

Interlayer coupling and c -axis quasiparticle transport in high- T_c cuprates

Wonkee Kim and J. P. Carbotte

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4M1

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The c -axis quasiparticle conductivity shows different behavior depending on the nature of the interlayer coupling. For coherent coupling with a constant hopping amplitude t_{\perp} , the conductivity at zero frequency and zero temperature $\sigma(0,0)$ depends on the direction of the magnetic field, but it does not for angle-dependent hopping $t(\phi)$ which removes the contribution of the nodal quasiparticles. For incoherent coupling, the conductivity is also independent of field direction and changes only when paramagnetic effects are included. The conductivity sum rule can be used to determine the admixture of coherent to incoherent coupling. The value of $\sigma(0,0)$ can be dominated by t_{\perp} while at the same time $t(\phi)$ dominates the temperature dependence of the superfluid density.

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I. INTRODUCTION

The nature of the interlayer coupling between two adjacent CuO_2 planes in the cuprates is an important issue that remains unresolved. Suggestions include effects of strong intralayer scattering,¹ non-Fermi-liquid ground states,² the general phenomenon of confinement,³ interplane, and in-plane charge fluctuations,⁴ indirect c -axis coupling through the particle-particle channel,⁵ as well as resonant tunneling on localized states in the blocking layer,⁶ and two band models.⁷ Coherent coupling originates from an overlap of the electronic wave functions between planes, and in-plane momentum is conserved in interlayer hopping. By contrast for impurity-mediated incoherent coupling, the in-plane momentum is not constraint. It has been shown that the c -axis conductivity sum rule depends on the nature of interlayer coupling.⁸ Coherent coupling obeys the conventional sum rule⁹ regardless of the angular dependence of the interlayer hopping amplitude. On the other hand, incoherent coupling violates the sum rule even if the in-plane dynamics can be described by a Fermi-liquid. However, in order to explain the violation observed in some experiments,¹⁰ it was also necessary to include the non-Fermi-liquid nature of the in-plane dynamics.¹¹ For $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ at optimum doping, a conventional c -axis sum rule is observed¹⁰ which is consistent with coherent c -axis coupling and an in-plane Fermi liquid. For the underdoped case, a pseudogap is observed and the sum rule is closer to $1/2$.^{10,12} This value can most easily be understood as a pseudogap effect with incoherent c -axis coupling.¹¹

Another interesting example of the interlayer coupling is the c -axis conductivity due to low-lying quasiparticles at zero temperature and zero frequency. A method for observing the quasiparticle current is to measure the hysteretic c -axis tunneling current-voltage characteristics of layered cuprates.¹³ For measurements of the optical conductivity which includes the pair tunneling, see Ref. 14 For coherent coupling with a constant hopping amplitude t_{\perp} , the c -axis quasiparticle conductivity $\sigma(\omega \rightarrow 0, T \rightarrow 0)$ shows a residual value independent of the in-plane scattering rate $\gamma \ll \Delta_0$ (gap) to leading order¹³ with the correction $-(\gamma/\Delta_0)^2$. The underlying physics in this case is the same as for the in-plane

quasiparticle conductivity,¹⁵⁻¹⁷ namely, the impurity-induced density of states at $\omega=0$ is canceled by the decrease of quasiparticle lifetime. A universal value has been observed in $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$ for the in-plane thermal conductivity.¹⁸ In some sense the thermal conductivity is an ideal probe of universal behavior since, as opposed to the electrical conductivity, it is not renormalized by vertex and by Fermi-liquid corrections,¹⁶ and so there is less ambiguity in its identification. The universal conductivity limit does not appear for coherent coupling with an angle-dependent hopping amplitude of the form $t(\phi) = t_{\phi} \cos^2(2\phi)$ which is believed to be appropriate for the copper oxides, where $\phi = \tan^{-1}(k_y/k_x)$ is an angle in the momentum space. In this case the c -axis quasiparticle conductivity is reduced by a factor of $(3/8) \times (\gamma/\Delta_0)^2$ as compared with the value of $\sigma(\omega \rightarrow 0, T \rightarrow 0)$ for a constant t_{\perp} assuming the same magnitude of hopping amplitude, which of course is not the case. This arises because the angular dependence $\cos^2(2\phi)$ eliminates the contribution of the quasiparticles on the nodal lines from interlayer transport. For incoherent coupling, the residual conductivity is proportional to $(\gamma \ln[\Delta_0/\gamma]/\Delta_0)^2$ in leading order. Consequently, the universal value of the c -axis quasiparticle conductivity is a characteristic only of coherent coupling with a constant hopping amplitude. Geometrical consideration of the Cu and O atomic arrangements^{19,20} from plane to plane leads one to expect that the dominant overlapping of orbitals would lead to a $t(\phi)$ form with possibly a subdominant constant piece t_{\perp} . In this case, t_{\perp} would still dominate the value of $\sigma(0,0)$ for very small γ , but the other terms would become important as $(\gamma/\Delta_0)^2$ corrections become larger in which case the term $t(\phi)$ or the incoherent part can become important.

In this paper we investigate effects of interlayer couplings on the c -axis quasiparticle transport in the absence, as well as in the presence of an in-plane magnetic field. It is our aim in this paper to look at all three contributions of interlayer coupling in detail, and our discussion of the magnetic-field effects will not be restricted to the constant t_{\perp} as it is in Ref. 21 We also investigate the role played by the conductivity sum rule in quasiparticle transport. We find that it can be used to fix the relative amount of coherent to incoherent coupling. We also find that $\sigma(\omega,0)/\sigma(0,0)$ shows a different

ω^2 coefficient for $\omega < \gamma$ and $\sigma(0, T)/\sigma(0, 0)$ a different T^2 coefficient ($T < \gamma$) depending on the nature of the coupling. In the presence of an in-plane magnetic field, $\sigma_{\mathbf{q}}(0, 0)$ for coherent coupling with a constant hopping amplitude t_{\perp} depends on the direction of the field²¹ as does the coefficient of the T^2 term in $\sigma_{\mathbf{q}}(0, T)$, but such a dependence is negligible for an angle-dependent coherent $t(\phi)$ because it removes the contribution from the nodal quasiparticles which otherwise manifest the effect of field direction. For incoherent coupling, $\sigma_{\mathbf{q}}(0, 0)$ is independent of field direction, and in fact changes only when a paramagnetic interaction is included. Here we mention that there exist, in the literature, studies which consider effects of oxygen doping on the c -axis transport and Coulomb charging effect⁶ associated with a pseudogap behavior.²² Such complication, however, is beyond the scope of the present paper.

