Interplay of superconductivity and magnetism in superconductor/ferromagnet structures

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We study the influence of the proximity effect on several superconductor/ferromagnet S/F layered structures. In the case of F/S/F sandwiches, we calculate the dependence of the superconducting critical temperature as a function of the mutual orientation of ferromagnetic exchange fields of the outer layers. In the case of atomic-scale S/F superlattices, we analyze the properties of the superconducting phase for parallel or antiparallel orientation of magnetic moments of ferromagnetic layers. In both cases the superconducting π-phase appears.

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I. INTRODUCTION

The presence of the exchange field acting on electrons in ferromagnets provokes an oscillatory behavior of a superconducting order parameter. First this phenomena has been discovered by Larkin and Ovchinnikov[1] and Fulde and Ferrell,[2] who predicted a new modulated superconducting state, the "FFLO" state, when the magnetic field acts on electron spins. Experimentally, the FFLO state has not been unambiguously detected yet. On the other hand oscillations of a superconducting order parameter in the ferromagnetic barrier of S/F/S junctions must lead to an oscillatory dependence of the critical temperature as a function of a ferromagnetic layer thickness[3,4] and so-called π-junction realization.[5,6] Recently these phenomena have been observed experimentally.[7–9] Note that the appearance of the π-junctions has been predicted in the presence of magnetic impurities in the barrier of the Josephson junction in Ref. 10.

In this paper, we present detailed calculations of the dependence of the critical temperature of the metallic F/S/F sandwiches on the mutual orientation of ferromagnetic moments of the outer layers. The situation of the parallel and antiparallel orientations of ferromagnetic moments has been considered for the case of the thin superconducting layer in Ref. 11, and the thick superconducting layer case has been treated in Ref. 12. Note that for the first time the coupling between ferromagnets through a superconducting layer has been treated theoretically in Ref. 13 for the case of ferromagnetic insulators and observed in experiment in Refs. 14 and 15. We analyze also the properties of the superconducting phase in the case of atomic-scale S/F superlattices. Using the approach of Ref. 16, we treat the case of the antiparallel orientation of moments in the adjacent ferromagnetic layers and demonstrate that the superconducting π phase appears, similar to the parallel orientation case.

II. SPIN-ORIENTATION DEPENDENCE OF SUPERCONDUCTIVITY

A. Case of thick ferromagnetic layers

In this section, we examine the F/S/F structure presented in Fig. 1, assuming that the dirty limit conditions are held in the superconducting layer. First, we consider the case of thick ferromagnetic layers and a thin superconducting layer, supposing that $\sigma_f \ll \sigma_s$, where σ_f and σ_s are the conductivities in the ferromagnetic and the superconducting layers. This condition assures a relatively weak proximity effect.[6] Indeed, the case of thick ferromagnetic layers and a strong proximity effect will always lead to the suppression of the superconductivity. Let the exchange field $\mathbf{h}$ for $x > d_s$ being tilted by an angle ϕ from the quantization axis $\mathbf{z}$, and for $x < -d_s$ by an angle $-\phi$. Then the parallel orientation of the magnetizations on both sides of the S layer corresponds to $\phi = 0$, and the antiparallel orientation corresponds to $\phi = \pi/2$. We may use, supposing that the dirty limit conditions are held, the Usadel's equations[17] in the case of a system of electrons with Cooper pairing in an external field $\mathbf{h}$ [lying in the (y,z) plane] acting on the electron spins.[18] In matrix form, the corresponding Usadel's equations are

$$-\frac{D_{s,f}}{2}\boldsymbol{\nabla}[\hat{G}(\mathbf{r},\omega_n)\boldsymbol{\nabla}\hat{F}(\mathbf{r},\omega_n)-\hat{F}(\mathbf{r},\omega_n)\boldsymbol{\nabla}\hat{G}(\mathbf{r},\omega_n)]$$
$$+[\omega_n\hat{1}+i\hat{H}(\mathbf{r})]\hat{F}(\mathbf{r},\omega_n)$$
$$=\Delta(\mathbf{r})\hat{I}\hat{G}(\mathbf{r},\omega_n), \qquad (1)$$

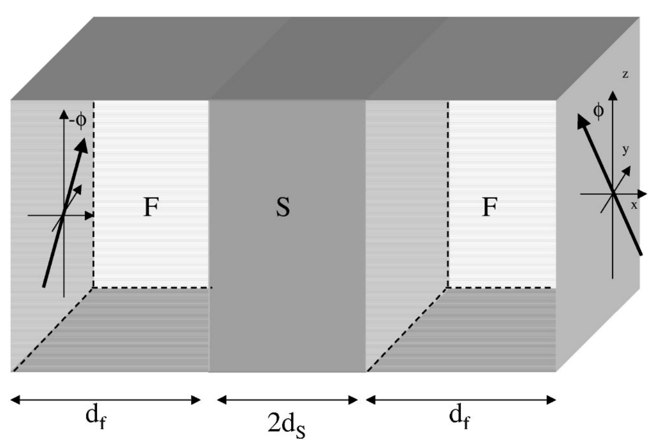

FIG. 1. Geometry of the F/S/F sandwich. Directions of the exchange field in F layers are given by the thick arrows lying in the (y,z) plane. The thickness of the S layer is $2d_s$ and d_f is the thickness of the F layers.

