# Sliding phases via magnetic fields

S. L. Sondhi
*Department of Physics, Princeton University, Princeton, New Jersey 08544*

Kun Yang
*National High Magnetic Field Laboratory and Department of Physics, Florida State University, Tallahassee, Florida 32306*
(Received 9 August 2000; published 11 January 2001)

We show that three-dimensional ''sliding'' analogs of the Kosterlitz-Thouless phase, in stacked classical two-dimensional $XY$ models and quantum systems of coupled Luttinger liquids, can be enlarged by the application of a parallel magnetic field, which has the effect of increasing the scaling dimensions of the most relevant operators that can perturb the critical sliding phases. Within our renormalization-group analysis, we also find that for the case of coupled Luttinger liquids, this effect is interleaved with the onset of the integer quantum Hall effect for weak interactions and fields. We comment on experimental implications for a conjectured smectic metal phase in the cuprates.

DOI: 10.1103/PhysRevB.63.054430 PACS number(s): 75.10.Hk, 71.10.Pm, 74.60.−w

## I. INTRODUCTION

In 1979, Efetov[1] suggested that it would be possible to extend the low-temperature Kosterlitz-Thouless (KT) phase of a two-dimensional superconductor to three dimensions by stacking two-dimensional systems in the presence of a parallel magnetic field. The underlying idea, most simply understood in a particular gauge for the field, that we specify below, is that the interlayer Josephson coupling which would ordinarily be relevant even when weak, is now spatially modulated and no longer gives rise to divergences. It turns out that this does not work. As pointed out by Korshunov and Larkin,[2] the modulated Josephson coupling gives rise to a coupling that is *not* modulated, and although it is of higher scaling dimension than the zero-field Josephson coupling, it is still relevant everywhere within the KT phase. However, recent works on cationic-lipid-DNA complexes by O'Hern and Lubensky[3] and Golubovic and Golubovic,[4] and then on $XY$ systems by O'Hern, Lubensky, and Toner[5] (OLT) have found a different way of obtaining analogs of KT phases in three dimensions. In this approach, additional derivative couplings leave the phases in the different planes free to rotate globally with respect to each other (hence ''sliding phases'') while extending the region of irrelevant vortex fugacity to a range where the interlayer Josephson coupling is now irrelevant. Emery *et al.*,[6] and Vishwanath and Carpentier[7] have applied this insight to quantum problems and obtained an analog of the Luttinger liquid in two dimensions.

Our purpose in this paper is to point out that one can combine Efetov's insight with the more recent works and considerably extend the domain of these sliding phases by reducing the dimension of a large class of relevant operators via the action of a parallel magnetic field. This is of considerable interest, for the full class of perturbations in such problems can be quite constraining,[7] even though it is reasonable that most of them are not realized with substantial amplitude.[6] We will be especially interested in ''sliding Luttinger liquids'' or ''smectic metals,'' which have been argued to arise in the cuprate superconductors on account of the stripe instability of a doped antiferromagnet.[6,8] We should note that there is a close connection between our work and that on the striped phases in high Landau levels[9] even though our point of departure (Efetov's conjecture) is very different. In the Landau level problem, the field is built in at the first step and is central in giving rise to the striped phase in the first instance, while for us it can be variable in magnitude and give rise to both gapped quantum Hall and gapless smectic behavior and an interesting phase transition between them. Nevertheless, in both the cases, the field serves to constrain the available set of relevant operators in a very similar fashion.

We will begin in Sec. II with a quick account of the ''dimensional reduction'' of the Josephson coupling produced by a parallel field, the genesis of the sliding phase, and its enlargement by the field. Next (Sec. III) we discuss the application of these ideas to coupled Luttinger liquids and present contrasting phase diagrams for a model studied by Emery *et al.* In this discussion we also show how the integer quantum-Hall states are rediscovered by perturbation theory about a smectic metal if the interactions are not too strong. We close with a brief summary and a discussion of possible experimental implications for the cuprates.