This paper is organized as follows: In Sec. II, we derive general formulas associated with the c -axis quasiparticle conductivity including paramagnetic effects. We describe, in Sec. III, that effects of coherent coupling with a constant as well as with an angle-dependent hopping amplitude on the c -axis quasiparticle transport with and without an in-plane magnetic field. In Sec. IV, impurity-mediated incoherent c -axis coupling is considered. We also illustrate the role of the conductivity sum rule in quasiparticle transport in Sec. V and draw conclusions in Sec. VI.

II. FORMALISM

The Hamiltonian H for a cuprate superconductor with interlayer coupling can be written as $H = H_0 + H_c$, where H_0 describes a d -wave superconductor and

$$H_c = \sum_{\sigma, \mathbf{k}, \mathbf{p}} [t_{\mathbf{k}-\mathbf{p}} C_{1\sigma}^+(\mathbf{k}) C_{2\sigma}(\mathbf{p}) + \text{H.c.}]. \quad (1)$$

The interlayer hopping amplitude $t_{\mathbf{k}-\mathbf{p}}$ depends on nature of c -axis couplings: (i) $t_{\mathbf{k}-\mathbf{p}} = t_{\perp} \delta_{\mathbf{k}-\mathbf{p}}$ for coherent coupling with a constant amplitude. (ii) For an angular dependent amplitude $t_{\mathbf{k}-\mathbf{p}} = t_{\phi} \delta_{\mathbf{k}-\mathbf{p}} \cos^2(2\phi)$. In the lattice, it can be seen from geometrical consideration¹⁹ that $t_{\phi} \delta_{\mathbf{k}-\mathbf{p}} [\cos(k_x a) - \cos(k_y a)]^2$ with an in-plane lattice constant a . (iii) For incoherent coupling $t_{\mathbf{k}-\mathbf{p}} = V_{\mathbf{k}-\mathbf{p}}$ and an impurity average is to be taken into account.

Applying perturbation theory for H_c , we obtain the c -axis quasiparticle conductivity $\sigma(\omega, T)$ in terms of the electronic spectral weight function $A(\mathbf{k}, \epsilon)$:

$$\sigma(\omega) = -\frac{C}{\omega} \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 \int d\epsilon [f(\epsilon + \omega) - f(\epsilon)] \times A(\mathbf{k}, \epsilon) A(\mathbf{p}, \epsilon + \omega), \quad (2)$$

where C is a constant which depends on the nature of interlayer coupling, $f(\epsilon)$ is the usual Fermi thermal factor, and the spectral function

$$A(\mathbf{k}, \epsilon) = \frac{\gamma(1 + \xi_{\mathbf{k}}/E_{\mathbf{k}})}{(\epsilon - E_{\mathbf{k}})^2 + \gamma^2} + \frac{\gamma(1 - \xi_{\mathbf{k}}/E_{\mathbf{k}})}{(\epsilon + E_{\mathbf{k}})^2 + \gamma^2} \quad (3)$$

with $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$. We assume $\Delta_{\mathbf{k}} = \Delta_0 [\cos(k_x a) - \cos(k_y a)]$ [or $\Delta_0 \cos(2\phi)$ in the continuum limit] and a cylindrical Fermi surface with $\xi_{\mathbf{k}} = k^2/(2m) - \epsilon_F$ measured from the Fermi energy ϵ_F . As $T \rightarrow 0$ and $\omega \rightarrow 0$, $\sigma(0, 0) = C \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 A(\mathbf{k}, 0) A(\mathbf{p}, 0)$. It is easy to calculate $\sigma(0, 0)$ ($\equiv \sigma_0$) for the couplings we mentioned earlier. To obtain the c -axis quasiparticle conductivity at finite frequency ω at zero temperature, we take $T \rightarrow 0$ limit and assume $\omega < \gamma$:

$$\begin{aligned} \sigma(\omega) &\simeq -C \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 \int d\epsilon \left[\partial_{\epsilon} f(\epsilon) + \left(\frac{\omega}{2} \right) \partial_{\epsilon}^2 f(\epsilon) \right. \\ &\quad \left. + \left(\frac{\omega^2}{6} \right) \partial_{\epsilon}^3 f(\epsilon) \right] A(\mathbf{k}, \epsilon) A(\mathbf{p}, \epsilon + \omega) \\ &\simeq C \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 A(\mathbf{k}, 0) A(\mathbf{p}, \omega) \\ &\quad - C \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 \left(\frac{\omega}{2} \right) \partial_{\epsilon} A(\mathbf{k}, \epsilon) A(\mathbf{p}, \epsilon + \omega) \Big|_{\epsilon=0} \\ &\quad + C \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 \left(\frac{\omega^2}{6} \right) \partial_{\epsilon}^2 A(\mathbf{k}, \epsilon) A(\mathbf{p}, \epsilon + \omega) \Big|_{\epsilon=0} + \dots, \end{aligned} \quad (4)$$

where an expansion in ω has been made. The finite T corrections at $\omega = 0$ will be considered later along with an in-plane magnetic field.

In the presence of an in-plane magnetic field, we assume that the field penetrates freely into the sample so that the field is uniform between the planes.²¹ This assumption should be valid for Bi- and Tl-based cuprates which are nearly two dimensional. For YBCO, however, there is a possibility that in-plane vortices form. This means that in such a case an average over vortices in a unit cell is required.²³ The interlayer coupling Hamiltonian is modified by the presence of a uniform in-plane magnetic field \mathbf{H} as follows:

$$H_c = \sum_{\sigma, \mathbf{k}, \mathbf{p}} [t_{\mathbf{k}-\mathbf{p}} C_{1\sigma}^+(\mathbf{k} + \mathbf{q}) C_{2\sigma}(\mathbf{p}) + \text{H.c.}], \quad (5)$$

where $\mathbf{q} = (ed/2)(\hat{\mathbf{z}} \times \mathbf{H})$ with an interlayer distance d .

