Coulomb effects on the transport properties of quantum dots in a strong magnetic field

V. Moldoveanu, A. Aldea, A. Manolescu, and M. Niţă
National Institute of Materials Physics, P.O. Box MG7, Bucharest-Magurele, Romania
(Received 3 April 2000; published 22 December 2000)

We investigate the transport properties of quantum dots placed in a strong magnetic field using a quantum-mechanical approach based on the two-dimensional tight-binding Hamiltonian with direct Coulomb interaction and the Landauer-Büttiker formalism. The electronic transmittance and the Hall resistance show Coulomb oscillations and also prove multiple addition processes. We identify this feature as the ''bunching'' of electrons observed in recent experiments and give an elementary explanation in terms of spectral characteristics of the dot. The spatial distribution of the added electrons may distinguish between the edge and bulk states and it has specific features for bunched electrons. The dependence of the charging energy on the number of electrons is discussed for a strong magnetic field. The crossover from the tunneling to quantum Hall regime is analyzed in terms of dot-lead coupling.

DOI: 10.1103/PhysRevB.63.045301 PACS number(s): 73.23.Hk, 73.50.−h, 73.43.−f

I. INTRODUCTION

The basic phenomena in quantum dots (QD) are usually described by the ''orthodox theory''[1,2] which explains the Coulomb blockade effects (charge quantization and the oscillations of the electrical resistance) in terms of the capacitive properties of the isolated (or very weakly coupled) dot. Generally speaking, the corresponding capacitance should depend on the dimension and the dot shape, number of electrons accommodated inside, and the electron-electron interaction (EEI). Nevertheless, the orthodox theory considers the dot capacitance as being a constant, independent of the number of electrons and the size quantization effects. This idea can be accepted for large metallic QDs when the charging energy due to the Coulomb interaction is larger than the level spacing of the one-electron energy spectrum. However, many specific properties of QDs remain beyond this model. Such aspects occur especially for small semiconductor quantum dots, when the level spacing is relatively large, the distribution of the energy levels depends visibly on the dot shape, and the number of electrons is smaller than in the metallic case. In small dots the interplay between the quantum aspects and the charging effects is important. This is why a more advanced description pretends to pay attention to the one-particle energy spectrum and to consider a more realistic Hamiltonian.

In spite of the general acceptance that the charging energy should depend on the number of electrons inside dot *N*, a fact which is proved experimentally by the irregular Coulomb blockade oscillations,[3] this effect has been simulated numerically only recently.[4,5] The situation becomes even more complicated when the coupling to the leads is taken into account carefully, going beyond the lowest order of the perturbation series in the tunneling matrix elements. This was done by mapping the scattering of electrons by the dot into a Kondo problem,[7] a method which, however, could not avoid the use of the constant capacitance model (CCM). But, as it is known,[8,1] CCM fails dramatically in the presence of a high magnetic field. For this reason, one of the aims of our paper is to find an alternative formalism able to consider the size effects and to describe the case of strong magnetic fields, replacing thus the constant capacitance model. The proposed approach is the quantum-mechanical Landauer-Büttiker (LB) formalism which has the advantage of simplicity, contains all the lead-dot tunneling processes (i.e., the full perturbation series), and is definitely valid in strong magnetic fields. Originally, it was considered that this formalism works only for noninteracting electrons. However, afterwards Meir and Wingreen[6] proved that at zero temperature and in the linear-response regime, the LB formalism remains valid even if an interaction is present in the dot, as long as the leads are free of interaction [see the discussion after Eq. (10) in Ref. 6]. In this paper, a one-particle approximation (namely, Hartree) is used, so that the applicability of the LB formalism is beyond any doubt. In a previous paper by Maccuci *et al*. it was shown that the differences between Hartree and better treatments of the interaction [local density approximation (LDA)] are not qualitatively significant for the transmittance problem (see Fig. 9 in Ref. 4).

In this framework we show that the lead-dot (LD) coupling plays an even more pregnant role in the presence of a strong magnetic field, in which case an interesting crossover from Coulomb oscillations to the quantum Hall regime can be noticed for the transverse resistance with increasing coupling. In fact, this coupling between dot and leads (or using other words: the degree of pinching and/or constriction at the contacts) decides the degree of quantization of the charge in QD and affects the electronic transmittance through the dot and the Hall resistance. The influence of the coupling between ideal terminals and noninteracting systems on the transport properties was emphasized some time ago; in the case of noninteracting QD Büttiker predicted oscillations of the Hall resistance induced by the pinching,[9] while for a smoothly tapered junction Kirczenow also obtained a resonant feature of R_H.[10]

The strong magnetic field perpendicular on the two-dimensional dot gives rise to edge states even if the dot is small. Then, for large LD coupling, quantum Hall effect–(QHE-) type effects appear, although, usually, only the first plateau is visible in small dots (and not always very clean). Such aspects were evidentiated in Ref. 11 by the use of a tight-binding (TB) model in the absence of the electron-

electron interaction. Due to the full polarization of the spin degree of freedom a spinless Hamiltonian can be used.