$$\hat{G}^2(\mathbf{r},\omega_n)+\hat{F}(\mathbf{r},\omega_n)\hat{F}^*(\mathbf{r},-\omega_n)=\hat{1}. \quad (2)$$

D_s and D_f are the diffusion coefficients in the superconducting and the ferromagnetic region, the Matsubara frequencies $\omega_n=(2n+1)\pi T$, the matrices

$$\hat{H}(\mathbf{r})=\begin{pmatrix} 0 & h_z(x)+ih_y(x) \\ h_z(x)-ih_y(x) & 0 \end{pmatrix}$$

$$\text{and } \hat{I}=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$\hat{F}=\begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \text{ and } \hat{G}=\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

are the Usadel's normal and anomalous functions (i.e., Green's functions integrated over angle and energy), and $\hbar=c=1$ throughout. In addition, we suppose that the Cooper pairing constant is equal to zero in F layers, then $\Delta\equiv 0$ for $|x|>d_s$.

As usual, Eq. (1) has to be supplemented by a self-consistency equation in the S layer

$$\Delta(\mathbf{r})\ln\frac{T_c}{T_c^*}=\pi T_c\sum_{\omega_n}\left[\frac{\Delta(\mathbf{r})}{|\omega_n|}-F_{12}(\mathbf{r},\omega_n)\right], \quad (3)$$

where T_c is the transition temperature of the S layer in the absence of a proximity effect, and T_c^* is the transition temperature in the presence of a proximity effect.

Considering the case of highly transparent F/S interfaces (it means a small boundary resistance or likewise a small potential barrier at the S/F interface), we can simplify the boundary conditions found in Ref. 19 as

$$\hat{F}_s=\hat{F}_f,\ \ \frac{\partial\hat{F}_s}{\partial x}=\gamma\frac{\partial\hat{F}_f}{\partial x}, \quad (4)$$

where γ is a phenomenological parameter characterizing the properties of the given S/F bilayer. In the case of an superconductor/normal metal (S/N) bilayer, $\gamma=\sigma_n/\sigma_s$ is the ratio of the normal states' conductivities. In this section we assume $\gamma\ll 1$, i.e., it has a relatively small proximity effect. For an experimental study of S/F interfaces of arbitrary transparency and a discussion of the transparency influence on the proximity effect see Refs. 20 and 21.

Near T_c^*, as Δ goes to 0, we can linearize Eq. (1) and then replace the normal Green's function by its value in the absence of superconductivity, i.e., put $G_{11}=G_{22}=\mathrm{sgn}(\omega_n)$ and $G_{12}=G_{21}=0$ as

$$-\frac{D_{s,f}}{2}\nabla^2\hat{F}(x,\omega_n)+[|\omega_n|\hat{1}+i\,\mathrm{sgn}(\omega_n)\hat{H}(x)]\hat{F}(x,\omega_n)$$
$$=\Delta(x)\hat{I}. \quad (5)$$

With the particular disposition of the magnetizations shown in Fig. 1, the exchange field is

$$\hat{H}(x<-d_s)=\hat{H}^*(x>d_s)=h\begin{pmatrix} 0 & \exp(i\phi) \\ \exp(-i\phi) & 0 \end{pmatrix}$$

$$\text{and } \hat{H}(-d_s<x<d_s)=0. \quad (6)$$

In the ferromagnetic regions, under the hypothesis $h\gg k_BT_c$, one can neglect the frequencies ω_n in comparison to h, hence the Usadel's equation for $x<-d_s$ can be written as

$$ih\,\mathrm{sgn}(\omega_n)\exp(i\phi)F_{12}-\frac{1}{2}D_f\frac{\partial^2F_{11}}{\partial x^2}=0,$$

