Exciton-exciton interaction in semiconductor quantum wells

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Analytical expressions of the effective exciton-exciton interaction in quantum wells are derived with a main focus on the dependence of the interactions on the quantum-well width. Any modification in the strength of the exciton-exciton interactions due to confinement is incorporated in α, a measure of dimensionality of the confined excitonic system. The flexibility of the derived expressions is shown in a systematic study of both the direct and exchange terms of the exciton-exciton interaction in CdTe/Zn$_x$Cd$_{1-x}$Te and GaAs/Al$_x$Ga$_{1-x}$As quantum wells. Results show the appreciable sensitivity of interexcitonic interaction to α due to changes in the extension of excitonic radial distribution and strength at which local charges are neutralized, perpendicular to the direction of the growth of well layers.

I. INTRODUCTION

The role of exciton-exciton interaction in determining the properties of low-dimensional systems and in introducing a variety of new physical effects has attracted much attention in recent years,[1–7] due in part to interest in applications that include quantum-well lasers and optical devices.[8] Some examples of novel physical effects include nonlinear light scattering in microcavities[3] caused by the exciton-exciton interaction, the increase in the exchange part of the exciton-exciton interaction with increasing confinement in semiconductor quantum wires,[4] and the notable reduction in strength of exciton-exciton interaction with the well width in quantum wells.[7]

Theories of excitons in low-dimensional heterostructures[9] have generally utilized models of the exciton in which no assumption of the fixed exciton dimensionality is made.[10–13] Such exact calculations of the excitonic problem in low-dimensional systems involve solving the Schrodinger equation either in K space or in real space using improved numerical techniques. For example, a set of coupled equations of a multicomponent envelope function based on a quadrature method[11,12] has been used to accurately predict low-dimensional excitonic energies under realistic experimental conditions. Recently, Iotti *et al.*[13] have also calculated accurate exciton binding energies and oscillator strengths by diagonalizing the Hamiltonian on a large nonorthogonal basis set.

It is interesting to note that experimental studies[14] clearly show the dimensional crossover of excitons as well widths are gradually decreased. In this regard, an alternative method based on the concept of fractional dimensionality can be used to study the effects of a changing exciton dimensionity and to simplify calculations of excitonic interactions in solid-state materials. The fractional-dimensional model of the exciton adopted in this paper is an approximate technique that simulates the exact results by focusing on the change in exciton dimensionality. This method is known to yield good estimates of the exciton interaction energy.[15–17]

The initial investigations that led to the concept of fractional-dimensional space in fact originated from the study of the axiomatic basis[18] in noninteger dimension in mathematics, but in recent years the developments have been more in physics. The use of a fractional-dimensional space in simplifying the evaluation of energy levels of excitons in quantum-well structures was first shown by He *et al.*[15] and Lefebvre *et al.*[16,17] Alternative methods of solving for the exciton energies in the absence of external fields were either sensitive to the assumed form of variational wave functions[19] or involved tedious and costly computations.[20] In recent years, the fractional-dimensional approach has been utilized in studies of shallow-donor, impurity, and excitonic states[21] in semiconducting quantum wells and in the analysis of excitonic Stark effects.[22]

In our work here, we present a unified approach to exciton-exciton interaction in quantum wells by treating the interaction between excitons as existing in fractional-dimensional space. The exciton-exciton interaction operators are derived in a fractional-dimensional space. As a result, the interaction operators are modified accordingly with changes in the quantum-well width. Thus only a single parameter, known as the degree of dimensionality (denoted by α), is needed to incorporate the effects due to changes in the widths of the well or barrier regions on the strength of exciton-exciton interaction as α increases from 2 in an exact two-dimensional system (e.g., infinite potential with zero well width) to 3 in an exact three-dimensional system (zero confinement). The origin of the reduction in strength of exciton-exciton interaction with well width, which has remained unclear in earlier works,[7] will be explained by the theory used in this work.

This paper is organized as follows. In Sec. II, we provide the theoretical basis used to obtain expressions for the exciton-exciton interactions in quasi-two-dimensional systems and we formulate the problem in fractional-dimensional space in Sec. III. We obtain analytical expressions of the exciton-exciton interaction in αD space in Sec. IV and present numerical results for CdTe/Zn$_x$Cd$_{1-x}$Te and GaAs/Al$_x$Ga$_{1-x}$As quantum wells, with comparison made with available experimental results in Sec. V. The conclusions of this work are presented in Sec. VI.