Our approach for calculating the transport properties of quantum dots is the following: the QD is coupled weakly to four semi-infinite leads supporting many channels. The number of degrees of freedom of the terminals is infinite while the QD has only a finite number of degrees of freedom, so that the Fermi level E_F of the whole system is imposed by terminals. At a given magnetic field, when a gate potential V_g is applied and varied, the fixed E_F scans the whole energy spectrum. We calculate the electronic transmittance matrix, the Hall resistance, and charging energy. The transmittance peaks — which correspond to the charge-degeneracy points — are distributed irregularly keeping track of the size quantization. We show that when the contacts are pinched, the quantum plateaus disappear and quantum oscillations of R_H are installed even in the absence of EEI. Next we show the characteristics of the R_H oscillations in the presence of the long-range direct electron-electron interaction in Hartree approximation. These oscillations as a function of V_g or the magnetic flux were already observed experimentally a long time ago,[12] but they were never simulated numerically on the basis of a theoretical model. Our calculations put into evidence the electronic *bunching* in the addition process, an effect which was found recently by single-electron capacitance spectroscopy[13] (SECS) and which cannot be explained in the framework of CCM.

The description of the formalism is made in Sec. II, while in Sec. III we discuss briefly some aspects of resonant transport through a noninteracting QD subject to a strong magnetic field. Our main results are established and commented in Sec. IV, the conclusions being isolated in Sec. V.

II. THE FORMALISM

We use a pure quantum-mechanical approach of the transport properties of open quantum dots, which is based on the Landauer-Büttiker formalism and the Hartree approximation for the electron-electron interaction. While the Hartree term is meaningless in an infinite homogeneous system it becomes important for finite systems like QD's. The method is complementary to the semiclassical master-equation approach,[2] goes beyond the constant-interaction model, and is able to account for size, tunneling and interaction effects in quantum dots in the presence of the magnetic field.

Our calculations are based on a lattice model. In spite of the fact that in recent years this model is extensively used for the study of various effects in quantum dots[14,15] we would like to say a few words of caution on this approach to small systems, in the presence of the EEI. When the lattice model is considered as the discretization of a continuous system with a rectangular grid of intersite distance a, the (direct) EEI reads $U\Sigma_{j>i}1/|i-j|c\dagger_i c\dagger_j c_j c_i$, where i,j are the lattice sites and $U=e^2/a$; this means that the strength parameter U depends on the grid which is meaningless. Also the hopping integral $t=\hbar^2/2ma^2$ and the radius $r_s=e^2/(at\sqrt{4\pi\nu})$ (ν being the filling factor) depend on the grid. One may say that the discretization works better at low filling factor (meaning small a or a large number of grid sites). Another way to avoid this paradox is to consider the lattice model as a tight-binding approximation, i.e., to assume that the overlap between the atomic orbitals (and usually, one considers only one type of orbital) located on different sites is small and the effective mass is big. In general, this is not the case for the semiconductors used in the experimental devices. In particular, the strength of the electron-electron Coulomb interaction depends on the dielectric constant of the semiconductor host material, which we incorporate here in our coupling constant U.

The discrete model allows the tailoring of different shapes and introducing the magnetic field as a phase of the hopping integral. The parameters controling the problem are (a) the strength of the LD coupling, (b) the size and shape of the dot and the magnetic flux which, all of them, determine the electronic spectrum in the absence of the EEI, and (c) the strength of the EEI (U).

We model the QD as a two-dimensional (2D) mesoscopic plaquette weakly coupled to four external semi-infinite leads. In the TB approximation the Hamiltonian is written as

$$H=H^D+\sum_\alpha H_\alpha^L+\sum_\alpha H_\alpha^{LD}. \tag{2.1}$$

In the above relation H^D is used to describe the isolated QD, H_α^L characterizes the lead α ($\alpha=1,\dots,4$), while the lead α is coupled to the dot by

$$H_\alpha^{LD}=t^{LD}(c_{0\alpha}^\dagger c_\alpha+c_\alpha^\dagger c_{0\alpha}), \tag{2.2}$$

where the operator $c_\alpha^\dagger$ creates an electron in the dot state $|\alpha\rangle$ and $c_{0\alpha}$ annihilates it in the neighboring lead state $|0\alpha\rangle$. Here t^{LD} is the hopping integral between dot and leads. Since the role of the leads is only to inject and drain the electrons or to probe the potential drop, the EEI will be included only in the Hamiltonian of the dot. In the ''orthodox theory'' the Coulomb effects are mostly studied in the constant-interaction model (see, for instance, Ref. 16) which considers the Hamiltonian $H_{ee}=(e^2/2C)(N-N_g)^2$ (C is the capacitance, N is the operator of the total number of particles, and N_g is the external parameter related to the gate potential). Here, we use a long-range direct Coulomb interaction. Expressed in terms of creation and annihilation operators on localized states indexed by $i\in QD$, the Hamiltonian of the dot reads

$$H^D=\sum_{i,j}\left(t_{ij}^D\ c_i^\dagger c_j+\frac{1}{2}U_{ij}c_i^\dagger c_j^\dagger c_j c_i\right)+\sum_i V_g c_i^\dagger c_i,\qquad i,j\in QD. \tag{2.3}$$