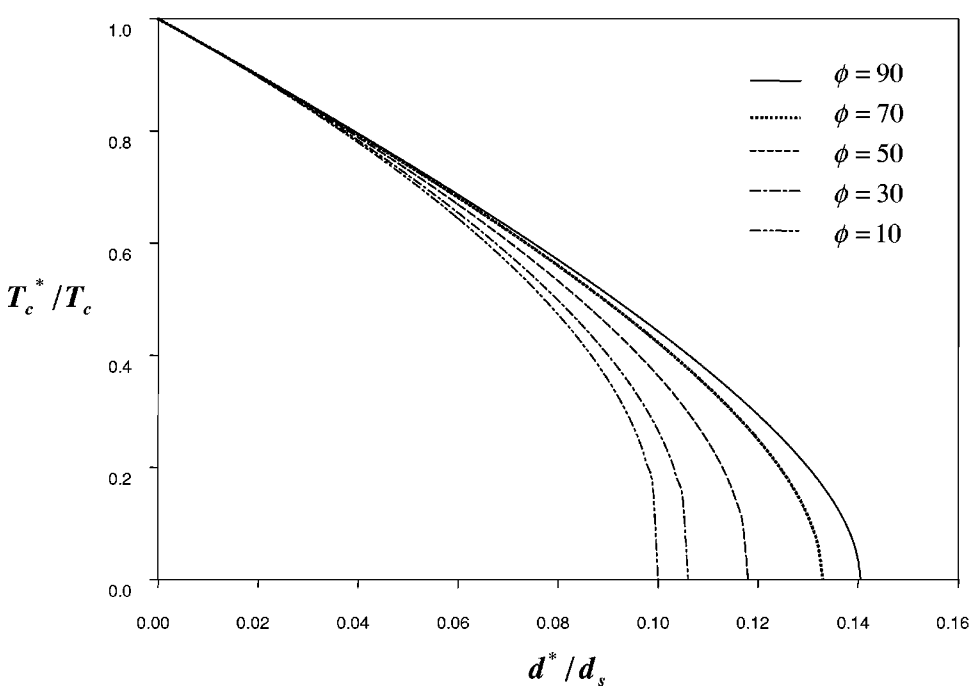

FIG. 2. Dependence of the superconducting reduced transition temperature T_c^*/T_c as a function of d^*/d_s for different values of the angle 2ϕ between the magnetization of F layers.

$$ih\,\mathrm{sgn}(\omega_n)\exp(-i\phi)F_{11}-\frac{1}{2}D_f\frac{\partial^2 F_{12}}{\partial x^2}=0. \tag{7}$$

For $x>d_s$, ϕ must be replaced by $-\phi$.

Solving these equations in both of the F layers, we obtain

$$x<-d_s,$$

$$F_{11}=e_{11}\exp(k_f x)+f_{11}\exp(k_f^* x),$$

$$F_{12}=\exp(-i\phi)[e_{11}\exp(k_f x)-f_{11}\exp(k_f^* x)],$$

$$x>d_s,$$

$$F_{11}=a_{11}\exp(-k_f x)+b_{11}\exp(-k_f^* x),$$

$$F_{12}=\exp(i\phi)[a_{11}\exp(-k_f x)-b_{11}\exp(-k_f^* x)], \tag{8}$$

where $k_f=(1+i)\sqrt{h/D_f}$, and $a_{11},b_{11},e_{11},f_{11}$ are coefficients to be determined using the boundary conditions.

In the superconducting region, the Usadel's equation takes the form

$$|\omega_n|F_{11}-\tfrac{1}{2}D_s\frac{\partial^2 F_{11}}{\partial x^2}=0, \tag{9}$$

$$|\omega_n|F_{12}-\tfrac{1}{2}D_s\frac{\partial^2 F_{12}}{\partial x^2}=\Delta(x). \tag{10}$$

A solution of Eq. (9) is $F_{11}=c_{11}\exp(-k_s x)+d_{11}\exp(k_s x)$, with $k_s^2=2|\omega_n|/D_s$ and two coefficients c_{11} and d_{11}.

In the case when $d_s\ll\xi_s=\sqrt{D_s/2\pi T_c}$ (the coherence length of the superconductor), the order parameter varies slowly on the distances of an order of the S-layer thickness. So, if we take into account the symmetry of the structure, we can seek a solution of Eq. (10) in the form[3,4]

$$F_{12}=\frac{\Delta\cos(kx)}{|\omega_n|+\rho}+f_{12}(x,\omega_n), \tag{11}$$

where $\rho=D_s k^2/2$ [a pair-breaking parameter that controls the transition temperature and that will be determined by the self-consistency Eq. (3)]. The function $f_{12}(x,\omega_n)$ yields the equation $|\omega_n|f_{12}(x,\omega_n)-1/2D_s f''_{12}(x,\omega_n)=0$, the solutions of which are $f_{12}(x,\omega_n)=c_{12}\exp(-k_s x)+d_{12}\exp(k_s x)$.

Using the boundary conditions given in Eq. (4), we finally find the system of equations that permits us to calculate all the coefficients. In the limit $d_s\ll\xi_s$, the self-consistency Eq. (3) can be written as

$$\sum_{\omega_n}(c_{12}+d_{12})=\sum_{\omega_n}(c_{12}+c_{12}^*)=2\sum_{\omega_n}\mathrm{Re}(c_{12})=0. \tag{12}$$

Note that the hypothesis $\gamma\ll 1$ (a weak proximity effect) enables us to neglect in Eq. (12) the terms proportional to the small parameter $d_s\gamma|k_f|$. The coefficient c_{12} can be found from the system provided by the boundary conditions in Eq. (4) and we may write the self-consistency Eq. (3) in the final form

$$\sum_{\omega_n}\frac{1}{(\omega_n+\rho)\left[\omega_n+\frac{\gamma|k_f|D_s}{2d_s}(1+i\cos\phi)\right]\left[\omega_n+\frac{\gamma|k_f|D_s}{2d_s}(1-i\cos\phi)\right]}=0. \tag{13}$$