II. HAMILTONIAN REPRESENTING EXCITON-EXCITON INTERACTION

The Hamiltonian that yields information about the nature of exciton-exciton interaction can be obtained by operating Usui transformation[23] on an electron-hole Hamiltonian. This method, first developed by Hanamura[24] to incorporate the interactions between excitons, is used to obtain an effective form for the exciton-exciton interaction Hamiltonian:

$$\hat{\mathcal{H}} = \sum_{k\mu\nu} \left\{ \epsilon_{\mu\nu}(k)\hat{b}_{\mu k}^{\dagger}\hat{b}_{\nu k} + \tfrac{1}{2} \sum_{k'\mu'\nu'q} V_{\mu\mu'\nu\nu'}(k,k',q) \right.$$
$$\left. \times \hat{b}_{\nu k+q}^{\dagger}\hat{b}_{\nu'k'-q}^{\dagger}\hat{b}_{\mu'k'}\hat{b}_{\mu k} \right\}. \tag{1}$$

Here $\hat{b}_{\mu k}^{\dagger}$ creates an exciton with wave number k, state μ, and $\epsilon_{\mu\nu}$ is the intraexcitonic interaction energy from the μth to νth state. $V_{\mu\mu'\nu\nu'}(k,k',q)$ denotes the interexcitonic interaction energy where μ and ν (μ' and ν') denote the initial (final) state of the excitons before (after) the interaction. q is the momentum transferred during the interaction between two excitons. In general, the evaluation of the various terms that constitute the interexcitonic interaction energy is complicated[25,26] and involves intensive numerical techniques for the evaluation of the magnitude of interactions for a given exciton momenta and effective mass.

In order to simplify the evaluation of the interaction term, $V_{\mu\mu'\nu\nu'}(k,k',q)$, we consider that the excitons remain in the $1s$ state throughout their interactions, i.e., $\mu=\mu'=\nu=\nu'=1s$. We also limit the theory to the low-density limit of the exciton density and thus assume the presence of only two interacting excitons. The $1s$ quasi-two-dimensional exciton state with wave vector $\mathbf{k}$ is expressed as[27]

$$|\mu=1s,\mathbf{k}\rangle = \sum_{\mathbf{k}_e,\mathbf{k}_h} \Phi_{1s}(\mathbf{k}_e,\mathbf{k}_h,\mathbf{k})\delta_{\mathbf{k}_e-\mathbf{k}_h,\mathbf{k}}a_{1,\mathbf{k}_e}^{\dagger}a_{0,\mathbf{k}_h}|0,n\rangle, \tag{2}$$

where A is the surface area of the unit cell and $a_{1,\mathbf{k}_e}^{\dagger}$ ($a_{0,\mathbf{k}_h}$) is the creation (annihilation) operator of an electron in the conduction (valence) band, denoted by 1 (0) with wave vector $\mathbf{k}_e$ ($\mathbf{k}_h$). The function $\Phi_{1s}(\mathbf{k}_e,\mathbf{k}_h,\mathbf{k})$ is given by

$$\Phi_{1s}(\mathbf{k}_e,\mathbf{k}_h,\mathbf{k}) = \frac{1}{A}\int d\mathbf{r}\Psi_{1s}(\mathbf{r}_e-\mathbf{r}_h)\exp[i(\gamma_e\mathbf{k}-\mathbf{k}_e)\cdot\mathbf{r}], \tag{3}$$

where

$$\gamma_e = \frac{m_e^*}{(m_e^*+m_h^*)} = 1 - \gamma_h \tag{4}$$

and m_e^* and m_h^* are the effective masses of the electron and hole, respectively, and $\Phi_{1s}(\mathbf{k}_e,\mathbf{k}_h,\mathbf{k})$ denotes the relative motion of the electron-hole pair in an exciton.

Utilizing the well-known commutation relations for hole and electron operators[25] in Eqs. (1)–(3), we obtain

$$V_{1s,1s,1s,1s}(k,k',q) = \sum_{i=1}^{6} V_{1s,1s,1s,1s}^{i}(k,k',q), \tag{5}$$

where the terms $V_{1s,1s,1s,1s}^{i}$ ($i=1,2,3$) correspond to exciton-exciton scattering due to the Coulomb interaction (dynamical scattering) between excitons. On the other hand, the terms $V_{1s,1s,1s,1s}^{i}$ ($i=4,5,6$) are due to exchange interactions (kinematic scattering) between excitons. These six terms are obtained as

$$V_{1s,1s,1s,1s}^{1}(k,k',q) = \frac{C_q}{(\Omega_3)^2}\sum_{jj'} \Phi_{1s}(j-\gamma_h k+\gamma_e q)$$
$$\times \Phi_{1s}(j'-\gamma_h k'-\gamma_h q)$$
$$\times \Phi_{1s}(j-\gamma_h k)\times\Phi_{1s}(j'-\gamma_h k'), \tag{6}$$