The external gate V_g is simulated by a site energy in H^D. In the Hartree approximation and nearest-neighbors model, it becomes

$$H^D=\sum_i\left(V_g+U\sum_{j>i}\frac{\langle n_j\rangle}{|i-j|}\right)c_i^\dagger c_i+t^D\sum_{\langle i,j\rangle}e^{i2\pi\phi_{ij}}\ c_i^\dagger c_j, \tag{2.4}$$

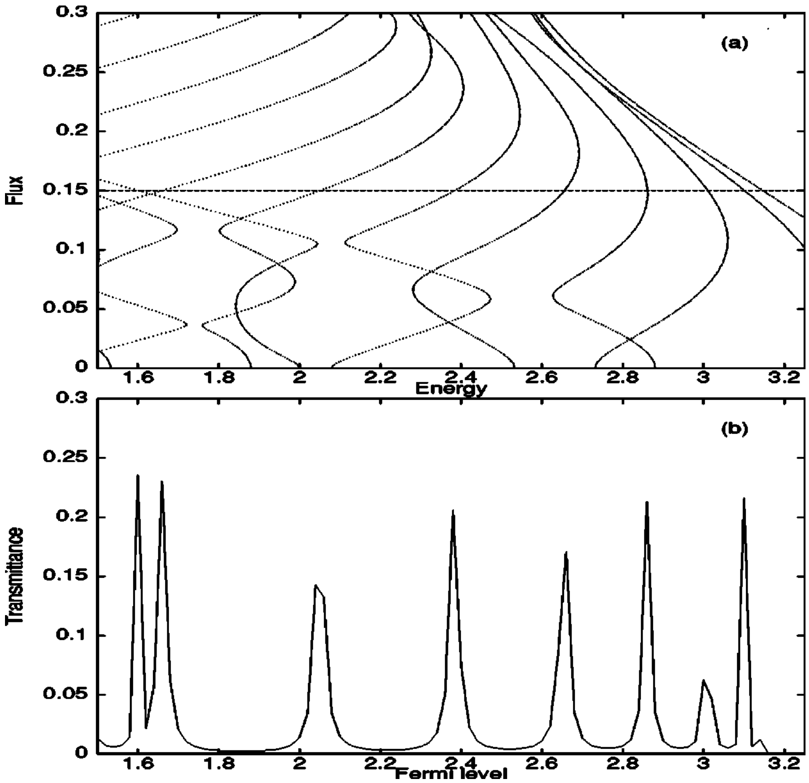

FIG. 1. The correspondence between the transmittance spectrum of a noninteracting QD in a strong magnetic field ($\phi = 0.15$) and the Hofstadter spectrum. Each transmittance peak arises when E_F equals an eigenvalue of the isolated spectrum.

where $\langle n_j \rangle = \langle c_j^\dagger c_j \rangle$ is the mean occupation number of the site j and U is the parameter describing the strength of the EEI. We have chosen $t^D = 1$, i.e., the energy unit is the hopping integral in QD, and we have denoted by $\langle \cdots \rangle$ the nearest-neighbors summation. The Peierls phase ϕ_{ij} is proportional to the magnetic flux through the unit cell $\phi = Ba^2$ measured in quantum flux units ϕ_0. The explicit calculation is made for a rectangular plaquette containing 5×8 sites and the phases correspond to the Landau gauge chosen as in Ref. 17.

At this point a useful ''trick'' is to describe the open dot by an effective Hamiltonian which includes the influence of the leads. Eliminating formally the degrees of freedom of the leads, one obtains a non-Hermitean Hamiltonian depending on the energy,

$$H_{eff}^D(z) = H^D + H^{DL} \frac{1}{z - H^L} H^{LD} = H^D + H^{DL} G^L(z) H^{LD}. \tag{2.5}$$

In the above equation, the Green function $G_L(z)$ of the semi-infinite lead can be calculated analytically,

$$G_{ij}^L(z) = \frac{1}{t_L(\zeta_2 - \zeta_1)} [\zeta_1^{|i-j|} - \zeta_1^{i+j+2}], \quad i,j \in \text{lead} \tag{2.6}$$

where ζ_1 and ζ_2 are the roots of the equation,

$$t_L \zeta^2 - z\zeta + t_L = 0, \quad |\zeta_1| < 1 < |\zeta_2|. \tag{2.7}$$

Let $\zeta_1(z)$ be analytic in the upper half plane. By approaching the real axis from above one obtains $\zeta_1(z) = \zeta(E + i0) = e^{-ik}$, where k is defined by $2t_L \cos k = E$, t_L being the hopping energy of leads. After straightforward manipulations the effective Hamiltonian of the dot is obtained explicitly,

$$H_{eff}^D = H^D + \tau^2 t_L \sum_\alpha e^{-ik} c_\alpha^\dagger c_\alpha. \tag{2.8}$$

