Exact solution for a disordered correlated electron model

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The first exact solution for a disordered ensemble of embedded magnetic impurities in a correlated electron chain is presented. We have studied nonlinear equations that exactly determine the thermodynamics of that disordered correlated electron system for arbitrary ranges of the temperature, external magnetic field, and number of electrons. We have shown how strong disorder divergences of the low-temperature susceptibility and specific heat appear, which drastically differ from the homogeneous case. Low-energy asymptotics for the behavior of several characteristics of the disordered correlated electron chain are calculated analytically; they reveal universal properties.

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Recently there has been a great interest in the low-energy behavior of some heavy-fermion compounds[1] and low-dimensional organic salts.[2] There strong correlations between electrons probably preclude a standard Fermi-liquid (FL) behavior. The low-energy non-FL features of these systems are logarithmic (or weak power law) divergences of their magnetic susceptibility χ and low temperature (T), specific heat (Sommerfeld) coefficient γ, nonuniform distribution of the Knight shifts, and linear in T resistivity. Several scenarios were proposed to explain those non-FL data: (a) The enhancement of quantum fluctuations in low-dimensional systems at low T causes itself the non-FL behavior of low-energy characteristics; (b) single magnetic impurity drastically renormalizes the behavior of electron systems, especially for a multichannel Kondo situation;[3] and (c) the non-FL behavior of dilute systems of magnetic impurities was explained by presence of a disorder.[4,1,5] However, the mentioned theories used approximations that can be hardly applied to some non-FL compounds (e.g., to concentrated alloys or to systems without large clusters of ordered phases). To explain some properties of disordered correlated electron systems the use of exact results is highly desirable.

Here we propose the first exactly solvable model of correlated electrons with disorder: The supersymmetric t-J chain (STJC) with the finite concentration of disordered magnetic impurities. The Hamiltonian of the system has the form $H=\Sigma_n H_{n,n+1}+H_{imp}+H_{imp,imp'}$, where we denote $H_{l,m}=-tP(c^{\dagger}_{l,\sigma}c_{m,\sigma}+\text{H.C.})P+J(\vec{S}_l\vec{S}_m-n_l n_m/4)$, where $c^{\dagger}_{l,\sigma}$ creates an electron with the z projection of spin σ at the site l, $\vec{S}_l=c^{\dagger}_{l,\sigma}\hat{S}_{\sigma,\sigma'}c_{l,\sigma'}$, $n_l=\Sigma_\sigma n_{l,\sigma}$, $n_{l,\sigma}=c^{\dagger}_{l,\sigma}c_{l,\sigma}$ and the multipliers $P=(1-n_{l,-\sigma})(1-n_{m,-\sigma})$ exclude the double occupation at each site. In the supersymmetric point the exchange constant is equal to $J=2t=2$ (we equate the hopping integral to unity).[6] By impurities we mean the electrons' sites, the values of hopping integrals and exchange couplings of which differ from the host's (unity). Suppose the impurity number j is situated at the link between the mth and $(m+1)$th site of the host. Then the integrable impurity-host Hamiltonian can be written as[7] $H_{imp}=\Sigma_j J^j_{imp}(H_{m,imp}+H_{imp,m+1}-H_{m,m+1}-i\theta_j[H_{m,imp},H_{imp,m+1}])$, where $J^j_{imp}=[\theta_j^2+1]^{-1}$, $[.,.]$ denotes the commutator, and the index imp denotes the position of an impurity. It is clear that the impurity-host part of the local Hamiltonian has the same form as for the host, with the different overall multiplier J^j_{imp}, i.e., the coupling of each impurity to the host is determined by a single constant θ_j. The case $\theta_j=0$ pertains just to the adding of a one-host site to the system. For $\theta_j\to\infty$, an impurity site is totally decoupled from the host. We have shown[7] that namely this constant determines the effective Kondo scale of a single impurity via $T^j_K\propto\exp(-\pi|\theta_j|)$. For energies higher than this crossover scale, an asymptotically free impurity spin results, whereas for lower energies, the spin of a single impurity is screened, and FL-like behavior persists with the finite magnetic susceptibility and linear temperature dependence of the specific heat at low T,[7] similar to the theory of a Kondo impurity in a host of free electrons.[3] Any number of such impurities can be incorporated in our model.[8,9] They will be characterized by their own couplings to the host, i.e., by their own T^j_K.[10] The disorder is quenched in our model. Previous studies of more generic one-dimensional correlated electron systems with disorder are valid only for relatively weak couplings in the one-loop approximation of the renormalization group and within the replica approach for a quenched disorder.[11] If our impurities are situated not at the nearest-neighbor links of the host, they do not interact with each other. But if the neighboring impurities are situated at the neighboring links of the host, they can interact *directly* via $H_{imp,imp'}=\Sigma_j\theta_j^2J^j_{imp}H_{imp,imp'}$. It does not destroy the integrability of the model.[12] These impurity-impurity couplings can model, e.g., a Ruderman-Kittel-Kasuya-Yosida interaction (being short range though) between impurities in concentrated metallic alloys.

