Nonlinear magnetostatic surface waves of magnetic multilayers: Effective medium theory

Xuan-Zhang Wang
*Institute for Condensed Matters and Department of Physics, Harbin Normal University, Harbin 150080, People's Republic of China;**
Laboratory of Excited State Process, Chinese Academy of Science, Changchun 130021, China;
and Changchun Institute of Optics Fine Mechanics and Physics, Chinese Academy of Science, Changchun 130021, China

Shu-Rong Xu
Laboratory of Excited State Process, Chinese Academy of Science, Changchun 130021, China
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We present a nonlinear effective medium method to investigate properties of nonlinear surface magnetostatic waves of magnetic superlattice films, or magnetic multilayers. In the third-order approximation, the nonlinear expressions of alternating effective magnetization in the systems are obtained. On this basis, the nonlinear coefficients, wave number, and frequency shifts of surface magnetostatic waves are calculated numerically. A very interesting result is that magnetostatic surface envelope solitons are possible to exist in a certain frequency range since the Lighthill criterion is satisfied in this range. In the meanwhile, we find that there is a basic mistake in the previous theory related to the nonlinear surface magnetostatic waves of a single magnetic film.

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I. INTRODUCTION

In magnetic mediums, generally speaking, the effects of exchange interaction on the dispersion properties of spin waves can be ignored safely as the wavelength $\lambda > 10^{-7}$ m, then the dipolar interaction dominates the properties of spin waves and the magneto-optic properties of the mediums.[1] In this situation, two kinds of spin-wave modes are more interesting, known as the magnetostatic modes and retarded modes (or electromagnetic modes). According to the present conditions of experiments and technologies, one is interested in magnetostatic modes in ferromagnets, and interested in retarded modes of antiferromagnets. The Brillouin light scattering technique can be used to study the magnetostatic modes[2,3] and the attenuated total refection (ATR) experimental method is available to the studies of retarded modes of antiferromagnets.[4] Theoretically, the Maxwell's equations and boundary conditions can describe completely these modes and determine their properties.[1,5] In ferromagnets, the motion of magnetization obeys the Landau–Lifshitz equation

$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \vec{H}, \tag{1}$$

where the magnetization $\mathbf{M} = \mathbf{M_0} + \mathbf{m}(t)$, and magnetic field $\mathbf{H} = \mathbf{H_0} + \mathbf{h}(t)$. We assume that the alternating magnetization $\mathbf{m}(t)$ and magnetic field $\mathbf{h}(t)$ are small quantities, comparing with $\mathbf{M_0}$ (static magnetization) and $\mathbf{H_0}$ (external magnetic field). For the linear approximation, there is the relation $\mathbf{b} = \overleftrightarrow{\mu} \cdot \mathbf{h}$ between $\mathbf{h}$ and $\mathbf{b}$, where $\overleftrightarrow{\mu}$ is called as the magnetic permeability tensor, obtained by linearizing Eq. (1), which is

$$\overleftrightarrow{\mu} = 1 + \overleftrightarrow{\chi} = \begin{pmatrix} 1+\chi_1 & i\chi_2 & 0 \\ -i\chi_2 & 1+\chi_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

with

$$\chi_1 = \frac{4\pi\gamma^2 M_0 H_0}{(\gamma H_0)^2 - \omega^2} = \frac{\omega_m \omega_0}{\omega_0^2 - \omega^2} \tag{3}$$

and

$$\chi_2 = \frac{4\pi\gamma M_0 \omega}{(\gamma H_0)^2 - \omega^2} = \frac{\omega_m \omega}{\omega_0^2 - \omega^2}, \tag{4}$$

and then the linear alternating magnetization $\mathbf{m} = \overleftrightarrow{\chi}\mathbf{h}$. In these formulas, γ is the gyromagnetic ratio.

Nowadays, the magnetic superlattices or multilayers composed of magnetic layers and nonmagnetic spacers are paid close attention. Magnetic layers and nonmagnetic layers in these kind of systems are actually very thin, with the thickness generally smaller than 1.0×10^{-8} m. Therefore dipolar spin-wave's wavelength is much larger than this numerical order, so the change of wave fields is very small over a magnetic layer and a nonmagnetic layer (or a period). In this case, the effective medium theory often is used to describe the superlattices as the relevant effective mediums.[6–9] The geometry and coordinate axes applied in this paper are shown in Fig. 1, with the y axis normal to the surface and the static magnetization parallel to the z axis. Surface waves propagate along the x axis. Applying the permeability tensor (2), the boundary conditions between adjoining magnetic and nonmagnetic layers, in a certain approximation, the effective medium permeability tensor can be shown as[6,7]

$$\overleftrightarrow{\mu}^e = 1 + \overleftrightarrow{\chi}^e = \begin{pmatrix} \mu_{xx} & i\mu_{xy} & 0 \\ -i\mu_{yx} & \mu_{yy} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

and then the linear alternating magnetization $\mathbf{m} = \overleftrightarrow{\chi}\mathbf{h}$. In formula (5)

$$\mu_{xx} = 1 + f_1\chi_1 - \frac{f_1 f_2 \chi_2^2}{1 + f_2\chi_1} = 1 + \chi_{xx}^e, \tag{6}$$

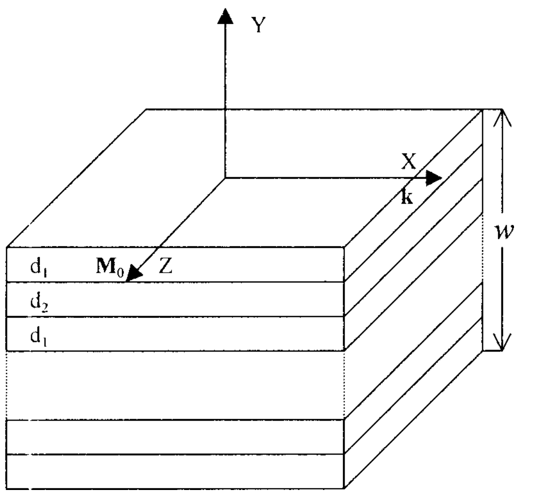

FIG. 1. Geometry and coordinate system applied in the paper $\mathbf{M}_0$ is the static magnetization parallel to the z axis and $\mathbf{k}$ the wave number along the x axis. The magnetic superlattice film is composed of magnetic layers with thickness d_1 and nonmagnetic layers with thickness d_2. The total thickness is w.

$$\mu_{yy}=1+\frac{f_1\chi_1}{1+f_2\chi_1}=1+\chi^e_{yy}, \tag{7}$$

$$\mu_{xy}=\mu_{yx}=\frac{f_1\chi_2}{1+f_2\chi_1}=\chi^e_{xy}. \tag{8}$$

Here f_1 and f_2 are used as the magnetic and nonmagnetic fractions in the multilayers, respectively. Therefore $f_1 = d_1/(d_1+d_2)$ and $f_2=d_2/(d_1+d_2)$, with d_1 representing the thickness of magnetic layers and d_2 the thickness of nonmagnetic layers. For a different geometry, formulas (6)–(8) are different.[8,9] In this theory, a magnetic superlattice film is changed into a homogeneous effective medium with obvious anisotropy, but it is linear. A lot of interesting results were found in the previous works based on this theory.

The investigation of nonlinear spin waves in ferromagnets or antiferromagnets is very interesting and practical since the nonlinearity, resulting from the Landau–Lifshitz equation, is intrinsic in magnets. The propagation properties of nonlinear spin waves in the exchange and dipole-exchange regions,[10–13] the properties of nonlinear magnetostatic waves and solitons (include the bistability and multistability that magnetostatic waves may exhibit),[14–26] are all interesting and valuable topics. In addition to the above references mentioned, there are still many references related to the nonlinear properties of exchange-interaction and dipole-interaction spin waves, which are not given here, but these works almost deal with the relevant properties of magnetic films too and we have not found any published work corresponding to the nonlinear magnetostatic waves of magnetic multilayers or superlattices. In magnetic superlattice films or multilayers, because of the presence of periodical interfaces, components layer thickness is selected arbitrarily to a certain degree, so the properties of these artificial systems may be designed. It is very possible to find some new features.