$$V_{1s,1s,1s,1s}^{2}(k,k',q) = \frac{C_q}{(\Omega_3)^2}\sum_{jj'} \Phi_{1s}(j-\gamma_h k-\gamma_h q)$$
$$\times \Phi_{1s}(j'-\gamma_h k'+\gamma_h q)$$
$$\times \Phi_{1s}(j-\gamma_h k)\times\Phi_{1s}(j'-\gamma_h k'), \tag{7}$$

$$V_{1s,1s,1s,1s}^{3}(k,k',q) = -\frac{2C_q}{(\Omega_3)^2}\sum_{jj'} \Phi_{1s}(j-\gamma_h k-\gamma_h q)$$
$$\times \Phi_{1s}(j'-\gamma_h k'-\gamma_e q)$$
$$\times \Phi_{1s}(j-\gamma_h k)\times\Phi_{1s}(j'-\gamma_h k'), \tag{8}$$

$$V_{1s,1s,1s,1s}^{4}(k,k',q) = -\frac{1}{(\Omega_3)^2}\sum_{jj'} C_j'\Phi_{1s}(j-j'-\gamma_h k-\gamma_h q)$$
$$\times \Phi_{1s}(j-k+\gamma_e k'-\gamma_e q)$$
$$\times \Phi_{1s}(j-j'-k-q+\gamma_e k')$$
$$\times \Phi_{1s}(j-\gamma_h k), \tag{9}$$

$$V_{1s,1s,1s,1s}^{5}(k,k',q) = -\frac{1}{(\Omega_3)^2}\sum_{jj'} C_j'\Phi_{1s}(j-j'-\gamma_h k$$
$$+\gamma_e q)\Phi_{1s}(j-\gamma_h k'+\gamma_h q)$$
$$\times \Phi_{1s}(j-j'+q-\gamma_h k')$$
$$\times \Phi_{1s}(j-\gamma_h k), \tag{10}$$

$$V_{1s,1s,1s,1s}^{6}(k,k',q) = \frac{2}{(\Omega_3)^2}\sum_{jj'} C_j'\Phi_{1s}(j-j'-\gamma_h k-\gamma_h q)$$
$$\times \Phi_{1s}(j-k+\gamma_e k'-\gamma_e q)$$
$$\times \Phi_{1s}(j-k-q+\gamma_e k')$$
$$\times \Phi_{1s}(j-\gamma_h k), \tag{11}$$

where C_q denotes the Coulombic interaction between electrons and holes in q space and Ω_3 denotes the three-dimensional crystal volume.

III. FORMULATION IN FRACTIONAL-DIMENSIONAL SPACE

The exciton-exciton interactions discussed in Sec. II can be generalized to fractional-dimensional space by considering that the exciton state vector [Eq. (1)] is operational in an α-dimensional space ($2 \leq \alpha \leq 3$). Thus all position ($\mathbf{r}$) and momentum vectors ($\mathbf{k}, \mathbf{q}$) are taken to exist in αD space and Ω_α, the volume in an αD space, is given by the Hausdorff measure,

$$\Omega_\alpha(|\mathbf{r}|) = \frac{\pi^{\alpha/2}}{\Gamma\left(1 + \frac{\alpha}{2}\right)} |\mathbf{r}|^\alpha, \qquad (12)$$

where $\Gamma[x]$ is Euler's gamma function.

It is to be noted that the Coulombic field lines pass through the semiconductor material that surrounds the well, and hence the Coulomb potential retains its three-dimensional expression $1/r$ in fractional-dimensional space. Using the spatial integral relation in αD space,[15]

$$\int_{\alpha D} dr = \frac{2\pi^{(\alpha-1)/2}}{\Gamma\left(\frac{\alpha-1}{2}\right)} \int_0^\infty r^{\alpha-1} dr \int_0^\pi d\theta \sin^{\alpha-2}\theta, \quad (13)$$

the Coulomb potential C_q^α in an αD space is derived (see Appendix A) as

$$C_q^\alpha = G(\alpha) \frac{1}{\Omega_\alpha} \frac{e^2}{\varepsilon_0 q^{\alpha-1}}, \qquad (14)$$

where $G(\alpha)$ is given by

$$G(\alpha) = \left[2^{\alpha-1} \pi^{(\alpha/2)-(1/2)} \Gamma\left(\frac{1}{2} + \frac{\alpha-2}{2}\right) \right]. \qquad (15)$$

Thus we have generalized C_q in Eq. (14) by introducing a noninteger dimension α. C_q^α reduces to the well established forms of $2\pi e^2/\Omega_2 \varepsilon_0 q$ and $4\pi e^2/\Omega_3 \varepsilon_0 q^2$, respectively, in the exact two-dimensional (2D) and 3D limits.

