Effect of rare locally ordered regions on a disordered itinerant quantum antiferromagnet with cubic anisotropy

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We study the quantum phase transition of an itinerant antiferromagnet with cubic anisotropy in the presence of quenched disorder, paying particular attention to the locally ordered spatial regions that form in the Griffiths region. We derive an effective action where these rare regions are described in terms of static annealed disorder. A one-loop renormalization-group analysis of the effective action shows that for order-parameter dimensions $p<4$, the rare regions destroy the conventional critical behavior, and the renormalized disorder flows to infinity. For order-parameter dimensions $p>4$, the critical behavior is not influenced by the rare regions; it is described by the conventional dirty cubic fixed point. We also discuss the influence of the rare regions on the fluctuation-driven first-order transition in this system.

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I. INTRODUCTION

Quenched disorder can have very drastic influences on the critical behavior of a system undergoing a continuous phase transition. According to the Harris criterion,[1] the critical behavior of a clean system is unaltered by disorder, if the correlation length critical exponent ν obeys the inequality $\nu>2/d$, where d is the spatial dimensionality of the system. In the opposite case, $\nu<2/d$, the clean critical behavior is unstable, and the disorder either leads to a new, different universality class or to an unconventional critical point or even to the destruction of the phase transition.

Another, less well-understood consequence of quenched disorder is the formation of rare locally ordered regions in the disordered phase. For a transition occurring at a finite temperature, this can be explained in the following way. In general, disorder leads to the suppression of the critical temperature from its clean value T_c^0 to T_c. In the temperature region between T_c^0 and T_c, the system does not show long-range order. However, there will be arbitrarily large regions that are devoid of impurities and thus order locally. The probability of finding such regions usually decreases exponentially with their size; they represent nonperturbative degrees of freedom. These locally ordered regions are known as rare regions, and the order-parameter fluctuations induced by them as local moments or instantons. Griffiths[2] showed that the rare regions lead to a nonanalytic free energy everywhere in the temperature region between T_c^0 and T_c, now called the Griffiths region or Griffiths phase. In generic classical systems, this is a very weak effect, and the nonanalyticity in the free energy is only an essential one. However, the Griffiths singularities become stronger if the disorder is spatially correlated. McCoy and Wu[3] studied a two-dimensional Ising model where the disorder is perfectly correlated in one spatial direction and uncorrelated in the other. In this model, the rare regions lead to the divergence of the susceptibility at some temperature T_χ within the Griffiths region.

A very interesting question is what is the influence of the rare regions on the critical behavior of a system? Dotsenko *et al.*[4] studied this question for a weakly disordered classical ferromagnet. They found that the conventional theory of critical behavior[5] in this system is unstable with respect to replica symmetry breaking. They also showed that the rare regions actually induce replica symmetry-breaking perturbations and thus destabilize the conventional critical fixed point. While so far no final conclusion about the fate of the transition in the weakly disordered ferromagnet could be reached, the occurrence of replica symmetry breaking raises the possibility of an unconventional transition with activated scaling, as is believed to occur in the random-field Ising model.[6]

For quantum phase transitions,[7] which occur at zero temperature as a function of some nonthermal control parameter, one expects an even stronger influence of the rare regions than for classical transitions. The reason is that a quantum model with uncorrelated quenched disorder is effectively equivalent to a classical model with the disorder being perfectly correlated in one dimension (the imaginary time dimension). Fisher[8] investigated the critical behavior of a one-dimensional quantum Ising spin chain in a transverse field, which is equivalent to the classical McCoy-Wu model. He found that due to the rare regions, the critical behavior is controlled by an infinite-disorder fixed point, which leads to activated scaling. Recently, analogous behavior was found in random quantum Ising systems in higher dimensions.[9] These results have been confirmed by numerical simulations in one[10] and two[11] dimensions. However, there are indications[9] that a continuous order-parameter symmetry weakens the effect of the rare regions. This could lead to a finite-disorder fixed point with more conventional scaling.