We see from the theoretical calculations for the nonlinear magnetostatic surface modes of magnetic films[23–25] that the theory is rather complicated. It is difficult to use this theory strictly to solve the nonlinear magnetostatic waves of multilayers containing a great number of magnetic and nonmagnetic layers. Therefore it is necessary to find an approximative method.

II. EFFECTIVE MEDIUM EXPRESSION OF NONLINEAR MAGNETIZATION

In the investigation of linear magnetostatic and retarded modes in magnetic superlattices or multilayers, with the effective medium method, one can easily obtain some analytical results.[6–9,28,29] In the case of nonlinearity, the question becomes very complicated. In this paper, we are going to establish an effective medium method available to calculations of dipolar spin waves and discussions of the relevant questions. Of course, such a theory has to be approximate. For the understanding of readers, we first present simply the theoretical process to get the nonlinear alternating magnetization in a single magnetic film, and then on this basis we derive the nonlinear alternating magnetization with an effective medium method. According to the perturbation method, we write the magnetic field and magnetization as

$$\vec{H}(\vec{R},t)=H_0\vec{e}_z+\vec{h}^{(1)}(\vec{R},t)+\vec{h}^{(2)}(\vec{R},t)+\vec{h}^{(3)}(\vec{R},t)+\cdots \tag{9a}$$

and

$$\vec{M}(\vec{R},t)=M_0\vec{e}_z+\vec{m}^{(1)}(\vec{R},t)+\vec{m}^{(2)}(\vec{R},t)+\vec{m}^{(3)}(\vec{R},t)+\cdots \tag{9b}$$

with $\mathbf{R}=x\mathbf{e_x}+y\mathbf{e_y}+z\mathbf{e_z}$, a position vector and the superscripts representing the various orders. Because the film is homogeneous and unbounded in the x-z plane, we can write the field and magnetization of different orders as

$$\vec{h}^{(1)}(\vec{R},t)=\vec{h}^{(1)}(\omega,y)e^{i(\vec{k}_\omega\cdot\vec{r}-\omega t)}+\text{c.c.}, \tag{10a}$$

$$\vec{h}^{(2)}(\vec{R},t)=\vec{h}^{(2)}(0,\vec{R})+\vec{h}^{(2)}(2\omega,y)e^{i(\vec{k}_{2\omega}\cdot\vec{r}-2\omega t)}+\text{c.c.}, \tag{10b}$$

$$\vec{h}^{(3)}(\vec{R},t)=\vec{h}^{(3)}(\omega,y)e^{i(\vec{k}_\omega\cdot\vec{r}-\omega t)}+\vec{h}^{(3)}(3\omega,y)e^{i(\vec{k}_{3\omega}\cdot\vec{r}-3\omega t)}+\text{c.c.}, \tag{10c}$$

$$\vec{m}^{(1)}(\vec{R},t)=\vec{m}^{(1)}(\omega,y)e^{i(\vec{k}_\omega\cdot\vec{r}-\omega t)}+\text{c.c.}, \tag{11a}$$

$$\vec{m}^{(2)}(\vec{R},t)=\vec{m}^{(2)}(0,\vec{R})+\vec{m}^{(2)}(2\omega,y)e^{i(\vec{k}_{2\omega}\cdot\vec{r}-2\omega t)}+\text{c.c.}, \tag{11b}$$

$$\vec{m}^{(3)}(\vec{R},t)=\vec{m}^{(3)}(\omega,y)e^{i(\vec{k}_\omega\cdot\vec{r}-\omega t)}+\vec{m}^{(3)}(3\omega,y)e^{i(\vec{k}_{3\omega}\cdot\vec{r}-3\omega t)}+\text{c.c.}, \tag{11c}$$

where $\mathbf{k}_\omega$, $\mathbf{k}_{2\omega}$, and $\mathbf{k}_{3\omega}$ are wave numbers associated with the wave frequencies ω, 2ω, and 3ω, respectively. $\mathbf{r}$ is the in-plane position vector. By substitution of formulas (9), (10), and (11) into (1), selecting terms with the same frequency components and the same order, one can obtain

$$m_j^{(2)}(0)+m_j^{(2)*}(0)=\frac{1}{H_0}\{M_0[h_j^{(2)}(0)+h_j^{(2)*}(0)]-m_j^{(1)}(\omega)h_z^{(1)*}(\omega)-m_j^{(1)*}(\omega)\times h_z^{(1)}(\omega)\}, \tag{12a}$$

$$m_z^{(2)}(0)+m_z^{(2)*}(0)=-\frac{1}{M_0}[|m_x^{(1)}(\omega)|^2+|m_y^{(1)}(\omega)|^2], \tag{12b}$$

$$m_j^{(2)}(2\omega)=\chi_1(2\omega)h_j^{(2)}(2\omega)\pm i\chi_2(2\omega)h_k^{(2)}(2\omega)-\frac{1}{M_0}[\chi_1(2\omega)m_j^{(1)}(\omega)\pm i\chi_2(2\omega)m_k^{(1)}(\omega)]\times h_z^{(1)}(\omega)e^{i(2\vec{k}_\omega-\vec{k}_{2\omega})\cdot\vec{r}}, \tag{12c}$$

$$m_z^{(2)}(2\omega)=-\frac{1}{2M_0}[m_x^{(1)^2}(\omega)+m_y^{(1)^2}(\omega)]e^{i(2\vec{k}_\omega-\vec{k}_{2\omega})\cdot\vec{r}}, \tag{12d}$$

where $j,k=x$, or y, but $j\neq k$. When $j=x$ and $k=y$, the positive signs should be selected in formula (12c), otherwise the negative signs should be taken.

The third-order components of nonlinear magnetization are represented by

$$\begin{aligned}m_j^{(3)}(\omega)=&\chi_1(\omega)h_j^{(3)}(\omega)\pm i\chi_2(\omega)h_k^{(3)}(\omega)-\frac{1}{M_0}\{[\chi_1(\omega)m_j^{(1)}(\omega)\pm i\chi_2(\omega)m_k^{(1)}(\omega)][h_z^{(2)}(0)+h_z^{(2)*}(0)]\\&+[\chi_1(\omega)(m_j^{(2)}(0)+m_j^{(2)*}(0))\pm i\chi_2(\omega)(m_k^{(2)}(0)+m_k^{(2)*}(0))]h_z^{(1)}(\omega)-[\chi_1(\omega)h_j^{(1)}(\omega)\pm i\chi_2(\omega)h_k^{(1)}(\omega)]\\&\times[m_z^{(2)}(0)+m_z^{(2)*}(0)]\}-\frac{e^{-i(2\vec{k}_\omega-\vec{k}_{2\omega})\cdot\vec{r}}}{M_0}\{-[\chi_1(\omega)h_j^{(1)*}(\omega)\pm i\chi_2(\omega)h_k^{(1)*}(\omega)]m_z^{(2)}(2\omega)\\&+[\chi_1(\omega)m_j^{(2)}(2\omega)\pm i\chi_2(\omega)m_k^{(2)}(2\omega)]h_z^{(1)*}(\omega)+[\chi_1(\omega)m_j^{(1)*}(\omega)\pm i\chi_2(\omega)m_k^{(1)*}(\omega)]h_z^{(2)}(2\omega)\}.\end{aligned} \tag{13}$$

The first-order magnetization is just of linearity and used repeatedly by previous works,[6–9,26–28] so it is not given here. The terms with frequency 3ω and $m_z^{(3)}(\omega)$ do not appear since they will not be used to solve the surface nonlinear magnetostatic waves at fundamental frequency ω, propagating along the x axis. These results (12) and (13) can be found in other references.[25,27,28]