Following a procedure similar to the one we applied in the zero-field case, we obtain the c -axis quasiparticle conductivity $\sigma_{\mathbf{q}}(\omega, T)$ in the presence of an in-plane magnetic field,

$$\sigma_{\mathbf{q}}(\omega) = -\frac{C}{\omega} \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 \int d\epsilon [f(\epsilon + \omega) - f(\epsilon)] A(\mathbf{k} + \mathbf{q}, \epsilon) A(\mathbf{p}, \epsilon + \omega). \quad (6)$$

In our consideration, the energy scale associated with the magnetic field are always much less than the gap Δ_0 . Due to an in-plane field, the quasiparticles gain an additional momentum \mathbf{q} on transferring to the next plane. In other words, the Bogoliubov-de Gennes (BdG) wave functions²⁴ $u_{\mathbf{k}}(\mathbf{r})$ and $v_{\mathbf{k}}(\mathbf{r})$ become $u_{\mathbf{k}} e^{i(\mathbf{k} + \mathbf{q}) \cdot \mathbf{r}}$ and $v_{\mathbf{k}} e^{i(\mathbf{k} + \mathbf{q}) \cdot \mathbf{r}}$, respectively.²⁵ The spectral weight function becomes

$$A(\mathbf{k} + \mathbf{q}, \epsilon) = \frac{\gamma(1 + \xi_{\mathbf{k}+\mathbf{q}}/E_{\mathbf{k}+\mathbf{q}})}{(\epsilon - E_{\mathbf{k}+\mathbf{q}})^2 + \gamma^2} + \frac{\gamma(1 - \xi_{\mathbf{k}+\mathbf{q}}/E_{\mathbf{k}+\mathbf{q}})}{(\epsilon + E_{\mathbf{k}+\mathbf{q}})^2 + \gamma^2}, \quad (7)$$

where $E_{\mathbf{k}+\mathbf{q}} \simeq \sqrt{\xi_{\mathbf{k}+\mathbf{q}}^2 + \Delta_{\mathbf{k}}^2}$ with $\xi_{\mathbf{k}+\mathbf{q}} \simeq \xi_{\mathbf{k}} + (\mathbf{k} \cdot \mathbf{q})/m$. Note that the angular dependence of $\sigma_{\mathbf{q}}$ comes from the factor $(\mathbf{k} \cdot \mathbf{q})/m \simeq E_q \cos(\phi - \theta_q)$, where $E_q = v_F q$ and θ_q is the direction of \mathbf{q} , which can be interpreted as the direction of the field (θ) because of the symmetry in the problem. The gap seen by quasiparticles with a momentum $\mathbf{k} + \mathbf{q}$ is not $\Delta_{\mathbf{k}}$ but $\Delta_{\mathbf{k}+\mathbf{q}}$; however, $\Delta_{\mathbf{k}+\mathbf{q}} = \Delta_{\mathbf{k}} + \delta_q$, where $\delta_q = q v_G \cos(\phi + \theta_q)$ with $v_G \simeq 2\Delta_0/k_F$, and the correction δ_q makes a negligible contribution to the quasiparticle energy spectrum. When the paramagnetic interaction is included, $E_{\mathbf{k}+\mathbf{q}} \rightarrow E_{\mathbf{k}+\mathbf{q}} \pm \mu_B H$ in the denominator of $A(\mathbf{k} + \mathbf{q}, \epsilon)$, where μ_B is the Bohr magneton. As $\omega \rightarrow 0$ and $T \rightarrow 0$,

$$\sigma_{\mathbf{q}}(\omega \rightarrow 0, T \rightarrow 0) = C \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 A(\mathbf{k} + \mathbf{q}, 0) A(\mathbf{p}, 0). \quad (8)$$

Based on symmetry consideration,¹⁵ one can easily deduce that the dependence of $\sigma_{\mathbf{q}}(\omega \rightarrow 0, T \rightarrow 0) \equiv \sigma(\theta)$ on field angle θ has a period $\pi/2$ because of the d -wave gap. The sign of the gap does not matter. Consequently,

$$\sigma(\theta) = \sum_{n=0} C_n \cos(4n\theta), \quad (9)$$

where the C_n 's depend on the ratio of the magnetic-field energy to γ , i.e., on E_q/γ and $\mu_B H/\gamma$. Since both E_q and $\mu_B H$ are linear to H , we can parameterize the ratio $(E_q/\mu_B H) \sim d/a$. We choose $(E_q/\mu_B H) = 6$ for the high- T_c Bi- and Tl-based cuprates.²⁶ Another relation can be deduced without detailed calculation. If we expand $\sigma(\theta)$ in terms of E_q/γ , then its angular dependence will appear for the first time from the term $(E_q/\gamma)^4 \cos^4(\phi - \theta)$ because the fourth harmonic $\cos(4\theta)$ comes from the $\cos^4(\theta)$ term. This means that $\gamma < E_q$ is required for any clear angular dependence to show up. For the eighth harmonic $\cos(8\theta)$ to enter, the field has to be even higher.

At a finite temperature $T < \gamma$, we apply the Sommerfeld expansion²⁷ to $\sigma_{\mathbf{q}}(\omega \rightarrow 0, T) \equiv \sigma(\theta, T)$ and obtain

$$\begin{aligned} \sigma(\theta, T) &= -C \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 \int d\epsilon \left(\frac{\partial f(\epsilon)}{\partial \epsilon} \right) A(\mathbf{k} + \mathbf{q}, \epsilon) A(\mathbf{p}, \epsilon) \\ &\simeq \sigma(\theta) + \frac{\pi^2}{6} C T^2 \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 \frac{\partial^2}{\partial \epsilon^2} A(\mathbf{k} + \mathbf{q}, \epsilon) A(\mathbf{p}, \epsilon) \Big|_{\epsilon=0} \\ &\equiv \sigma(\theta) \left[1 + \frac{\pi^2}{6} \left(\frac{T}{\gamma} \right)^2 \alpha(\theta) \right], \end{aligned} \quad (10)$$

