Magnetically mediated superconductivity in quasi-two and three dimensions

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We compare predictions of the mean-field theory of superconductivity for nearly antiferromagnetic and nearly ferromagnetic metals in two and three dimensions. The calculations are based on a parametrization of the effective interaction arising from the exchange of magnetic fluctuations. The results show that for comparable parameters, magnetic pairing is more robust in quasi-two-dimensions than in three dimensions, for either *p*-wave (spin triplet) pairing in nearly ferromagnetic metals or *d*-wave (spin singlet) pairing in nearly antiferromagnetic metals. Moreover we find higher mean-field transition temperatures for *d*-wave pairing than for *p*-wave pairing (for comparable parameters), regardless of dimensionality. We present intuitive arguments for why quasi-two-dimensional *d*-wave pairing is a particularly favorable case.

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I. INTRODUCTION

As a metal is driven to the border of long range ferromagnetic or antiferromagnetic order one would expect that a magnetic or spin-spin interaction will become the dominant channel for interaction between fermion quasiparticles. It has been shown that such a magnetic interaction treated at the mean-field level can produce anomalous normal state properties and superconducting instabilities to anisotropic pairing states. In the simplest cases, the magnetic interaction is attractive in the *p*-wave spin triplet channel for nearly ferromagnetic metals and in the *d*-wave spin singlet channel for nearly antiferromagnetic metals.

There exists a wide range of compounds that are on or can be tuned to a metallic state on the border of long range magnetic order. If the predictions of the mean-field theory of the magnetic interaction model are correct then one would not expect magnetically mediated superconductivity to be a rare phenomenon. In an increasing number of experimental studies in carefully prepared specimens anisotropic superconductivity is indeed observed although in some cases only in very narrow regions of the phase diagram close to long range magnetic order.

Superconducting transition temperatures ranging from the low millikelvin range to liquid nitrogen temperature and beyond have been found in systems with strong short range magnetic correlations. It is interesting to consider whether the magnetic interaction model can account for this behavior and also explain why superconductivity has not yet been observed in a number of nearly magnetic metals.

A first step toward answering these questions is to consider which factors are favorable to magnetically mediated superconductivity. In this paper we consider within a unified phenomenological framework the following factors. The role of dimensionality and whether one is close to a ferromagnetic or commensurate antiferromagnetic instability. This extends our earlier analysis[1] for quasi-two-dimensional metals and confirms our speculations that such systems are more likely to exhibit magnetic pairing than three-dimensional metals having otherwise similar properties.

II. MODEL

We consider quasiparticles on a two-dimensional (2D) square or three-dimensional (3D) cubic lattice. We assume that the dominant scattering mechanism is of magnetic origin and postulate the following low-energy effective action for the quasiparticles

$$S_{\text{eff}}=\sum_{\mathbf{p},\alpha}\int_0^\beta d\tau\,\psi^\dagger_{\mathbf{p},\alpha}(\tau)(\partial_\tau+\epsilon_{\mathbf{p}}-\mu)\psi_{\mathbf{p},\alpha}(\tau) -\frac{g^2}{6}\sum_{\mathbf{q}}\int_0^\beta d\tau\int_0^\beta d\tau'\,\chi(\mathbf{q},\tau-\tau')\mathbf{s}(\mathbf{q},\tau)\cdot\mathbf{s}(-\mathbf{q},\tau'). \quad (1)$$

The spin density $\mathbf{s}(\mathbf{q},\tau)$ is given by

$$\mathbf{s}(\mathbf{q},\tau)\equiv\sum_{\mathbf{p},\alpha,\gamma}\psi^\dagger_{\mathbf{p}+\mathbf{q},\alpha}(\tau)\sigma_{\alpha,\gamma}\psi_{\mathbf{p},\gamma}(\tau), \quad (2)$$

where σ denotes the three Pauli matrices. The quasiparticle dispersion relation is

$$\epsilon_{\mathbf{p}}=-2t[\cos(p_xa)+\cos(p_ya)]-4t'\cos(p_xa)\cos(p_ya) \quad (3)$$

in quasi-2D and

$$\epsilon_{\mathbf{p}}=-2t[\cos(p_xa)+\cos(p_ya)+\cos(p_za)] -4t'[\cos(p_xa)\cos(p_ya)+\cos(p_xa)\cos(p_za) +\cos(p_ya)\cos(p_za)] \quad (4)$$

in 3D, with hopping matrix elements t and t' and lattice spacing a. μ denotes the chemical potential, β the inverse temperature, g^2 the coupling constant, and $\psi^\dagger_{\mathbf{p},\sigma}$ and $\psi_{\mathbf{p},\sigma}$ are Grassmann variables. In the following we shall measure temperatures, frequencies and energies in the same units. In order to reduce the number of independent parameters we shall take the nearest neighbor hopping $t'=0.45t$ and an electron filling factor $n\approx1.1$ as in our earlier work for 2D alone.[1] The effects of ϵ'/t and n will be discussed in Sec. III B.

The retarded generalized magnetic susceptibility $\chi(\mathbf{q},\omega)$ that defines the effective interaction, Eq. (1), is assumed to take the phenomenological form

$$\chi(\mathbf{q},\omega)=\frac{\chi_0\kappa_0^2}{\kappa^2+\hat{q}^2-i\dfrac{\omega}{\eta(\hat{q})}}, \tag{5}$$

where κ and κ_0 are the inverse correlation lengths (in units of a^{-1}) with and without strong magnetic correlations, respectively. Let

$$\hat{q}_{\pm}^2=4\pm2[\cos(q_xa)+\cos(q_ya)], \tag{6}$$

$$\hat{q}_{\pm}^2=6\pm2[\cos(q_xa)+\cos(q_ya)+\cos(q_za)] \tag{7}$$

in quasi-2D and in 3D, respectively. In the case of a nearly ferromagnetic metal, the parameters $\hat{q}^2$ and $\eta(\hat{q})$ in Eq. (5) are defined as

$$\hat{q}^2=\hat{q}_-^2, \tag{8}$$

$$\eta(\hat{q})=T_{\mathrm{SF}}\hat{q}_-, \tag{9}$$

where T_{SF} is a characteristic spin-fluctuation temperature. We shall also investigate nearly antiferromagnetic metals with commensurate incipient ordering wave vectors $\mathbf{Q}_{2\mathrm{D}}=(\pi/a,\pi/a)$ in quasi-2D and $\mathbf{Q}_{3\mathrm{D}}=(\pi/a,\pi/a,\pi/a)$ in 3D. In this case we have

$$\hat{q}^2=\hat{q}_+^2, \tag{10}$$

$$\eta(\hat{q})=T_{\mathrm{SF}}\hat{q}_-. \tag{11}$$

As in our previous work,[1] the band structure and generalized magnetic susceptibility are modeled independently. This choice may be inconsistent when all of the contributions to $\chi(\mathbf{q},\omega)$ come from the chosen band. However, it allows us, in principle, to deal with the case where there are other important contributions to the generalized magnetic susceptibility. It has been argued that the latter case is of relevance to the ruthenates.[5]

The spin-fluctuation propagator on the imaginary axis $\chi(\mathbf{q},i\nu_n)$ is related to the imaginary part of the response function $\mathrm{Im}\chi(\mathbf{q},\omega)$, Eq. (5), via the spectral representation

$$\chi(\mathbf{q},i\nu_n)=-\int_{-\infty}^{+\infty}\frac{d\omega}{\pi}\,\frac{\mathrm{Im}\,\chi(\mathbf{q},\omega)}{i\nu_n-\omega}. \tag{12}$$

To get $\chi(\mathbf{q},i\nu_n)$ to decay as $1/\nu_n^2$ as $\nu_n\to\infty$, as it should, we introduce a cutoff ω_0 and take $\mathrm{Im}\,\chi(\mathbf{q},\omega)=0$ for $\omega\geqslant\omega_0$. A natural choice for the cutoff is $\omega_0=\eta(\hat{q})\kappa_0^2$. We have checked that our results for the critical temperature are not sensitive to the particular choice of ω_0 used.