In order to establish a nonlinear effective medium method, we propose three assumptions. First, the wavelength of magnetostatic or retarded modes $\lambda\gg d_1+d_2$ to guarantee that the change of the wave fields is very small over a superlattice period. Second, the static magnetization $\mathbf{M_0}$ is homogeneous and the same in all magnetic layers, but the alternating magnetization **m** is given by expressions (13), (12), and $\mathbf{m}^{(1)}=\overleftrightarrow{\chi}\cdot\mathbf{h}^{(1)}$. The third assumption is that the nonmagnetic layers are a linear medium, so the nonlinear effective magnetization results from the magnetic layers. According to these assumptions, we can introduce the following equations:

$$h_{1x}=h_{2x}=h_x, \tag{14a}$$

$$h_{1z}=h_{2z}=h_z, \tag{14b}$$

$$b_{1y}=b_{2y}=b_y, \tag{14c}$$

$$f_1h_{1y}+f_2h_{2y}=h_y, \tag{14d}$$

$$f_1b_{1x}+f_2b_{2x}=b_x, \tag{14e}$$

$$f_1b_{1z}+f_2b_{2z}=b_z, \tag{14f}$$

where subscripts 1 and 2 correspond to the adjoining magnetic layer and nonmagnetic layer in a superlattice period, respectively. The fields without subscript 1 or 2 are the relevant effective fields in the superlattice. These equations imply that a field continuous at the interfaces is equal in these two adjoining different layers, and considered as the corresponding effective field, say, Eqs. (14a)–(14c). The average value (over a superlattice period) of a field discontinuous at the interface is defined as the corresponding effective field, or Eqs. (14d)–(14f). The nonlinear parts of alternating magnetization **m** are equal to

$$\vec{m}^{(2)}=f_1\vec{m}_1^{(2)}, \tag{15a}$$

$$\vec{m}^{(3)}=f_1m_1^{(3)}. \tag{15b}$$

Our aim is to describe the effective nonlinear magnetization in terms of the effective magnetic field **h**, and for that, applying Eqs. (14a), (14c), and (7), we obtain

$$h_{1y}^{(1)}=\frac{1}{1+f_2\chi_1}[h_y^{(1)}+if_2\chi_2h_x^{(1)}]=\gamma h_y^{(1)}+i\beta h_x^{(1)}, \tag{16a}$$

which also defines γ and β. Equations (14c), (15a), and (15b) lead to

$$h_{1y}^{(i)}=h_y^{(i)}-\frac{f_2}{f_1}m_y^{(i)}, \tag{16b}$$

where $i=2$ or 3. The effective alternating magnetization is equal formally to

$$\vec{m}^{\rm NL}=\vec{m}^{(1)}+\vec{m}^{(2)}+\vec{m}^{(3)} \tag{17}$$

with NL meaning the nonlinearity. $\mathbf{m}^{(2)}=\mathbf{m}^{(2)}(0)+\mathbf{m}^{(2)}(2\omega)$, then we obtain the terms of the second-order magnetization as

$$m_x^{(2)}(0)+m_x^{(2)*}(0)=\frac{f_1}{H_0}\{M_0h_x^{(2)}(0)-[(\chi_1\text{-}\chi_2\beta)h_x^{(1)}+i\chi_2\gamma h_y^{(1)}]h_z^{(1)*}\}+\text{c.c.}, \tag{18a}$$

$$m_x^{(2)}(2\omega)=f_1[\chi_1(2\omega)h_x^{(2)}(2\omega)+i\chi_2(2\omega)h_y^{(2)}(2\omega)]-\frac{f_1}{M_0}\times\{[\chi_1(2\omega)(\chi_1-\beta\chi_2)+\chi_2(2\omega)(\chi_2-\chi_1\beta)]\times h_x^{(1)}+i\gamma[\chi_2\chi_1(2\omega)+\chi_1\chi_2(2\omega)]h_y^{(1)}\}\times h_z^{(1)}F(\vec{r})-if_2\chi_2(2\omega)m_y^{(2)}(2\omega), \tag{18b}$$

$$m_y^{(2)}(0)+m_y^{(2)*}(0)=\frac{f_1}{H_0+f_2M_0}\{M_0h_y^{(2)}(0)-[i(\beta\chi_1-\chi_2)\times h_x^{(1)}+\gamma\chi_1h_y^{(1)}]h_z^{(1)*}\}+\text{c.c.}, \tag{19a}$$

$$m_y^{(2)}(2\omega)=\frac{f_1}{1+f_2\chi_1(2\omega)}\left\{[\chi_1(2\omega)h_y^{(2)}(2\omega)-i\chi_2(2\omega)h_x^{(2)}(2\omega)]-\frac{1}{M_0}(i[(\beta\chi_1-\chi_2)\times\chi_1(2\omega)+(\chi_2\beta-\chi_1)\chi_2(2\omega)]h_x^{(1)}+\gamma[\chi_1\chi_1(2\omega)+\chi_2\chi_2(2\omega)]h_y^{(1)})h_z^{(1)}F(\vec{r})\right\}, \tag{19b}$$

$$m_z^{(2)}(0)+m_z^{(2)*}(0)=-\frac{f_1}{M_0}\{[(\chi_1^2+\chi_2^2)(1+\beta^2)-4\chi_1\chi_2\beta]\times|h_x^{(1)}|^2+\gamma^2(\chi_1^2+\chi_2^2)|h_y^{(1)}|^2+i\gamma[\beta(\chi_1^2-\chi_2^2)-2\chi_1\chi_2]\times(h_x^{(1)}h_y^{(1)*}-h_x^{(1)*}h_y^{(1)})\}, \tag{20a}$$

$$m_z^{(2)}(2\omega)=-\frac{f_1(\chi_1^2-\chi_2^2)}{2M_0}[(1-\beta^2)h_x^{(1)}h_x^{(1)}+\gamma^2h_y^{(1)}h_y^{(1)}+2i\gamma\beta h_x^{(1)}h_y^{(1)}]F(\vec{r}), \tag{20b}$$

and $F(\vec{r})=\exp[i(2\vec{k}_\omega-\vec{k}_{2\omega})\cdot\vec{r}]$. The fields with argument 0 do not change with time, but the fields with ω change with time, according to factor $\exp(-i\omega t)$. We also can imagine the fields χ_1 and χ_2 with argument 2ω. For notational simplicity, the argument ω is not pointed out in the field functions. The results of the second-order approximation, Eqs. (18)–(20), tell us that dc magnetization $\mathbf{m}^{(2)}(0)$ and frequency-doubling magnetization $\mathbf{m}^{(2)}(2\omega)$ can excited not only by a dc field $\mathbf{h}^{(2)}(0)$ and 2ω-field $\mathbf{h}^{(2)}(2\omega)$, respectively, but also by the interaction of $\mathbf{h}^{(1)}(\omega)$ and $\mathbf{m}^{(1)}(\omega)$, at the fundamental frequency ω.