where

$$\sigma(\theta) = t_{\perp}^2 C \mathcal{J} \sum_{\text{node}} \int d\varphi \int \frac{p dp}{p^2 + 1} \frac{1}{p^2 + 2(E_q/\gamma) \cos(\phi - \theta) p \cos(\varphi) + (E_q/\gamma)^2 \cos^2(\phi - \theta) + 1}. \quad (15)$$

$$\alpha(\theta) = \frac{\frac{\partial^2}{\partial \epsilon'^2} \sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 A(\mathbf{k} + \mathbf{q}, \epsilon') A(\mathbf{p}, \epsilon') \Big|_{\epsilon'=0}}{\sum_{\mathbf{k}, \mathbf{p}} t_{\mathbf{k}-\mathbf{p}}^2 A(\mathbf{k} + \mathbf{q}, 0) A(\mathbf{p}, 0)}, \quad (11)$$

with $\epsilon' = \epsilon/\gamma$.

We use the nodal approximation^{15,16} to describe the low-temperature physics associated with the quasiparticle transport because quasiparticles near the nodal lines dominantly contribute to the resistive transport. Linearizing $\xi_{\mathbf{k}}$ and $\Delta_{\mathbf{k}}$ near the nodal points on the Fermi surface (FS), we obtain $\xi_{\mathbf{k}} = v_F (\mathbf{k} \cdot \hat{\mathbf{k}}_{\perp} - k_F)$ and $\Delta_{\mathbf{k}} = v_G \mathbf{k} \cdot \hat{\mathbf{k}}_{\parallel}$, where $v_F(k_F)$ is the Fermi velocity (momentum) and $v_G = \sqrt{2} a \Delta_0$ is the gap velocity.²⁸ The unit vector $\hat{\mathbf{k}}_{\perp}$ ($\hat{\mathbf{k}}_{\parallel}$) is perpendicular (parallel) to the FS. These unit vectors will change depending on the nodal line. For example, on the nodal line of $\phi = \pi/4$, $\hat{\mathbf{k}}_{\perp} = (\hat{\mathbf{k}}_x + \hat{\mathbf{k}}_y)/\sqrt{2}$ and $\hat{\mathbf{k}}_{\parallel} = (-\hat{\mathbf{k}}_x + \hat{\mathbf{k}}_y)/\sqrt{2}$. Since we include quasiparticles only near the nodal points on the FS in our considerations, we will use the following procedure in the actual calculation:

$$\sum_{\mathbf{k}} \rightarrow \sum_{\text{node}} \int \frac{dk_{\perp} dk_{\parallel}}{(2\pi)^2} \rightarrow \sum_{\text{node}} \int \mathcal{J} dp_1 dp_2, \quad (12)$$

where $p_1 = v_F k_{\perp}$, $p_2 = v_G k_{\parallel}$ and $\mathcal{J} = [(2\pi)^2 v_F v_G]^{-1}$. The coordinate transformations we made are rotation, translation and dilation. In the coordinate of (p_1, p_2) , the energy dispersion of the quasiparticle $E_{\mathbf{k}}$ becomes $\sqrt{p_1^2 + p_2^2} < p_0$, where $p_0 \sim \mathcal{O}(\Delta_0)$.

III. COHERENT COUPLING

In this section, we consider the effects of coherent coupling on c -axis quasiparticle transport. Since $t_{\mathbf{k}-\mathbf{p}} = t_{\perp} \delta_{\mathbf{k}-\mathbf{p}}$ for a constant hopping amplitude, as $\omega \rightarrow 0$ and $T \rightarrow 0$, $\sigma_0 = C \sum_{\mathbf{k}} t_{\perp}^2 A(\mathbf{k}, 0)^2$. For $\omega < \gamma$, we use the nodal approximation for Eq. (4). Then, up to order $(\omega/\gamma)^2$,

$$\frac{\sigma(\omega)}{\sigma_0} = 1 + \frac{1}{18} \left(\frac{\omega}{\gamma} \right)^2, \quad (13)$$

where σ_0 is a constant independent of γ .

In the presence of a uniform in-plane field, the quasiparticle conductivity becomes

$$\sigma(\theta) = C \sum_{\mathbf{k}} t_{\perp}^2 A(\mathbf{k} + \mathbf{q}, 0) A(\mathbf{k}, 0). \quad (14)$$

Applying the substitution Eq. (12) to the above equation, we obtain without the paramagnetic part

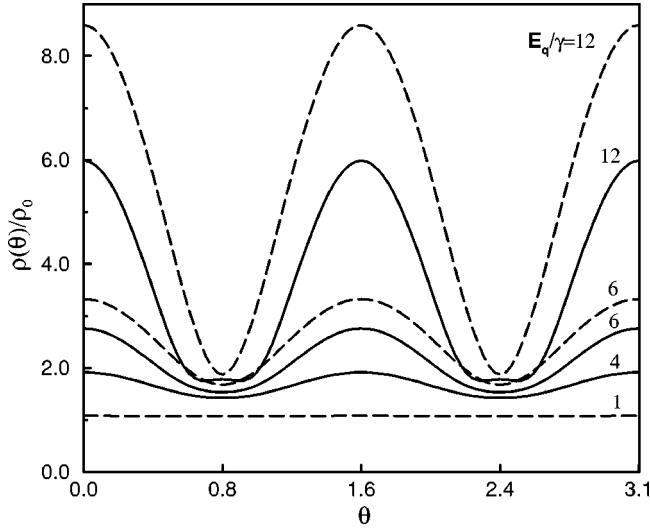


FIG. 1. For various values of $E_q/\gamma (=1,4,6,12)$, resistivity $\rho(\theta)/\rho_0 = \sigma_0/\sigma(\theta)$ as a function of a direction of the in-plane field θ is plotted with (solid curve) and without (dashed curve) the paramagnetic interaction. As E_q/γ is increased, $\rho(\theta)/\rho_0$ is increased, and for a high field ($E_q/\gamma=12$), the paramagnetic interaction unambiguously reduces the amplitude of $\rho(\theta)$ and its anisotropy