The ratio $\tau = t_{LD}/t_L$ defines the degree of constrinction at the contacts and represents an input parameter that can be varied continuously. It is important to observe that the influence of the leads is expressed as a non-Hermitean diagonal term proportional to τ^2, which produces a shift in the real part of the eigenvalues of H^D and introduces also an imaginary part. If $\tau \ll 1$, i.e., for a weakly coupled dot, these shifts become negligible and the spectrum of H_{eff} approaches the spectrum of the isolated dot. This behavior has important consequences on the conductance matrix $g_{\alpha\beta}$, seen as the transmittance $T_{\alpha\beta}$, which can be expressed in terms of the retarded Green function $G^+(E) = (E - H_{\text{eff}} + i0)^{-1}$ by the LB formula

$$g_{\alpha\beta} = \frac{e^2}{h} T_{\alpha\beta} = 4 \frac{e^2}{h} \tau^4 t_L^2 \sin^2 k |G_{\alpha\beta}^+(E_F)|^2, \quad \alpha \neq \beta. \tag{2.9}$$

Once the conductance matrix $g_{\alpha\beta}$ is known, the Hall resistance can be calculated immediately,[20]

$$R_H = (g_{21} g_{43} - g_{12} g_{34})/D, \tag{2.10}$$

where D is a 3×3 subdeterminant of the 4×4 matrix $g_{\alpha\beta}$. The matrix elements of the Green's function, $G_{ij}^+(E)$, are calculated numerically using the self-consistency condition:

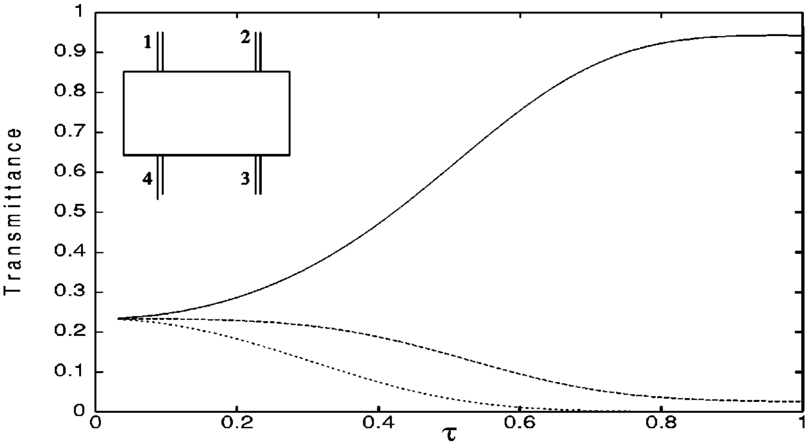

FIG. 2. The dependence of transmittances on constrinction (T_{12}, full line; T_{13}, dashed line; T_{14}, dotted line; $\phi=0.15$). The pinching effect is obvious: at large coupling the conditions for the QHE are fulfilled, while by continuously decreasing LD coupling, T_{ij} evolves smoothly to the same order of magnitude. The four-probe plaquette is shown in the inset.

$$\langle n_j\rangle=\frac{1}{\pi}\int_{-\infty}^{E_F}\mathrm{Im}\,G_{jj}^{+}(E)dE. \tag{2.11}$$

In the weak-coupling limit $\tau\ll1$, the transport problem reduces to a tunneling problem. Indeed, from Eq. (2.9) it follows that the poles of the Green function will induce a series of peaks in the transmittance, and Eq. (2.11) shows that the mean number of electrons in the QD changes abruptly by 1 at every peak, indicating a charge addition process. So, the correspondence between the peaks observed in the transmittance and the charge accumulation in the dot is manifestly established.

Taking into account the strong conditioning of the transport properties (electronic transmittance and Hall resistance) by the energy spectrum of the dot, we have to perform a comparative analysis of spectral properties of H^D, with and without EEI.

III. NONINTERACTING DOT IN STRONG MAGNETIC FIELD

In this section we address the resonant transport through a noninteracting QD, mainly because this simple framework gives a clear picture of the constrinction effects. Moreover, some data about the noninteracting spectrum will be needed in Sec. IV.

As it is known, when periodic boundary conditions are imposed to a *noninteracting* 2D electronic system subjected to a perpendicular magnetic field, the tight-binding approach yields the Harper equation associated with the usual Hofstadter-butterfly spectrum. When the periodic boundary conditions are replaced by the Dirichlet conditions, a ''quasi-Hofstadter'' spectrum is obtained,[18] since the hard-wall potential lifts the degeneracy and the gaps get filled with eigenvalues that correspond to the so-called edge states (which are extended along the edges of the system and are responsible for the quantization of the Hall conductance). Another type of states is the ''bulk states,'' which are grouped in energy bands and geometrically concentrated in the middle of the dot. The nature—bulk or edge—of a given state $\Psi_n(\phi)$ can be checked also by its chirality,[19] i.e., by the sign of the current carried by that state, defined as the slope of the energy level

$$I_n=\frac{dE_n(\phi)}{d\phi}. \tag{3.1}$$

Since the eigenvalues $E_n(\phi)$ are not monotonic functions of the magnetic flux [as can be noticed in Fig. 1(a)], it follows that the nature of the corresponding eigenstate may change from bulk to edge or vice versa when the flux is varied.