The third-order terms satisfy

$$[1+f_2\chi_1]m_y^{(3)}=f_1[\chi_1h_y^{(3)}-i\chi_2h_x^{(3)}]-\frac{f_1}{M_0}\{i[\beta(\chi_1^2+\chi_2^2)-2\chi_1\chi_2]h_x^{(1)}+\gamma(\chi_1^2+\chi_2^2)h_y^{(1)}\}[h_z^{(2)}(0)+h_z^{(2)*}(0)]-\frac{1}{M_0}\{\chi_1[m_y^{(2)}(0)+m_y^{(2)*}(0)]-i\chi_2[m_x^{(2)}(0)+m_x^{(2)*}(0)]\}h_z^{(1)}+\frac{1}{M_0}[\chi_1\gamma h_y^{(1)}+i(\beta\chi_1-\chi_2)h_x^{(1)}]\times[m_z^{(2)}(0)+m_z^{(2)*}(0)]+\frac{1}{M_0}\{[\chi_1\gamma h_y^{(1)*}-i(\beta\chi_1+\chi_2)h_x^{(1)*}]m_z^{(2)}(2\omega)-[\chi_1m_y^{(2)}(2\omega)-i\chi_2m_x^{(2)}(2\omega)]h_z^{(1)*}-f_1(\chi_1^2-\chi_2^2)(-i\beta h_x^{(1)*}+\gamma h_y^{(1)*})h_z^{(2)}(2\omega)\}F(-\vec{r}), \tag{21a}$$

$$m_x^{(3)}=f_1[\chi_1h_x^{(3)}+i\chi_2h_y^{(3)}]-\frac{f_1}{M_0}[(\chi_1^2+\chi_2^2-2\beta\chi_1\chi_2)h_x^{(1)}+2i\chi_1\chi_2\gamma h_y^{(1)}][h_z^{(2)}(0)+h_z^{(2)*}(0)]-\frac{1}{M_0}\{\chi_1[m_x^{(2)}(0)+m_x^{(2)*}(0)]+i\chi_2[m_y^{(2)}(0)+m_y^{(2)*}(0)]\}h_z^{(1)}+\frac{1}{M_0}[(\chi_1-\chi_2\beta)h_x^{(1)}+i\chi_2\gamma h_y^{(1)}][m_z^{(2)}(0)+m_z^{(2)*}(0)]+\frac{1}{M_0}\{[(\chi_1+\beta\chi_2)h_x^{(1)*}+i\chi_2\gamma h_y^{(1)*}]m_z^{(2)}(2\omega)-[\chi_1m_x^{(2)}(2\omega)+i\chi_2m_y^{(2)}(2\omega)]h_z^{(1)*}-f_1(\chi_1^2-\chi_2^2)h_x^{(1)*}h_z^{(2)}(2\omega)\}F(-\vec{r})-if_2\chi_2m_y^{(3)}. \tag{21b}$$

Equations (21) reveal that in the third-order approximation, the magnetization $\mathbf{m}^{(3)}(\omega)$ at the foundational frequency can be produced not only by a field $\mathbf{h}^{(3)}(\omega)$ that is of the same order, but also by the interaction between the fields and magnetizations with different frequencies and different orders.

These results can be used for one to investigate both nonlinear magnetostatic modes and retarded modes of magnetic superlattices or multilayers. In addition, these formulas imply that our nonlinear effective medium system is obviously anisotropic. For $f_2=0$, this superlattice film becomes just a single magnetic film, and the different order expressions of the alternating magnetization return to those given by Eqs. (12) and (13).

III. PROPAGATION OF NONLINEAR MAGNETOSTATIC SURFACE WAVES

In this section, we shall discuss properties of nonlinear magnetostatic surface waves in magnetic superlattices with the effective medium method. As a starting work, here we only study the surface wave propagating along the x axis in the Voigt geometry given by Fig. 1. Thus a linear magnetostatic potential φ is introduced as

$$\varphi=\psi(e^{py}+\alpha e^{-py})e^{ikx}\quad(0>y>-w)\tag{22a}$$

in the superlattice and

$$\varphi=\psi_1 e^{-ky}e^{ikx}\quad(y>0),\tag{22b}$$

$$\varphi=\psi_2 e^{ky}e^{ikx}\quad(y<-w)\tag{22c}$$

above and below it, where ψ, ψ_1, and ψ_2 are the amplitude of the magnetostatic potential in different spaces. p and k are positive for the surface mode. As Ref. 25 did, we also prove easily that $\mathbf{h}^{(2)}(2\omega)$ and $\mathbf{h}^{(2)}(0)$ vanish without external driving fields with frequency 2ω and 0, respectively, for the surface waves. $m_x^{(2)}(2\omega)$, $m_y^{(2)}(2\omega)$, $m_x^{(2)}(0)$, and $m_y^{(2)}(0)$ are equal to zero in the Voigt geometry since $\partial/\partial z=0$ ($\mathbf{h}=\nabla\varphi$) and $\mathbf{h}^{(2)}=0$. Thus the nonzero components of the second-order magnetizations are reduced to

$$\begin{aligned}\vec{m}_z^{(2)}(0)+m_z^{(2)*}(0)&=-\frac{f_1}{M_0}\Bigg\{[(\chi_1^2+\chi_2^2)(1+\beta^2)-4\chi_1\chi_2\beta]\left|\frac{\partial\varphi}{\partial x}\right|^2\\&\quad+\gamma^2(\chi_1^2+\chi_2^2)\left|\frac{\partial\varphi}{\partial y}\right|^2+i\gamma[\beta(\chi_1^2+\chi_2^2)-2\chi_1\chi_2]\\&\quad\times\left(\frac{\partial\varphi}{\partial x}\frac{\partial\varphi^*}{\partial y}-\frac{\partial\varphi^*}{\partial x}\frac{\partial\varphi}{\partial y}\right)\Bigg\},\end{aligned}\tag{23a}$$

$$\begin{aligned}m_z^{(2)}(2\omega)=-\frac{f_1(\chi_1^2-\chi_2^2)}{2M_0}\Bigg[&(1-\beta^2)\left(\frac{\partial\varphi}{\partial x}\right)^2+\gamma^2\left(\frac{\partial\varphi}{\partial y}\right)^2\\&+2i\gamma\beta\frac{\partial\varphi}{\partial x}\frac{\partial\varphi}{\partial y}\Bigg]F(\vec{r}).\end{aligned}\tag{23b}$$

The other components of the second-order magnetization are equal to zero. We find that the z component of the second-order magnetization, both the dc and double-frequency components, can be excited by the first-order field, linear field. Useful components of the third-order magnetization are

$$\begin{aligned}m_y^{(3)}=&\frac{f_1}{1+f_2\chi_1}\left[\chi_1\frac{\partial\varphi^{(3)}}{\partial y}-i\chi_2\frac{\partial\varphi^{(3)}}{\partial x}\right]+\frac{1}{(1+f_2\chi_1)M_0}\\&\times\left[\chi_1\gamma\frac{\partial\varphi}{\partial y}+i(\beta\chi_1-\chi_2)\frac{\partial\varphi}{\partial x}\right][m_z^{(2)}(0)+m_z^{(2)*}(0)]\\&+\frac{1}{(1+f_2\chi_1)M_0}\left[\chi_1\gamma\frac{\partial\varphi^*}{\partial y}-i(\beta\chi_1+\chi_2)\frac{\partial\varphi^*}{\partial x}\right]\\&\times m_z^{(2)}(2\omega)F(-\vec{r}),\end{aligned}\tag{24a}$$

$$\begin{aligned}m_x^{(3)}=&f_1\left[\chi_1\frac{\partial\varphi^{(3)}}{\partial x}+i\chi_2\frac{\partial\varphi^{(3)}}{\partial y}\right]+\frac{1}{M_0}\Bigg[(\chi_1-\chi_2\beta)\frac{\partial\varphi}{\partial x}\\&+i\chi_2\gamma\frac{\partial\varphi}{\partial y}\Bigg][m_z^{(2)}(0)+m_z^{(2)*}(0)]\\&+\frac{1}{M_0}\left\{\left[(\chi_1+\beta\chi_2)\frac{\partial\varphi^*}{\partial x}+i\chi_2\gamma\frac{\partial\varphi^*}{\partial y}\right]m_z^{(2)}(2\omega)\right\}\\&\times F(-\vec{r})-if_2\chi_2 m_y^{(3)}.\end{aligned}\tag{24b}$$