We describe the termodynamics of an ensemble of magnetic impurities randomly coupled to the one-dimensional (1D) STJC by the quantum transfer matrix (QTM) technique.[13] It is based on the mapping of a 1D quantum system to a statistical 2D model (the second coordinate determines the T behavior) by means of Trotter-Suzuki decomposition. The width of that statistical 2D vertex lattice coin-

cides with the length L of the quantum chain, while the height N is the Trotter number. Taking the trace over the auxiliary space of the product of standard (row-to-row from the viewpoint of the vertex model) R matrices,[13] we construct the transfer matrix τ of the inhomogeneous quantum 1D STJC. R matrices satisfy the Yang-Baxter relations, hence the transfer matrices with different spectral parameters commute. The Hamiltonian of our disordered STJC has been constructed as usual, as the logarithmic derivative of the transfer matrix; it commutes with the latter (i.e., it is integrable by construction). The only difference from the homogeneous case is that for the R matrices of impurities their spectral parameter u is shifted by θ_j. Rotating the initial R matrices in the clockwise and anticlockwise directions we construct the transfer matrix $\overline{\tau}$ from those new $\tilde{R}$ matrices in a way similar to τ. Then we substitute $u=-1/NT$, where N is the Trotter number. We find $[\tau(u)\overline{\tau}(u)]^{N/2}=\exp(-H/T)+\mathcal{O}(1/N)$. Hence, the partition function Z of the quantum 1D disordered STJC is identical to the partition function of an inhomogeneous classical vertex model with alternating rows on a square lattice of size $L\times N$, $Z=\lim_{N\to\infty}\mathrm{Tr}[\tau(u)\overline{\tau}(u)]^{N/2}$. Interactions of the vertex model are (alternating) homogeneous in each column, but can vary along the row. It permits us to study exactly the termodynamics of the disordered electron chain.[14,15] Then we construct the (column-to-column) QTM as an alternating product of $R(x+u)$ and their rotated partners $\tilde{R}(x-u)$. It describes the T behavior of the disordered STJC. The free energy per site f is given by only the largest eigenvalue of the QTM Λ as $f=-\lim_{L\to\infty}T/L\,\Sigma_{i=1}^{L}\lim_{N\to\infty}\ln\Lambda(x,\theta_i,u)$ taken at $x=0$. The free energy of the total STJC with impurities is $F=L^{-1}\Sigma_j f(\theta_j/2)$, where the sum is taken over all the sites [for sites without impurities we get $f(0)$].[16] The random distribution of the values θ_j can be described by a distribution function $P(\theta_j)$. An advantage of our model has to be emphasized. Contrasting to the models of disordered dilute Kondo impurities in free-electron hosts used before[1,4] we can study *finite* concentrations of impurities in the thermodynamic limit unlike the *single* impurity nature of the studies.[1,4] In this context, our model shares such a feature with the Griffiths' phase theory.[5] On the other hand, our approach does not demand neither formation of large clusters, nor magnetic anisotropy, in contrast to the Griffiths' phase approach.[5] Impurities are coupled to the *interacting lattice host* (and to the neighboring impurities); all the interactions are exactly taken into account. Hence, our model provides the *nonperturbative* description of the *random ensemble* of impurities in the correlated electron host.[17] Instead of solving Bethe ansatz equations for the STJC, the QTM method suggests to study analytic properties of the eigenvalue of the QTM.[13] From those properties the finite set of nonlinear integral equations for the ''energy density'' functions, which at low T are closely related to Gibbs' exponents of ''dressed energies'' of spin, $a(x)$ and $\overline{a}(x)$, and charge, $c(x)$, excitations of the STJC[6,13] (x is the spectral parameter) is constructed absolutely similar to the analogous procedure