The two- or three-dimensional Eliashberg equations for the critical temperature T_c in the Matsubara representation reduce, for the effective action Eq. (1), to

$$\Sigma(\mathbf{p},i\omega_n)=g^2\frac{T}{N}\sum_{\Omega_n}\sum_{\mathbf{k}}\chi(\mathbf{p}-\mathbf{k},i\omega_n-i\Omega_n)G(\mathbf{k},i\Omega_n), \tag{13}$$

$$G(\mathbf{p},i\omega_n)=\frac{1}{i\omega_n-(\epsilon_{\mathbf{p}}-\mu)-\Sigma(\mathbf{p},i\omega_n)}, \tag{14}$$

$$\Lambda(T)\Phi(\mathbf{p},i\omega_n)=\begin{bmatrix}\dfrac{g^2}{3}\\ -g^2\end{bmatrix}\frac{T}{N}\sum_{\Omega_n}\sum_{\mathbf{k}}\chi(\mathbf{p}-\mathbf{k},i\omega_n-i\Omega_n)\times|G(\mathbf{k},i\Omega_n)|^2\Phi(\mathbf{k},i\Omega_n),$$

$$\Lambda(T)=1\to T=T_c, \tag{15}$$

where $\Sigma(\mathbf{p},i\omega_n)$ is the quasiparticle self-energy, $G(\mathbf{p},i\omega_n)$ the one-particle Green's function, and $\Phi(\mathbf{p},i\omega_n)$ the anomalous self-energy. $\epsilon_{\mathbf{p}}$ is the bare quasiparticle spectrum, Eq. (3,4), μ the chemical potential that is adjusted to give an electron density of $n=1.1$, and N the total number of allowed wavevectors in the Brillouin zone. In Eq. (15), the prefactor $g^2/3$ is for triplet pairing while the prefactor $-g^2$ is appropriate for singlet pairing. Only the longitudinal spin-fluctuation mode contributes to the pairing amplitude in the triplet channel and gives rise to an attractive interaction. Both transverse and longitudinal spin-fluctuation modes contribute to the pairing amplitude in the singlet channel and give an interaction which is repulsive in reciprocal space with a peak at $\mathbf{Q}_{2\mathrm{D}}$ or $\mathbf{Q}_{3\mathrm{D}}$. When Fourier transformed, such a potential is repulsive on one sublattice (even sites) and attractive on the other (odd sites). All three modes contribute to the quasiparticle self-energy.

The momentum convolutions in Eqs. (13), (15) are carried out with a fast Fourier transform algorithm on a 128×128 lattice in two dimensions and $48\times48\times48$ lattice in three dimensions. The frequency sums in both the self-energy and linearized gap equations are treated with the renormalization group technique of Pao and Bickers.[2] We have kept between 8 and 16 Matsubara frequencies at each stage of the renormalization procedure, starting with an initial temperature $T_0=0.4t$ and cutoff $\Omega_c\approx20t$ in two dimensions while the corresponding values for three dimensions were $T_0=0.6t$ and $\Omega_c\approx30t$. The renormalization group acceleration technique restricts one to a discrete set of temperatures $T_0>T_1>T_2\cdots$. The critical temperature at which $\Lambda(T)=1$ in Eq. (15) is determined by linear interpolation. The savings in computer time and memory requirements afforded by this technique allowed us to study a wide range of temperatures and spin-fluctuation spectrum parameters in two and three dimensions on a desktop workstation.

III. RESULTS

A. Solution of the Eliashberg equations

The dimensionless parameters at our disposal are $g^2\chi_0/t$, T_{SF}/t, κ_0, and κ. For comparison with the results of our earlier work,[1] we take $T_{\mathrm{SF}}=\frac{2}{3}t$ and $\kappa_0^2=12$. In 2D, this T_{SF} corresponds to about 1000 K for a bandwidth of 1 eV while our choice of of κ_0^2 is a representative value.

The parameters of the model can in principle be inferred from the electronic structure, the dynamical magnetic susceptibility, and the resistivity in the normal state. The resis-

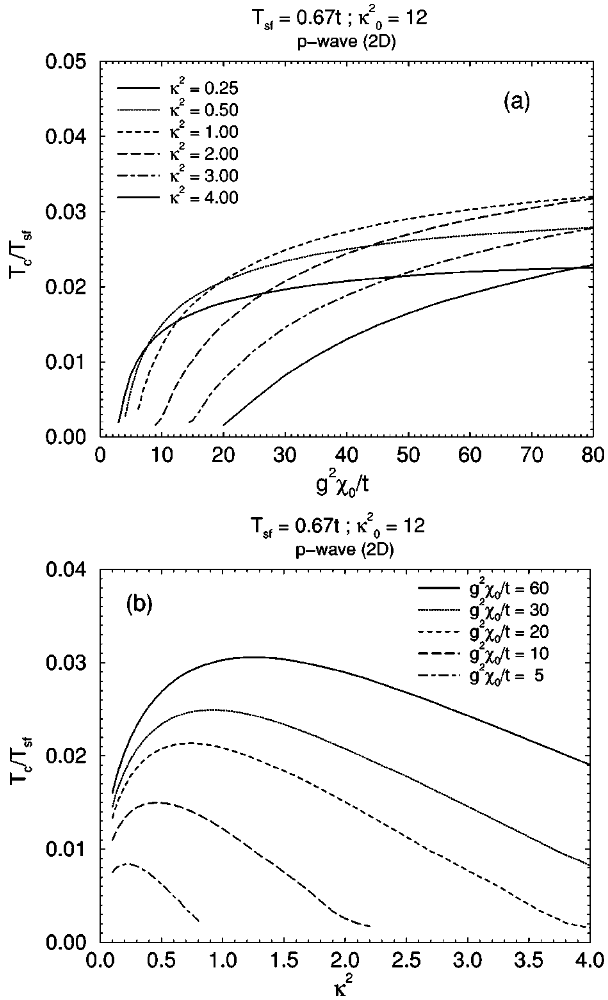

FIG. 1. The mean-field critical temperature T_c to the p-wave superconducting state in quasi-2D versus $g^2\chi_0/t$ for $\kappa^2=0.25$, 0.50, 1.0, 2.0, 3.0, 4.0 (a) and versus κ^2 for $g^2\chi_0/t=60$, 30, 20, 10, 5 (b). The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ with $\kappa_0^2=12$.

tivity in particular may be used to estimate the dimensionless coupling parameter $g^2\chi_0/t$, the value of which is between 10 and 20 for the simplest RPA approximation for the magnetic interaction potential.

The results of our numerical calculations of the mean-field critical temperature T_c in the case of a nearly ferromagnetic metal are shown in Figs. 1 and 2 for 2D and 3D, respectively. We find an instability for a p-wave gap function $\Phi(\mathbf{p},i\omega_n)$ transforming as $\sin(p_xa)$ (or a symmetry related function).

Figures 1(a) and 2(a) show T_c versus the dimensionless coupling parameter $g^2\chi_0/t$ for several values of the square of the inverse correlation length parameter κ^2 while Figs. 1(b), and 2(b) show T_c versus κ^2 for several values of the coupling parameter $g^2\chi_0/t$. The parameter κ^2 can be varied experimentally, for example, by applying pressure to the sample. The T_c versus κ^2 graphs can be interpreted as T_c versus pressure plots, with the critical pressure corresponding to the quantum critical point at $\kappa^2=0$. The critical temperature saturates, in the strong coupling limit, to a value of