After summation over ω_n we get, in the implicit form, the equation for the pair-breaking parameter ρ

$$\Psi\left(\frac{1}{2}+\frac{\rho}{2\pi T_c^*}\right)=\mathrm{Re}\,\Psi\left[\frac{1}{2}+\frac{\gamma D_s(1+i\cos\phi)}{4\pi d_s T_c^*}\sqrt{\frac{h}{D_f}}\right]. \tag{14}$$

According to Eq. (3), the equation giving the critical temperature T_c^* in the presence of an anisotropic proximity effect is

$$\ln\left(\frac{T_c^*}{T_c}\right)=\Psi\left(\frac{1}{2}\right)-\Psi\left(\frac{1}{2}+\frac{\rho}{2\pi T_c^*}\right)$$
$$=\Psi\left(\frac{1}{2}\right)-\mathrm{Re}\,\Psi\left[\frac{1}{2}+\frac{d^*T_c}{d_sT_c^*}(1+i\cos\phi)\right], \tag{15}$$

where d^* is the characteristic length

$$d^*=\frac{\gamma D_s}{4\pi T_c}\sqrt{\frac{h}{D_f}}. \tag{16}$$

In Fig. 2, we plot the solutions of Eq. (15) for different values of the angle ϕ as a function of the dimensionless parameter d^*/d_s. Note that if we assume $\phi=0$ (i.e., parallel alignments of the magnetizations or *P* phase), or $\phi=\pi/2$ (i.e., antiparallel alignments of the magnetizations or *AP* phase), we find the same dependence of T_c^* as that in Ref. 11.

The critical value d_{sc} of the thickness of the superconducting layer (below which the superconductivity is always destroyed by the proximity effect) is found in the limit $T_c^*\to 0$,

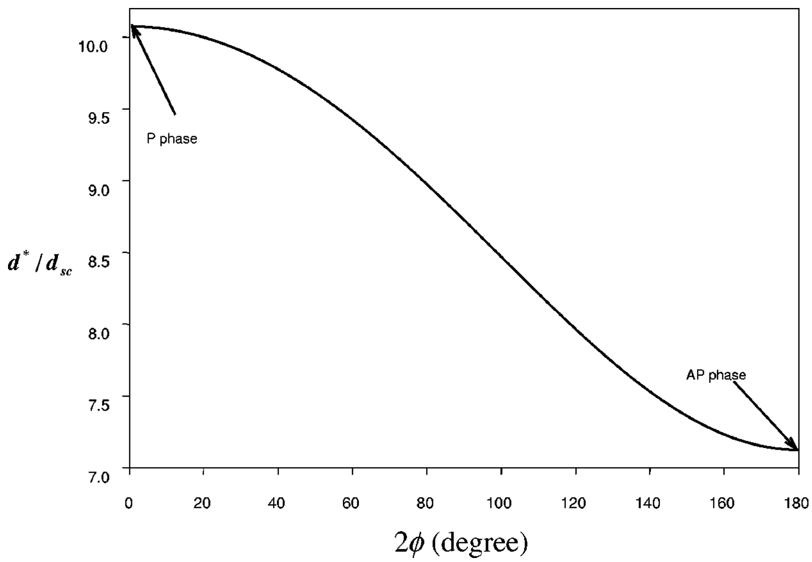

FIG. 3. The critical value d_{sc} of the thickness of the S layer as a function of the angle 2ϕ. The angle $2\phi=180°$ corresponds to antiparallel alignment and $\phi=0°$ corresponds to parallel alignment.

$$\frac{d_{sc}}{d^*}=\exp\left[-\Psi\left(\frac{1}{2}\right)+\mathrm{Re}\ln(1+i\cos\phi)\right]. \tag{17}$$

This critical thickness as a function of the angle ϕ is presented in Fig. 3, and we see that superconductivity is more easily suppressed by the proximity effect in the *P* phase than in the *AP* phase. Indeed in the *P* phase, the depairing effect of the exchange magnetic field is much stronger than in the *AP* phase.

B. Case of thin ferromagnetic layers

In this section, we consider the model when the electronic parameters of superconducting and ferromagnetic layers are the same, referring to the equality of the conductivities and of the Fermi velocities in all layers. Under this condition, the pair-breaking effect of the ferromagnetic layers is strong. Thus, the superconducting phase can only appear if the ferromagnetic layers are thin (compared to the coherence length ξ_s of the superconductor), which weakens the proximity effect. We shall examine the F/S/F structure assuming that the exchange field in the F layers may be either parallel or antiparallel. The set of equations describing an inhomogeneous superconductor has been developed by Eilenberger.[22] They are transportlike equations for the energy-integrated Green's functions f and g, assuming that relevant length scales are much larger than atomic length scales. If we consider the Cooper pairing of electrons in the presence of an exchange field $\mathbf{h}(\mathbf{r})=[0,0,h(r)]$, the Eilenberger equations take the form[23]