For the expressions in Eqs. (6)–(11) to apply in an αD space, we use the $1s$ state of an isolated exciton in an αD space (see Appendix B):

$$\Phi_{1s}(k) = (4\pi)^{(\alpha/4)-(1/4)} (\alpha-1)^{(\alpha/2)+(1/2)}$$

$$\times \sqrt{\Gamma\left[\frac{\alpha-1}{2}\right]} \frac{a_B^{\alpha/2}}{\left(1 + \left[\frac{\alpha-1}{2} k a_B\right]^2\right)^{(\alpha+1)/2}}, \qquad (16)$$

where a_B is the three-dimensional Bohr radius of the exciton. Once again, Eq. (16) yields the expected forms in the exact three-dimensional and two-dimensional limits.[28] The direct use of Eq. (16) in Eqs. (6)–(11) would, however, result in computational difficulties due to the multiple integrals that are involved. In order to simplify the evaluation, we apply

the relation $1/(1+x)^y \approx \exp(-xy)$ for $x \leq \frac{1}{4}$ to Eq. (16) so that the excitonic wave function is reduced to an exponential form:

$$\Phi_{1s}(k) = F'(\alpha) a_B^{\alpha/2} \exp\left[\left(\frac{\alpha-1}{2} k a_B\right)^2 \frac{\alpha+1}{2}\right], \quad (17)$$

where $F'(\alpha)$ can be easily determined using the normalization relation,

$$\sum_{\mathbf{k}} |\Phi_{1s}(k)|^2 = \Omega_\alpha. \qquad (18)$$

It is important to note that the range of exciton wave vector $[k \leq (1/2a_B)]$ for which Eq. (17) is valid generally lies close to the band edge where excitons are initially created[29,30] at low carrier temperatures.

IV. EXCITON-EXCITON INTERACTION IN αD SPACE

In order to compare the direct and exchange energy terms of the exciton-exciton interaction in αD space, we define the exciton-exciton interaction strength, W^i, as

$$W^i = \left(\frac{\Omega_\alpha}{(a_B)^\alpha}\right) \frac{V^i_{1s,1s,1s,1s}}{E_\alpha}, \qquad (19)$$

where the exciton binding energy E_α is given by[15,16]

$$E_\alpha = \frac{4\,\text{Ry}}{(\alpha-1)^2} \qquad (20)$$

and Ry is the effective exciton Rydberg.

It is to be noted that the quantity W^i in Eq. (19) can be shown to be dimensionless using Eqs. (6)–(11), (14), and (16). The magnitude of W^i is dependent on the ratio $(\Omega_\alpha/(a_B)^\alpha)$, which is determined by the degree of proximity of the two interacting excitons. It is difficult to study the effect of exciton density on W^i, as we are considering only two interacting excitons. Thus W^i increases with increased closeness of the interacting excitons, as expected. It is thus possible to obtain a wide range of values for W^i depending on how close the two interacting excitons are, as will be shown in subsequent calculations.

Substituting Eq. (17) in Eqs. (6)–(8), and transforming the discrete sum over the wave vectors into a spatial integral using

$$\sum_{\mathbf{q}} \to \frac{\Omega_\alpha}{(2\pi)^\alpha} \int_{\alpha D} d\mathbf{q}, \qquad (21)$$

we obtain an explicit analytical expression for the sum of direct energy terms, $W^1 + W^2 + W^3$:

$$W^1 + W^2 + W^3 = \frac{(\alpha-1) 2^{(\alpha-1)}}{(q a_B)^{(\alpha-1)}} \left[\exp\left(-\frac{\alpha+1}{4} (\gamma_h q a_B)^2\right) \right.$$

$$\left. - \exp\left(-\frac{\alpha+1}{4} (\gamma_e q a_B)^2\right) \right]^2. \qquad (22)$$

The derivation of the exchange energy terms is less straightforward and requires use of the integral[31]

$$\int_0^1 \exp(bx^2) = \frac{1}{2}\sqrt{\frac{\pi}{b}}\,\mathrm{erfi}(\sqrt{b}), \qquad (23)$$

where $\mathrm{erf}(z)$ is the error function. Using Eqs. (9), (10), (17), and (23), we derive the sum $W^4 + W^5$ as

$$W^4 + W^5 = -\left[\frac{3\pi 2^{(\alpha-2)}(\alpha-1)^4}{\alpha}\right]^{(\alpha-1)/2}\sqrt{\frac{\pi}{2(\alpha+1)}}$$

$$\times \exp\left(-\frac{\alpha+1}{2}[(\gamma_h q a_B)^2 + (\gamma_e q a_B)^2]\right)$$

$$\times \left[\frac{\mathrm{erfi}\left(\sqrt{\frac{\alpha+1}{2}}\gamma_e q a_B\right)}{\gamma_e q a_B}\right.$$

$$\left. + \frac{\mathrm{erfi}\left(\sqrt{\frac{\alpha+1}{2}}\gamma_h q a_B\right)}{\gamma_h q a_B}\right] \qquad (24)$$

and W^6 is obtained as

$$W^6 = -[2^{(3\alpha-3)}\pi]^{(\alpha-1)/2}\sqrt{\frac{\pi\alpha}{2(\alpha-1)}}$$

$$\times \exp\left[-\frac{\alpha+1}{2}[(\gamma_h q a_B)^2 + (\gamma_e q a_B)^2]\right]$$

$$\times \frac{\mathrm{erfi}\left(\sqrt{\frac{\alpha-1}{\alpha}}(\gamma_e - \gamma_h)q a_B\right)}{(\gamma_e - \gamma_h)q a_B}. \qquad (25)$$