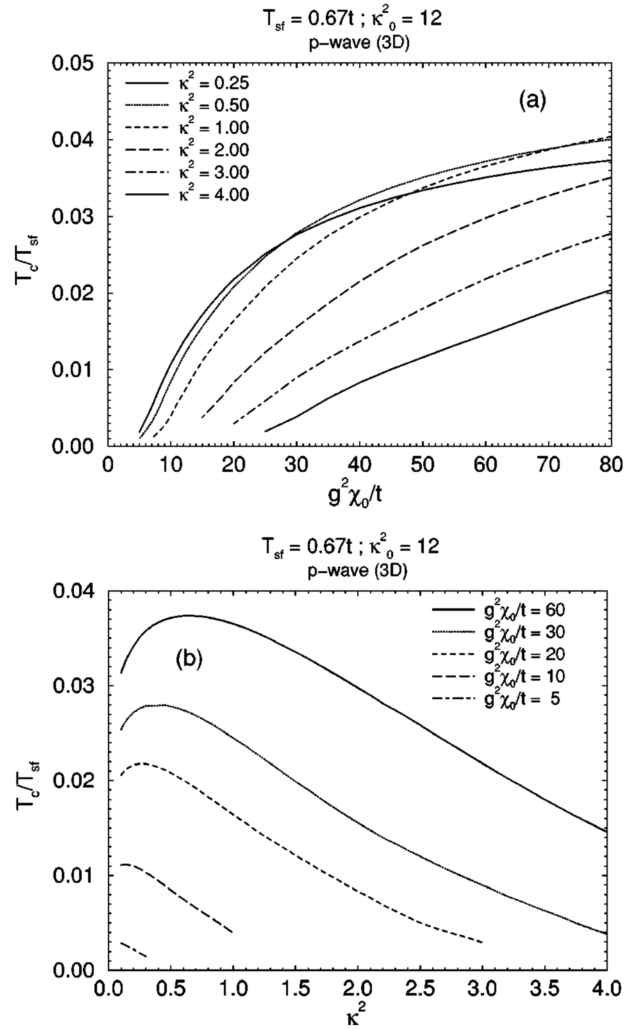

FIG. 2. The mean-field critical temperature T_c to the p-wave superconducting state in 3D versus $g^2\chi_0/t$ for $\kappa^2=0.25$, 0.50, 1.0, 2.0, 3.0, 4.0 (a) and versus κ^2 for $g^2\chi_0/t=60$, 30, 20, 10, 5 (b). The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ with $\kappa_0^2=12$.

about $T_{\mathrm{SF}}/30$ for values of κ^2 of 0.5 to 1.0 in both 2D and 3D. However, for realistic coupling constants ($g^2\chi_0/t$ of the order of 10) T_c is higher in 2D than in 3D except for small κ^2 of the order of 0.2 or lower. In particular the decrease of T_c with increasing κ^2 for $\kappa^2\geqslant 0.2$ is much weaker in 2D than in 3D.

The calculations of the Eliashberg renormalization factor $Z(\mathbf{p},i\omega_n)=1-\mathrm{Im}\,\Sigma(\mathbf{p},i\omega_n)/\omega_n$ in the case of a nearly ferromagnetic metal are presented in Figs. 3 and 4 for 2D and 3D, respectively. The main point to note is that $Z(\mathbf{p},i\pi T)$ is consistently higher in 2D than in 3D.

Our results for the mean-field transition temperature T_c in the case of a nearly antiferromagnetic metal are shown in Figs. 5 and 6 for 2D and 3D, respectively. We find an instability for a d-wave gap function $\Phi(\mathbf{p},i\omega_n)$ transforming as $\cos(p_xa)-\cos(p_ya)$ (or a symmetry related function).

Figures 5(a) and 6(a) show T_c versus the dimensionless coupling parameter $g^2\chi_0/t$ for several values of κ^2 while Figs. 5(b) and 6(b) show T_c versus κ^2 for several values of

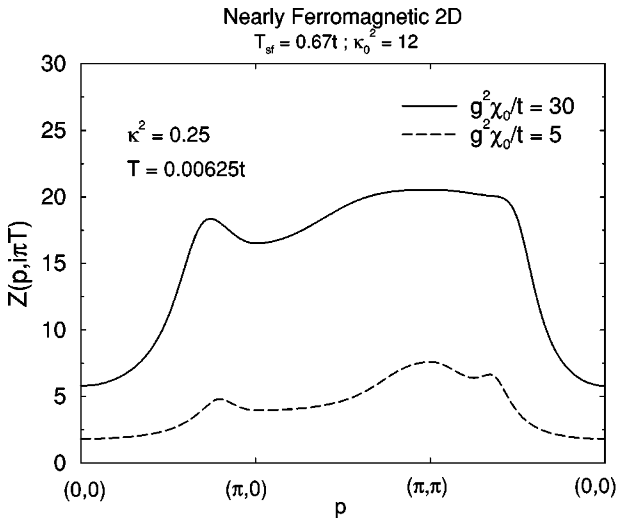

FIG. 3. The Eliashberg renormalization factor $Z(\mathbf{p},i\pi T)=1-\mathrm{Im}\,\Sigma(\mathbf{p},i\pi T)/\pi T$ versus wave vector $\mathbf{p}$ for ferromagnetic spin-fluctuations in quasi-2D for $g^2\chi_0/t=5{,}30$, $\kappa^2=0.25$, and $T=0.00625t$. The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ and $\kappa_0^2=12$.

the coupling parameter $g^2\chi_0/t$. The critical temperature saturates, in the strong coupling limit, to a value of about $T_{\mathrm{SF}}/2$ for values of κ^2 of 0.5 to 1.0 in both 2D and 3D. However, as in the case of nearly ferromagnetic systems, for realistic coupling constants ($g^2\chi_0/t$ of the order of 10) T_c is higher in 2D than in 3D. Also, the decrease of T_c with increasing κ^2 is much weaker in 2D than in 3D. These effects are even more pronounced than in nearly ferromagnetic systems (Figs. 1 and 2). Note also that the values of T_c are everywhere higher for a nearly antiferromagnetic than a nearly ferromagnetic metal for otherwise equivalent parameters.

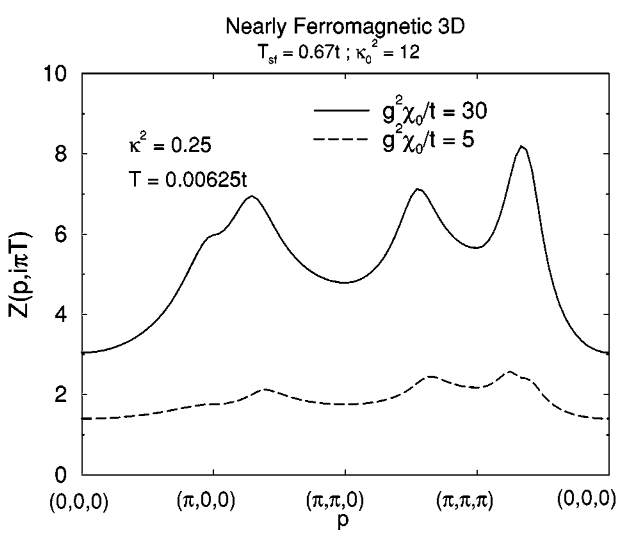

FIG. 4. The Eliashberg renormalization factor $Z(\mathbf{p},i\pi T)=1-\mathrm{Im}\,\Sigma(\mathbf{p},i\pi T)/\pi T$ versus wave vector $\mathbf{p}$ for ferromagnetic spin-fluctuations in 3D for $g^2\chi_0/t=5{,}30$, $\kappa^2=0.25$, and $T=0.00625t$. The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ and $\kappa_0^2=12$.

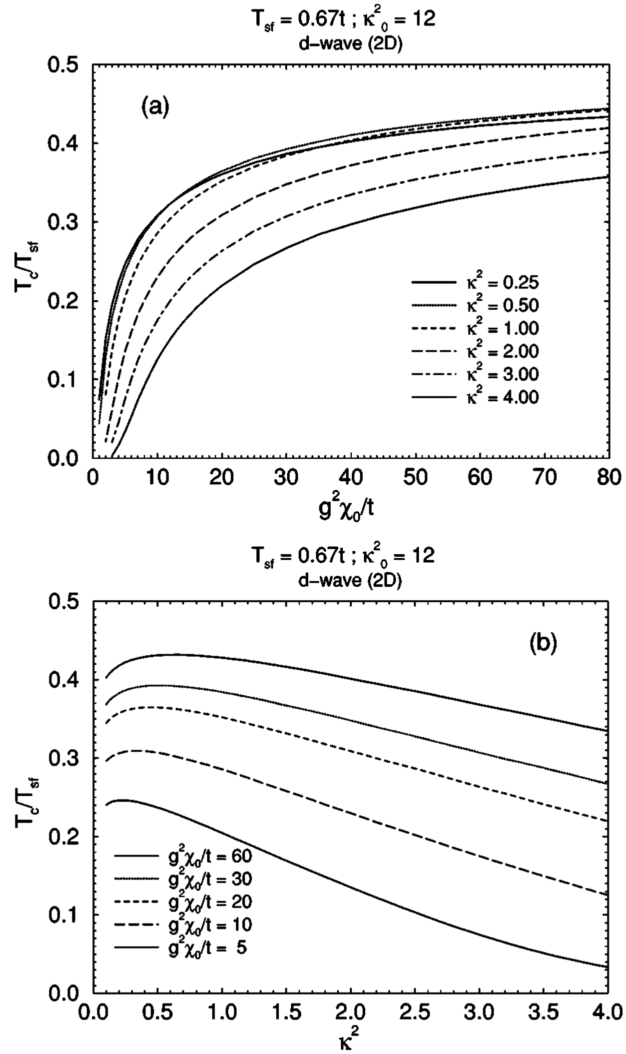

FIG. 5. The mean-field critical temperature T_c to the d-wave superconducting state in quasi-2D versus $g^2\chi_0/t$ for $\kappa^2=0.25$, 0.50, 1.0, 2.0, 3.0, 4.0 (a) and versus κ^2 for $g^2\chi_0/t=60$, 30, 20, 10, 5 (b). The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ with $\kappa_0^2=12$.