For strongly pinched contacts, a continuous variation of the gate potential (or, equivalently, of the Fermi level) gives rise to a resonance peak whenever the Fermi level is aligned to an eigenvalue of the isolated dot (the width of the peak is determined only by the strength of the LD coupling in the noninteracting case). This can be seen in Fig. 1(b) which depicts the transmittance spectrum of the noninteracting quantum dot as function of E_F mapped onto the corresponding piece of the quasi-Hofstadter spectrum.

The modifications in the transmittance induced by pinching is shown in Fig. 2 for T_{12}, T_{13}, and T_{14} in the case of strong magnetic field. One remarks that for completely open QD's (at $\tau=1.0$) the transmittances take the values which describe the quantum Hall regime: all $T_{\alpha,\beta}$ with $\alpha\neq\beta$ vanish except $T_{\alpha,\alpha+1}$.[20] On the other hand, for very weakly coupled QD's ($\tau\ll1$), which corresponds to the resonant tunneling regime, the dwell time of the electron inside the dot increases and all $T_{\alpha,\beta}$ become of the same order of magnitude.

While the transmission spectrum identifies the positions of the levels it cannot specify whether the corresponding states are edge or bulk type. This can, however, be evidentiated by the Hall resistance for simple reasons: if the leads are strongly coupled to the dot, the Hall resistance exhibits quantum Hall plateaus in the range of the spectrum occupied by edge states. At strong constrinction, interference effects occur when the electron travels along the edge states, resulting in oscillations of the resistance in the region of the former plateau. This is shown in Fig. 3(a) where each minimum in the Hall resistance corresponds to a resonance condition (when the Fermi level equals an eigenenergy belonging to an edge state). Note the sudden drop of the Hall resistance between the QH plateaus indicating a narrow bulk domain.

When the electron-electron interaction is considered, some features appear that can be traced from Fig. 3(b), which depicts R_H versus V_g for $U=0.5$. The discussion of this

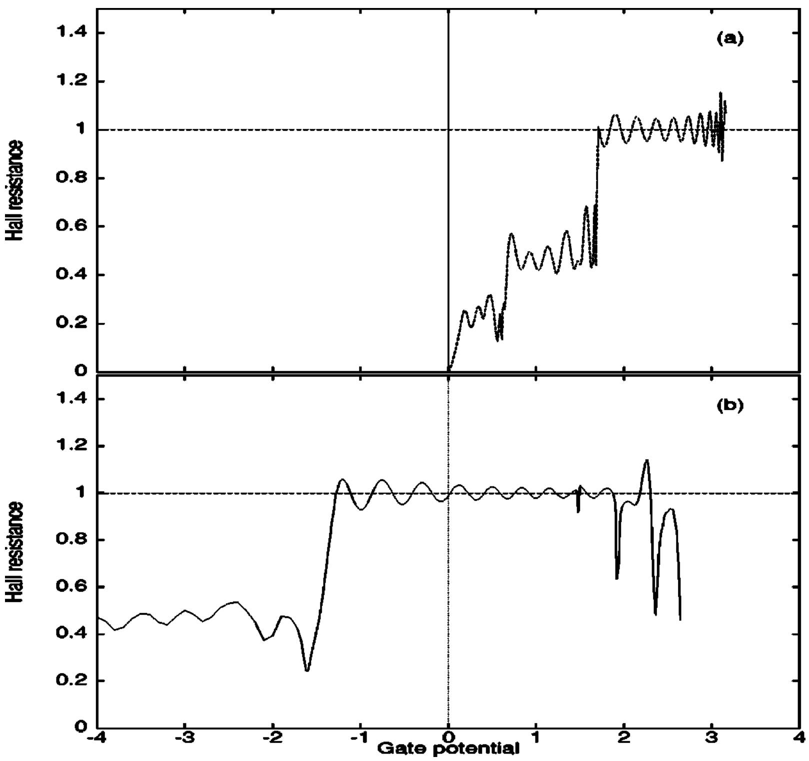

FIG. 3. (a) Quantum oscillations of the Hall resistance for pinched contacts ($\tau=0.5$, $\phi=0.15$, $U=0.0$). Note the sudden drop of R_H between different Hall plateaus. (b) Interaction effects on the Hall resistance ($U=0.5$): the drop of R_H is slower and the oscillations in the range of edge states are widened but their amplitude is poorely affected by EEI.

figure is postponed to the next section where the influence of the EEI on the edge and bulk states will be analyzed.