In two recent papers,[12] we developed a systematic approach to rare regions at quantum phase transitions of itinerant electrons in $d>1$. In this approach, the rare regions were identified nonperturbatively as the inhomogeneous saddle-point solutions of the order-parameter field theory. The interaction between the rare regions and the order-parameter fluctuations led to a new term in the effective action that was of the form of annealed static disorder. The resulting effective field theory was then studied using renormalization-group methods. In the case of the quantum antiferromagnetic transition, this new term resulted in the destruction of the conventional critical fixed point if the number p of order-parameter components was smaller than 4. No new fixed point could be identified, and the system displayed runaway flow to large disorder strength. On the other hand, for the quantum ferromagnetic transition, the rare regions did not affect the critical behavior, since a self-induced long-range interaction suppressed all fluctuations including those produced by the local moments.

In this paper, we apply the approach developed in Ref. 12 to a model of an itinerant antiferromagnet with an additional interaction term with cubic symmetry. This model is equivalent to a weakly disordered classical ferromagnet with cubic anisotropy, in which the disorder is perfectly correlated in some of the spatial dimensions but uncorrelated in the remaining dimensions. The conventional theory for this model (without taking rare regions into account) has been developed by Yamazaki, Holz, Ochiai, and Fukuda.[13]

The purpose for this work is threefold. We want to investigate (i) whether the conventional critical fixed point is stable under the influence of the rare regions. If it is unstable, we want to find out (ii) whether a new stable fixed point exists that describes a rare region-driven transition. Finally we want to study (iii) the influence of the rare regions on the fluctuation-driven first-order transition occurring in our system. The layout of the paper is as follows. In Sec. II we derive the effective field theory by taking into account the disorder-induced rare regions. In Sec. III, we carry out the renormalization-group analysis. Finally, Sec. IV is left for a summary of our results.

II. AN EFFECTIVE ACTION FOR DISORDERED ANTIFERROMAGNETS WITH CUBIC ANISOTROPY

A. The model

In 1976, Hertz[14] derived an order-parameter field theory for the description of the antiferromagnetic quantum phase transition of itinerant electrons. Later this model was generalized to the dirty case by making the distance from the critical point a random function of position.[12,15] Here we consider an extension of this order-parameter field theory by incorporating an additional ϕ^4 term which possesses a (hyper)cubic symmetry.

In terms of the p-component order-parameter field $\boldsymbol{\phi}$ (with components ϕ_i), the total action can be written as

$$S[\boldsymbol{\phi}]=S_{\mathrm{G}}[\boldsymbol{\phi}]+S_{\mathrm{int}}[\boldsymbol{\phi}]+S_{\mathrm{cubic}}[\boldsymbol{\phi}], \tag{2.1a}$$

with the Gaussian part, the interaction part, and the cubic anisotropic part given by

$$S_{\mathrm{G}}[\boldsymbol{\phi}]=\frac{1}{2}\int dx\, dy \sum_i \phi_i(x)\Gamma(x-y)\phi_i(y), \tag{2.1b}$$

$$S_{\mathrm{int}}[\boldsymbol{\phi}]=u\int dx \sum_{i,j} \phi_i(x)\phi_i(x)\phi_j(x)\phi_j(x), \tag{2.1c}$$

$$S_{\mathrm{cubic}}[\boldsymbol{\phi}]=\lambda\int dx \sum_i \phi_i^4(x). \tag{2.1d}$$