The calculations of the Eliashberg renormalization factor $Z(\mathbf{p},i\omega_n)=1-\mathrm{Im}\,\Sigma(\mathbf{p},i\omega_n)/\omega_n$ in the case of a nearly antiferromagnetic metal are presented in Figs. 7 and 8 for 2D and 3D, respectively. As for nearly ferromagnetic metals, we find that $Z(\mathbf{p},i\pi T)$ is consistently higher in 2D than in 3D.

B. Mass renormalization and interaction parameters

In order to make a comparison with the corresponding electron-phonon problem it is instructive to define a mass renormalization parameter λ_Z and interaction parameter λ_Δ. We define

$$\lambda_Z=\frac{\displaystyle\int_{-\infty}^{+\infty}\frac{d\omega}{\pi}\left\langle\frac{1}{\omega}\mathrm{Im}\,V_Z(\mathbf{p}-\mathbf{p}',\omega)\right\rangle_{\mathrm{FS}(\mathbf{p},\mathbf{p}')}}{\langle 1\rangle_{\mathrm{FS}(\mathbf{p})}}, \tag{16}$$

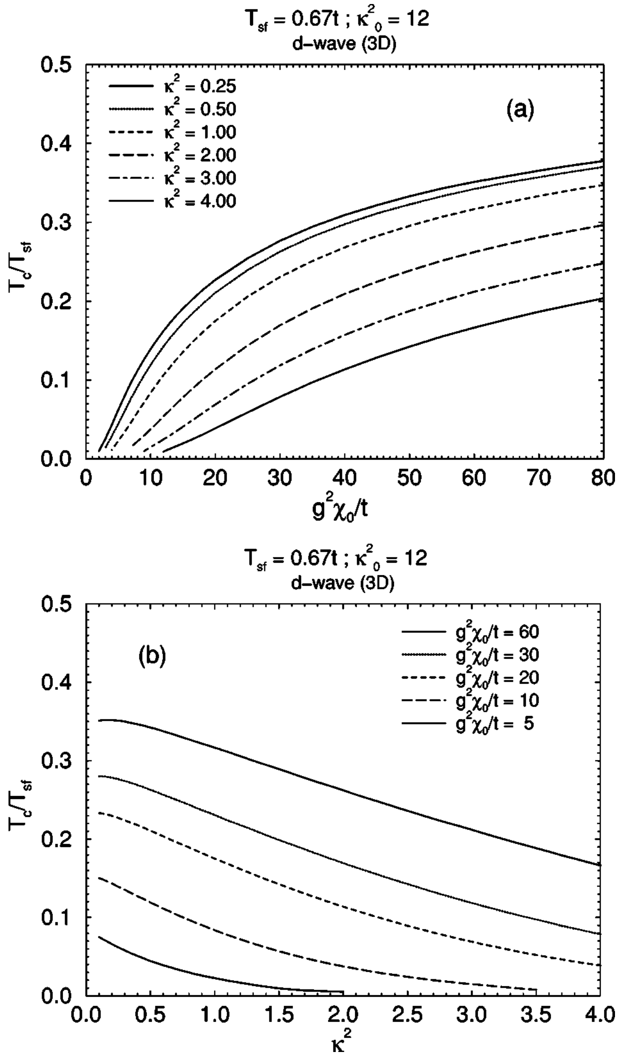

FIG. 6. The mean-field critical temperature T_c to the d-wave superconducting state in 3D versus $g^2\chi_0/t$ for $\kappa^2=0.25$, 0.50, 1.0, 2.0, 3.0, 4.0 (a) and versus κ^2 for $g^2\chi_0/t=60$, 30, 20, 10, 5 (b). The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ with $\kappa_0^2=12$.

$$\lambda_\Delta=-\frac{\left\langle \int_{-\infty}^{+\infty}\frac{d\omega}{\pi}\left\langle\frac{1}{\omega}\,\mathrm{Im}\,V_\Delta(\mathbf{p}-\mathbf{p}',\omega)\,\eta(\mathbf{p})\,\eta(\mathbf{p}')\right\rangle_{\mathrm{FS}(\mathbf{p},\mathbf{p}')}\right.}{\langle\eta^2(\mathbf{p})\rangle_{\mathrm{FS}(\mathbf{p})}},\tag{17}$$

where

$$V_Z(\mathbf{q},\omega)=g^2\chi(\mathbf{q},\omega)\tag{18}$$

and

$$V_p(\mathbf{q},\omega)=-\frac{g^2}{3}\chi(\mathbf{q},\omega),\tag{19}$$

$$\eta(\mathbf{p})=\sin(p_x a)\tag{20}$$

for p-wave spin triplet pairing ($\Delta\equiv p$) while

$$V_d(\mathbf{q},\omega)=g^2\chi(\mathbf{q},\omega)\tag{21}$$

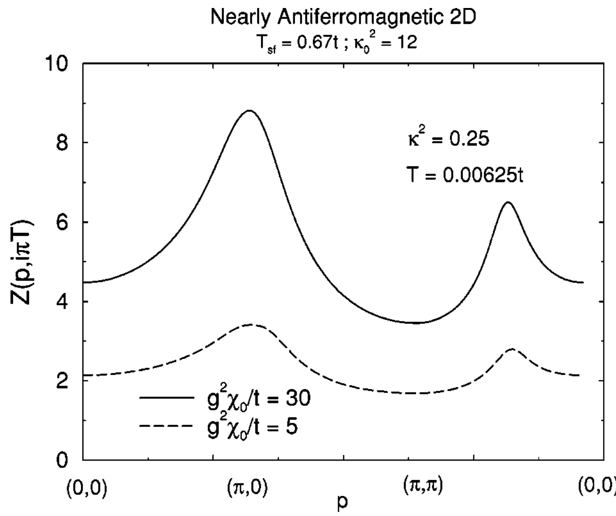

FIG. 7. The Eliashberg renormalization factor $Z(\mathbf{p},i\pi T)=1-\mathrm{Im}\,\Sigma(\mathbf{p},i\pi T)/\pi T$ versus wave vector $\mathbf{p}$ for antiferromagnetic spin-fluctuations in quasi-2D for $g^2\chi_0/t=5{,}30$, $\kappa^2=0.25$, and $T=0.00625t$. The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ and $\kappa_0^2=12$.

$$\eta(\mathbf{p})=\cos(p_x a)-\cos(p_y a)\tag{22}$$

in the case of d-wave spin-singlet pairing ($\Delta\equiv d$). The Fermi surface averages are given by

$$\langle\cdots\rangle_{\mathrm{FS}(\mathbf{p})}=\int\frac{d^d p}{(2\pi)^d}\cdots\delta(\epsilon_{\mathbf{p}}-\mu),\tag{23}$$

$$\langle\cdots\rangle_{\mathrm{FS}(\mathbf{p},\mathbf{p}')}=\int\frac{d^d p}{(2\pi)^d}\frac{d^d p'}{(2\pi)^d}\cdots\delta(\epsilon_{\mathbf{p}}-\mu)\,\delta(\epsilon_{\mathbf{p}'}-\mu).\tag{24}$$