It can be shown within the nodal approximation that $\phi = \phi_n \pm \tan^{-1}[p_2/v_G k_F]$, where ϕ_n is a direction on the nodal line; however, to a good approximation, we can replace ϕ by ϕ_n . For the weak field case $E_q < \gamma$, we expand $\sigma(\theta)$ in terms of E_q/γ and obtain

$$\frac{\sigma(\theta)}{\sigma_0} \approx 1 - \frac{1}{12} \left(\frac{E_q}{\gamma} \right)^2 + \frac{1}{80} \left[1 - \frac{1}{3} \cos(4\theta) \right] \left(\frac{E_q}{\gamma} \right)^4, \quad (16)$$

where σ_0 is the c -axis quasiparticle conductivity in the absence of the in-plane magnetic field. As we see, the angular dependence of $\sigma(\theta)$ is small for a weak field. $\sigma(\theta)$ is maximum (minimum) when the in-plane field is along a nodal (antinodal) line. It can be physically understood as follows: When \mathbf{H} is along a nodal line, for example, $\theta = \pi/4$, the angle of \mathbf{q} is $3\pi/4$ because $\mathbf{q} \propto (\hat{\mathbf{z}} \times \mathbf{H})$. Then, while the shifted momenta $\mathbf{k} + \mathbf{q}$ of the quasiparticles with $\phi = \pi/4$ and $\phi = 5\pi/4$ deviate from the nodal regions when q is compatible with k , quasiparticles with $\phi = 3\pi/4$ and $\phi = 7\pi/4$ remain in the nodal areas. This means the remaining quasiparticles govern the c -axis transport when $\theta = \pi/4$. On the other hand, for $\theta = 0$, all quasiparticles move away from the nodal regions due to the momentum shift \mathbf{q} . Therefore $\sigma(\theta) < \sigma_0$ and $\sigma(0) < \sigma(\pi/4)$ because of a mismatch between \mathbf{k} and $\mathbf{k} + \mathbf{q}$ in the interlayer transport.

In Fig. 1, we plot resistivity as a function of direction of the in-plane magnetic field, $\rho(\theta) = 1/\sigma(\theta)$ for various values of E_q with (solid line) and without (dashed line) the paramagnetic interaction included. We reproduced results of Ref. 21 for $E_q/\gamma = 1, 2$, and 4. As shown in Fig. 1, $\rho(\theta)$ is increased as the magnetic field increases, and it is decreased for a give E_q when the paramagnetic interaction is considered because the interaction is pair breaking. If we consider

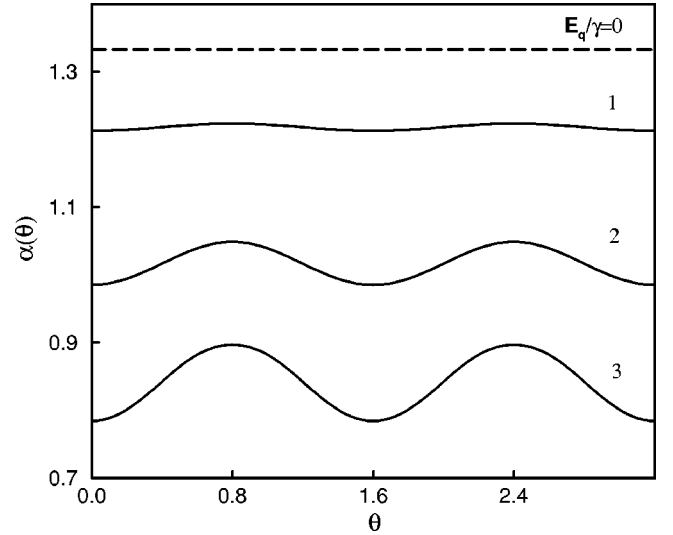


FIG. 2. $\alpha(\theta)$ in Eq. (10) shows finite temperature effects on the c -axis quasiparticle conductivity for $E_q/\gamma = 1, 2$, and 3. The paramagnetic interaction is included. In the absence of the field, $\alpha(\theta) = 4/3$ (dashed line).

only the paramagnetic interaction, then the quasiparticle conductivity σ has no θ dependence and increases as follows:

$$\sigma \rightarrow \sigma_0 \left[1 + \left(\frac{\mu_B H}{\gamma} \right) \arctan \left(\frac{\mu_B H}{\gamma} \right) \right]. \quad (17)$$

The eighth harmonic clearly appears for $E_q/\gamma = 12$. Actually, it begins to appear when $E_q/\gamma \geq 10$. Interestingly, the paramagnetic interaction unambiguously reduces the amplitude of $\rho(\theta)$ for a higher field ($E_q \gg \gamma$). We also plot $\alpha(\theta)$ of Eq. (10) for a finite $T < \gamma$ in Fig. 2. For reference in the absence of the magnetic field, $\alpha(\theta) = 4/3$ (dashed line).

In the case of an angle-dependent hopping amplitude $t_{\mathbf{k}-\mathbf{p}} = t(\phi) \delta_{\mathbf{k}-\mathbf{p}}$, the c -axis quasiparticle conductivity in zero field is $\sigma_0 = C \sum_{\mathbf{k}} t(\phi)^2 A(\mathbf{k}, 0)^2$. For $\omega < \gamma$, using the nodal approximation we obtain

$$\frac{\sigma(\omega)}{\sigma_0} = 1 + \frac{34}{9} \ln \left(\frac{\Delta_0}{\gamma} \right) \left(\frac{\omega}{\Delta_0} \right)^2, \quad (18)$$

where $\sigma_0 \propto (\gamma/\Delta_0)^2$ in this case. Effects of the in-plane field on the c -axis quasiparticle conductivity can be seen in the same way as before, namely,

$$\sigma(\theta) = C \sum_{\mathbf{k}} t(\phi)^2 A(\mathbf{k} + \mathbf{q}, 0) A(\mathbf{k}, 0). \quad (19)$$