## II. SLIDING $XY$ PHASES IN PARALLEL FIELDS

We begin with a brief summary of the genesis of the sliding phase in a three-dimensional stack of layers characterized by an $XY$-order parameter. We largely follow OLT and their notation for ease of comparison. The Hamiltonians of the sliding phase fixed points (the plural is warranted) belong to the family

$$H_S=\frac{1}{2}\sum_{nn'}\int d^2r\, K_{nn'}\nabla_\perp\theta_n(\mathbf{r})\nabla_\perp\theta_{n'}(\mathbf{r}), \tag{1}$$

where $K_{nn'}=Kf_{n-n'}$ with $f_n=(1+\Sigma_m\gamma_m)\delta_{n,0}-\frac{1}{2}\Sigma_m\gamma_m(\delta_{n,m}+\delta_{n,-m})$ and and $\nabla_\perp\theta_n(\mathbf{r})$ denotes the in-layer gradient of the $XY$ variable in layer $n$. We take $\mathbf{r}\equiv(x,y)$ and set the separation of successive layers along the $z$ axis to 1. One can check that $H_S$ is invariant under shifts

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$\theta_n(\mathbf{r}) \rightarrow \theta_n(\mathbf{r}) + \psi_n$, for any choice of $\psi_n$. This freedom to globally rotate the angle in one layer relative to another, even in the presence of interlayer couplings in $H_S$, is the hallmark of the sliding phase.

Note that $H_S$ returns to itself under a renormalization group (RG) transformation that ''lives'' in two dimensions and treats the layer index $n$ as an internal or flavor index on the fields $\theta_n$. In order to identify functions $K_{nn'}$ that would govern *stable* fixed points under this RG, we need to examine the behavior of vortex fugacities and Josephson couplings. The former yields, for a vortex configuration $\{\sigma_n\}$ in which a net vorticity $\sigma_n$ occurs in layer $n$, the scaling dimension

$$\Delta_v[\sigma_n] = \frac{\pi K}{T} \sum_{n,n'} f_{n-n'} \sigma_n \sigma_{n'}, \tag{2}$$

which signals a KT unbinding transition at a temperature $T_{KT}[\sigma_n]$, upon exceeding the value 2 appropriate to a two-dimensional RG. The generalized Josephson couplings,

$$H_J[s_n] = -V_J[s_n] \int d^2r \cos\left[\sum_p s_p \theta_{n+p}(\mathbf{r})\right], \tag{3}$$

where the $s_n$ are integers that satisfy $\Sigma_n s_n = 0$, are readily shown to have the scaling dimension,

$$\Delta_J[s_n] = \frac{T}{4\pi K} \sum_{n,n'} s_n s_{n'} f^{-1}_{n-n'}, \tag{4}$$

where the inverse couplings

$$f_p^{-1} = \frac{1}{\pi} \int_0^{\pi} dk \frac{\cos kp}{f(k)} \tag{5}$$

are defined via the Fourier transform

$$f(k) = 1 + \sum_m \gamma_m (1 - \cos km) \tag{6}$$

of the scaled couplings $f_n$.

The Josephson couplings are irrelevant above a decoupling temperature $T_d[s_n]$, at which $\Delta_J[s_n] = 2$. If $\min_{\sigma_n} T_{KT}[\sigma_n] > \max_{s_n} T_d[s_n]$ for some choice of $K_{nn'}$, then we obtain a sliding phase. In the sliding phase, the spin correlations are algebraically long ranged in a given layer and vanish between layers,

$$\langle \cos[\theta_n(\mathbf{r}) - \theta_m(0)] \rangle \sim \frac{\delta_{nm}}{r^{\eta}}, \tag{7}$$

where $\eta = (T/2\pi K) f_0^{-1}$.

### *Including a parallel magnetic field*

We now consider the inclusion of a magnetic field parallel to the layers, appropriate to instances where $\theta_n$ are phases of a superconducting order parameter; without loss of generality, we take $\mathbf{B} = B\hat{y}$. It is convenient to work in the gauge $A_z(\mathbf{r}) = Bx$. In this gauge, the sliding phase Hamiltonians are of the same form, and the computation of the scaling of the vortex fugacity is unchanged. However, the Josephson couplings are modified by the replacements

$$\theta_n \rightarrow \theta_n + 2nq_B x, \tag{8}$$

where $q_B = eB/\hbar c$ is a characteristic wave vector introduced by the field.