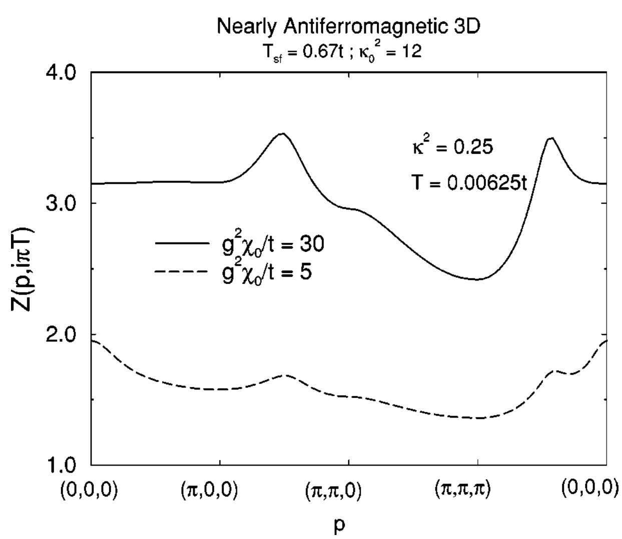

FIG. 8. The Eliashberg renormalization factor $Z(\mathbf{p},i\pi T)=1-\mathrm{Im}\,\Sigma(\mathbf{p},i\pi T)/\pi T$ versus wave vector $\mathbf{p}$ for antiferromagnetic spin-fluctuations in 3D for $g^2\chi_0/t=5{,}30$, $\kappa^2=0.25$, and $T=0.00625t$. The characteristic spin-fluctuation temperature is $T_{\mathrm{SF}}=0.67t$ and $\kappa_0^2=12$.

In practice, we compute the Fermi surface average with a discrete set of momenta on an N^d lattice and we replace the delta function by a finite temperature expression

$$\int \frac{d^d p}{(2\pi)^d} \to \frac{1}{N^d} \sum_{\mathbf{p}} , \tag{25}$$

$$\delta(\epsilon_{\mathbf{p}} - \mu) \to \frac{1}{k_B T} f_{\mathbf{p}}(1 - f_{\mathbf{p}}), \tag{26}$$

where $f_{\mathbf{p}}$ is the Fermi function. Note that $(1/k_B T) f_{\mathbf{p}}(1-f_{\mathbf{p}}) \to \delta(\epsilon_{\mathbf{p}} - \mu)$ as $T \to 0$. We have used $T = 0.1t$ and $N = 128$ in all of our calculations. The finite temperature effectively means that van Hove singularities will be smeared out.

Note that the Fermi surface average that appears in λ_Z, Eq. (16) plays a role similar to that of $\alpha^2 F(\omega)/\omega$ in the case of phonon mediated superconductivity. From the definitions of the parameters $\lambda_{Z,\Delta}$ Eqs. (16), (17) and our model for $\chi(\mathbf{q},\omega)$ Eq. (5), we see that $\lambda_{Z,\Delta}$ are directly proportional to the dimensionless factor $g^2 \chi_0 \kappa_0^2/t$. Thus we will consider the quantities

$$\lambda^*_{Z,\Delta} \equiv \lambda_{Z,\Delta}/(g^2 \chi_0 \kappa_0^2/t) \tag{27}$$

which are functions only of n, t'/t, and κ^2.

The quantity $1 + \lambda_Z$ is the Fermi surface average of the mass renormalization calculated with the bare Green's function to lowest order in perturbation theory. In the previous sections we presented out results for the Eliashberg renormalization factor $Z(\mathbf{p}, i\pi T)$. The quantity $Z(\mathbf{p}, i\pi T)_{T\to 0}$ is closely related to the mass renormalization calculated with the dressed Green's function. Due to negative feedback in dressing the Green's function, the Fermi surface average of $Z(\mathbf{p}, i\pi T)_{T\to 0}$ is always smaller than λ_Z and should approach λ_Z in the weak coupling limit $g \to 0$.

The quantity λ_Δ is a measure of the effectiveness of the interaction $V_\Delta(\mathbf{q},\omega)$ in pairing quasiparticles in the Cooper pair state of symmetry described by $\eta(\mathbf{p})$. For the special case of s-wave phonon mediated pairing λ_Δ reduces to λ_Z. However, for magnetically mediated superconductivity λ_Δ is typically substantially less than λ_Z and becomes comparable to λ_Z only under very special conditions. The ratio λ_Δ/λ_Z is of particular interest. Magnetically mediated superconductivity may be expected to be difficult to detect in systems with low values of this quantity.

Calculations of $\lambda^*_{Z,\Delta}$ vs κ^2 for $t'/t = 0.45$ and n = 1.1 are presented in Figs. 9 and 10 for a nearly ferromagnetic and a nearly antiferromagnetic system, respectively. We note that both λ^*_Z and λ^*_Δ increase with decreasing κ^2 and that their values in 2D are everywhere greater than in 3D. The latter may be seen directly from Eqs. (16) and (17) and is a consequence of a general tendency of fluctuations to be stronger in 2D than in 3D.

The higher values of λ^*_Δ in 2D than in 3D might lead us to expect that T_c would always be higher in 2D than in 3D, for otherwise equivalent conditions. This agrees with the results of the Eliashberg calculations for weak to moderate coupling and not too low values of κ^2. However, at very small κ^2 and strong coupling the full Green's function used in the Eliashberg calculations but not in λ^*_Z and λ^*_Δ lead, in a nearly ferromagnetic metal, to a suppression of T_c which is more pronounced in 2D than in 3D Figs. 1, 2. Therefore, a simple McMillan type formula relating T_c/T_{SF} to λ_Z and λ_Δ

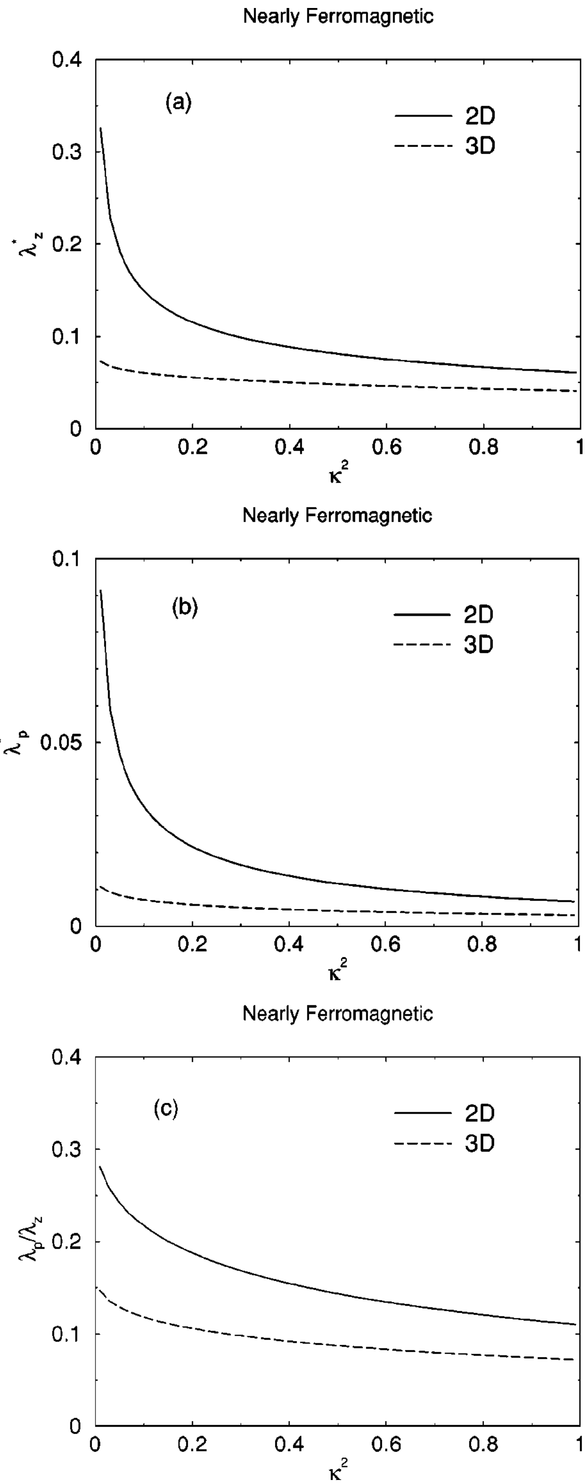

FIG. 9. The interaction parameters $\lambda^*_Z = \lambda_Z/(g^2\chi_0\kappa_0^2/t)$ (a), $\lambda^*_p = \lambda_p/(g^2\chi_0\kappa_0^2/t)$ (b), and the ratio λ_p/λ_Z (c) versus κ^2 for a quasi-2D and a 3D nearly ferromagnetic system. The next nearest neighbor hopping $t' = 0.45t$, the band filling $n = 1.1$ and the spin-fluctuation temperature $T_{\mathrm{SF}} = 0.67t$.