$$\left[\widetilde{\omega_n}+ih(r)+\frac{1}{2}\mathbf{v}_F\cdot\nabla\right]f(h,\mathbf{v}_F,\mathbf{r})=\widetilde{\Delta}(\mathbf{r})g(h,\mathbf{v}_F,\mathbf{r}),$$

$$\widetilde{\Delta}(\mathbf{r})=\Delta(\mathbf{r})+\frac{1}{2\tau}\int\frac{d\Omega}{4\pi}f(h,\mathbf{v}_F,\mathbf{r}),$$

$$\widetilde{\omega_n}(\mathbf{r})=\omega_n+\frac{1}{2\tau}\int\frac{d\Omega}{4\pi}g(h,\mathbf{v}_F,\mathbf{r}). \tag{18}$$

The Eilenberger Green's functions f and g depend on Matsubara frequencies $\omega_n=\pi T(2n+1)$ (note the omitted ω dependence of these functions), the elastic scattering time $\tau=l/v_F$, and the Fermi velocities $\mathbf{v}_F$. They obey the normalization condition $g^2(h,\mathbf{v}_F,\mathbf{r})+f(h,\mathbf{v}_F,\mathbf{r})f^*(-h,-\mathbf{v}_F,\mathbf{r})=1$.

We suppose $d_s\ll\xi_s$, thus Δ varies slowly on distances of the order of magnitude of ξ_s. An analytical solution of Eq. (18) has been derived in Ref. 23 for a periodic domain magnetic structure in the superconductor. The *AP* phase of our F/S/F structure is analogous to this magnetic structure if we assume that the period of the magnetic structure is $2(2d_s+d_f)$. For an exchange magnetic field of the form $h(r)=\Sigma_n h_{2n+1}\sin(2n+1)\mathbf{Qr}$, with $\mathbf{Q}=\pi\hat{\mathbf{x}}/(2d_s+d_f)$ the inverse lattice vector of the ferromagnetic structure, this solution for $h\tau\ll 1$ leads to the following self-consistency equation:

$$\ln\frac{T_c^*}{T_c}=\Psi\left(\frac{1}{2}\right)-\Psi\left(\frac{1}{2}+\frac{1}{\pi\tau_m T_c^*}\right), \tag{19}$$

$$\tau_m^{-1}=\frac{1}{2}\sum_n\int\frac{d\Omega}{4\pi}\frac{\tau\tilde{h}_{2n+1}\tilde{h}_{-(2n+1)}}{1+(2n+1)^2\tau^2(\mathbf{v}_F\cdot\mathbf{Q})^2},$$

$$\tilde{h}_{2n+1}=h_{2n+1}\left\{1-\frac{\arctan[Ql(2n+1)]}{Ql(2n+1)}\right\}^{-1}. \tag{20}$$

Note that to concentrate on the effect of the exchange field in the F layers, for simplicity, we supposed that Cooper pairing is the same everywhere. In the general case, we simply have to consider T_c as a critical temperature of an N/S/N sandwich with an average coupling constant $\overline{\lambda}=\lambda_s[d_s/(d_s$

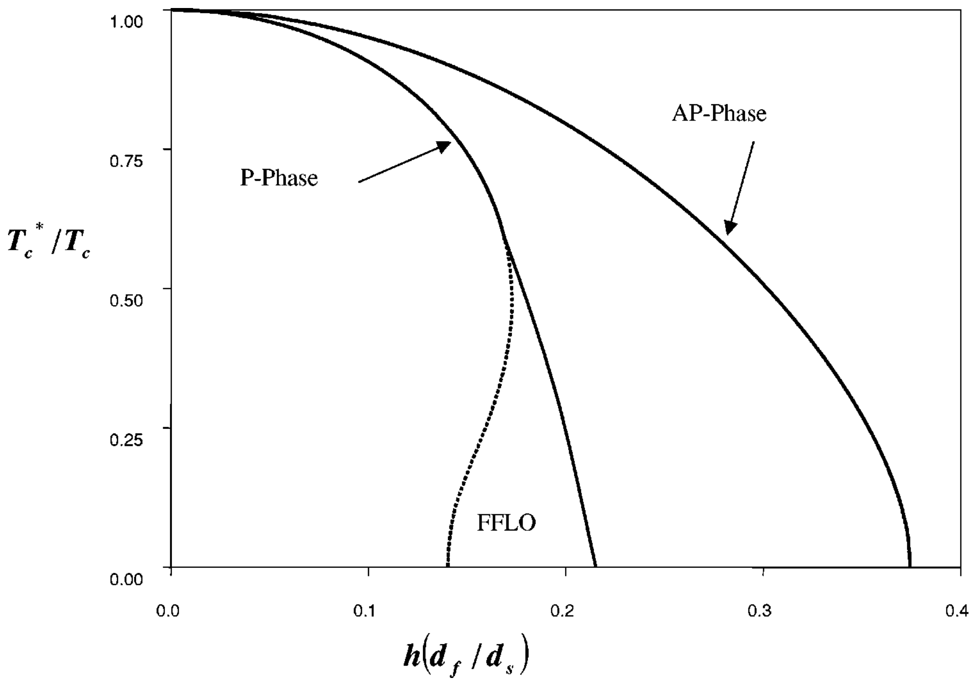

FIG. 4. Phase (h,T) diagram of the superconducting state for both parallel and antiparallel alignment of the exchange field in the case of thin ferromagnetic layers. Note the possibility of the appearance of an FFLO state in clean superconductors in the case of parallel orientation of the magnetic moments in F layers.