The key observation regarding the effect of the field is this: for those Josephson couplings for which $\Sigma_p p s_p \neq 0$, there is an explicit oscillating term in the argument of the cosine that will render them less relevant. Most straightforwardly, consider treating such a term in the perturbation theory. In zero field, we would discover that the term was relevant upon finding divergences in perturbation theory. The inclusion of the field will attenuate these divergences due to the oscillation of the correlation functions of the perturbation.

However, the net result is not always to render the perturbation theory convergent. Higher-order graphs can involve regions where products of the oscillating couplings nevertheless give rise to operators that do not oscillate. For example, the Josephson coupling for layers at distance $p$,

$$\cos[\theta_{n+p}(\mathbf{r}) - \theta_n(\mathbf{r}) + 2pq_B x] \sim e^{i[\theta_{n+p}(\mathbf{r}) - \theta_n(\mathbf{r})]} e^{i2pq_B x} + \text{c.c.} \tag{9}$$

will give rise to

$$\cos[\theta_{n+p}(\mathbf{r}) - 2\theta_n(\mathbf{r}) + \theta_{n-p}(\mathbf{r})], \tag{10}$$

which can then produce divergences of its own. Indeed, this particular generation is exactly what invalidates Efetov's original conjecture, for the operators (10) are relevant everywhere in the KT phase of decoupled $XY$ layers. Nevertheless, the application of the field does effect a ''dimensional reduction'' in that ''charged'' operators that have a net $\Sigma_p p s_p$ (microscopically these arise from hopping processes that move a net charge up or down the stack) can only affect the result through the generation of ''neutral'' operators for which $\Sigma_p p s_p = 0$. At the (unstable) decoupled two-dimensional (2D) $XY$ fixed line, the latter have higher dimension and we may expect that this will be true at sliding fixed points as well. While that is not always the case, as will be clear by the following example, it is still true that knocking out the charged operators improves the stability of the sliding phase—after all, the neutral operators were present anyway.

To illustrate this effect, we consider the example used by OLT with first and second neighbor couplings. The coupling function $f_n$ has a Fourier transform

$$f(k) = 1 + \gamma_1(1 - \cos k) + \gamma_2(1 - \cos 2k) \tag{11}$$

that is required to take its minimum value at $k = k_o$:

$$f(k_o) = \delta, \quad f'(k_o) = 0 \quad \text{and} \quad f''(k_o) = 2C. \tag{12}$$

Sliding phases arise when $\delta$ and $k_o$ are chosen so that the system is close to an incommensurate transverse ordering instability, as has been discussed nicely by Vishwanath and Carpentier.[7] At small $\delta$, the asymptotic form,

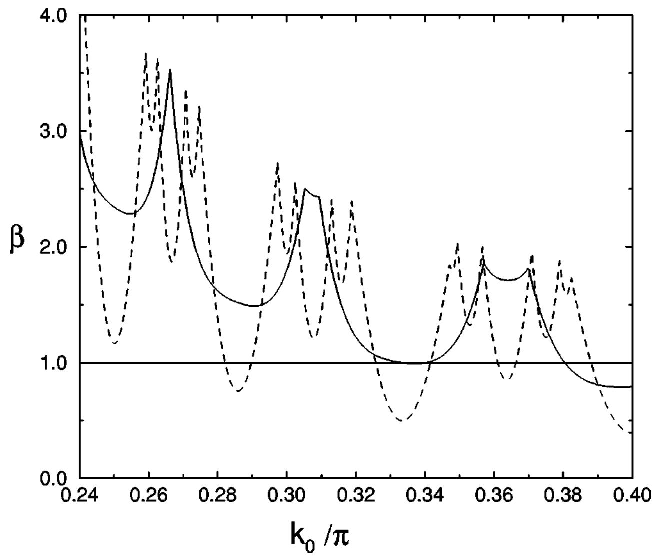


FIG. 1. A plot of $\beta = T_{KT}/T_d$ against $k_o/\pi$ at $\delta = 10^{-5}$. The dashed line is due to charged two-layer couplings while the solid line is due to the leading, three-layer, neutral couplings. The $B=0$ sliding phase exists when the minimum of the two curves exceeds 1. The $B\neq 0$ sliding phase requires only that the solid curve exceeds unity and hence leads to a larger sliding phase.