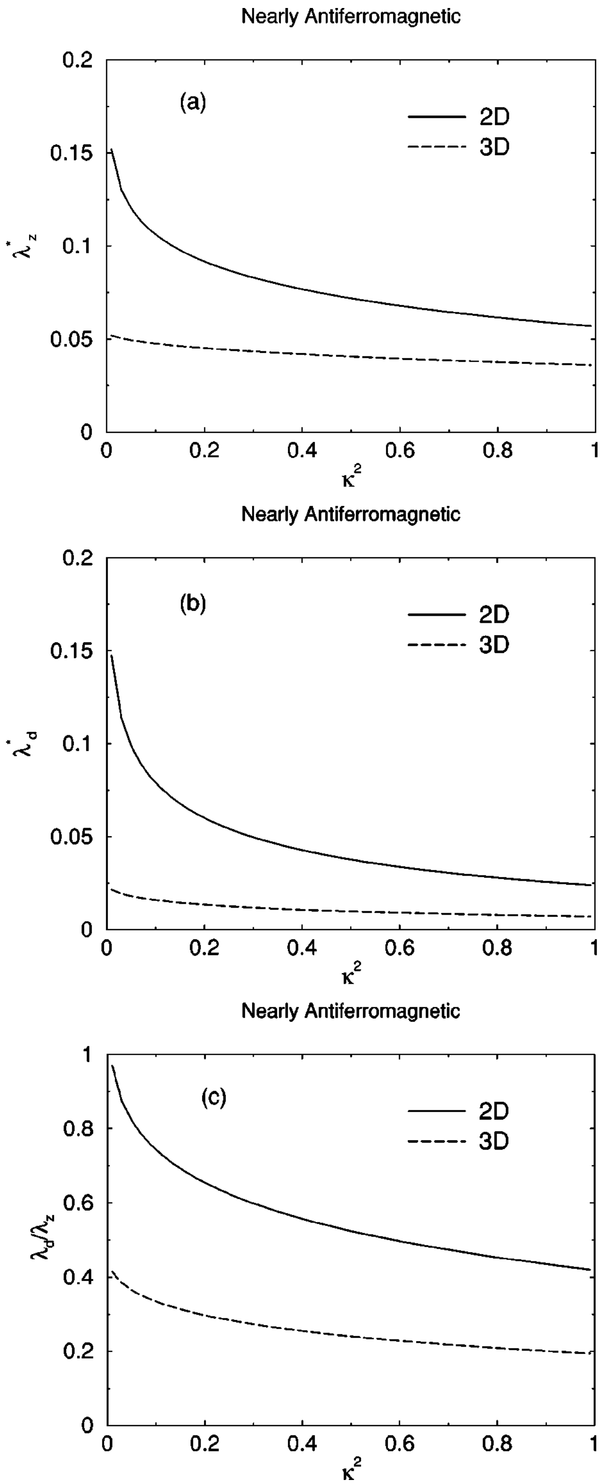

FIG. 10. The interaction parameters $\lambda_Z^* = \lambda_Z/(g^2\chi_0\kappa_0^2/t)$ (a), $\lambda_d^* = \lambda_d/(g^2\chi_0\kappa_0^2/t)$ (b), and the ratio λ_d/λ_Z (c) versus κ^2 for a quasi-2D and 3D nearly antiferromagnetic system. The next nearest neighbor hopping $t' = 0.45t$, the band filling $n = 1.1$ and the spin-fluctuation temperature $T_{\rm SF} = 0.67t$.

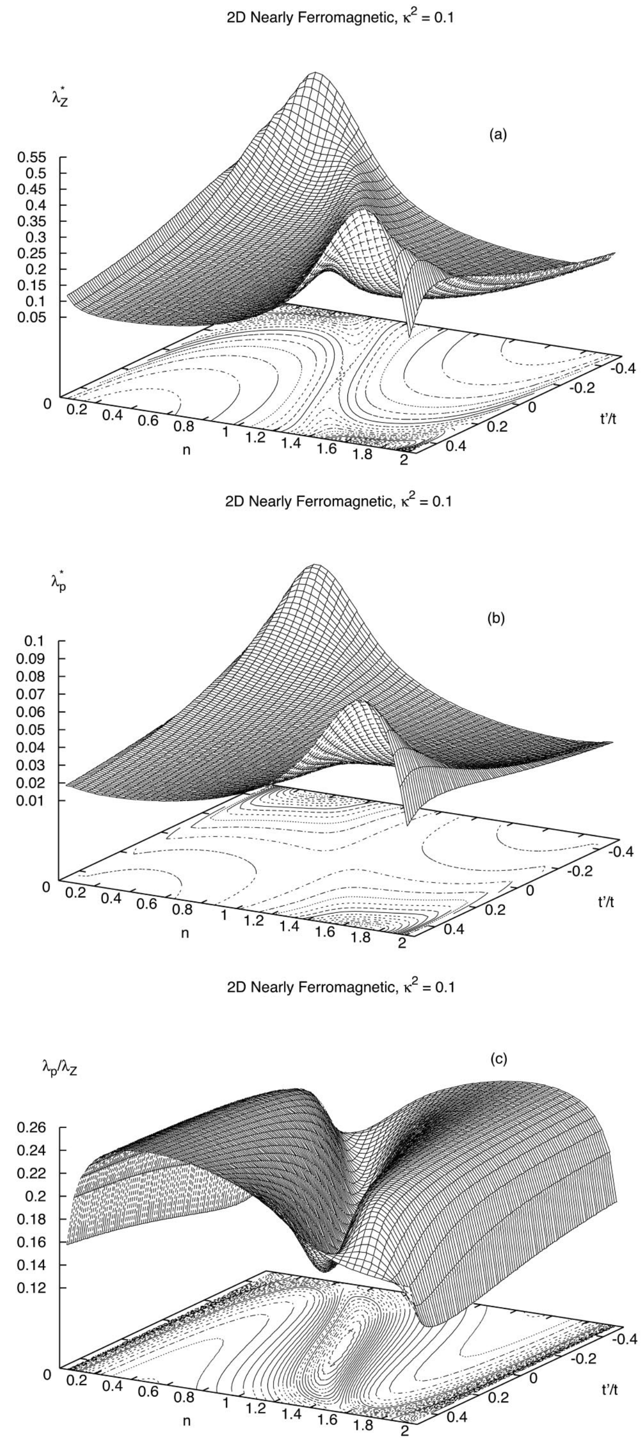

FIG. 11. The interaction parameters $\lambda_Z^* = \lambda_Z/(g^2\chi_0\kappa_0^2/t)$ (a), $\lambda_p^* = \lambda_p/(g^2\chi_0\kappa_0^2/t)$ (b), and the ratio λ_p/λ_Z (c) versus band filling n and ratio of next nearest to nearest hopping t'/t for a quasi-2D nearly ferromagnetic system, for $\kappa^2 = 0.1$ and $T_{\rm SF} = 0.67t$.

for a wide range of κ^2 and coupling constants does not seem to exist. This conclusion also applies, albeit less noticeably, to a nearly antiferromagnetic metal.