$+d_f)]$,[24] where λ_s is the BCS coupling constant in superconducting regions, while it is zero in ferromagnetic regions.

1. Case of antiparallel exchange fields

In the case of antiparallel exchange fields, the Fourier transform of the magnetic exchange field $h(r)$ is $h_{2n+1}=4h(-1^n)\sin[d_f/(8d_s(2n+1)\pi)]/[(2n+1)i\pi]$, and $\mathbf{Q}\approx(\pi/2d_s)\mathbf{x}$. As $\tilde{h}_{2n+1}$ doesn't depend on the direction of the Fermi velocity, we can perform the integration over the Fermi surface

$$\tau_m^{-1}=\frac{d_s}{2v\pi}\sum \tilde{h}_{2n+1}^2\frac{1}{(2n+1)}\arctan\left[\frac{(2n+1)\pi}{4}\left(\frac{l}{d_s}\right)\right]. \tag{21}$$

Depending on the relation between the mean-free-path l and the thickness of the superconducting layer $2d_s$, we have to distinguish the ballistic and the dirty diffusive regime of propagation of the electrons in the superconductor.

a. The ballistic regime. The condition $l\gg d_s$ ensures the ballistic propagation of the electrons over the thickness of the superconducting layer (and over the ferromagnetic layer because $d_s\gtrsim 10d_f$). Under this condition, $\tilde{h}_{2n+1}=h_{2n+1}$, thus we can give an analytical solution of Eq. (20):

$$\ln\frac{T_c^*}{T_c}=\Psi\left(\frac{1}{2}\right)-\Psi\left[\frac{1}{2}+\frac{\tau h^2}{4\pi T_c^*}\left(\frac{d_f}{d_s}\right)^2\right]. \tag{22}$$

b. The dirty limit. If $l\ll d_s$, scattering by impurities dominates in the superconducting layer. And the critical temperature is given by

$$\ln\frac{T_c^*}{T_c}=\Psi\left(\frac{1}{2}\right)-\Psi\left[\frac{1}{2}+\frac{\tau h^2}{4\pi T_c^*}\left(\frac{d_s}{l}\right)^4\left(\frac{d_f}{d_s}\right)^2\right]. \tag{23}$$

Note that we find the same critical temperature for $l\ll d_f$ and $l\gg d_f$. If we compare Eqs. (22) and (23), we notice that the equation giving the critical temperature in the dirty limit is the same as in the clean limit if we substitute h by $h(d_s/l)^2$. It means that in the dirty limit the effective exchange field acting on the electron spins is quite higher than in the clean limit, so superconductivity is quite easily suppressed.

2. Case of parallel exchange fields

The case of parallel exchange fields is quite different. The Fourier transform h_n of the magnetic exchange field h can be expressed as $h_n=h(d_f/d_s)\cos[n\pi d_s/(d_s+d_f)]$, so this exchange field has a $n=0$ nonvanishing component, which gives the most important contribution to the interaction with superconductivity in the clean limit (i.e., $l\gg\xi_s$). Thus, at first approximation, the case of parallel exchange fields is equivalent to the situation of a superconductor in a ''mean'' magnetic field $h_{eff}=h(d_f/d_s)$ acting on electron spins. The paramagnetic effect in superconductors has been discussed in detail in Ref. 25. Basing on these results, we may write directly the equation giving the critical temperature T_c^* of the S layer in the presence of the proximity effect (T_c is the critical temperature of the S layer in the absence of the proximity effect) as

$$\ln\left(\frac{T_c^*}{T_c}\right)=\Psi\left(\frac{1}{2}\right)-\mathrm{Re}\,\Psi\left[\frac{1}{2}+i\,\frac{h}{2\pi T_c^*}\left(\frac{d_f}{d_s}\right)\right]. \tag{24}$$

Using the analysis presented in Ref. 25, we complete Fig. 4, and predict in the clean limit the appearance of the FFLO state, with a modulation of the order parameter in the (y,z) plane $\{\Delta(y,z)=\Delta\exp[i(q_z z+q_y y)]\}$. Comparing Eqs. (22) and (24), we see that for $h\tau\ll 1$, T_c^* is quite higher for the antiparallel configuration than for the parallel one, as in the case of the thick ferromagnetic layers.