$$f_p^{-1} \sim \frac{\cos(pk_0)e^{-p\sqrt{\delta/C}}}{\sqrt{C\delta}} \tag{13}$$

enables easy numerical calculation of the scaling dimensions of the Josephson couplings and thence of the temperatures $T_d$. In Fig. 1, we plot the ratio $\beta = \min_{\sigma_n} T_{KT}[\sigma_n]/\max_{s_n} T_d$ where the $s_n$ are restricted to the two layer Josephson couplings (9) and the three-layer terms that they generate (10). The value of $k_o$ where *both* are greater than 1 supports a sliding phase in zero field,[10] while the latter alone determines the sliding phase in a magnetic field. The expansion of the phase is clear. (We have not attempted to include all operators that might be allowed by symmetry. As noted in Ref. 7, in the Luttinger liquid context, higher-order operators allow increasingly finer instabilities. We do not know of a proof that all such operators allow or exclude a connected sliding phase, but assume that in a given system, a finite set will be important over some reasonable range of length scales. Regardless, the magnetic field will improve matters by knocking out all the charged operators.)

## III. SMECTIC METALS IN TRANSVERSE FIELDS

In this section we discuss coupled one-dimensional (1D) Luttinger liquids (LL's) in the presence of a magnetic field, with the field **B** transverse to the plane in which the 1D chains are placed. It was known that at the *decoupled* LL fixed points, the transverse interchain coupling is always relevant in one of three channels: single electron hopping, (Cooper) pair hopping, and interchain $2k_F$ back scattering. As a consequence, the decoupled LL phase is always unstable and driven toward the Fermi liquid,[11] superconducting, or charge/spin density wave (CDW/SDW) phases. It was recently pointed out[6,7] that adding strong interchain forward-scattering terms (which are exactly marginal) to the decoupled LL fixed point can drive all these interchain couplings irrelevant. The resulting stable, non-Fermi liquid, smectic metal phase is the quantum analog of the classical sliding phase.[5]

Since the single electron and Cooper pair hopping processes involve charge transfer between neighboring chains, the presence of a magnetic field has a similar effect, as before, of increasing the scaling dimensions of the operators corresponding to these processes, that can perturb the smectic metal fixed points, and hence increasing the range of stability of the smectic metal phase (for simplicity we will neglect the Zeeman effect of the field in this paper). In the following, we present an explicit analysis of this effect. Following Emery *et al.*[6] three different types of smectic metal fixed points need to be distinguished and analyzed in turn: (i) a spinful smectic metal with a spin gap; (ii) a spinful smectic metal without a spin gap; and (iii) a spinless smectic metal. For simplicity we will only include nearest-neighbor interchain couplings and their immediate descendants.

### A. Spin-gapped smectic metal

In this case the fixed point action in Euclidean space takes the form[6]

$$S = \frac{1}{2}\sum_Q \,[W_0(k_\perp)\omega^2 + W_1(k_\perp)k^2]|\phi(Q)|^2$$
$$= \frac{1}{2}\sum_Q \left[\frac{\omega^2}{W_0(k_\perp)} + \frac{k^2}{W_1(k_\perp)}\right]|\theta(Q)|^2, \tag{14}$$

where $Q=(\omega,k,k_\perp)$, for each chain the 2-current is $j_\mu = (1/\sqrt{\pi})\epsilon_{\mu\nu}\partial^\nu\phi$, and $\theta$ is the dual field of $\phi$. The scaling dimensions of various local operators are determined by the dimensionless Luttinger coupling function

$$w(k_\perp) = \sqrt{W_0(k_\perp)W_1(k_\perp)}, \tag{15}$$

which is periodic in $(k_\perp)$ with period $2\pi$ as we have set the interchain distance to 1. As in Ref. 6, we consider the simplified model in which $w(k_\perp)$ takes the form

$$w(k_\perp) = K_0 + K_1\cos(k_\perp) = K_0[1+\lambda\cos(k_\perp)]. \tag{16}$$

Stability requires $|\lambda|<1$. In the presence of a spin gap, single electron hopping is irrelevant and the magnetic field has no effect on $2k_F$ backscattering that does not involve charge transfer between chains. We thus focus on the singlet pair hopping process, which in the presence of a magnetic field is described by the following perturbing Hamiltonian (near-neighbor hopping only):