Since $t(\phi) = t_\phi (p_2/\Delta_0)^2$ within the nodal approximation, $\sigma(\theta)$ becomes without the paramagnetic interaction

$$\sigma(\theta) = t_\phi^2 C \mathcal{J} \sum_{\text{node}} \int d\varphi \int \frac{p dp}{p^2+1} \left(\frac{p}{\Delta_0} \right)^4 \sin^4(\varphi) \frac{1}{p^2 + 2(E_q/\gamma) \cos(\phi - \theta) p \cos(\varphi) + (E_q/\gamma)^2 \cos^2(\phi - \theta) + 1}. \quad (20)$$

In this case, we find that $\sigma(\theta)/\sigma_0 \approx 1$ without clear dependence on the direction of the in-plane field, and $\alpha(\theta) \sim \mathcal{O}[(\gamma/\Delta_0)^2 \ln(\Delta_0/\gamma)]$ in the leading order and the next order is $\mathcal{O}[(E_q/\Delta_0)^2]$; therefore $\alpha(\theta)$ is negligible. The c -axis quasiparticle conductivity is insensitive to the in-plane magnetic field because the angle-dependent hopping amplitude $t_\perp(\phi)$ eliminates contributions of nodal quasiparticles, which otherwise manifest the effects of the field.

IV. INCOHERENT COUPLING

For impurity-mediated incoherent coupling $t_{\mathbf{k}-\mathbf{p}} = V_{\mathbf{k}-\mathbf{p}}$, we need a model for the impurity scattering potential and need to carry out an average over impurity configurations. We use a simple model²⁹ for the scattering potential $|V_{\mathbf{k}-\mathbf{p}}|^2 = |V_0|^2 + |V_1|^2 \cos(2\phi_k) \cos(2\phi_p)$. One may expand $|V_{\mathbf{k}-\mathbf{p}}|^2$ with respect to scattering symmetry so that it is decomposed

$$|V_{\mathbf{k}-\mathbf{p}}|^2 = |V_0|^2 + \sum_{l,l'} |V_{ll'}|^2 \cos[(4l+2)\phi_k] \times \cos[(4l'+2)\phi_p], \quad (21)$$

where l and l' are integers. However, for the c -axis quasiparticle transport, only the $|V_0|^2$ term gives a nonzero value to the conductivity. As $\omega \rightarrow 0$ and $T \rightarrow 0$, $\sigma_0 = C|V_0|^2 \sum_{\mathbf{k}, \mathbf{p}} A(\mathbf{k}, 0) A(\mathbf{p}, 0)$. For $\omega < \gamma$, it can be shown that

$$\frac{\sigma(\omega)}{\sigma_0} = 1 + \frac{1}{3} \frac{(\omega/\gamma)^2}{\ln(\Delta_0/\gamma)}, \quad (22)$$

where $\sigma_0 \propto [\gamma \ln(\Delta_0/\gamma)/\Delta_0]^2$ for impurity-mediated incoherent coupling.

In the presence of an in-plane magnetic field, the c -axis quasiparticle conductivity becomes

$$\sigma(\theta) = C|V_0|^2 \sum_{\mathbf{k}, \mathbf{p}} A(\mathbf{k} + \mathbf{q}, 0) A(\mathbf{p}, 0). \quad (23)$$

As we see, the angular dependence of $\sigma(\theta)$ is determined by $\sum_{\mathbf{k}} A(\mathbf{k} + \mathbf{q}, 0)$. It can be shown numerically and analytically

that the field effect appears only as the paramagnetic contribution. We present some analytic results, for example, in the weak-field limit

$$\sum_{\mathbf{k}} [A(\mathbf{k} + \mathbf{q}, 0) - A(\mathbf{k}, 0)] \approx \frac{2\gamma}{\pi v_G v_F} h \left(\frac{\mu_B H}{\gamma} \right), \quad (24)$$

where $h(x) = x^2 - x^4/6 + x^6/15$. Therefore the c -axis quasiparticle conductivity for a weak field becomes

$$\frac{\sigma(\theta)}{\sigma_0} \approx 1 + \frac{h(\mu_B H/\gamma)}{\ln(\Delta_0/\gamma)}. \quad (25)$$

Consequently, for incoherent coupling the in-plane magnetic field has no effect on the c -axis quasiparticle transport if the paramagnetic interaction is not considered. Physically, the momentum of the transport quasiparticle is not constraint in incoherent coupling so that the change in momentum \mathbf{q} does not matter. Since the paramagnetic interaction is pair breaking, the magnitude of $\sigma(\theta)$ is increased but shows no angular dependence when this interaction is included. At a finite temperature ($T < \gamma$), we use Eq. (10) and for a weak field, $\alpha(\theta)$ becomes

$$\alpha(\theta) \approx 1 + \frac{(\pi^2/3)}{[\ln(\Delta_0/\gamma) + h(\mu_B H/\gamma)][1 - (\mu_B H/\gamma)^2]}. \quad (26)$$

V. c -AXIS CONDUCTIVITY SUM RULE

From the c -axis conductivity sum rule, the superfluid density ρ_s can be written in terms of the missing spectral weight $(N_n - N_s)$ and the thermal averages of kinetic energy $\langle H_c \rangle^{s(n)}$ of a superconducting (normal) state as follows:

$$\rho_s = (N_n - N_s) - 4\pi e^2 d [\langle H_c \rangle^s - \langle H_c \rangle^n], \quad (27)$$

where ω_c is the cutoff frequency for the interband transitions that H_c does not describe. Using the above equation as well as the Kramers-Kronig relation between the conductivity and the penetration depth, we obtain the normalized missing spectral weight $(N_n - N_s)/\rho_s$:

$$\frac{(N_n - N_s)}{\rho_s} = \frac{1}{2} + \frac{1}{2} \frac{\sum_{\omega} \sum_{\mathbf{k}, \mathbf{p}} |t_{\mathbf{k}-\mathbf{p}}|^2 [G(\mathbf{k}, \omega) G(\mathbf{p}, \omega) - G_0(\mathbf{k}, \omega) G_0(\mathbf{p}, \omega)]}{\sum_{\omega} \sum_{\mathbf{k}, \mathbf{p}} |t_{\mathbf{k}-\mathbf{p}}|^2 F(\mathbf{k}, \omega) F^+(\mathbf{p}, \omega)}, \quad (28)$$

where $G(\mathbf{k}, \omega)$ and $F(\mathbf{k}, \omega)$ are superconducting Green functions and $G_0(\mathbf{k}, \omega)$ is in the normal state.^{8,11}