For substitution of solutions (23) into equations (24), the useful components of nonlinear magnetization can be shown as

$$\begin{aligned}m_y^{\rm NL}=&\chi_{yy}^e h_y^{\rm NL}-i\chi_{xy}^e h_x^{\rm NL}-\left(A\left|\frac{\partial\varphi}{\partial x}\right|^2+B\left|\frac{\partial\varphi}{\partial y}\right|^2\right)\frac{\partial\varphi}{\partial y}\\&-i\left(C\left|\frac{\partial\varphi}{\partial x}\right|^2+D\left|\frac{\partial\varphi}{\partial y}\right|^2\right)\frac{\partial\varphi}{\partial x},\end{aligned}\tag{25a}$$

$$\begin{aligned}m_x^{(\rm NL)}=&\chi_{xx}^e h_x^{\rm NL}+i\chi_{xy}^e h_y^{\rm NL}+if_2\chi_2\Bigg[\left(A\left|\frac{\partial\varphi}{\partial x}\right|^2+B\left|\frac{\partial\varphi}{\partial y}\right|^2\right)\frac{\partial\varphi}{\partial y}\\&+i\left(C\left|\frac{\partial\varphi}{\partial x}\right|^2+D\left|\frac{\partial\varphi}{\partial y}\right|^2\right)\frac{\partial\varphi}{\partial x}\Bigg]-i\Bigg(a\left|\frac{\partial\varphi}{\partial x}\right|^2\\&+b\left|\frac{\partial\varphi}{\partial y}\right|^2\Bigg)\frac{\partial\varphi}{\partial y}-\left(c\left|\frac{\partial\varphi}{\partial x}\right|^2+d\left|\frac{\partial\varphi}{\partial y}\right|^2\right)\frac{\partial\varphi}{\partial x}\end{aligned}\tag{25b}$$

with $\mathbf{h}^{\rm NL}=\nabla(\varphi+\varphi^{(3)})=\nabla\varphi^{\rm NL}$ and the coefficients given by

$$\begin{aligned}A=&\frac{f_1\gamma}{2M_0^2(1+f_2\chi_1)}[2(\chi_1^2+\chi_2^2)(\chi_1-2\beta\chi_2+3\beta^2\chi_1)\\&-(\chi_1^2-\chi_2^2)(\chi_1-2\beta\chi_2-3\beta^2\chi_1)\\&-8\chi_1\chi_2(2\beta\chi_1-\chi_2)],\end{aligned}\tag{26a}$$

$$B=\frac{f_1\gamma^3}{2M_0^2(1+f_2\chi_1)}\chi_1(3\chi_1^2+\chi_2^2),\tag{26b}$$

$$C=\frac{f_1}{2M_0^2(1+f_2\chi_1)}\{2[(\chi_1^2+\chi_2^2)(1+\beta^2)-4\chi_1\chi_2\beta]\times(\beta\chi_1-\chi_2)-(\chi_1^2-\chi_2^2)(1-\beta^2)(\beta\chi_1+\chi_2)\},\tag{26c}$$

$$D=\frac{f_1\gamma^2}{2M_0^2(1+f_2\chi_1)}[2(\chi_1^2+\chi_2^2)(3\beta\chi_1-\chi_2)+(\chi_1^2-\chi_2^2)\times(3\beta\chi_1+\chi_2)-8\chi_1^2\chi_2],\tag{26d}$$

$$a=\frac{f_1\gamma}{2M_0^2}[2(\chi_1^2+\chi_2^2)(\chi_2-2\beta\chi_1+3\beta^2\chi_2)-(\chi_1^2-\chi_2^2)\times(\chi_2-2\beta\chi_1-3\beta^2\chi_2)-8\chi_1\chi_2(2\beta\chi_2-\chi_1)],\tag{27a}$$

$$b=\frac{f_1\gamma^3}{2M_0^2}\chi_2(\chi_2^2+3\chi_1^2),\tag{27b}$$

$$c=\frac{f_1}{2M_0^2}\{2[(\chi_1^2+\chi_2^2)(1+\beta^2)-4\chi_1\chi_2\beta](\chi_1-\beta\chi_2)+(\chi_1^2-\chi_2^2)(1-\beta^2)(\beta\chi_2+\chi_1)\},\tag{27c}$$

$$d=\frac{f_1\gamma^2}{2M_0^2}[2(\chi_1^2+\chi_2^2)(\chi_1-3\beta\chi_2)-(\chi_1^2-\chi_2^2)(3\beta\chi_2+\chi_1)+8\chi_1\chi_2^2].\tag{27d}$$

As we know $\mathbf{b}=\nabla\varphi+\mathbf{m}$ and $\nabla\cdot\mathbf{b}=0$ for magnetostatic waves, so

$$\nabla^2\varphi^{\mathrm{NL}}=-\nabla\cdot\vec{m}^{\mathrm{NL}}.\tag{28}$$

The nonlinear effects on φ^{NL} are included in the right side of Eq. (28). One will see that the nonlinear magnetostatic potential can be obtained from the relevant linear potential. Therefore, applying solution (22a) and substitution of Eqs. (25a) and (25b) into (28), we can get an equation satisfied by the nonlinear magnetostatic potential, i.e.,

$$\frac{\partial^2}{\partial y^2}\varphi^{\mathrm{NL}}-p^2\varphi^{\mathrm{NL}}=\frac{1+f_2\chi_1}{1+\chi_1}N_{\mathrm{NL}},\tag{29}$$

where

$$p^2=\frac{1+\chi_1+f_1f_2(\chi_1^2-\chi_2^2)}{1+\chi_1}k^2=\lambda^2k^2\tag{30}$$

and N_{NL} is equal to

$$N_{\mathrm{NL}}=A_0e^{py}+B_0e^{-py}+C_0e^{3py}+D_0e^{-3py}.\tag{31}$$

The coefficients in this expression are

$$A_0=\alpha|\psi|^2k^4\{\lambda[A(\lambda+f_2\chi_2)-a]-3\lambda^3[B(\lambda+f_2\chi_2)-b]-3[C(\lambda+f_2\chi_2)+c]+\lambda^2[D(\lambda+f_2\chi_2)+d]\},\tag{32a}$$

$$B_0=\alpha^2|\psi|^2k^4\{\lambda[A(\lambda-f_2\chi_2)+a]-3\lambda^3[B(\lambda-f_2\chi_2)+b]+3[C(\lambda-f_2\chi_2)-c]-\lambda^2[D(\lambda-f_2\chi_2)-d]\},\tag{32b}$$

$$C_0=|\psi|^2k^4\{\lambda[A(3\lambda+f_2\chi_2)-a]+\lambda^3[B(3\lambda+f_2\chi_2)-b]-[C(3\lambda+f_2\chi_2)+c]-\lambda^2[D(3\lambda+f_2\chi_2)+d]\},\tag{32c}$$

$$D_0=\alpha^3|\psi|^2k^4\{\lambda[A(3\lambda-f_2\chi_2)+a]+\lambda^3[B(3\lambda-f_2\chi_2)+b]+[C(3\lambda-f_2\chi_2)-c]+\lambda^2[D(3\lambda-f_2\chi_2)-d]\}.\tag{32d}$$

Thus the solution of Eq. (29) is represented by

$$\varphi^{\mathrm{NL}}=\psi e^{ikx}\{(1+ykL_1)e^{py}+\alpha(1+ykL_2)e^{-py}+L_3e^{-3py}+L_4e^{3py}\}\tag{33}$$

with L_{1-4}, the nonlinear coefficients of the magnetostatic surface mode. We see easily that

$$L_1=\frac{1+f_2\chi_1}{2pk(1+\chi_1)}A_0,\tag{34a}$$

$$L_2=-\frac{1+f_2\chi_1}{2p\alpha k(1+\chi_1)}B_0,\tag{34b}$$

$$L_3=\frac{1+f_2\chi_1}{8p^2(1+\chi_1)}D_0,\tag{34c}$$

$$L_4=\frac{1+f_2\chi_1}{8p^2(1+\chi_1)}C_0,\tag{34d}$$

with α presented by the linear solution and boundary conditions as follows:

$$\alpha=\frac{(1+\chi_{yy}^e)\lambda+\chi_{xy}^e+1}{(1+\chi_{yy}^e)\lambda-\chi_{xy}^e-1}.\tag{35}$$