$$H_{sc} = -t_J\int dx\; h_{sc}(x),$$

$$h_{sc}(x) = \sum_j \;\cos[\sqrt{2\pi}\{\theta_j(x) - \theta_{j+1}(x)\} + 2q_Bx], \tag{17}$$

where $t_J$ is the Josephson coupling strength and $q_B = eB/\hbar c$ as before. As in the previous section, the field adds an oscillatory phase to the pair hopping term, which renders $h_{sc}$ irrelevant by itself. However, as it flows, it again generates terms in which the oscillatory phases cancel. The most relevant of these is

$$\tilde{H}_{sc} \propto t_J^2 \int dx \ \tilde{h}_{sc}(x),$$

$$\tilde{h}_{sc}(x) = \sum_j \cos[\sqrt{2\pi}\{2\theta_j(x) - \theta_{j+1}(x) - \theta_{j-1}(x)\}], \tag{18}$$

which is generated at second order in $t_J$. The scaling dimension of this term is

$$\tilde{\Delta}_{sc} = \frac{2K_0}{2\pi}\int_0^{2\pi} dk_\perp \{1 + \lambda \cos(k_\perp)\}\{1 - \cos(k_\perp)\}^2 = (3 - 2\lambda)K_0. \tag{19}$$

Combining the knowledge of $\tilde{\Delta}_{sc}$ with the scaling dimension of the $2k_F$ backscattering operator[6]

$$\Delta_{CDW} = \frac{2}{K_0(1 - \lambda + \sqrt{1-\lambda^2})}, \tag{20}$$

we can determine the phase diagram of the model (14) in the presence of a magnetic field and near-neighbor interchain couplings, and subject to weak, generic perturbations, using the criteria that the smectic phase is stable when $\tilde{\Delta}_{sc} > 2$ and $\Delta_{CDW} > 2$; otherwise the system is in the stripe crystal/superconducting phase for $\Delta_{CDW}$ smaller/bigger than $\tilde{\Delta}_{sc}$. [In this identification we have made the natural assumption that the coupling (18) will govern the properties of the phase when it grows most rapidly. By itself, it will produce a vortex lattice.[12]] The phase diagram is plotted in Fig. 2. For comparison we have also included the phase boundaries separating the superconducting phase from the smectic metal and stripe crystal phases in the *absence* of a magnetic field[6] as dotted lines. It is quite obvious that both the smectic metal and stripe crystal phases get expanded by the magnetic field, which suppresses interchain Josephson coupling and increases the scaling dimension of operators involving pair hopping.

### B. Spin-ungapped smectic metal

In this case the fixed point action has contributions from both the charge and spin sectors: $S = S_\rho + S_\sigma$, where we take $S_\rho$ to have the same form as Eq. (14), and

$$S_\sigma = \frac{K_\sigma}{2}\sum_j \left[\frac{1}{v}(\partial\tau\phi_{j\sigma})^2 + v(\partial_x\phi_{j\sigma})^2\right], \tag{21}$$

in which we assume that there is no interchain coupling among spin fields, as in Ref. 6. Spin rotation invariance (also assumed here) requires $K_\sigma = 1$. The analysis of pair hopping is similar to the previous case and it is easy to show that the

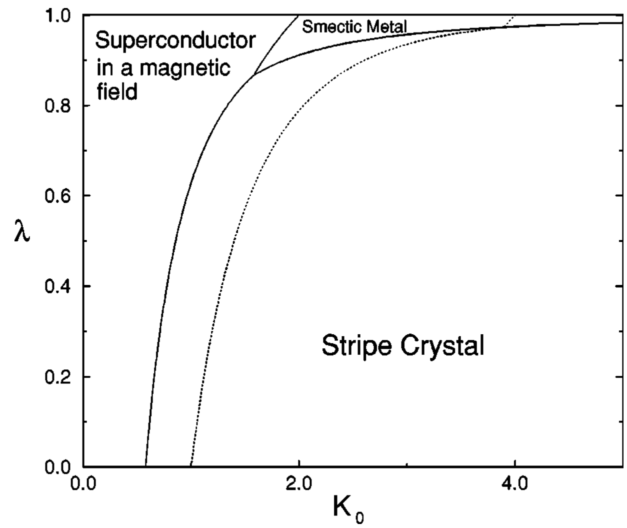


FIG. 2. Phase diagram for coupled Luttinger liquids with a spin gap, in the presence of a magnetic field. For comparison, we have also included the phase boundaries separating the superconducting phase from the smectic metal and the stripe crystal phases in the *absence* of a magnetic field (dotted line), as reported by Emery *et al.* (Ref. 6). The presence of a magnetic field significantly expands the region of both the smectic metal and the stripe crystal.