Since, in principle, all possible interlayer couplings might be present in a given sample, we need to consider coherent and incoherent coupling at the same time. The actual c -axis transport of quasiparticles is determined from competing effects between couplings. Based on the conductivity sum rule, we can estimate the ratio $(\sigma_{co}/\sigma_{in})$, where σ_{co} (σ_{in}) is the c -axis quasiparticle conductivity for coherent (incoherent) coupling. Since the sum rule does not distinguish between t_{\perp} and $t(\phi)$, we consider two limiting cases: (i) $t_{\mathbf{k}-\mathbf{p}} = t_{\perp} \delta_{\mathbf{k}-\mathbf{p}} + V_{\mathbf{k}-\mathbf{p}}$ and (ii) $t_{\mathbf{k}-\mathbf{p}} = t(\phi) \delta_{\mathbf{k}-\mathbf{p}} + V_{\mathbf{k}-\mathbf{p}}$. As $\omega \rightarrow 0$ and $T \rightarrow 0$, for the case with t_{\perp}

$$\begin{aligned} \sigma &= \sigma_{co} + \sigma_{in} = \sigma_{co} \left[1 + \frac{\sigma_{in}}{\sigma_{co}} \right] \\ &= \sigma_{co} \left[1 + \frac{4\Delta_0 \sigma_{cn}}{\pi e^2 dt_{\perp}^2 N(0)} \left(\frac{\gamma}{\Delta_0} \ln(\Delta_0/\gamma) \right)^2 \right], \end{aligned} \quad (29)$$

where we have used $\sigma_{co} = 2e^2 dt_{\perp}^2 N(0)/(\pi \Delta_0)$ and $\sigma_{in} = (8/\pi^2) \sigma_{cn} [\gamma \ln(\Delta_0/\gamma)/\Delta_0]^2$ with $\sigma_{cn} = 4\pi n_i d [e V_0 N(0)]^2$. One may think that we need to know the ratio of $|V_0|^2$ to t_{\perp}^2 in order to compare σ_{co} with σ_{in} . However, the conductivity sum rule helps us to estimate the ratio $(\sigma_{co}/\sigma_{in})$ without *ad hoc* information.

Let us define $\eta = \rho_{s,co}/\rho_{s,in} = (1/\lambda_{co}^2)/(1/\lambda_{in}^2)$, where ρ_s and λ are the corresponding superfluid density and penetration depth, respectively. For coherent coupling

$$\frac{1}{\lambda_{co}^2} = 32\pi e^2 dt_{\perp}^2 N(0) \left[\frac{1}{2} - \frac{1}{\pi} \mathbf{K}(i\Delta_0/\gamma) \right], \quad (30)$$

where \mathbf{K} is the complete elliptic integral of the first kind. For incoherent coupling, we assume $|V_0|^2 = |V_1|^2$ for simplicity and to illustrate possible effects. Then,

$$\frac{1}{\lambda_{in}^2} = 32\sigma_{cn} \sum_{\omega} [(\kappa'^2/\kappa) \mathbf{K}(\kappa) - \mathbf{E}(\kappa)/\kappa]^2, \quad (31)$$

where $\kappa = \Delta_0/\sqrt{\Delta_0^2 + (\omega + \gamma \text{sgn} \omega)^2}$, $\kappa' = \sqrt{1 - \kappa^2}$, and \mathbf{E} is the complete elliptic integral of the second kind.¹¹ Since $\gamma \ll \Delta_0$, $\eta \approx 16\pi e^2 dt_{\perp}^2 N(0)/[12\Delta_0 \sigma_{cn}]$ and we obtain

$$\sigma = \sigma_{co} \left[1 + \frac{(16/3)}{\eta} \left(\frac{\gamma}{\Delta_0} \ln(\Delta_0/\gamma) \right)^2 \right]. \quad (32)$$

Since the total superfluid density $\rho_s = \rho_{s,co} + \rho_{s,in}$, it can be shown that

$$\frac{N_n - N_s}{\rho_s} \approx \frac{1 + 2\eta}{2(1 + \eta)} + \frac{1.08}{1 + \eta}. \quad (33)$$

Note that η in Eq. (32) can be determined from the violation of the conductivity sum rule Eq. (33). $\eta \rightarrow \infty$ corresponds to pure coherent c -axis coupling in which case the sum rule Eq. (33) is conventional and equal to 1. On the other hand, $\eta \rightarrow 0$ means pure incoherent coupling and the sum rule is

larger than 1 and equal to 1.58 in our model with $|V_0/V_1| = 1$. A finite η corresponds to an admixture of coherent and incoherent c -axis transport. For example, $\eta \approx 5$ applies when $(N_n - N_s)/\rho_s = 1.1$, that is, there is a 10% violation of the sum rule upwards. The sum rule itself determines the admixture of $\rho_{s,co}$ to $\rho_{s,in}$, but cannot differentiate between t_{\perp} and $t(\phi)$ for the coherent part. Limits on the relative size of these two overlap integrals can only be set from consideration of the chemistry of the CuO_2 planes and their overlap or, alternatively, from experimental information such as the behavior of $\sigma(0,0)$ when the impurity content is increased.

For the case with $t(\phi)$

$$\frac{1}{\lambda_{co}^2} \approx 32\pi e^2 dt_{\phi}^2 N(0) \left[\frac{3}{16} - \frac{2}{3\pi} \frac{\gamma}{\sqrt{\gamma^2 + \Delta_0^2}} \mathbf{E} \left(\frac{\Delta_0}{\sqrt{\gamma^2 + \Delta_0^2}} \right) \right], \quad (34)$$

where \mathbf{E} is the complete elliptic integral of the second kind, and η as above with $6 \leftrightarrow 16$ and $t_{\phi} \leftrightarrow t_{\perp}$. Following a similar way as before, we obtain

$$\sigma = \sigma'_{co} \left[1 + \frac{(16/3)}{\eta} \ln^2(\Delta_0/\gamma) \right], \quad (35)$$

where $\sigma'_{co} = \sigma_{co}(\gamma/\Delta_0)^2$ with $(3/4) \leftrightarrow 2$ and $t_{\phi} \leftrightarrow t_{\perp}$.