III. PROPERTIES OF THE SUPERCONDUCTING PHASE OF ATOMIC-SCALE FERROMAGNET/SUPERCONDUCTOR SUPERLATTICES

In this section, we consider an atomic-scale multilayer F/S system, where the superconducting (S) and the ferromagnetic (F) layers alternate. Recently a layered superconductor of this type ($RuSr_2GdCu_8$) has been discovered (see, for example, Ref. 26, and references cited therein). In $RuSr_2GdCu_8$, the magnetic transition occurs at $T_M \sim 130-140$ K and superconductivity appears at $T_c \sim 30-40$ K. When the electron transfer integral between the S and F layers is quite small, superconductivity can coexist with ferromagnetism in the adjacent layers.[16] However, the strong exchange field in the F layers favors the π-phase behavior of superconductivity (the superconducting order parameter alternates its sign on the adjacent *S* layers).[16,27] The very recent neutron-diffraction data on $RuSr_2GdCu_8$ (Ref. 28) revealed the antiferromagnetic ordering in all three directions, thus the question if there exists any layered compound with alternating S and F layers is still open. However, as it follows from the results of Ref. 28, in an external magnetic field, an induced ferromagnetic moment appears and $RuSr_2GdCu_8$ occurs to be a suitable candidate for π-phase observation.

In principle, it is possible to have a ferromagnetic ordering inside the F layers and an antiferromagnetic ordering between the adjacent F layers. Namely, this situation is realized in $Sm_{1.85}Ce_{0.15}CuO_4$, which reveals the superconductivity at $T_c = 23.5$ K and the magnetic order appears below $T_N = 5.9$ K.[29] Adopting the model in Ref. 16, we treat this situation in more detail. For simplicity, we assume that the electron's motion inside the F and S layers is described by the same energy spectrum $\xi(\mathbf{p})$. Three basic parameters characterize the system: t is the transfer energy between the F and S layers, λ is the Cooper pairing constant that is assumed to be nonzero in the S layers only, and h is the constant exchange field in the F layers. The Hamiltonian of the system can be written as

$$H = \sum_{\mathbf{p},n,i,\sigma} \xi(\mathbf{p}) a^{+}_{ni\sigma}(\mathbf{p}) a_{ni\sigma}(\mathbf{p}) + t[a^{+}_{ni\sigma}(\mathbf{p}) a_{n,-i,\sigma}(\mathbf{p}) + a^{+}_{n+1,-i,\sigma}(\mathbf{p}) a_{ni\sigma}(\mathbf{p}) + hc] + H_{int1} + H_{int2},$$

$$H_{int1} = \frac{\lambda}{2} \sum_{\mathbf{p}_1,\mathbf{p}_2,n,\sigma} a^{+}_{n1\sigma}(\mathbf{p}_1) a^{+}_{n,1,-\sigma}(-\mathbf{p}_1) a_{n,1,-\sigma} \times(-\mathbf{p}_2) a_{n1\sigma}(\mathbf{p}_2),$$

$$H_{int2} = -h \sum_{\mathbf{p},n,\sigma} (-1)^n \sigma a^{+}_{n,-1,\sigma}(\mathbf{p}) a_{n,-1,\sigma}(\mathbf{p}),$$

where $a^{+}_{ni\sigma}$ is the creation operator of an electron with spin σ in the nth elementary cell and momentum $\mathbf{p}$ in the layer i, where $i=1$ for the S layer, and $i=-1$ for the F layer.

Similar to the case of ferromagnetic interlayer ordering,[16] we may find the exact Green's functions in the case of antiferromagnetic interlayer ordering. Avoiding the simple but cumbersome calculations, we may readily write the expression for the modified superconducting critical temperature T_c^{*AF}, in the limit of small interlayer coupling $t \ll T_c$

$$\ln\frac{T_c^{*AF}}{T_c} = -\pi T_c \sum_{\omega_n} \frac{4t^2}{|\omega_n|(h^2+4\omega_n^2)} \times \left[1 - \frac{t^2(36\omega_n^4 + 17\omega_n^2 h^2 - h^4)}{4|\omega_n|^2(h^2+\omega_n^2)(h^2+4\omega_n^2)} - \cos k \frac{t^2(12\omega_n^4 - 7h^2\omega_n^2 - h^4)}{|\omega_n|^2(h^2+\omega_n^2)(h^2+4\omega_n^2)} \right], \tag{25}$$

where T_c is the critical temperature of the S layers at $t=0$ in the mean-field approximation. It was assumed that the superconducting order parameter may change from one S layer to another in the following manner: $\Delta_n = \Delta \exp(ikn)$ (n is the number of the S layer).