As $f_1=1$, the system becomes the relevant ferromagnetic film, then the nonlinear coefficients of the nonlinear magnetostatic surface mode are reduced to

$$L_1=\frac{4|\psi|^2k^2\alpha}{(1+\chi_1)M_0^2}\chi_1(\chi_2^2-\chi_1^2),\tag{36a}$$

$$L_2=-\frac{4|\psi|^2k^2\alpha}{(1+\chi_1)M_0^2}\chi_1(\chi_2^2-\chi_1^2),\tag{36b}$$

$$L_3=\frac{|\psi|^2k^2\alpha^3}{2(1+\chi_1)M_0^2}(\chi_1-\chi_2)^3,\tag{36c}$$

and

$$L_4=\frac{|\psi|^2k^2}{2(1+\chi_1)M_0^2}(\chi_1+\chi_2)^3.\tag{36d}$$

Expressions (36a) and (36b) tell us $L_1=-L_2$. Unfortunately, these results are completely different from those given previously.[23,27] After carefully checking our derivations and the results in these references, we find that the problem may come from that during expressing the second-order magnetization $m_z^{(2)}(0)$ with the first-order alternating field $\mathbf{h}^{(1)}$, the term $-2i\chi_1\chi_2(h_yh_x^*-h_xh_y^*)$ is lost, which produces the differences between our results and those in these references. These nonlinear coefficients form a basis on which the properties of nonlinear magnetostatic surface waves are investigated, so these differences may influence the relevant previous results, at least quantitatively. Expression (33) is the magnetostatic potential of the nonlinear surface mode, which will be applied when the nonlinear dispersion relations of the magnetostatic surface mode are derived and discussed.

Now we apply expressions (22b), (22c), and (33), as well the boundary conditions, φ and $b_y=h_y+m_y$ continuous at the surfaces ($y=0$ and $y=-w$), to solve the nonlinear dispersion relations of the surface mode. For simplicity, we rewrite expression (33) as

$$\varphi^{\mathrm{NL}}=\psi e^{ikx}[e^{py}+\alpha' e^{-py}+\eta(y)] \tag{37}$$

and m_y^{NL} is shown as

$$m_y^{\mathrm{NL}}=m_y^{(1)}+m_y^{(3)}=-i\chi_{xy}^e h_x^{\mathrm{NL}}+\chi_{yy}^e h_y^{\mathrm{NL}}+\psi e^{ikx}\theta(y), \tag{38}$$

where $\mathbf{h}^{\mathrm{NL}}=\nabla\varphi^{\mathrm{NL}}$. $\theta(y)$ and $\eta(y)$ are of the second order, determined with expressions (33) and (25a), respectively, and presented by

$$\eta(y)=kye^{py}L_1+ky\alpha e^{-py}L_2+e^{-3py}L_3+e^{3py}L_4, \tag{39}$$

$$\begin{aligned}\theta(y)=&-|\psi|^2k^3[\alpha(\lambda A-3\lambda^3B-3C+\lambda^2D)e^{py}\\&+\alpha^2(-\lambda A+3\lambda^3B-3C+\lambda^2D)e^{-py}\\&-\alpha^3(\lambda A+\lambda^3B+C+\lambda^2D)e^{-3py}\\&+(\lambda A+\lambda^3B-C-\lambda^2D)e^{3py}].\end{aligned} \tag{40}$$

With these simple notations, the boundary conditions at $y=0$ lead to the following equations:

$$\psi_1=\psi[1+\alpha'+\eta(0)], \tag{41a}$$

$$-\psi_1=\psi\left\{\lambda\mu_{yy}^e(1-\alpha)+\mu_{xy}^e(1+\alpha)+\left[\mu_{yy}^e\frac{\partial\eta(y)}{k\partial y}\bigg|_{y=0}+\mu_{xy}^e\eta(0)+\frac{\theta(0)}{k}\right]\right\}, \tag{41b}$$

however, the conditions at $y=-w$ offer

$$\psi_2e^{-kw}=\psi[e^{-pw}+\alpha' e^{pw}+\eta(-w)], \tag{42a}$$

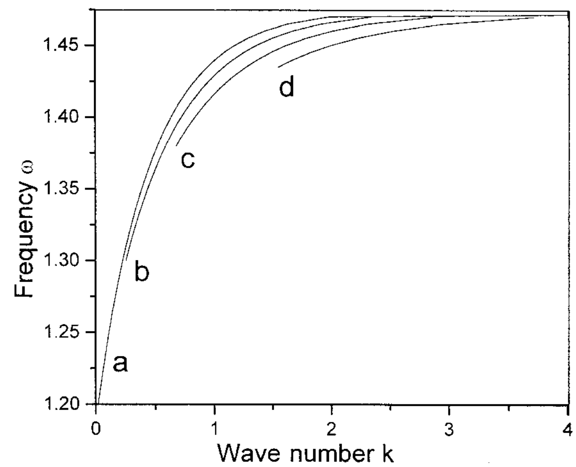

FIG. 2. Dispersion curves of the linear magnetostatic surface mode. Curves a to d correspond to $f_1=1.0$, 0.9, 0.8, and 0.7, respectively.

$$\psi_2e^{-kw}=\psi\left\{\lambda\mu_{yy}^e(e^{-pw}-\alpha' e^{pw})+\mu_{xy}^e(e^{-pw}+\alpha' e^{pw})+\left[\mu_{yy}^e\frac{\partial\eta(y)}{k\partial y}\bigg|_{y=-w}+\mu_{xy}^e\eta(-w)+\frac{\theta(-w)}{k}\right]\right\}. \tag{42b}$$

The elimination of α', ψ, ψ_1, and ψ_2 in these equations leads to the dispersion relation of the nonlinear magnetostatic surface mode,

$$(1+\lambda^2\mu_{yy}^{e^2}-\mu_{xy}^{e^2})\sinh(\lambda kw)+2\lambda\mu_{yy}^e\cosh(\lambda kw)=\frac{F}{2} \tag{43}$$

with the condition that λ is real and $\lambda>0$. This is the most important theoretical result in this paper, which determines dispersion properties of the nonlinear magnetostatic surface mode. In Eq. (43), the expression of F is given by

$$\begin{aligned}F=&(\lambda\mu_{yy}^e-\mu_{xy}^e-1)\left[(\mu_{xy}^e-1)\eta(-w)+\mu_{yy}^e\frac{\partial\eta(y)}{k\partial y}\bigg|_{y=-w}+\frac{\theta(-w)}{k}\right]-e^{pw}(\lambda\mu_{yy}^e-\mu_{xy}^e+1)\\&\times\left[(\mu_{xy}^e+1)\eta(0)+\mu_{yy}^e\frac{\partial\eta(y)}{k\partial y}\bigg|_{y=0}+\frac{\theta(0)}{k}\right]\end{aligned} \tag{44}$$

which results from the contribution of the nonlinearity and is directly proportional to $|\psi|^2$. The frequency and wave number included in Eq. (43) should be considered as nonlinear quantities. When the linear approximation is taken, $F=0$, then Eq. (43) is just the dispersion relation of the relevant linear surface mode.[8] Therefore, so long as $|\psi|^2$ is given, we can calculate numerically the coefficients, the frequency, and wave-number shifts caused by the nonlinearity.