Thus far, we have considered fixed values of t'/t and electron filling n. In Figs. 11–14 we present calculations of $\lambda_{Z,\Delta}^*$, λ_Δ/λ_Z at fixed $\kappa^2 = 0.1$ as functions of t'/t and n. The variation of these quantities with t'/t and n are significant in nearly ferromagnetic and antiferromagnetic systems in both 2D and 3D. This suggests that magnetically mediated superconductivity may be quite sensitive not only to dimensionality and nature of the spin fluctuations, but also to details of

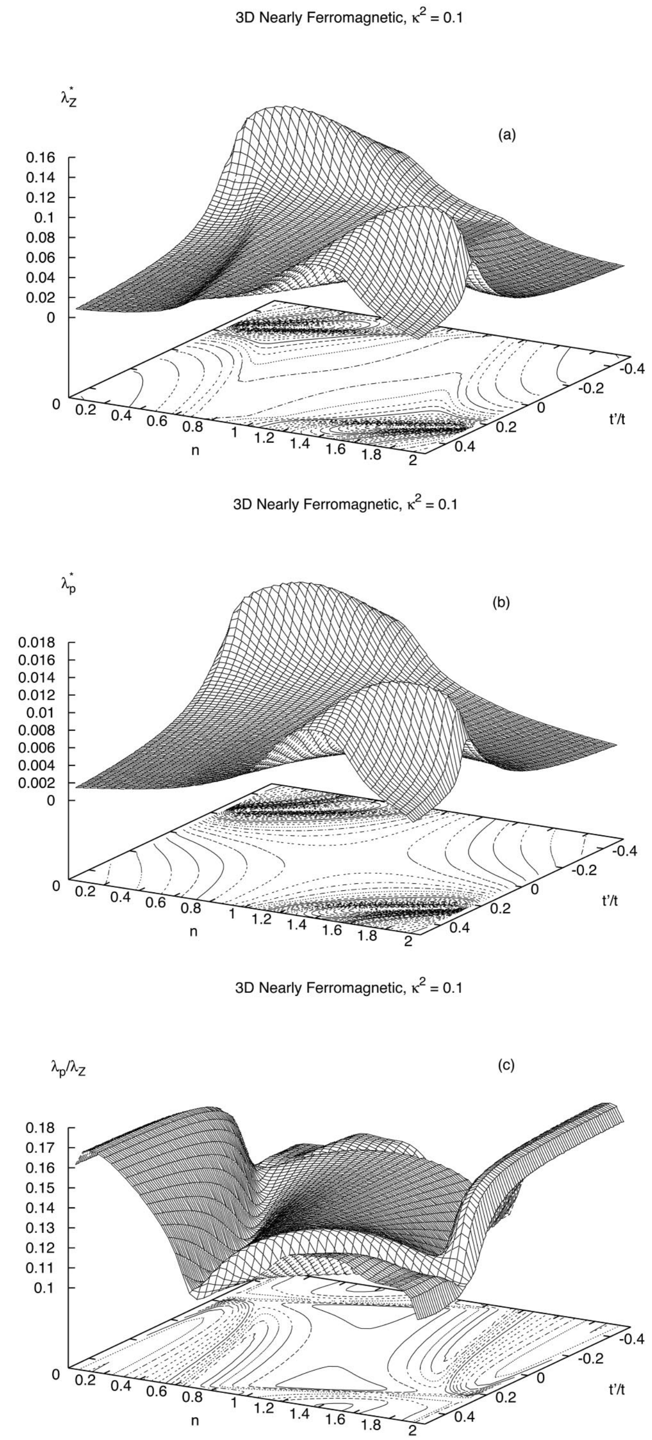

FIG. 12. The interaction parameters $\lambda_Z^* = \lambda_Z/(g^2\chi_0\kappa_0^2/t)$ (a), $\lambda_p^* = \lambda_p/(g^2\chi_0\kappa_0^2/t)$ (b), and the ratio λ_p/λ_Z (c) versus band filling n and ratio of next nearest to nearest hopping t'/t for a 3D nearly ferromagnetic system, for $\kappa^2 = 0.1$ and $T_{\mathrm{SF}} = 0.67t$.

the band structure. We note, however, that the use of the full Green's function in the Eliashberg equations tends to reduce this sensitivity.

IV. DISCUSSION

The above results and those of our previous work[1] suggest that magnetically mediated superconductivity could to be

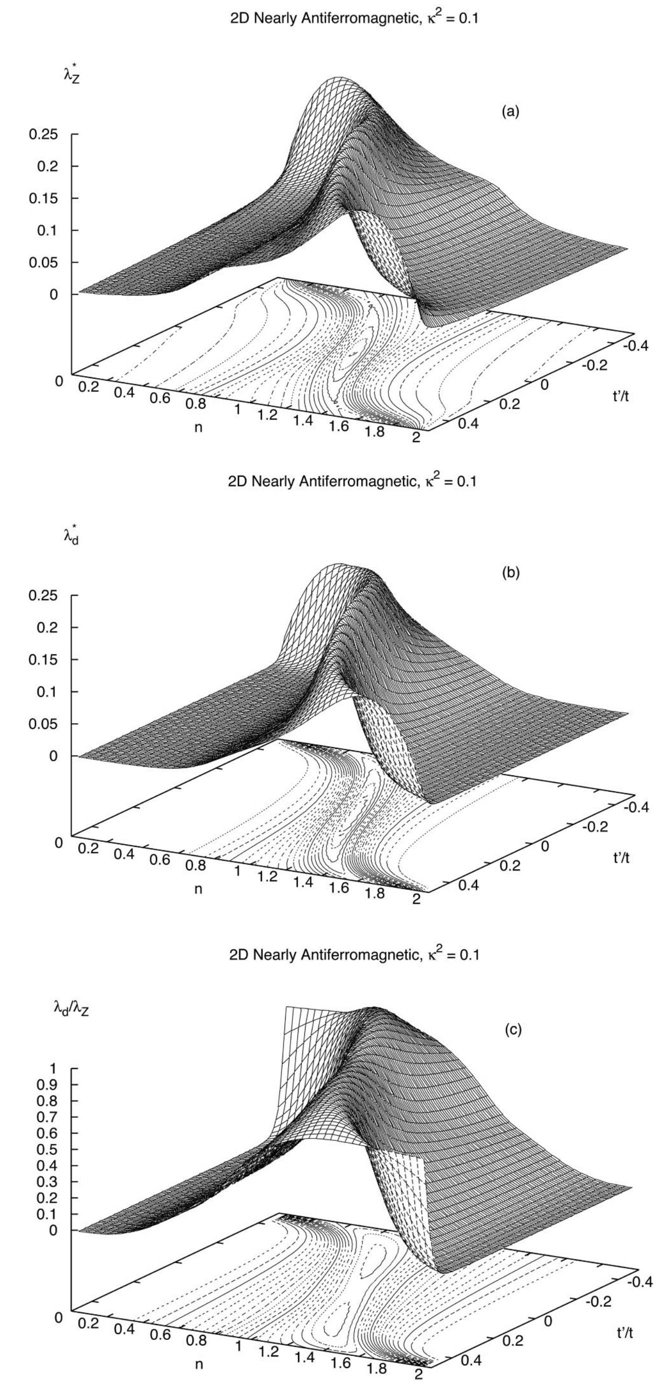

FIG. 13. The interaction parameters $\lambda_Z^* = \lambda_Z/(g^2\chi_0\kappa_0^2/t)$ (a), $\lambda_d^* = \lambda_d/(g^2\chi_0\kappa_0^2/t)$ (b), and the ratio λ_d/λ_Z (c) versus band filling n and ratio of next nearest to nearest hopping t'/t for a quasi-2D nearly antiferromagnetic system, for $\kappa^2 = 0.1$ and $T_{\mathrm{SF}} = 0.67t$.

particularly robust in a quasi-2D metal with a simple band near half-filling and on the border of antiferromagnetism. Our considerations have been limited to a square or cubic lattice, a single band, isotropic spin-spin coupling, and commensurate spin fluctuations. We have chosen to model the magnetic interaction potential independently of the single band. This allows us, in principle, to take account of contributions to $\chi(\mathbf{q},\omega)$ from other bands. Solutions of the Eliash-