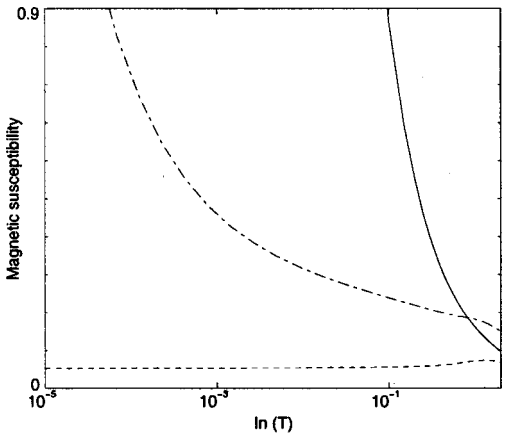

FIG. 1. The magnetic susceptibility versus the logarithm of T for the STJC with the electron density per site 0.9 and $h=0$ for the homogeneous case (dashed line), Lorentzian distribution of θ_j (solid line), and log-normal distribution (dashed-dotted line).

for the homogeneous STJC.[13] It turns out that the parameters of impurities θ_j do not enter those nonlinear equations, which read[13]

$$
\begin{aligned}
T\ln a(x) &= 2\pi\Psi_a(x+i\epsilon)+\mu+(h/2)\\
&\quad -T\Psi_a^{*}\ln(1+\overline{a})\big|_{x+2i\epsilon}-T\Psi_a^{*}\ln(1+c)\big|_{x+i\epsilon},\\
T\ln c(x) &= 2\pi\Psi_c+2\mu-T\Psi_c^{*}\ln(1+c)\\
&\quad -T\Psi_{\bar{a}}^{*}\ln(1+\overline{a})\big|_{x+i\epsilon}-T\Psi_{\bar{a}}^{*}\ln(1+a)\big|_{x-i\epsilon},
\end{aligned}
\tag{1}
$$

where $*$ means convolution, $2\pi\Psi_a(x)=[x(x-i)]^{-1}$, $2\pi\Psi_{\bar{a}}(x)=[x(x+i)]^{-1}$, $\pi\Psi_c(x)=[x^2+1]^{-1}$, $0<\epsilon<1$, μ is the chemical potential, and h is an external magnetic field. The equation for $\overline{a}(x)$ is obtained from the one for a by the replacements $i\to-i$, $h\to-h$ and $a\leftrightarrow\overline{a}$. However the free energy per site of the STJC with impurities *does* depend on θ_j and is given by $f(\theta_j)=2\mu-T\ln c(\theta_j)$ (cf. Ref. 13). The numerical solution of Eqs. (1) shows that for narrow distributions (weak disorder) the disordered STJC is in a singlet state, i.e., the Kondo screening persists. For broad distributions (strong disorder) non-Fermi-liquid behavior is manifested — low-T characteristics like the magnetic susceptibility diverge, i.e., there is no Kondo quenching (see Fig. 1). The low-T divergences disappear upon applying a finite magnetic field, which restores the screening of impurities.

We can analytically solve Eqs. (1) in several important limiting cases. First, for low T the free energy per site is given by $f(\theta_j)\approx e_0(\theta_j)-(\pi T^2/6)[v_c^{-1}(\theta_j)+v_s^{-1}(\theta_j)]+\cdots$, where $e_0(\theta_j)\equiv e_0^j$ is the ground-state energy per site (cf. Ref. 7) and $v_{c,s}(\theta_j)$ are velocities of the charge and spin low-lying excitations of the STJC taken at the associated Fermi points shifted by θ_j. For $\theta_j=0$, it is the known low-T conformal limit of the homogeneous host (cf. Ref. 13). It turns out that the central charges of the semidirect product of charge and spin conformal algebras are equal to one and do not depend on the parameters of impurities θ_j, i.e., *universal*. The only low-energy parameters that get renormalized