As we are mainly interested in the effects of confinement on the exciton-exciton interaction, the excitons involved in the interactions are assumed to possess equal wave vectors, i.e., $k = k'$ in Eqs. (6)–(11), for the purpose of simplification. However, the calculations can be extended to excitons with unequal wave vectors, but numerical methods will be required to evaluate the strength in exciton-exciton interaction.

V. RESULTS AND DISCUSSION

A. CdTe/$\mathrm{Zn}_x\mathrm{Cd}_{1-x}$Te quantum

In Figs. 1 and 2, we have plotted the direct ($W^1 + W^2 + W^3$) [Eq. (22)] and exchange terms ($W^4 + W^5 + W^6$) [Eq. (24) plus Eq. (25)] of the exciton-exciton interaction as a function of ($q a_B$) for $\alpha = 2$, 2.3, 2.6, and 3 in CdTe/$\mathrm{Zn}_x\mathrm{Cd}_{1-x}$Te quantum wells using[32] $m_e = 0.1 m_o$ and $m_h = 0.76 m_o$, where m_o is the free-electron mass. ($q a_B$) is a measure of the momentum transferred between two colliding excitons. The total contribution $W_{\mathrm{tot}} = W^1 + W^2 + W^3 + W^4 + W^5 + W^6$ is plotted in Fig. 3. The figures clearly show that the strength of exciton-exciton interaction is reduced as the dimensionality α is decreased. The notable sensitivity of the interexcitonic interactions to α can be attributed to changes in the extension of excitonic radial distribution as the well
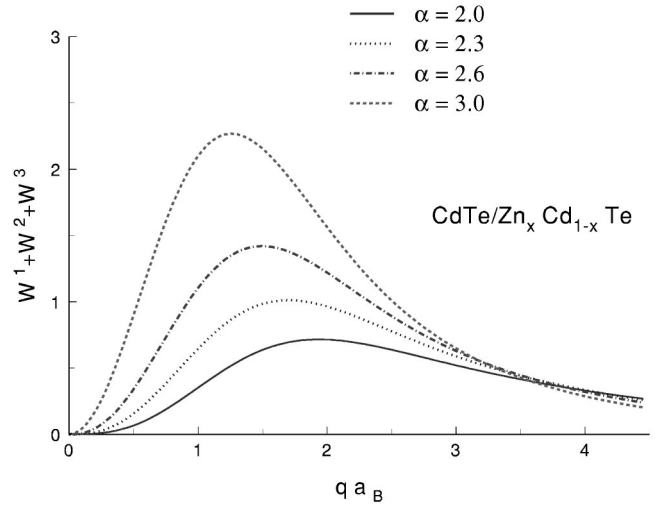

FIG. 1. Direct interaction terms ($W^1 + W^2 + W^3$) [Eq. (22)] as a function of ($q a_B$) for $\alpha = 2$, 2.3, 2.6, and 3 in CdTe/$\mathrm{Zn}_x\mathrm{Cd}_{1-x}$Te quantum wells. q is the momentum transferred between two interacting excitons.

width is altered. It is expected that the strength at which local charges are neutralized changes with α, which in turn influences the interaction between excitons.

It is important to note that while the individual terms of W^i diverge, the sum of the contribution remains finite for all values of $q a_B$ as shown in Figs. 1–3. The interaction terms give no contributions beyond $q a_B = 4$. The sum of the exchange terms of the exciton-exciton interaction are generally larger than the sum of the direct terms. From Eq. (22), it can be seen that the direct channel of exciton-exciton interaction vanishes for $q = 0$ as well as for the specific case of $m_e = m_h$. However, the direct terms should not be neglected in materials where $\gamma_e \to 1$ or $\gamma_e \to 0$.