IV. THE INTERACTING CASE

When the electron-electron interaction is taken into account, important differences appear in the positions and widths of the transmittance peaks and simultaneously in the Hall resistance. This is due to the Coulomb blockade effect, meaning that the addition of an extra electron needs some energy which is not simply the difference between two consecutive one-electron levels, but has also a contribution coming from the electron repulsion. We shall start with some considerations on the addition spectrum which will be useful

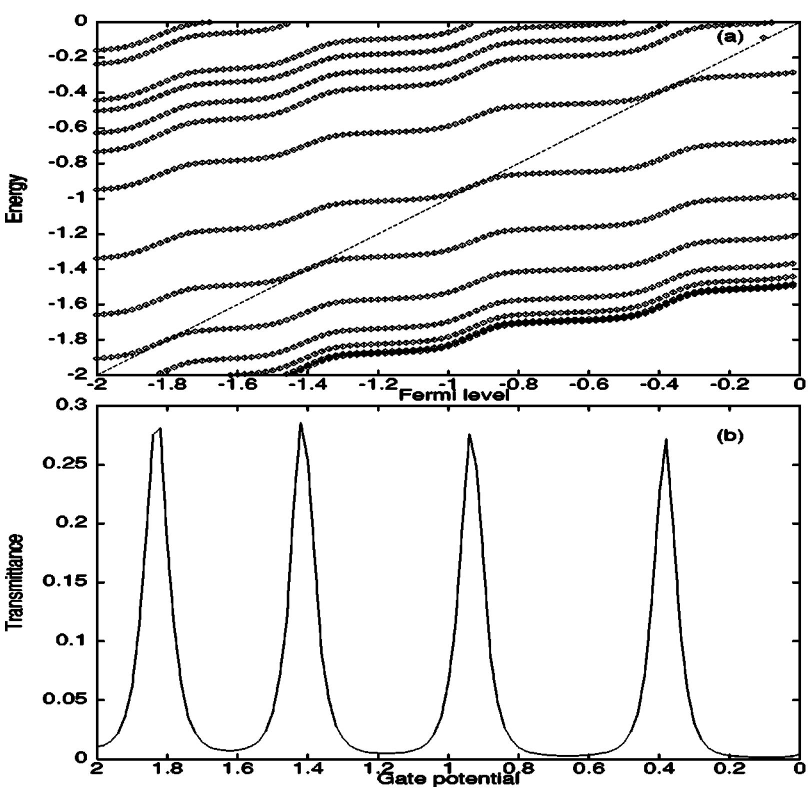

FIG. 4. (a) The Hartree spectrum of an isolated dot in strong magnetic field ($\phi=0.15$, $U=0.5$). (b) The transmittance of a weakly coupled QD ($\tau=0.1$) as a function of the gate potential. The charging energy can be obtained as the width at the bottom of the peaks.

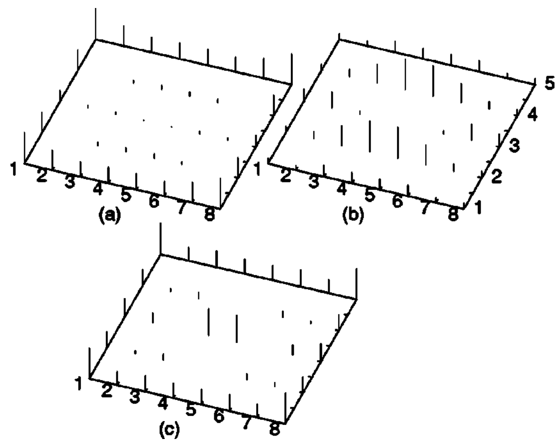

FIG. 5. The spatial distribution of the added electron inside the interacting dot: (a) $\Delta n_i(7,8)$—the eighth electron is added strictly on the edge. (b) $\Delta n_i(8,9)$—the ninth electron is distributed almost uniformly. (c) $\Delta n_i(9,9+2)$—a multiple addition process in which the tenth and eleventh electrons are added together in the dot; a clear addition is made in the bulk, but some maxima are reached also at the corners.

for understanding the features of the resonant transport through QDs in the presence of the interaction. Let $E_n(N)$ be the nth eigenvalue of the system containing N electrons. $E_n(N)$ has a monotonic dependence on the Fermi energy. When E_F (the diagonal line in Fig. 4) approaches $E_n(N)$ the addition of the $N+1$th electron becomes possible and the whole spectrum raises with the *charging energy*: $E_{ch}(N,N+1)=E_n(N+1)-E_n(N)$. One notices from Fig. 4 that the charging energy is not supplied steplike but linearly (with slope=1.0) along an interval $\delta E_F = E_{ch}$; the addition of the extra electron occurs at the *charge degeneracy point* situated in the middle of this interval, where $E_F=[E_n(N+1)-E_n(N)]/2$. The transmittance shows the addition spectrum properties: the peaks point versus the degeneracy points; their widths—measured at the bottom—equals the charging energy and are due to the so-called ‘‘co-tunneling’’ near the degeneracy points.[21]).

