1|^2 - |\eta_2|^2) + g_{B2g} \epsilon_{xy} (\eta_1^* \eta_2 + \eta_1 \eta_2^*), \quad (3)$$

where the parameters  $g_\Gamma$  denote the coupling constants. According to the small change in  $C_{33}$  below  $T_c$  mentioned before,<sup>4</sup> the amplitude mode  $|\eta|$  relating to  $A_{1g}$  does not make a dominant contribution. It is noted that  $|\eta|$  plays an important role in UPt<sub>3</sub> and URu<sub>2</sub>Si<sub>2</sub>, where the noticeable jump is observed at  $T_c$  in longitudinal modes.<sup>21</sup> From the 2nd term, the coupling of  $\epsilon_{xx} - \epsilon_{yy}$  to the order parameter originates from the phase mode  $\theta$ . The large change in  $\Delta C_T$  below  $T_c^*$ , which is a direct evidence for the two-component order parameter and the chiral  $p$ -wave superconducting state,<sup>20</sup> results from the significant contribution of  $\theta$  to the superconducting state. The phase mode  $\phi$  in the 3rd term could be clarified by the measurement of  $C_{66}$ .

Finally we discuss the additional attenuation around  $T_a$  (Fig. 3), where the whole volume of the sample completes the transition to the superconducting state below  $T_c^{\text{offset}}$ . Moreover,  $\Delta C_T$  around  $T_a$  is larger than the calculated curve in Fig. 1(b), and thus there should exist some mechanism supplying the excess entropy. Such anomaly, however, is absent in  $C_{33}$  and  $C_{44}$ . These facts could be interpreted as an excitation of the superconducting collective mode by the transverse wave in  $(C_{11} - C_{12})/2$ . From the previous discussions on the strain-order parameter coupling, the phase mode  $\theta$  associated with a chirality should correspond

to the collective mode. Several collective order-parameter modes are theoretically discussed for a  $p$ -wave pairing with  $z\Delta_0(k_x + ik_y)$ .<sup>22</sup> In longitudinal modes of UPt<sub>3</sub>, a clear peak is emerged just below  $T_c$ , which could originate from the density correlation.<sup>23</sup> From the viewpoint of ultrasonic studies, it is evident that the  $p$ -wave superconducting state in Sr<sub>2</sub>RuO<sub>4</sub> is quite different from that in UPt<sub>3</sub>. Since the present measurements have been performed under hydrodynamic condition with large damping, further ultrasonic ex-

periments with high frequency are encouraged to clarify the collective mode.

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