To evaluate a mean value for $W_{\mathrm{tot}}(q a_B)$ over the range $0 \leq q a_B \leq 4$, we use

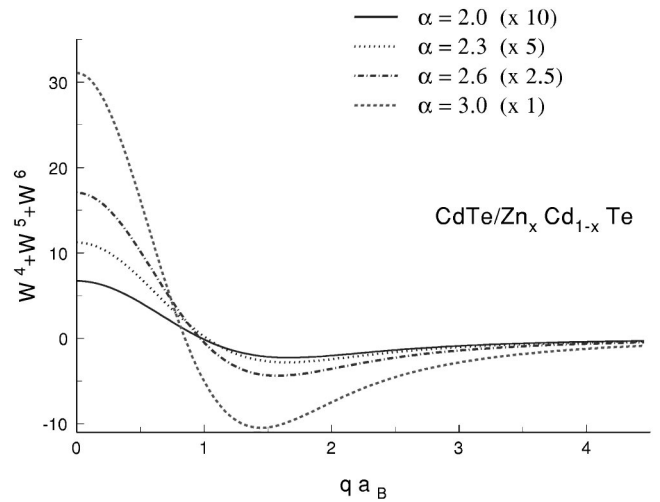
FIG. 2. Exchange interaction terms ($W^4 + W^5 + W^6$) [Eq. (24) plus Eq. (25)] as a function of ($q a_B$) for $\alpha = 2$, 2.3, 2.6, and 3 in CdTe/$\mathrm{Zn}_x\mathrm{Cd}_{1-x}$Te quantum wells. For the sake of clarity, we have multiplied the interaction terms corresponding to each different α by the scale factor indicated in the figure.
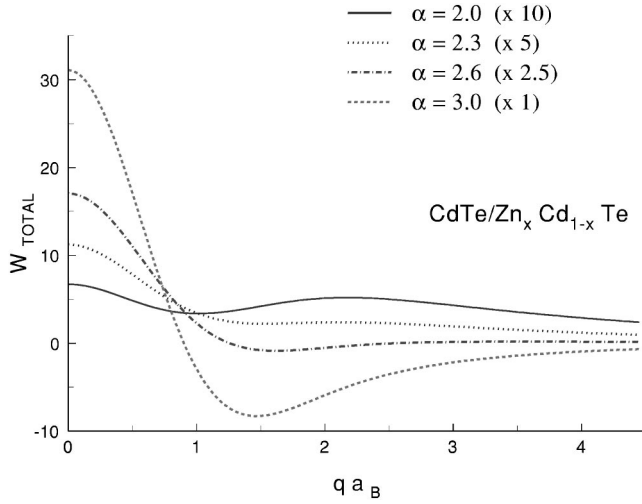
FIG. 3. Total of direct and exchange terms W_{tot} [Eq. (22) plus Eq. (24) plus Eq. (25)] as a function of (qa_B) for $\alpha=2$, 2.3, 2.6, and 3 in CdTe/Zn$_x$Cd$_{1-x}$Te quantum wells.

$$\langle W_{\text{tot}}\rangle = a_B \int_0^4 W_{\text{tot}}(qa_B)dq. \qquad (26)$$

We evaluate Eq. (26) using Eqs. (22)–(25) so as to compare our theoretical results with those of Mayer et al.,[7] who measured the strength of exciton-exciton interaction of the lowest exciton state in CdTe/Zn$_x$Cd$_{1-x}$Te quantum wells as a function of the quantum-well width using degenerate-four-wave mixing (DFWM) at low temperatures.

It is important to note that the definition of W^i in Eq. (19) that determines $\langle W_{\text{tot}}\rangle$ in Eq. (26) is consistent with the quantity used by Mayer et al.[7] to measure the strength of exciton-exciton interaction. For low density of excitons, Mayer et al.[7] considers that the homogeneous linewidth Γ is proportional to $\gamma a_B^2 E_\alpha n$, where γ is the quantity that measures the strength of exciton-exciton interaction and n is the exciton density. γ (defined for a strict two-dimensional system) is similar to W^i in Eq. (19) (applicable to a generalized dimension). Other than Γ and W^i having the same dimensions, our method of computing the matrix elements of exciton-exciton scattering, $V^i_{1s,1s,1s,1s}$, in Eq. (19) is similar to the approach of Braun et al.[4] as adopted by Mayer et al.[7] Although γ and W^i are similar, they are not exactly equivalent and may differ by a small factor, which is expected not to affect the significance of comparison of our results with the experimental results of Mayer et al.[7]

The values of α at the various well widths, needed to compute W_{tot}, were obtained using experimental values[33] of exciton binding energies at known values of the well width, in CdTe/Zn$_x$Cd$_{1-x}$Te quantum wells and Eq. (20). While we have used empirical values of α to obtain Fig. 4, other effective theoretical methods of determining this important parameter have been described in earlier works.[16,21,22]