Here we use a four-vector notation to combine the real-space coordinate $\mathbf{x}$ and imaginary time τ, $x=(\mathbf{x},\tau)$, $\int dx =\int d\mathbf{x}\int_0^{1/T} d\tau$. The bare two-point function

$$\Gamma(\mathbf{x}-\mathbf{y},\tau-\tau')=\Gamma_0(\mathbf{x}-\mathbf{y},\tau-\tau')+\delta(\mathbf{x}-\mathbf{y})\delta(\tau-\tau')\delta t(\mathbf{x}) \tag{2.2}$$

consists of the deterministic part derived by Hertz,[14] whose Fourier transform reads

$$\Gamma_0(\mathbf{q},\omega_n)=t_0+\mathbf{q}^2+|\omega_n|, \tag{2.3}$$

and a disorder part in the form of a ''random mass'' term. Here, $\mathbf{q}$ is the wave vector, ω_n is a bosonic Matsubara frequency, and $\delta t(\mathbf{x})$ is a random function of position and is endowed with the following statistical properties:

$$\langle \delta t(\mathbf{x})\rangle=0, \tag{2.4a}$$

$$\langle \delta t(\mathbf{x})\delta t(\mathbf{y})\rangle=\Delta\,\delta(\mathbf{x}-\mathbf{y}). \tag{2.4b}$$

B. Inhomogeneous saddle points and annealed disorder

In the conventional approach to critical behavior in systems with quenched disorder,[5] the disorder average is carried out at the beginning of the calculation by means of the replica trick.[16] A subsequent perturbative analysis of the resulting, spatially homogeneous effective theory misses the rare regions we are interested in since they are nonperturbative degrees of freedom.

We therefore follow the approach developed in Ref. 12, and work with a particular realization of the disorder rather than integrating it out. Let us consider spatially inhomogeneous, but time-independent saddle-point solutions of the action (2.1) (time-dependent saddle-point solutions—if any—will always have a higher free energy since the disorder is static). Depending on the sign of the cubic interaction term, the structure of the saddle points in the p-dimensional order-parameter space will be different. When $\lambda>0$, the free energy is minimized by saddle-point solutions that lie on the diagonals of a p-dimensional hypercube, while when $\lambda<0$, the free energy is minimized by solutions that lie on the axis of the hypercube. In either case, the modulus ϕ_{sp} of these minimizing saddle-point solutions fulfills the equation

$$[t_0+\delta t(\mathbf{x})-\partial_{\mathbf{x}}^2]|\boldsymbol{\phi}_{\mathrm{sp}}(\mathbf{x})|+4u_{\mathrm{eff}}|\boldsymbol{\phi}_{\mathrm{sp}}(\mathbf{x})|^3=0, \tag{2.5a}$$

$$u_{\mathrm{eff}}=\begin{cases} u+\dfrac{\lambda}{p} & \text{for } \lambda>0\\ u+\lambda & \text{for } \lambda<0. \end{cases} \tag{2.5b}$$

Although $\phi_{\mathrm{sp}}(\mathbf{x})=0$ is always a solution, there will be spatially inhomogeneous solutions if $\delta t(\mathbf{x})$ has sufficiently deep and wide troughs.[12] Let us now consider the Griffiths region, i.e., the region where the average distance t_0 from the critical point is positive but where there are isolated islands that support a nonzero $\phi_{\rm sp}$. If we have N such islands that are sufficiently apart from each other, the global saddle-point solutions may be written as

$$\boldsymbol{\phi}_{\rm sp}^{\{\sigma_I\}}(\mathbf{x}) \equiv \Phi^{\{\sigma_I\}}(\mathbf{x}) = \sum_{I=1}^{N} \psi_I(\mathbf{x}) \sigma_I, \qquad (2.6)$$

where $\psi_I(\mathbf{x})$ is a solution of Eq. (2.5) on the island I, and σ_I is a unit vector in spin space (on one of the axes for $\lambda<0$ or on one of the diagonals for $\lambda>0$).