most relevant operator generated by the pair hopping has scaling dimension $\tilde{\Delta}_{sc}^{nogap} = \tilde{\Delta}_{sc}^{gap} + 3 > 2$, i.e., operators generated by pair hopping are *always irrelevant* here.

Low-energy single-electron hopping, which is allowed, is on the other hand, more complicated and interesting. In terms of the original electron operators it takes the form

$$H_e = -t_e \int dx \ h_e(x),$$

$$h_e(x) = \sum_{j\sigma} \{\psi_{j\sigma}^\dagger(x)\psi_{j+1\sigma}(x)e^{iq_Bx} + \text{H.c.}\}. \tag{22}$$

We need to distinguish two different cases here.

(i) $k_F$ and $q_B$ are incommensurate. In this case, single-electron processes all involve an oscillating phase, and the most relevant process without an oscillating phase generated by $H_e$ is

$$\tilde{H}_e \propto t_e^2 \int dx \, \tilde{h}_e(x),$$

$$\tilde{h}_e(x) = \sum_i [\psi_{j\uparrow}^{L\dagger}(x)\psi_{j\downarrow}^{R\dagger}(x)\psi_{j+1\uparrow}^{L}\psi_{j-1\downarrow}^{R} + \text{H.c.} + \cdots], \tag{23}$$

where $L/R$ stands for left/right mover, and ‘‘$\cdots$’’ stands for terms of similar structure. In bosonized form,

$$\tilde{h}_e(x) \propto \cos\{\sqrt{2\pi}[2(\theta_{\rho i} + \phi_{\sigma i}) - \theta_{\rho i+1} - \theta_{\rho i-1} - \phi_{\sigma i+1} - \phi_{\sigma i-1}]\}, \tag{24}$$

which has the scaling dimension

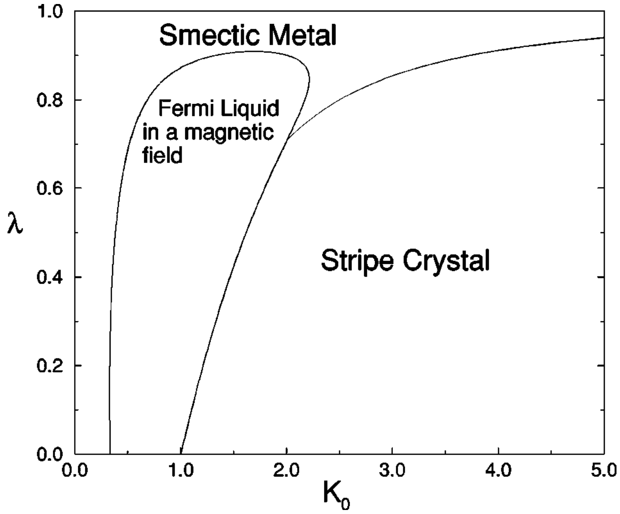


FIG. 3. Phase diagram for coupled spinful Luttinger liquids without a spin gap, in the presence of a magnetic field, assuming there is no commensuration between $k_F$ and $q_B$.

$$\tilde{\Delta}_e = 1 + \frac{K_0}{2}\left(\frac{3}{2} - \lambda\right) + \frac{1-\sqrt{1-\lambda^2}}{2K_0\lambda^2\sqrt{1-\lambda^2}}. \tag{25}$$

We assume that the system behaves as a Fermi liquid in a magnetic field when this term dominates. This identification is suggested if we note that at the noninteracting point, $\lambda = 0$ and $K_0 = 1$, this term is marginal. This leads to the phase diagram Fig. 3, which is qualitatively different from the phase diagram in the absence of the field, Fig. 2 of Ref. 6. There are two particularly interesting differences: (i) The superconducting phase gets completely squeezed out by the field; (ii) the smectic metal phase now extends all the way to $\lambda = 0$, which corresponds to the *decoupled* LL fixed point, a situation impossible without the field.