In Fig. 4, the experimental data show a notable reduction of the strength of exciton-exciton interaction with well width, which is in good qualitative agreement with our theoretical results. This reduction of interaction strength with well thickness can be attributed to the decrease in cross sec-
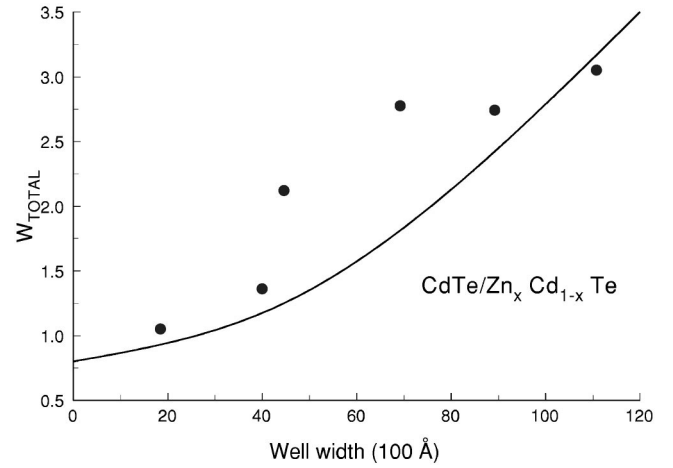

FIG. 4. Solid circles denote experimental values (Ref. 7) of the exciton-exciton interaction as a function of well width. The solid line represents the numerical values of $\langle W_{\text{tot}}\rangle$ [Eq. (26)] in CdTe/Zn$_x$Cd$_{1-x}$Te quantum wells, evaluated using known values of exciton binding energies as function of the well width (Ref. 30).

tions of excitons via scattering by other excitons due to a reduced spatial extension of the excitonic wave function in thinner wells, as is seen in Fig. 3. It is interesting to note that while the exciton-exciton interaction decreases with decreasing well width in quantum wells, it increases with decreasing wire width as shown in recent experimental results.[4] It is to be noted that unlike quantum wells, quantum wires have larger values of exciton binding energies. This is the result of the enhanced Coulomb coupling between electrons and holes due to their localization in the nanostructure. It is likely that the presence of this additional confining potential may play an important role in interaction between excitons. However, the study of the effect of strong confinement effects on exciton-exciton interaction involves a theoretical approach[35] that is based on a generalization of the well-known semiconductor Bloch equations to the case of a multisubband wire structure. This approach requires the incorporation of a full three-dimensional multisubband description of the electron-hole Coulomb interaction. This is not a trivial procedure and, therefore, a theoretical relation between W_{tot} and wire width has not been explored in this work.

The quantitative agreement between the theoretical and experimental results in Fig. 4 is reasonable, in view of several underlying assumptions used in order to obtain analytical expressions for W_{tot}. For instance, we have neglected the distortion of the Coulombic interaction due to differences in the dielectric constants between the well and barrier regions. The effects of exciton scattering by acoustic and optical phonons as well as the influence of phase-space filling effects and Coulomb screening effects[34] on the process of interexcitonic scattering have not been included in our work. Also our calculated results would be influenced by the assumption, in Sec. IV, that the interacting excitons are assumed to possess equal wave vectors, i.e., $k=k'$ so as to obtain analytical solutions in Eqs. (22), (24), and (25). Though these factors are expected to introduce a small degree of error in our calculated values of the strength of
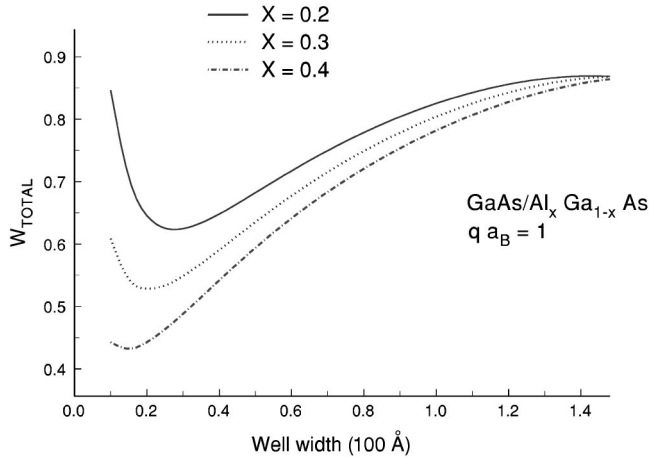
FIG. 5. W_{tot} as a function of well width in GaAs/Al$_x$Ga$_{1-x}$As quantum wells at $(qa_B)=1$, and for aluminum concentration, $x=0.2$, 0.3, and 0.4 in the barrier region.

exciton-exciton interaction, the overall gross features as illustrated in Fig. 4 generally remain intact.