Since the direction of the order parameter on each of the N islands can be chosen independently, Eq. (2.6) describes an exponentially large number of degenerate saddle points, $(2p)^N$ for $\lambda<0$ and $(2^p)^N$ for $\lambda>0$. To be precise, the saddle points are not exactly degenerate due to the residual interaction of the (exponentially small) tails of the order parameter between the islands. The complicated structure of the free-energy landscape connected with the existence of an exponentially large number of almost degenerate saddle points will finally turn out to be responsible for the failure of the conventional approach.

We now consider fluctuations around the saddle points (2.6). Since the saddle points are separated by large free-energy barriers, an expansion around one of them will not give a good representation of the partition function of the entire system. Instead we will restrict ourselves to small fluctuations and simply add the contributions coming from *all* of the saddle points. Thus the partition function for a particular realization $\delta t(\mathbf{x})$ of the disorder can be written as

$$Z[\delta t(\mathbf{x})] \approx \sum_{\{\sigma_I\}} \int_< D[\varphi(x)] e^{-S[\Phi^{\{\sigma_I\}}(\mathbf{x})+\varphi(x),\,\delta t(\mathbf{x})]}. \qquad (2.7)$$

Here, $\int_<$ indicates that the integration is restricted to small fluctuations $\boldsymbol{\varphi}$ only.

We now carry out the sum over the saddle-point configurations. The residual interaction between the islands will lead to slight deviations of the saddle-point function from the ideal one given in Eq. (2.6). This is taken into account by replacing the sum over the saddle points by an integral over a probability distribution

$$P[\Phi] \sim \exp\left(-\frac{1}{T}\int dx \mathcal{L}^{\rm sp(\Phi)} \right). \qquad (2.8)$$

The temperature factor in the exponent reflects the fact that the saddle points are classical (static) degrees of freedom.[18] Expanding in powers of the fluctuations, we obtain the following effective action for the fluctuations $\boldsymbol{\varphi}$ (still for a particular disorder realization):

$$\begin{aligned} S_{\rm eff} - S^{\rm SP} &= S_{\rm G}[\boldsymbol{\varphi}] + S_{\rm int}[\boldsymbol{\varphi}] + S_{\rm cubic}[\boldsymbol{\varphi}] \\ &\quad + T\bar{w}\int dx dy\, C(x,y) \sum_{i,j} \varphi_i^2(x)\varphi_j^2(y) \\ &\quad + \text{higher-order terms}. \end{aligned} \qquad (2.9)$$

The correlation function $C(x,y)$ measures, up to a constant factor determined by the precise form of $\mathcal{L}$, whether $\mathbf{x}$ and $\mathbf{y}$ belong to the same island, and $\bar{w} = [(2+4/p)u + 6\lambda/p]$ is a positive constant. The $\bar{w}$ term is produced by the interaction of the fluctuations with the rare regions. It is our approximation of the effect of these nonperturbative degrees of freedom. Terms of higher than fourth order in $\boldsymbol{\varphi}$ also arise, but they are renormalization-group irrelevant at both the Gaussian and the nontrivial fixed points of the conventional theory (see below).

Having identified the effects of the rare regions, we now use the replica trick[16] to perform the quenched disorder average over $\delta t(\mathbf{x})$, which implies an average over position and size of the rare regions. The resulting effective action reads

$$\begin{aligned} S_{\rm eff}[\boldsymbol{\varphi}^\alpha(x)] &= \frac{1}{2}\sum_\alpha \sum_i \int dx\, dy\, \Gamma_0(x-y) \boldsymbol{\varphi}_i^\alpha(x)\boldsymbol{\varphi}_i^\alpha(y) \\ &\quad + u\sum_\alpha \sum_{i,j} \int d\mathbf{x}\, d\tau [\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)]^2[\boldsymbol{\varphi}_j^\alpha(\mathbf{x},\tau)]^2 \\ &\quad + \lambda\sum_\alpha \sum_i \int d\mathbf{x}\, d\tau [\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)]^4 \\ &\quad - \Delta\sum_{\alpha,\beta} \sum_{i,j} \int d\mathbf{x}\, d\tau d\tau' [\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)]^2[\boldsymbol{\varphi}_j^\beta(\mathbf{x},\tau')]^2 \\ &\quad - T\bar{w}\sum_{\alpha,\beta} \sum_{i,j} \int d\mathbf{x}\, d\tau d\tau' [\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)]^2[\boldsymbol{\varphi}_j^\alpha(\mathbf{x},\tau')]^2. \end{aligned} \qquad (2.10)$$