Now we turn to discuss Fig. 3(b) and to make the comparison with Fig. 3(a) (the interacting vs the noninteracting case). The similarities of the two figures suggest that the edge states are present also in the interacting case, giving rise to oscillations of R_H on different quantum Hall plateaus. There are, however, qualitative differences: for $U\neq 0$, both the edge and bulk regions are much expanded, so that the whole picture is pushed upwards on the energy scale (in the numerical calculation this is equivalent to large negative V_g). This means that the Coulomb interaction increases the level spacing and a striking consequence is the slower drop of R_H in the region of the bulk states. However, in order to fully establish the nature of the states we have to observe the changing of the local electronic distribution n_i, when exactly one more electron is added, i.e., to calculate

$$\Delta n_i(N,N+1)=n_i(N+1)-n_i(N), \quad i\in QD, \ N=\text{integer} \tag{4.1}$$

In order to make sure that N is an integer, one has to calculate Eq. (4.1) for those values of V_g that ensure an integer number of electrons in QD; they correspond to two consecutive valleys in the transmittance spectrum Fig. 4. The interest in Eq. (4.1) follows from the fact that the map of Δn_i shows how the $(N+1)$th electron is added, namely on the edge or in the bulk. For instance, Fig. 5(a) gives clear proof that the

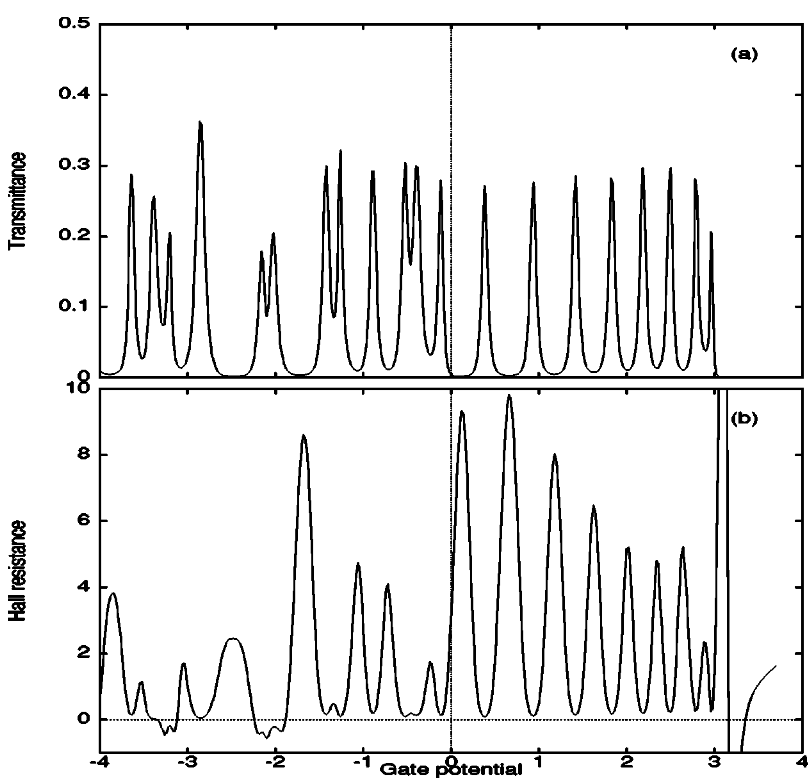

FIG. 6. (a) The transmittance spectrum in strong magnetic field ($\phi=0.15$, $U=0.5$). Note the appeareance of the double peaks associated with multiple addition processes. (b) Oscillations of the Hall resistance induced by strong pinching ($\tau=0.1$). Note that whenever a multiple addition process is allowed the oscillation amplitude is nearly vanished.

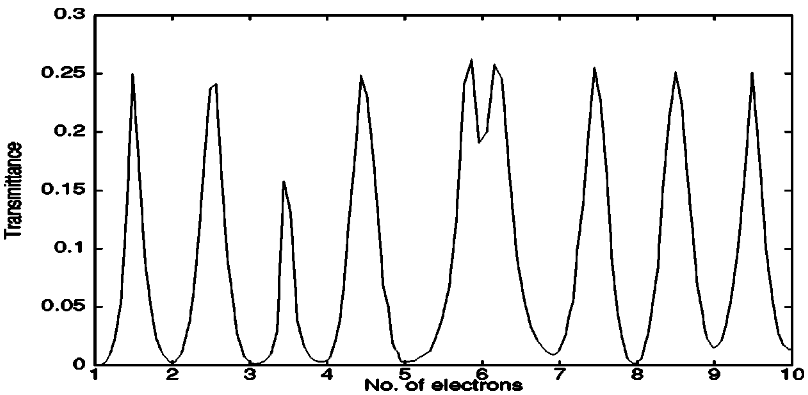

FIG. 7. The dependence of transmittance on the number of electrons at $\phi = 0.15$. Most of the the maximas are reached at half-integer numbers of electrons, but this condition is not obeyed when the bunching appears.