For comparison, we also write the equation for the critical temperature T_c^{*F} for ferromagnetic interlayer ordering[16]

$$\ln\frac{T_c^{*F}}{T_c} = -\pi T_c \sum_{\omega_n} \frac{4t^2}{|\omega_n|(h^2+4\omega_n^2)} \times \left[1 - \frac{t^2(36\omega_n^4 - 5\omega_n^2 h^2 - 5h^4)}{4|\omega_n|^2(h^2+\omega_n^2)(h^2+4\omega_n^2)} - \cos k \frac{t^2(12\omega_n^4 - 7h^2\omega_n^2 - h^4)}{|\omega_n|^2(h^2+\omega_n^2)(h^2+4\omega_n^2)} \right]. \tag{26}$$

We see that the difference between the transition temperatures of *F* and *AF* orientations appears only in terms proportional to t^4, which is quite natural because this effect is related to the interference of electrons coming from different magnetic layers. The terms proportional to $\cos k$ are the same in both *F* and *AF* cases. This means that at the critical temperature, the π phase appears when the exchange field exceeds some critical value $h_c = 3.77 T_c$ (Ref. 16), which is the same for *F* and *AF* phases. Note that the π-phase formation is simply related to the coupling of two S layers through a ferromagnetic one. At $T=0$, the critical value of the exchange field for the π-phase formation is somewhat smaller and is equal to $h_c^0 = 0.87 T_c$.

In the limit of a weak exchange field ($h \ll T_c$) one obtains

$$\ln\frac{T_c^{*F}}{T_c}\simeq\ln\frac{T_c^{*AF}}{T_c}$$
$$\simeq-\pi T_c\sum_{\omega_n}\frac{4t^2}{|\omega_n|h^2}\left[1-3\frac{|\omega_n|^2t^2}{h^4}(3+4\cos k)\right]. \tag{27}$$

If we call T_c^{*0} the critical temperature for the 0-phase formation and $T_c^{*\pi}$ the critical temperature for the π-phase formation, we have

$$\ln\frac{T_c^{*\pi}}{T_c^{*0}}\simeq-\pi T_c\sum_{\omega_n}\frac{96t^4}{h^6}|\omega_n|<0. \tag{28}$$

So in the limit of a weak exchange field, the 0 phase is energetically favorable for both *F* and *AF* interlayer ordering and the difference between T_c^{*AF} and T_c^{*F} is given by the expression

$$\frac{T_c^{*AF}-T_c^{*F}}{T_c}\simeq\ln\frac{T_c^{*AF}}{T_c^{*F}}\simeq3\,\frac{127}{4}\,\zeta(7)\,\frac{t^4h^2}{(2\pi T_c)^6}>0. \tag{29}$$

In the limit of a strong exchange field ($h\gg T_c$), we have to distinguish the ferromagnetic and the antiferromagnetic ordering.

For the ferromagnetic ordering we find

$$\ln\frac{T_c^{*F}}{T_c}\simeq-\pi T_c\sum_{\omega_n}\frac{t^2}{|\omega_n|^3}\left[1+\frac{t^2h^4}{|\omega_n|^2\omega_n^4}\left(\tfrac{5}{4}+\cos k\right)\right], \tag{30}$$

$$\ln\frac{T_c^{*\pi,F}}{T_c^{*0,F}}\simeq\pi T_c\sum_{\omega_n}\frac{5t^4h^4}{2|\omega_n|^5\omega_n^4}>0. \tag{31}$$

For the antiferromagnetic ordering we find

$$\ln\frac{T_c^{*AF}}{T_c}\simeq-\pi T_c\sum_{\omega_n}\frac{t^2}{|\omega_n|^3}\left[1+\frac{t^2h^4}{4|\omega_n|^2\omega_n^4}\left(\tfrac{1}{4}+\cos k\right)\right], \tag{32}$$

$$\ln\frac{T_c^{*\pi,AF}}{T_c^{*0,AF}}\simeq\pi T_c\sum_{\omega_n}\frac{t^4h^4}{2|\omega_n|^5\omega_n^4}>0. \tag{33}$$

So in the limit of a strong exchange field (like Fe, Ni, Co), the π phase is energetically favorable for both *F* and *AF* ordering and the difference between T_c^{*AF} and T_c^{*F} is given by the expression

$$\frac{T_c^{*AF}-T_c^{*F}}{T_c}\simeq7\zeta(3)\frac{t^4}{h^2\pi^2T_c^2}>0. \tag{34}$$

Then we may conclude that in all cases the critical temperature is higher for *AF* ordering.

IV. CONCLUSION

We have demonstrated that in the F/S/F sandwiches, when the thickness of the superconducting layer is smaller than the superconducting coherence length ξ_s, its critical temperature is controlled by the relative orientation of the ferromagnetic moments of the outer layers. Recently the fabrication of the so-called spin-valve sandwiches has been reported in Ref. 30. Such devices provide the possibility to change the relative orientation of ferromagnetic layer magnetizations by applying a weak magnetic field (less than 50 G). Then the spin-valve-type F/S/F sandwiches can be very interesting for application as a small magnetic field could trigger transition from a normal to a superconducting state. Finally the analysis of an atomic-scale S/F multilayered structure reveals the robustness of the superconducting π phase (where the sign of the superconducting order parameter in adjacent S layers is alternated) in strong ferromagnets, towards the relative orientation of the magnetic moments in F layers.

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