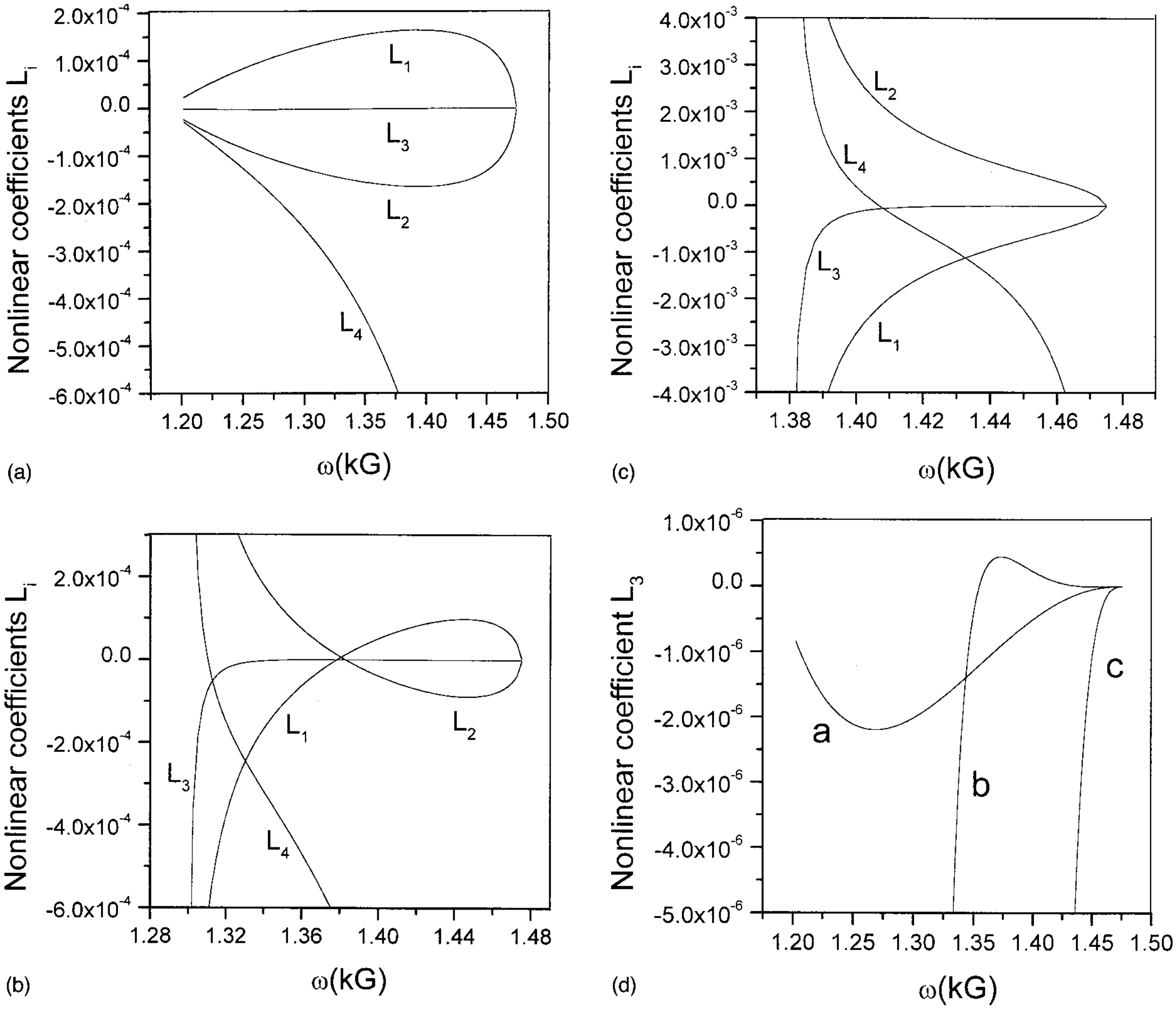

FIG. 3. Nonlinear coefficients versus frequency. (a) to (c) are related to $f_1 = 1.0$, 0.9, and 0.8. In order to examine the intensity of L_3, (d) is presented, where curves *a* to *c* correspond to $f_1 = 1.0$, 0.9, and 0.8, respectively.

IV. NUMERICAL RESULTS AND DISCUSSIONS

For numerical calculations, we take YIG/nonmagnetic superlattice film as an example. The parameters are $H_0 = 600\,\mathrm{G}$, $\gamma = 1.75\times10^7\,\mathrm{rad\,s^{-1}}$, $4\pi M_0 = 1750\,\mathrm{G}$, and $w = 10\,\mu\mathrm{m}$. When frequency is measured in the unit kG, there is a conversion relation, $1\,\mathrm{kG} = 2.9\times10\,\mathrm{rad\,s^{-1}}$. The wave-number unit is taken as $10^5\,\mathrm{m^{-1}}$. From $\mathbf{h} = \nabla\varphi$, we can estimate a suitable numerical range of $|\psi|$ for these given parameters, and here let $|\psi| = 2\times10^{-7}$.

For the magnetic superlattice films or multilayers changed into effective medium films, the magnetic fraction f_1 determines the frequency window of the linear surface modes.[8] In the geometry taken in this paper, if $f_1 < 0.5$, the window disappears, then the surface modes cannot exist.[6–8,29,30] We illustrate the dispersion curves of the linear surface mode in Fig. 2. One can see from this figure that the frequency window narrows rapidly as f_1 is reduced from 1, the bottom end of the window moves up and the upper end does not change. In the wave-number space, the existence of the surface mode requires a wave-number condition, say, the wave number must be larger than a fixed value, when f_1 is given. We consider properties of the corresponding nonlinear surface wave only in this frequency window.

The nonlinear coefficients are very important and they form a basis for investigating properties of the nonlinear magnetostatic surface waves, even they have to be used for studies of magnetostatic surface envelope solitons. Figure 3 presents the coefficients as a function of frequency for various values of f_1. These curves of $f_1 = 1.0$ [see Fig. 3(a)] correspond to the magnetic film and show four coefficients versus frequency. Among these coefficients, L_3 is the least and smaller by about two numerical orders than the others [also see Fig. 3(d)]. The absolute value of L_4 is very large at higher frequencies. Further, L_4 diverges in the end of high frequency. Therefore, at a higher frequency, the nonlinear effects should mainly come from the contribution of L_4.

For a magnetic superlattice film with $f_1 < 1.0$, Figs. 3(b) and 3(c) show that the absolute values of the coefficients are very large at lower frequencies. For a given frequency, the smaller f_1 is, the larger their absolute values are. The absolute value of L_3 still is the smallest in the frequency range of the surface modes. These coefficients all diverge at the bot-

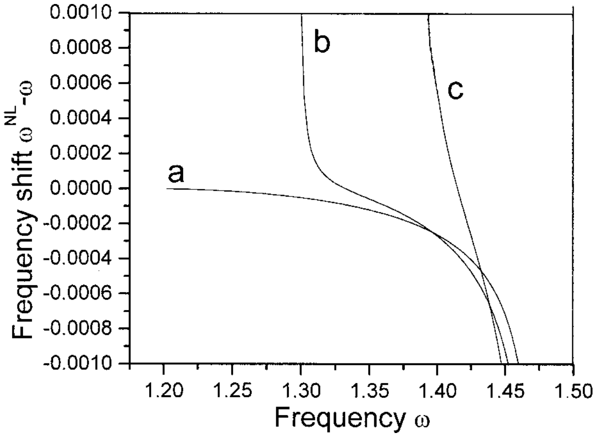

FIG. 4. Nonlinear frequency shift versus frequency. Curves *a* to *c* correspond to $f_1=1.0$, 0.9, and 0.8, respectively.