Note that t_{\perp} and t_{ϕ} case both contribute in the same order to the zero-temperature c -axis superfluid density. For the pure case, assuming t_{\perp} and t_{ϕ} are of the same magnitude, the ratio of the two contributions is 16/3. This is in sharp contrast to what we found in the case of $\sigma(0,0)$ where t_{\perp} contributes of order t_{\perp}^2 while the contribution from t_{ϕ} is of order $t_{\phi}^2(\gamma/\Delta_0)^2$ and so vanishes in the clean limit. Recently, Gaifullin *et al.*³⁰ have measured the temperature dependence of the c -axis superfluid density in Bi2212 and found that at low temperature it is well fit by a form $[1 - A(T/T_c)^{\alpha}]$ with α of order 4–6 close to the values reported for Hg1201.³¹ This finding favors a pure $t(\phi)$ model which is known to give a T^5 law (Ref. 20) and leaves little room for a subdominant t_{\perp} contribution because in this case the c -axis penetration depth will mirror perfectly³² its in-plane temperature dependence which goes like T . The data certainly preclude a linear T contribution to the superfluid density of more than a few percent implying a ratio (t_{\perp}/t_{ϕ}) of order $< 10^{-1}$ at most. In this instance, the ratio (γ/Δ_0) can easily be comparable to (t_{\perp}/t_{ϕ}) and can even be the larger of the two. As an illustration in the experimental work establishing the in-plane universal limit for the thermal conductivity¹⁸ the values of (γ/Δ_0) are of order $\lesssim 10^{-1}$. Latyshev *et al.*¹³ quoted $\gamma \approx 3$ meV for their single crystals, which is of similar order. This implies that in the experiments on Bi2212, the contribution to $\sigma(0,0)$ of each of t_{\perp} and t_{ϕ} terms are likely to be close in magnitude. Nevertheless, it is concluded in Ref. 13 that the universal limit is observed so that t_{\perp} presumably still dominates. Additional work with various values of γ , somewhat larger as well as smaller than used so far, should allow one to establish the size of the important ratio (t_{\perp}/t_{ϕ}) . This information, with the c -axis optical sum rule, would allow an estimate of each of the three possible contributions to the

c -axis transport discussed in this paper. These parameters are important because they control all c -axis transport. We point out here that we have considered separately the case t_{\perp} and $t(\phi)$. If amplitudes are added and squared, a cross term of the form $2t_{\perp}t_{\phi}\cos^2(2\phi)$ will enter but this will not change qualitatively the conclusion made above. The contribution of such a term to σ_0 is $[4e^2dN(0)/\pi\Delta_0]t_{\perp}t_{\phi}(\gamma/\Delta_0)^2\ln(\Delta_0/\gamma)$ and to the superfluid density is $32\pi e^2dN(0)t_{\perp}t_{\phi}[1/2 - (2/\pi)(\gamma/\sqrt{\gamma^2 + \Delta_0^2})\mathbf{E}(\gamma/\sqrt{\gamma^2 + \Delta_0^2})]$.

VI. CONCLUSIONS

We have discussed how the nature of the interlayer coupling influences the c -axis quasiparticle conductivity $\sigma_q(\omega, T)$ in the absence and in the presence of an in-plane magnetic field. In zero field, $\sigma(0, 0)$ is independent of the in-plane scattering rate γ , to leading order, only for coherent coupling with a constant hopping amplitude. Also, $\sigma(\omega, 0)$ shows a different behavior for different c -axis coupling. Similarly, the field effects depend crucially on the nature of the interlayer coupling. For coherent coupling with a constant hopping amplitude, an angular dependence of the conductivity $\sigma(\theta)$ appears in high field. The resistivity $\rho(\theta)$ increases because the mismatch between \mathbf{k} and $\mathbf{k} + \mathbf{q}$ decreases $\sigma(\theta)$, and $\rho(\theta)$ is maximum (minimum) when the field is along the antinodal (nodal) line. This confirms previous work.²¹ When the paramagnetic interaction is included, $\rho(\theta)$ is decreased because this interaction is pair breaking and the anisotropy also decreases. For high field ($E_q \gg \gamma$), the eighth harmonic appears and can be seen in the θ dependence of $\rho(\theta)$. For coherent coupling with an angle-dependent hopping amplitude, $\sigma(\theta)$ is insensitive to field direction because this hopping amplitude eliminates the contribution of the nodal quasiparticles which manifest the angular dependence. Also, the paramagnetic interaction does

not significantly change $\sigma(\theta)$. For incoherent coupling, only the paramagnetic interaction has an effect on the c -axis quasiparticle transport and $\sigma(\theta)$ has no dependence on field direction. The coefficient of the first temperature correction which goes like T^2 is anisotropic only in the constant t_{\perp} case and shows the same minima (maxima) as does the leading contribution.

The c -axis conductivity sum rule helps estimate separately the contributions from coherent and incoherent coupling to the quasiparticle transport without microscopic information on hopping amplitudes. However, it cannot, by itself, differentiate between contributions from constant or angular dependent coherent hopping amplitude. To get separate information on these two contributions, impurity studies could be used. Consideration of the temperature dependence of the c -axis superfluid density measured in Josephson plasma resonance experiments already gives evidence that t_{ϕ} is much larger than t_{\perp} , but do not unambiguously rule out a small t_{\perp} contribution which could still be large enough to dominate the universal limit for $\sigma(0, 0)$ in relatively pure samples ($t_{\perp}/t_{\phi} > \gamma/\Delta_0$). At the same time, t_{ϕ} would dominate the temperature dependence of the superfluid density. We mention that the sum rule of Eq. (33) was derived under the assumption that the normal state is Fermi-liquid like. However, if it has a non-Fermi-liquid nature such as a pseudogap, the sum rule has to be modified to account for this; therefore it is necessary to explore the competing effects of interlayer coupling in a more fundamental theory which goes beyond a Fermi-liquid normal state.

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