(ii) $2k_F = nq_B$ where $n$ is an integer. In this case $H_e$, or its higher-order descendents in the low-energy theory, can turn a left mover on the Fermi point of the $j$th chain to a right mover on the Fermi point of the $(j+n)$th chain; this is a low-energy single-electron hopping process that *does not* involve an oscillatory phase, which takes the form

$$H'_e = -t_e \int dx\, h'(x),$$

$$h'(x) = \sum_{j\sigma} (\psi^\dagger_{jL}\psi_{j+nR} + \text{H.c.})$$
$$\propto \cos\sqrt{\frac{\pi}{2}}(\theta_{\rho i} - \theta_{\rho i+n} + \phi_{\rho i} + \phi_{\rho i+n})$$
$$\times \cos\sqrt{\frac{\pi}{2}}(\theta_{\sigma i} - \theta_{\sigma i+n} + \phi_{\sigma i} + \phi_{\sigma i+n}). \tag{26}$$

The scaling dimension of this operator $\Delta'_{e,n}$ for $n=1$ is

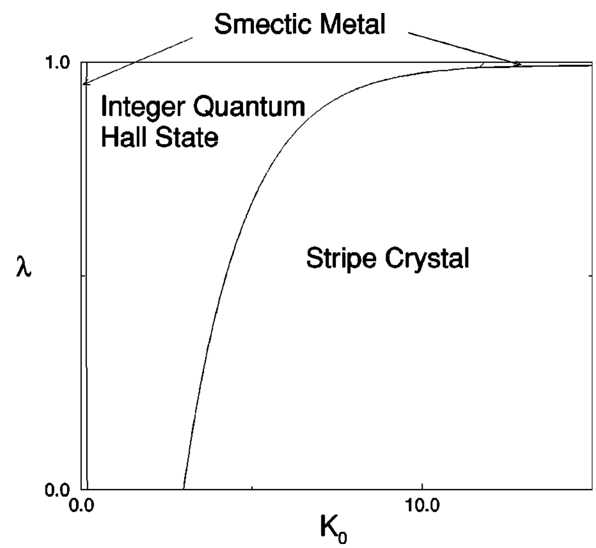


FIG. 4. Phase diagram for coupled spinful Luttinger liquids without a spin gap, in the presence of a magnetic field, in the commensurate case $2k_F = nq_B$.

$$\Delta'_{e,1} = \frac{K_0}{4}\left(1 - \frac{\lambda}{2}\right) + \frac{1}{2K_0(1+\lambda+\sqrt{1-\lambda^2})} + \frac{1}{2}. \tag{27}$$

In regions of parameter space where this is the most relevant operator, we expect that the system develops a gap that is largely single particle in character. The identification of the resulting state is easy once we recognize that the condition $2k_F = nq_B$ is precisely that the Landau level filling of the system is $\nu = 2n$, i.e., the electrons (inclusive of their spin degeneracy) occupy $n$ Landau bands and form an integer quantum Hall state.

In Fig. 4 we show the phase diagram for the case $n = 1$ ($\nu = 2$). As the transition between the quantum Hall state and the smectic metal happens via the hopping going irrelevant, it is a continuous transition. To our knowledge, this is the first instance of a continuous transition between a quantum Hall state and a metallic state. We should note that the persistence of the quantum Hall phase up to the upper boundary at $\lambda = 1$ is non-generic; it arises in the particular model studied upon a cancellation between numerator and denominator that will not typically take place.

Finally, higher-order commensurations between $k_F$ and $q_B$ are possible when lattice effects are strong on the chains and the electron operator has pieces oscillating at higher multiples of $k_F$. We have not investigated these.

### C. Spinless smectic metal

In this case we only have charged fields as in the spin gapped case, but single-electron processes need to be considered as in the spin ungapped case. The analysis of perturbing operators is very similar to the spin ungapped case, which leads to the phase diagram in Fig. 5, when $k_F$ and $q_B$ are incommensurate. Integer quantum Hall cases are, of course, allowed here as well, when $2k_F = nq_B$.