eighth electron is trapped by an edge state. On the other hand, the bulk states are much damaged: this can be seen in Fig. 5(b) where the added electron is more or less distributed everywhere. Equation (4.1) contains the *a priori* assumption that the electrons are added individually. However, recent experimental results suggest that the electrons may enter the dot not only one by one but also in bunches, a fact that can be seen in the addition spectrum.[13] This very ''nonorthodox'' feature has to be noticed also in the transmittance properties of the dot. So, let us discuss the double peaks existing in the calculated transmittance in Fig. 6(a). They become possible when two degeneracy points are very close and the cotunneling effect does not permit their resolution. The origin of this effect consists of two close poles of the resolvent $(E - H_{\rm eff})^{-1}$ fact that yields a multiple addition process, which is nothing else but the ''bunching.'' While the bunching is not allowed in the range of edge states (which are well separated), it appears in the bulk region due to the existence of quasidegenerate states. The way in which the two grouped electrons are distributed in the dot is shown by $\Delta n_i(9,9+2)$ in Fig. 5(c). We stress that, in this case, the distribution has evident maxima at the corners. This corroborates the results obtained recently by Canali.[15] The bunching has important consequences on the oscillation amplitude of R_H, in the sense that the amplitude is suppressed whenever a bunching appears [see Fig. 6(b)].

When the transmittance is plotted against a number of electrons (Fig. 7), interesting features can be noticed: some maxima are reached at a half-integer number of electrons, in accordance with the constant capacitance model in which the additions becomes costless at the charge degeneracy points satisfying the relationship $CV_g/e = N + 1/2$.[22,23] This is clearly violated, however, any time the multiple addition process occurs (see the bunching of the sixth and seventh electrons in the same figure).

Another interesting effect in the presence of the magnetic field is the dependence of the charging energy on the number of particles shown in Fig. 8. Without the field, it was shown in Ref. 4 that the charging energy depends irregularly on N. This behavior changes in strong magnetic field showing a monotonic increase of the charging energy as long as the electrons are added on the edges (this occurs for $3 \leqslant N \leqslant 8$, which also correspond to the quantum oscillations of the Hall resistance on the first plateau).

V. CONCLUSIONS

We have studied the transport properties of interacting quantum dots pierced by a strong magnetic field. The transmittance and the Hall resistance were calculated in the Landauer-Büttiker formalism, in the tight-binding picture (which contains explicitly the dot-lead coupling), while the electron-electron interaction was considered in the Hartree approximation. This approach is able to describe the competition between the size, interaction, and tunneling mechanisms in QD. After proving the essential role of the dot-lead coupling we have obtained the Coulomb oscillations of the transmittance and Hall resistance for various degrees of constriction. The charge degeneracy points were shown to coincide with the minima of Hall resistance and the peaks of

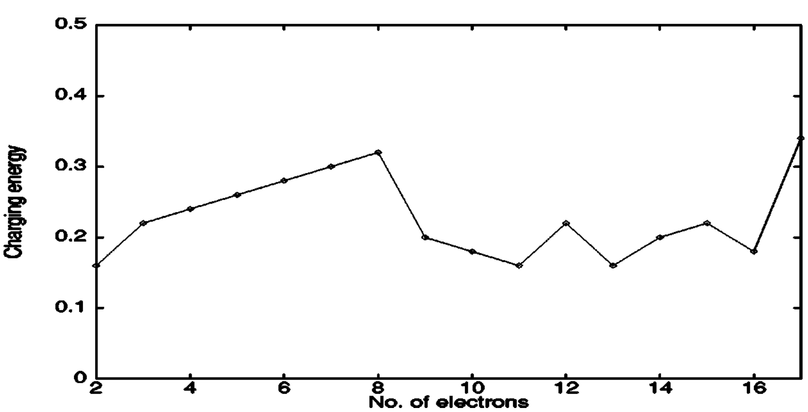

FIG. 8. The charging energy vs the number of electrons at $\phi = 0.15$. E_{ch} shows a monotonic increase as long as the electrons are added on the edge, followed by an irregular behavior.

the transmittance. It turns out that the edge states are stable against interaction, only the interlevel spacing is increased. On the contrary, the bulk states are much more damaged. The labeling of states into edgelike or bulklike was done by an explicit mapping of the spatial distribution for each added electron. We have also presented an elementary explanation for the bunching of electrons recently revealed by SECS experiments. In what concerns the dependence of the charging energy on the number of electrons, it is shown that, in strong magnetic field, E_{ch} increases monotonically with N as long as the Fermi level lies in the region of edge states.

ACKNOWLEDGMENTS

A.M. and A.A. benefited from the hospitality and support of the International Center for Theoretical Physics, Trieste, Italy, at the Research Workshop on Mesoscopic Systems, and through the Associateship Scheme (A.M.). A.A. is very grateful to Professor Johannes Zittartz for his hospitality at the Institute of Theoretical Physics, University of Cologne, where part of this work was performed under Grant No. SFB-341. Valuable discussions with Dr. P. Gartner are acknowledged.

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