by the disorder are the *effective velocities* of low-lying charge and spin excitations. For low densities of electrons (where $\mu \ll T$) for $h=0$ we obtain the free energy per site (we put $\epsilon=1/2$) $f(\theta_j) \approx e_0^j - T\ln(1+2e^{-1/T(\theta_j^2+1)})$. For the high-density regime $\mu \gg T$ one can use the approximation $\ln c \approx \ln(1+c)$. This yields $T\ln a = (\pi/\cosh \pi x) + (h/2) + Tk(x)*\ln(1+a) - Tk(x+i)*\ln(1+\overline{a})$ and similar for $\overline{a}$, with the kernel $k(x)=(2\pi)^{-1}\int d\omega e^{i\omega x}[1+e^{|\omega|}]^{-1}$. The free energy per site becomes $f(x)=e_0(x)+\mu - T\int dy \ln(1+a)(1+\overline{a})/\cosh \pi(x-y)$. One can recognize in these nonlinear equations the ones of Ref. 18 for the disordered Heisenberg spin $\frac{1}{2}$ chain. It is clear because in the limit of large μ the electron density per site is equal to one (the largest possible value for the STJC).[19] In the low-T (conformal) regime the lattice effects are non-essential and the couplings of impurities to the host can be considered as contact ones.[7] Typically the corrections to the low-T asymptotics of thermodynamic characteristics of, e.g., the Heisenberg spin chain[20] and of a single magnetic impurity[3,7] manifest logarithmic behavior (singularities). Its origin can be traced back to the marginal operators existing for models with SU(2) spin symmetry (present in the STJC). To know how logarithmic singularities in the low-T susceptibility and specific heat get renormalized for the disordered STJC in the high-density regime (which is most important because it pertains to the Kondo magnetic behavior for impurities[7]), we perform an analytic low-T study of Eqs. (1). We introduce scaling functions[3] $a_{\pm}(x) \equiv a(\pm x \pm Ln)$, where $Ln=\ln(\alpha T_K^j/T)$ (α is some constant)[7] etc. Equation (1) are transformed, so that for the new set of the scaling functions the only known asymptotic behavior of ''energy density'' functions $1+a_{\pm}$ and $1+\overline{a}_{\pm}$ at large spectral parameter enters. Then, following Ref. 13 we obtain (at $h=0$) the free energy of the dense limit of the STJC per site $f(T_K^j)=e_0^j+\mu-(\pi T^2/6T_K^j)[1+3(2Ln)^{-3}]+\ldots$. For nonzero field $h \ll T$ we calculate the logarithmic T corrections for the free energy per site $f(T_K^j)=e_0^j(h)+\mu-(\pi T^2/6T_K^j) - (h^2/4\pi T_K^j)[1+(2Ln)^{-1}-(2Ln)^{-2}\ln(2Ln)]+O(T^2)$. The groundstate magnetization per site in a weak external field is $M \sim h/T_K^j$ for $h \ll T_K^j$, cf. Ref. 7. Notice that for the dense limit of the low-T behavior of the STJC, the dependence on θ_j enters only as T_K^j, i.e., as the distributions of the characteristic velocities of spin excitations (or crossover scales, which pertain to each impurity). (It is not the case for higher energies and for lower densities, but those are not important for low-T disorder-driven divergencies.) Hence for low energies we can use distributions of T_K^j, which are also more appropriate in connection to experiments.[1,2] For a single impurity $P(T_K^j)=\delta_{T_K^j,T_K}$ we recover the logarithmic Kondo behavior of an asymtotically free spin (which is characteristic both to a Kondo impurity in a free electron host[3] and to a single magnetic impurity in the STJC).[7] For the case of the homogeneous STJC, we put $\theta_j=0$. It means $T_K^j \to v_s$, where v_s is the (Fermi) velocity of low-lying spin excitations (spinons) of the STJC.