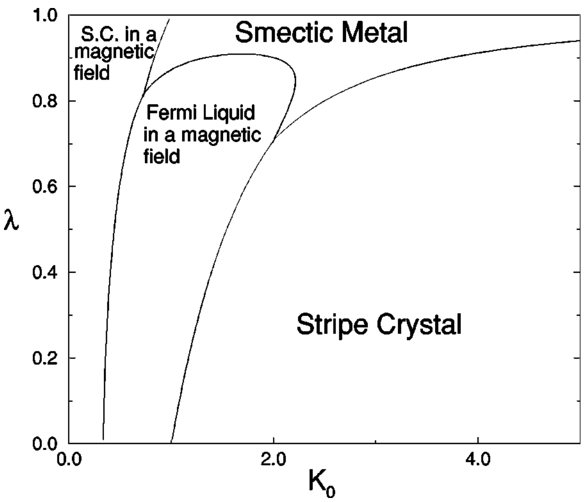


FIG. 5. Phase diagram for coupled spinless Luttinger liquids, in the presence of a magnetic field, assuming there is no commensuration between $k_F$ and $q_B$.

### D. Disorder

Following Giamarchi and Schulz,[13] one can also analyze the scaling of weak single-particle randomness. We have not done this systematically, but will content ourselves with a couple of remarks. First, in all cases it is possible to find subsets of the smectic metal where both intrachain random backscattering and interchain random hopping are *irrelevant*—hence the system is a perfect, albeit completely anisotropic, metal in the long-wavelength limit. Second, it is possible to find sections of the phase boundary between the quantum Hall states and the smectic metal where disorder is still irrelevant, e.g., in the spin ungapped problem this happens both near $\lambda=0$ and near $\lambda=1$. In these cases, we find an analytically tractable fixed point governing a transition out of a quantum Hall state in the presence of interactions *and* disorder that warrants further analysis.[14]

## IV. SUMMARY

Achieving a ''dimensional continuation'' of strong correlation physics from low dimensions by weakly coupling an infinite set of systems is an appealing strategy in the study of higher dimensional systems.[15] The application of a magnetic field has been conjectured previously to be useful in this task. In addition to the work of Efetov, we should also mention the suggestion of Strong, Clarke, and Anderson[16] that a two-dimensional non-Fermi liquid phase could be induced in this fashion in a layered system. Striking experiments in the organic superconductors that are evidence for this point of view have been discussed at some length.[17]

In this paper, we have shown that this decoupling effect of the magnetic field can be given a precise meaning in the context of two-dimensional sliding phases, via its reduction of the dimension of the most relevant charged operators that perturb them. This significantly expands the size of the sliding phases. As a bonus we find, in the quantum version of the problem, quantum Hall phases at commensurate fields that undergo a continuous transition to a smectic metal.

In the underdoped region of the cuprates, it has been argued that the stripe instability leads to a smectic metal state and that it may already have been observed.[6] In this setting, the spin gapped phase discussed here is the one at issue, whence we anticipate that the Zeeman coupling (ignored in our analysis) will not be important. We suggest that the field sensitivity of the phase diagram in this region would be an interesting test of the smectic hypothesis—essentially, one should look for the expansion of the metal or the onset of a CDW. The parameters needed to see this effect should ensure that the interchain hopping is weaker than the field, $t_e < v_F q_B$ ($v_F$ is the on-chain Fermi velocity) and that the temperature does not wash out the phases induced by the field. The latter condition can be translated, via the on-chain smectic correlation length $\xi \sim \epsilon_F / nT$ ($\epsilon_F$ is the on-chain Fermi energy and $n$ is the linear density of electrons), to the statement $Ba\xi \sim \phi_o$ where $a$ is the interchain spacing and $\phi_o$ is the flux quantum.

## ACKNOWLEDGMENTS

We are grateful to Ashvin Vishwanath, Tom Lubensky, and Steve Kivelson for useful discussions. We are especially grateful to David Huse for many illuminating discussions and also for producing a lucid, intuitive explanation of the content of Ref. 2. This work was supported by NSF DMR-9971541 and the Sloan Foundation (K.Y.), and NSF DMR-9978074 and the Sloan and Packard Foundations (S.L.S.).

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