It is to be noted that due to the shallow potential barriers of CdTe/Zn$_x$Cd$_{1-x}$Te quantum wells, the exciton dimensionality, for a fixed well width, is larger compared to CdTe/Mn$_x$Cd$_{1-x}$Te quantum wells where excitons are better localized due to higher potential barriers.[36] This provides a suitable explanation for the smaller strength of exciton-exciton interaction in CdTe/Mn$_x$Cd$_{1-x}$Te quantum wells as measured in an earlier experimental work.[36]

B. GaAs/Al$_x$Ga$_{1-x}$As quantum well

In Fig. 5, W_{tot} is plotted as a function of well width in GaAs/Al$_x$Ga$_{1-x}$As quantum wells at $qa_B=1$, and for aluminum concentration, $x=0.2$, 0.3, and 0.4 in the barrier region, using $m_e=0.067m_o$ and $m_h=0.38m_o$. The values of α at the various well widths needed to compute W_{tot} were obtained using experimental values[37] of exciton binding energies, E_b, in GaAs/Al$_x$Ga$_{1-x}$As quantum wells and Eq. (20). It is to be noted that the value of $(qa_B)=1$ is chosen arbitrarily, and the gross features in the figure remain the same for a wide range of values of q.

While the interexcitonic interaction W_{tot} increases gradually with α, the quantum-well width does not experience a similar monotonic increase with α, due to the spreading of excitonic wave functions into the barrier regions. This spreading effect becomes increasingly significant at narrow well widths as tunneling becomes prominent. In the ultimate limit of zero width, the exciton assumes a three-dimensional character. The dimensionality has been shown to reach its lowest value $(\sim 2.2-2.3)$ at some intermediate well width $(\sim 15-40$ Å$)$ that depends on the well and barrier materials[16] in GaAs/Al$_x$Ga$_{1-x}$As quantum wells. Accordingly, W_{tot} in Fig. 5 decreases with the well width to a minimum at a critical well width, before increasing at further increase in the well size. The critical well width becomes smaller as the concentration x is increased. This is because

the potential depth increases as x is increased, which results in a corresponding decrease in dimensionality α of the confined exciton.

VI. CONCLUSION

In conclusion, in this paper we have presented analytical expressions of the effective exciton-exciton interaction in quantum wells. We have focused on the dependence of the interactions on the quantum-well width by using a parameter α, a measure of the dimensionality of the confined excitonic system. The use of α is convenient as it incorporates the tunneling effect. Hence excitons assume a more three-dimensional character either at very small or large well widths, with a more distinct two-dimensional character at a critical well width, depending on the material composition.

Results show the notable sensitivity of interexcitonic interaction to α due to changes in the extension of excitonic radial distribution and strength at which local charges are neutralized, perpendicular to the direction of the growth of well layers. Numerical results of the exciton-exciton interaction in GaAs/Al$_x$Ga$_{1-x}$As quantum wells show that a minimum in the strength of exciton-exciton scattering occurs at a critical size of the well width, depending on the quantum well and barrier material composition.

In conclusion, we have shown that dimensionality α of excitons can effectively be used to determine the strength of exciton-exciton interaction in semiconductor quantum wells. Our work is expected to provide useful insights and stimulate further developments in experimental work involving excitons in low-dimensional systems.

APPENDIX A

The derivation of Eq. (14) is based on two integrals:[31]

$$\int_0^{\pi} e^{ia \cos x} \sin^{2b} x \, dx = \pi^{1/2}\left(\frac{2}{a}\right)^b \Gamma[b+\tfrac{1}{2}] J_b(a);$$

$$\text{Re } b \geqslant -\tfrac{1}{2} \tag{A1}$$

and

$$\int_0^{\infty} x^c J_b(ax) \, dx = 2^c a^{-(c+1)} \frac{\Gamma\left(\dfrac{1}{2}+\dfrac{b}{2}+\dfrac{c}{2}\right)}{\Gamma\left(\dfrac{1}{2}+\dfrac{b}{2}-\dfrac{c}{2}\right)}. \tag{A2}$$

APPENDIX B:

In order to derive Eq. (16), we use the form of $U_{1s}(r)$ for an isolated exciton in an α-dimensional space:

$$U_{1s}(r) = F(\alpha) \exp\left[-\frac{2}{\alpha-1}\frac{r}{a_B}\right], \tag{B1}$$

where $F(\alpha)$ is

$$F(\alpha) = \left[\frac{2^{\alpha+1}\pi^{(1-\alpha)/2}}{\Gamma\left(\frac{\alpha-1}{2}\right)(\alpha-1)^{\alpha+1}} \frac{1}{a_B^\alpha} \right]^{1/2}. \qquad (B2)$$

Equation (16) is then evaluated using Eq. (B1) and the relation[38,39]

$$\int_{\alpha D} d\mathbf{r} e^{-2\pi|\mathbf{r}|c} e^{-2\pi i \,\mathbf{q}\cdot\mathbf{r}} = \frac{\Gamma\left(\frac{\alpha+1}{2}\right)}{\pi^{(\alpha+1)/2}} \frac{c}{(c^2+q^2)^{(\alpha+1)/2}}. \qquad (B3)$$

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