Here, the first four terms are identical to the result of the conventional treatment. The fifth term has the form of static, annealed disorder and represents the interaction of the fluctuations with the rare regions in the Griffiths phase. For more details of this derivation, see Ref. 12.

III. RENORMALIZATION-GROUP ANALYSIS

A. Flow equations

We first consider the effective action (2.10) at tree level. As usual, let us define the scale dimension of a length L to be $[L]=-1$, and that of imaginary time τ to be $[\tau]=-z$, with z being the dynamical critical exponent. We first analyze the Gaussian fixed point. From the Gaussian part of the action (2.10), we see that ω_n scales as q^2, implying that $z=2$. The scale dimension of the field is $[\boldsymbol{\varphi}]=d/2$. Power counting for the interaction and disorder terms of the action gives the scale dimensions of u, λ, Δ, and $\bar{w}$ as $[u]=[\lambda]=[\bar{w}]=2-d$, and $[\Delta]=4-d$. Here we have used the fact that in Matsubara formalism the temperature scales like a frequency $[T]=z$. Consequently, u,λ, and $\bar{w}$ are irrelevant for $d>2$, while Δ is irrelevant only for $d>4$. This implies that in the physical dimension $d=3$, the Gaussian fixed point is unstable, and we must carry out a loop expansion of the effective action (2.10) close to $d=4$. All terms of higher order in $\boldsymbol{\varphi}$ that arise in addition to those given in Eq. (2.10) have negative scale dimensions at and close to $d=4$. Thus, they

TABLE I. Fixed points of the flow equations; p is the number of order-parameter components.

No.	FP values			
	u^*	λ^*	Δ^*	$\bar{w}^*$
1	0	0	0	0
2	$\tilde{\epsilon}/4(p+8)$	0	0	0
3	0	$\tilde{\epsilon}/36$	0	0
4	$\tilde{\epsilon}/12p$	$\tilde{\epsilon}(p-4)/36p$	0	0
5	0	0	$-\epsilon/32$	0
6	$(3\epsilon-2\tilde{\epsilon})/16(p-1)$	0	$[(p+8)\epsilon-2(p+2)\tilde{\epsilon}]/64(p-1)$	0
7	0	$O(\epsilon^{1/2})$	$O(\epsilon^{1/2})$	0
8	$(3\epsilon-2\tilde{\epsilon})/24(p-2)$	$[(3\epsilon-2\tilde{\epsilon})(p-4)]/72(p-2)$	$[3p\epsilon-4(p-1)\tilde{\epsilon}]/96(p-2)$	0
9	0	0	0	$-\tilde{\epsilon}/4p$
10	$\tilde{\epsilon}/4(p+8)$	0	0	$[(p-4)\tilde{\epsilon}]/4p(p+8)$
11	0	$\tilde{\epsilon}/36$	0	$-\tilde{\epsilon}/12p$
12	$\tilde{\epsilon}/12p$	$[(p-4)\tilde{\epsilon}]/36p$	0	$(p-4)\tilde{\epsilon}/12p^2$
13	0	0	$(\epsilon-2\tilde{\epsilon})/64$	$(2\tilde{\epsilon}-3\epsilon)/16p$
14	$(3\epsilon-2\tilde{\epsilon})/8(10-p)$	0	$[(p+8)\epsilon-12\tilde{\epsilon}]/32(10-p)$	$[(3\epsilon-2\tilde{\epsilon})(p-4)]/8p(10-p)$
15	0	$(3\epsilon-2\tilde{\epsilon})/72$	$(9\epsilon-12\tilde{\epsilon})/288$	$-3(3\epsilon-2\tilde{\epsilon})/72p$
16	$(3\epsilon-2\tilde{\epsilon})/48$	$(3\epsilon-2\tilde{\epsilon})(p-4)/144$	$[3p\epsilon-2(p+2)\tilde{\epsilon}]/192$	$(3\epsilon-2\tilde{\epsilon})(p-4)/48p$