Let us show how low-T divergences can result from wide distributions of T_K. Suppose one has the (strong disorder) distribution beginning with the term $P(T_K^j) \propto G^{-\lambda}(T_K^j)^{\lambda-1}$ ($\lambda<1$) valid until some energy scale G.[1] Then averaging, e.g., the low-T parts of χ and γ we obtain $\langle\chi\rangle \propto \langle\gamma\rangle \sim T^{\lambda-1}$. They are divergent, in a drastic contrast to the homogeneous STJC (cf. Ref. 13). The ground-state average magnetization of the disordered dense STJC reveals $M^z \sim (h/G)^{\lambda}$ behavior, also different from the homogeneous case. The average compressibility for the high-density limit also reveals the low-T divergence. We calculate the average dynamic magnetic susceptibility $\langle\chi''\rangle(\omega,T)$. The standard ansatz for the relaxational form of that susceptibility of a single magnetic impurity[4] $\chi''(\omega,T)=\chi(T)\Gamma(T)\omega[\omega^2+\Gamma^2(T)]^{-1}$ can be used, in which one supposed that the relaxation rate Γ does not depend on the frequency ω. (That ansatz satisfies the Kramers-Kronig relations.) At low T, the use of the Shiba approximation[4] determines the first ($T=0$) term in the series of $\Gamma(T)$ via $\lim_{\omega\to\infty}\chi''(\omega,0)/\pi\omega = 2\chi^2(0)$. That yields the low-$T$ dependence of the relaxation rate per site for the disordered chain $\Gamma(T) \sim T_K^j$. Hence, we get $\langle\chi''\rangle(\omega,T) \sim G^{-1}(G/T)^{\lambda-1}g(\omega/T)$ with g being the universal scaling function $g(x)=x\int_1^{\infty}dy/y^{\lambda-1}(x^2+y^2)$. It drastically differs from the homogeneous case. Similar calculations, e.g., for the variation of the Knight shift and for the NMR relaxation rate yield $\delta K/K \propto \delta\chi/\chi \sim T^{-\lambda/2}$ (where δA denotes the mean square deviation of A due to the distribution of T_K^j) and $T_1^{-1} \sim G^{-1}(G/T)^{\lambda-2}g(\omega/T)$. Consider the important *marginal* case $\lambda=1$. Here, one has the distribution with $P(T_K^j=0)=G^{-1}\neq 0$. Then, averaging the low-T part of the susceptibility and Sommerfeld coefficient, we obtain $\langle\chi\rangle \propto \langle\gamma\rangle \sim -(1/2\pi G)[\ln(G/T)+\ln\sqrt{\ln(\alpha G/T)}+\ldots]$. Here we see the logarithmic (more weak) divergencies of $\langle\chi\rangle$ and $\langle\gamma\rangle$. The low-field average ground-state magnetization is $\langle M^z\rangle \sim -(h/G)\ln(h/G)$. The weak power-law or logarithmic dependences pertain to the Griffiths' singularities in the proximity to the critical point $T=0$ (cf. Refs. 5 and 15). The dynamic magnetic susceptibility reveals the scaling behavior $\langle\chi''\rangle(\omega,T) \sim G^{-1}[(\pi/2)-\tan^{-1}(2GT/0.41\pi\omega)]$ (which is again in a drastic contrast to the homogeneous STJC). Notice that for these distributions of T_K^j, the Wilson ratio at $T=0$ is a constant, like in a FL case.

In conclusion, in this work we have introduced the first exactly solvable model of disordered strongly correlated electrons. We have studied the finite set of nonlinear integral equations, which totally determine the thermodynamics of the system. Analytic expressions for low-temperature, magnetic-field and frequency dependences of several important characteristics of our model were calculated. We found the universal features for the low-energy behavior of the model: The only parameters that get renormalized due to the disorder are effective velocities of low-lying excitations (crossover scales connected with each site of the model). The finite-size corrections to the ground-state behavior of the disordered STJC can be obtained by replacing $(G/T) \to L$. Our results for the low-energy behavior of the integrable 1D supersymmetric t-J model with disordered impurities surprisingly coincide with theoretical results for a single multichannel Kondo impurity,[3] dilute disordered Kondo impurities, the

Griffiths' phase theory,[21] and qualitatively agree with the previous approximate approaches for more generic models.[11] They also agree to the data of experiments on the disordered 1D organic conductors and, surprisingly, on 3D heavy metallic alloys.[22] This probably manifests generic features of concentrated disordered electron systems above critical points in our 1D exact solution for the disordered correlated electron model.[23]

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[8] Impurities of our model are only *elastic*, i.e., any excitation of the system only scatters off each of these *dynamic* impurities but does not reflect (notice, however that backscattering, i.e., the transfers from one Fermi point to the other due to impurities, is taken into account in our lattice model exactly). It is in a drastic contrast to the approach[9] in which a magnetic impurity was modeled by a *static boundary potential* at the edge of an open chain. We have proved that the magnetic behavior of such bulk impurities and of the impurity situated at the edge of the open chain (where an impurity is connected to the host only via $J_{imp}H_{L,imp}$) coincide (Ref. 7).