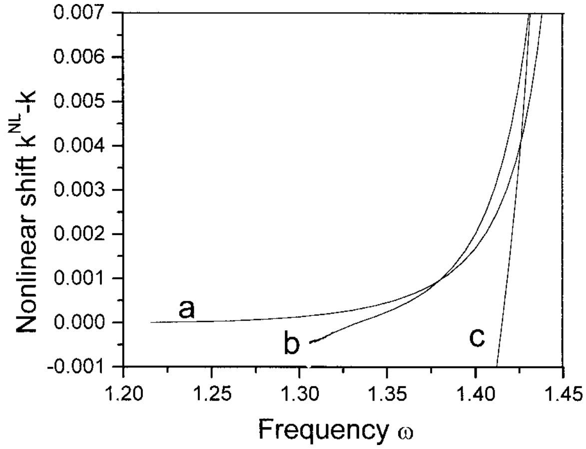

FIG. 5. Nonlinear wave-number shift versus frequency. Curves *a* to *c* correspond to parameters as the same as those for Fig. 4.

tom end of the window. Although $L_1=-L_2$ for the nonlinear surface modes of a single magnetic film, a small difference between them can be seen for this magnetic superlattice film, specially from Fig. 3(b). In order to see L_3 versus frequency in detail, Fig. 3(d) is given, where we can find that for $f_1=1.0$, L_3 has its minimum at about $\omega=1.27$, but it is equal to 0 at the two ends of the window. For $f_1=0.9$, L_3 increases from a negative value as frequency is increased and reaches its positive maximum at $\omega=1.36$, and then decreases as frequency is increased further and equal to 0 at the upper end of the window. For $f_1=0.8$, the curve has a completely different feature, or L_3 increases monotonously with frequency.

As we know, the frequency shift caused by the nonlinearity is defined as $\Delta\omega=\omega^{NL}-\omega$, and the nonlinear wave-number shift $\Delta k=k^{NL}-k$, where ω^{NL} and k^{NL} can be calculated numerically with Eq. (42). The following calculation method can be used. First, we calculate *k* for a given ω, applying the linear dispersion relation. Second, substitution of *k* in Eq. (42) and substitution of this given ω only in *F* on the right side of this equation, then ω included implicitly by the left side is just ω^{NL} that we want to solve. Following this process, we can get the frequency shift $\Delta\omega$ as a function of ω. By means of similar method, Δk can be obtained. Of course, we also can solve directly the nonlinear frequency or wave number with Eq. (42). The results obtained with the two different methods are nearly the same.

Figure 4 illustrates the frequency shift $\Delta\omega$ versus frequency ω. Curve *a* in the figure shows that $\Delta\omega$ is always negative for a single ferromagnetic film ($f_1=1.0$). Curves *b* and *c* corresponding to $f_1=0.9$ and 0.8 show that $\Delta\omega$ changes from positive to negative as frequency is increased, or the nonlinear frequency shift $\Delta\omega$ is positive for lower frequencies and negative for higher frequencies in the frequency window of the surface modes. In contrast to ferromagnetic films where the frequency shift is always negative, for our systems the frequency shift can be positive, but also can be negative, depending on frequency ω. The nonlinearity causes an increase in frequency of the surface wave in the low frequency region and a decrease in the high frequency region in the surface-mode window. This feature of superlattices is very distinctive. We also are interested in the wave-number shift as a function of frequency.

Figure 5 shows that the nonlinear shift of wave number increases with wave frequency and becomes positive from negative over a certain frequency value, especially in the vicinity of the high frequency end of the surface-mode window. This shift increases rapidly with frequency. This figure shows that for a single magnetic film, the nonlinearity does produce an increase in the wave number of the surface mode. For a magnetic superlattice ($f_1<1.0$), it causes a decrease in the wave number in the low frequency range, but an increase in the high frequency region. We are going to see that magnetostatic surface envelope solitons may exist in ferromagnetic superlattice films since the frequency or wave-number shift cannot only be negative, but also can be positive, depending on frequencies in the frequency window of the linear magnetostatic surface mode. Figures 4 and 5 also show that the nonlinear effects are rather more obvious for the magnetic-nonmagnetic superlattice than for the relevant single magnetic film, specially in the vicinities of the two ends of the surface-mode frequency window.

Applying the linear dispersion relation, we calculate numerically the second-order derivative of the linear frequency with respect to wave number $\omega''=(\partial^2\omega/\partial k^2)_L$. Figure 6 presents our results and shows that this derivative is always negative. Figure 7, presenting the calculation results of the linear group velocity $\omega'=(\partial\omega/\partial k)_L$, shows us that the group velocity is always positive.

Combining Fig. 4 with Fig. 6, we conclude that the formation of magnetostatic surface envelope solitons is possible in ferromagnetic superlattice films. The reason is that the necessary condition for the existence of the surface envelope solitons $\Delta\omega\times\omega''<0$ (the Lighthill criterion) is satisfied for the low frequency part of the frequency range of linear magnetostatic surface modes. We also note that for a single ferromagnetic film, this kind of surface solitons cannot exist since $\Delta\omega\times\omega''\geqslant 0$, and this point agrees with the given conclusion.[27,28]

V. CONCLUSIONS

In this paper, we have presented a nonlinear effective medium method and used it to investigate the properties of non-

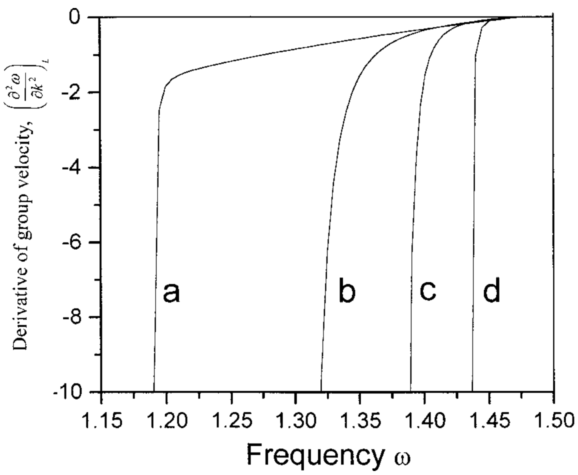

FIG. 6. In the case of linearity, the second derivative of frequency with respect to wave number versus frequency. Curves *a* to *d* are related to $f_1=1.0$, 0.9, 0.8 and 0.7, respectively.

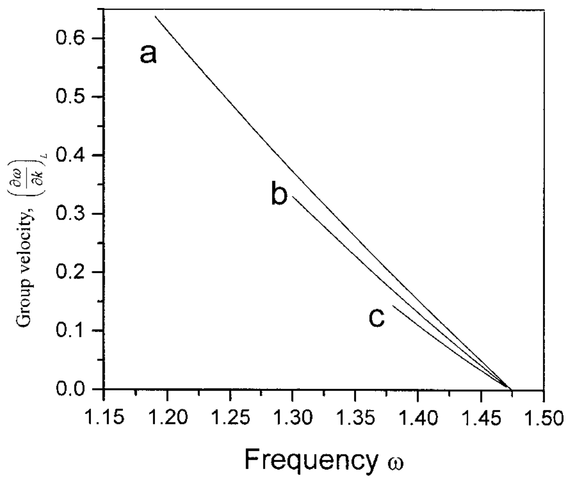

FIG. 7. The variance of linear group velocity with frequency. Curves *a* to *d* are related to $f_1=1.0$, 0.9, 0.8, and 0.7.

linear magnetostatic surface waves of magnetic multilayers or superlattice films. The expressions of the dynamic nonlinear magnetization are not only available to investigating the magnetostatic waves, but also to the retarded waves of magnetic superlattices and multilayers. Use of this theory requires a condition that the wavelength of modes is much larger than the period of the superlattice, $\lambda \gg d_1+d_2$. For the magnetic fraction $f_1=1.0$, our system becomes a single magnetic film. Comparing the previous results with ours, we see some differences resulting from the mistake in the previous theory. The Lighthill criterion is not satisfied for nonlinear magnetostatic surface waves in a single magnetic film, so the relevant envelope solitons cannot exist. It is very interesting that this criterion can be satisfied in magnetic superlattice films or magnetic multilayers, which means that the magnetostatic envelope surface solitons can be found in these kind of systems. The nonlinearity cannot only cause an increase in the surface-mode frequency, but also can cause a decrease, depending on the value of the surface-mode frequency, and the nonlinear effects are more obvious for the superlattice than the single magnetic film.

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*Corresponding address. Email address: xzwang@hrbnu.edu.cn

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