are irrelevant by power counting with respect to both the Gaussian and the conventional nontrivial fixed points.

As in the conventional theory,[13,15,17] we carry out the perturbation theory in $d=4-\epsilon$ spatial dimensions and ϵ_τ time dimensions. In this way, the perturbation expansion becomes a double expansion in terms of ϵ and ϵ_τ. The renormalization-group flow equations are obtained by performing a frequency momentum shell renormalization-group (RG) procedure.[14] To one-loop order, we obtain the following flow equations:

$$\frac{du}{dl}=\tilde{\epsilon}u-4(p+8)u^2+48u\Delta-24u\lambda, \tag{3.1a}$$

$$\frac{d\lambda}{dl}=\tilde{\epsilon}\lambda-36\lambda^2+48\lambda\Delta-48u\lambda, \tag{3.1b}$$

$$\frac{d\Delta}{dl}=\epsilon\Delta+32\Delta^2-8(p+2)u\Delta+8p\Delta\bar{w}-24\Delta\lambda, \tag{3.1c}$$

$$\frac{d\bar{w}}{dl}=\tilde{\epsilon}\bar{w}+4p\bar{w}^2-8(p+2)u\bar{w}+48\Delta\bar{w}-24\lambda\bar{w}. \tag{3.1d}$$

Here we have defined $\tilde{\epsilon}=\epsilon-2\epsilon_\tau$. Of course, the distance t from the critical point will also be renormalized. However, we only consider the flow on the critical surface $t=0$ since we are interested in the stability of the critical fixed points. Note that the coefficient of the rare region term $\bar{w}$ only couples to Δ. The flow of u and λ is only indirectly influenced by the rare regions (via a modification of the flow of Δ). This will be important later on.

B. Fixed points and their stability

The flow equations (3.1) possess 16 fixed points. Their fixed point (FP) values are given in Table I, and the eigenvalues of the corresponding linearized renormalization-group transformations are listed in Table II.
For eight of the 16 fixed points (Nos. 1–8 in Table I), the fixed-point value of the rare region term is $\bar{w}^*=0$. These fixed points have already been studied in Ref. 13 using the conventional approach. In the following, we concentrate on the case $\epsilon>0$ and $\tilde{\epsilon}=\epsilon-2\epsilon_\tau<0$ relevant for the itinerant quantum antiferromagnet.

We first consider the dirty Heisenberg fixed point (No. 6) and the dirty cubic fixed point (No. 8). These are the stable fixed points of the conventional theory for the cases of $p<4$ and $p>4$, respectively. Analyzing the stability matrix for the dirty Heisenberg fixed point shows that it is unstable since the eigenvalue e_4 is positive for $p<4$. In contrast, the dirty cubic fixed point remains stable for $p>4$ since all eigenvalues of the stability matrix are negative. Thus, we conclude that the rare regions destroy the conventional dirty Heisenberg critical behavior for $p<4$ while they do not influence the conventional dirty cubic critical behavior for $p>4$.