[9] See, e.g., P. Fendley, Phys. Rev. Lett. **71**, 2485 (1993); P. Fendley, A. W. W. Ludwig, and H. Saleur, *ibid.* **74**, 3005 (1995); *ibid.* **75**, 2196 (1995).

[10] Eigenstates of our integrable model do not depend on spatial distribution of impurities. However real systems often exhibit a large degeneracy as a function of the distribution of impurities for stoichiometric compounds (Ref. 1).

[11] See, e.g., G. Theodorou, Phys. Rev. B **16**, 2254 (1977); T. Giamarchi and H. J. Schulz, Phys. Rev. B **37**, 325 (1988); D. S. Fisher, *ibid.* **50**, 3799 (1994).

[12] Such an interaction between neighboring impurities produces impurity-impurity scattering matrices, which satisfy Yang-Baxter relations with impurity-host and host-host ones (the latters as well as the ''standard'' Bethe ansatz equations, with the trivial generalization for the finite concentration case, can be found, e.g., in Ref. 7) and mutually. Here we limit ourselves by a small concentration of those neighboring impurities' sites.

[13] See, e.g., G. Jüttner, A. Klümper, and J. Suzuki, Nucl. Phys. B **487**, 650 (1997); A. Klümper, Eur. Phys. J. B. **5**, 677 (1998;), and references therein.

[14] Namely the same property permitted to integrate the only previously known integrable model with disorder — the McCoy-Wu model (Ref. 15). However the McCoy-Wu model in its 1D quantum realization corresponds to a disordered Ising chain in a transverse magnetic field. The latter can be mapped by use of the Jordan-Wigner transformation onto the model of *noninteracting spinless* lattice fermions, whereas in our case there is an essential coupling between spinful electrons.

[15] B. M. McCoy and T. T. Wu, Phys. Rev. **176**, 631 (1968).

[16] Yang-Baxter relations for any Bethe ansatz-solvable model imply the factorization of all scattering processes. This manifests itself in the additive character of the free energy in the QTM scheme (the sum over all sites, impurities, and hosts). This additive character also exists for the homogeneous host (i.e., without impurities) (Ref. 13). Hence our impurities are as generic as host sites themselves, including the half filled case of a well-known spin-$\frac{1}{2}$ Heisenberg chain.

[17] An advantage of the QTM method is revealed in the *finite* number of nonlinear integral equations that exactly describe the thermodynamics. It is in a contrast with the use of the ''string hypothesis'' technique (see, e.g., Ref. 3), where the number of nonlinear integral equations is *infinite*. One needs some truncation procedure when solving that infinite set of equations numerically.

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[19] Close to half-filling the most important contribution to the low-energy properties of electron system with magnetic impurities should be connected with low-lying spin excitations (Ref. 3). The behavior of charge low-lying excitations pertains to higher energy scales and is nonuniversal (Ref. 7). Naturally, at half filling our results for the STJC coincide with the ones for the Heisenberg chain (Ref. 18).

[20] I. Affleck *et al.*, J. Phys. A **22**, 511 (1989).

[21] In recent studies, the distributions of T_K (used here) were derived either from the proximity to a phase transition point in the Griffiths' phase approximation (claster percolation), or from the Anderson localization effects in the infinite-dimension statistical dynamical mean-field approximation: A. H. Castro Neto and B. A. Jones, cond-mat/0003085 (unpublished); E. Miranda and V. Dobrosavljević, cond-mat/0003209 (unpublished).

[22] Comparing data of recent experiments (Refs. 1 and 2) with our results we find qualitative agreement for some types of disorder: $UCu_{5-x}Pd_x$ exhibits logarithmic (as well as in $CeNi_2Ge_2$) or weak power-law divergences with $\lambda \sim 0.66-0.84$; in TCNQ salts the exponents are $0.26-0.42$, for $CeRhRuSi_2$ it was 0.88, and for $U_{1-x}Th_xPd_2Al_3$ it was 0.9.

[23] The localization is a subtle point in our approach, though: bound states (present both due to interactions and impurities) are not localized because of the periodic boundary conditions used.