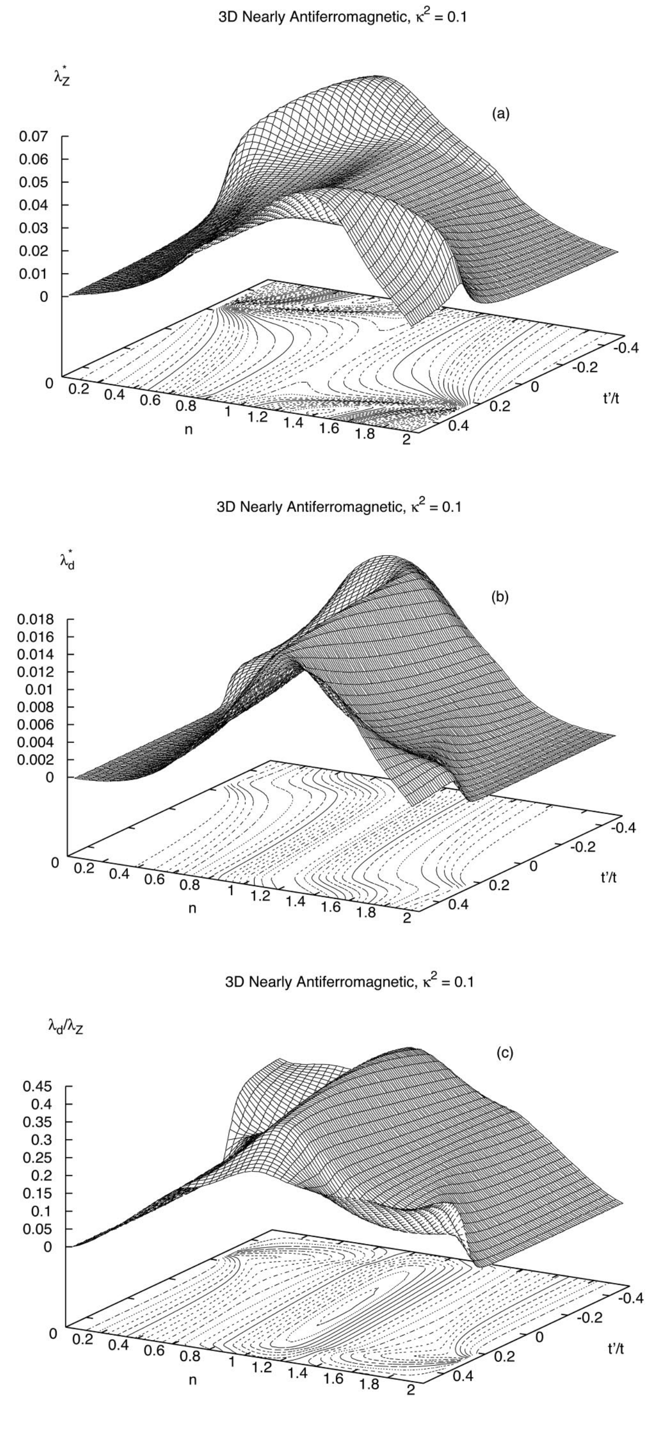

FIG. 14. The interaction parameters $\lambda_Z^* = \lambda_Z/(g^2\chi_0\kappa_0^2/t)$ (a), $\lambda_d^* = \lambda_d/(g^2\chi_0\kappa_0^2/t)$ (b), and the ratio λ_d/λ_Z (c) versus band filling n and ratio of next nearest to nearest hopping t'/t for a 3D nearly antiferromagnetic system, for $\kappa^2 = 0.1$ and $T_{\mathrm{SF}} = 0.67t$.

berg equations where the effective interaction is derived from the single band have been presented recently in Ref. 3. The main conclusion of this latter work is also that a quasi-2D nearly antiferromagnetic metal is a particularly favorable case for magnetically mediated superconductivity.

All of the above conclusions are based on solutions of the mean field Eliashberg equations which do not include effects of fluctuations of the superconducting order parameter. These fluctuations are expected to become increasingly more important as the interlayer coupling in the quasi-2D systems and the superconducting coherence length become small. Also, we note that our mean field treatment may be expected to eventually break down as the border of the Mott metal-insulator transition is approached. These effects can suppress the superconducting transition temperatures below the values calculated in the mean field Eliashberg model.

Our finding that magnetically mediated superconductivity can be particularly robust in quasi-2D nearly antiferromagnetic systems may at first sight seem paradoxical. In a nearly antiferromagnetic metal, the magnetic interaction potential is oscillatory in space and on average tends to cancel. On the other hand, the corresponding potential in the nearly ferromagnetic case is everywhere attractive in our model. This comparison is misleading, however, if it is possible to construct a low-energy Cooper pair wave function that either vanishes or is small in regions in space where the magnetic pairing potential is repulsive and is large where the potential is attractive. This is indeed the case in the square lattice in our model near half-filling. In the case of the cubic lattice it is not possible to choose a Cooper pair wave function that avoids as effectively the repulsive regions of the potential. These conclusions are corroborated by our finding that the effective interaction parameter λ_Δ/λ_Z in a nearly antiferromagnetic metal is usually significantly larger in 2D than in 3D.

Our analyses show that the oscillatory character of the magnetic interaction in a nearly antiferromagnetic system need not be a disadvantage. There is a further argument that tends to favor spin singlet over spin triplet pairing. For a spin triplet state only the longitudinal spin fluctuations contribute to pairing while all three components of the magnetic interaction contribute to pairing for a spin singlet Cooper state. For the spin triplet case, the transverse spin fluctuations do affect T_c, but adversely in contributing to the quasiparticle self-energy [i.e., to an increase in λ_Z, Eq. (16), and of the quasiparticle scattering rate, both of which are pair breaking].

We have assumed that the spin-spin interaction is isotropic. For an ''Ising'' spin-spin interaction where only longitudinal modes survive, the distinction between spin singlet and spin triplet pairing may be expected to be less pronounced.[1] We note that the spin-spin interaction may be highly anisotropic in systems with strong spin-orbit coupling. For example, in the actinide UGe_2 the magnetic anisotropy field is estimated to be as high as 100 T. This system provides us with the first example of coexistence of superconductivity (probably with spin triplet pairing) and itinerant electron ferromagnetism.[4] The above discussion assumes the spins are strongly constrained along a particular direction, as a result of the strong spin-orbit interactions. However, even weak spin-orbit interactions can lead to interesting consequences on the border of long-range magnetic order.[5] In this limit, a small anisotropy of the Lindhard function gets amplified by the large Stoner enhancement factor, leading to a strong anisotropic magnetic response. Under these circumstances spin-triplet pairing may be favored over a much wider range in **Q**, the wave vector of maximum magnetic response, than for isotropic spin-spin interactions considered in this paper.

A general advantage of quasi-2D versus 3D systems predicted by our model is that superconductivity can be observed in a wider region of the phase diagram in the former than in the latter. In fact our results show that for realistic coupling constants (i.e., of the order of the RPA value) superconductivity in 3D can be expected to be observed only very close to the magnetic boundary (see Fig. 2). The greater stability of T_c to changes in κ^2 in quasi-2D than in 3D is partly connected with the fact that the effective interaction parameter λ_Δ/λ_Z can be much larger in quasi-2D than in 3D for otherwise similar conditions. This leads to the saturation of T_c versus κ^2 over a wider range in quasi-2D than in 3D. Also, the higher values of both λ_Z and λ_Δ in quasi-2D leads to a reduced sensitivity of magnetically mediated superconductivity to effects from other channels of quasiparticle interaction and from disorder.

Our goal has been to compare the relative stability of magnetically mediated superconductivity in quasi-2D and in 3D both for nearly ferromagnetic and nearly antiferromagnetic systems. Recently the case of quasi-2D nearly antiferromagnetic metals has also been examined in Ref. 7 and that of 3D nearly ferromagnetic systems in Ref. 6.

V. CONCLUSIONS

The results of our calculations indicate that quasi 2D nearly antiferromagnetic metals may provide fertile ground for finding new examples of magnetically mediated superconductivity. However, because of the oscillatory character of the magnetic interaction in incipient antiferromagnets and other factors as discussed above, the magnitude of T_c even in quasi-2D may depend sensitively on details of the spin-fluctuations and of the band structure. Among the quasi-2D nearly antiferromagnetic metals the most promising candidates appear to be those with a single nearly half-filled band in a square lattice, strong antiferromagnetic correlations and high T_{SF}. In the simplest model T_{SF} is proportional to k_F^2/m where k_F is a characteristic Fermi wave vector and m is some effective mass. Systems with either a low electron density (e.g., organic metals) or narrow bands (e.g., *f*-electron systems) are expected to yield low values of T_{SF} and correspondingly low T_c. More favorable values of T_{SF} can be expected to be achieved in layered *d*-electron systems of moderate electron densities and band widths. High values of T_c might be obtained in cases where the proximity to the antiferromagnetic state can be tuned by some control parameter such as chemical doping or more generally hydrostatic pressure. We have shown how the results of the numerical calculations that have led to the above conclusions can be understood with simple intuitive arguments.

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