We now turn to the new fixed points with $\bar{w}^*\neq 0$ (Nos. 9–16 in Table I). Fixed points 9, 11, 13, and 15 are unphysical because their fixed-point values $\bar{w}^*$ are negative. Since the bare $\bar{w}$ is positive, and according to Eq. (3.1d) the flow cannot cross the $(\bar{w}=0)$ plane, these fixed points can never

TABLE II. Eigenvalues of the corresponding linearized RG transformation. p is the number of order-parameter components. A, B, C, and D are defined as $A=(p+8)\epsilon-2(p-4)\tilde{\epsilon}$, $B=16(p-1)(3\epsilon-2\tilde{\epsilon})[(p+8)\epsilon-2(p+2)\tilde{\epsilon}]$, $C=(p+8)\epsilon-2(p-4)\tilde{\epsilon}$, and $D=8(10-p)(3\epsilon-2\tilde{\epsilon})[8\epsilon-12\tilde{\epsilon}+p\epsilon]$. Analogously, $E=3p\epsilon+2(p-4)\tilde{\epsilon}$, $F=24(p-2)(3\epsilon-2\tilde{\epsilon})[4\tilde{\epsilon}+3p\epsilon-4p\tilde{\epsilon}]$, $G=8\tilde{\epsilon}+3p\epsilon-2p\tilde{\epsilon}$, and $H=48(3\epsilon-2\tilde{\epsilon})[-4\tilde{\epsilon}+3p\epsilon-2p\tilde{\epsilon}]$.

No.	Eigenvalues			
	e_1	e_2	e_3	e_4
1	$\tilde{\epsilon}$	$\tilde{\epsilon}$	ϵ	$\tilde{\epsilon}$
2	$-\tilde{\epsilon}$	$(p-4)\tilde{\epsilon}/(p+8)$	$\epsilon-2(p+2)\tilde{\epsilon}/(p+8)$	$-(p-4)\tilde{\epsilon}/(p+8)$
3	$\tilde{\epsilon}/3$	$-\tilde{\epsilon}$	$\epsilon-2\tilde{\epsilon}/3$	$\tilde{\epsilon}/3$
4	$-\tilde{\epsilon}$	$-\tilde{\epsilon}(p-4)/3p$	$\epsilon-4\tilde{\epsilon}(p-1)/3p$	$-\tilde{\epsilon}(p-4)/3p$
5	eigenvalues not calculated since FP is unphysical			
6	$\frac{-A+\sqrt{A^2-B}}{p-1}$	$\frac{-A-\sqrt{A^2-B}}{p-1}$	$(p-4)(3\epsilon-2\tilde{\epsilon})/4(p-1)$	$-(p-4)(3\epsilon-2\tilde{\epsilon})/4(p-1)$
7	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{1/2})$
8	$\frac{-E+\sqrt{E^2-F}}{12(p-2)}$	$\frac{-E-\sqrt{E^2-F}}{12(p-2)}$	$-(3\epsilon-2\tilde{\epsilon})(p-4)/6(p-2)$	$-(3\epsilon-2\tilde{\epsilon})(p-4)/6(p-2)$
9	eigenvalues not calculated since FP is unphysical			
10	$-\tilde{\epsilon}$	$(p-4)\tilde{\epsilon}/(p+8)$	$\epsilon-12\tilde{\epsilon}/(p+8)$	$(p-4)\tilde{\epsilon}/(p+8)$
11	eigenvalues not calculated since FP is unphysical			
12	$-\tilde{\epsilon}$	$-\tilde{\epsilon}(p-4)/3p$	$\epsilon-2\tilde{\epsilon}(p+2)/3p$	$\tilde{\epsilon}(p-4)/3p$
13	eigenvalues not calculated since FP is unphysical			
14	$\frac{-C+\sqrt{C^2-D}}{4\,(10-p)}$	$\frac{-C-\sqrt{C^2-D}}{4\,(10-p)}$	$(p-4)(3\epsilon-2\tilde{\epsilon})/2(10-p)$	$(p-4)(3\epsilon-2\tilde{\epsilon})/2(10-p)$
15	eigenvalues not calculated since FP is unphysical			
16	$\frac{-G+\sqrt{G^2-H}}{24}$	$\frac{-G-\sqrt{G^2-H}}{24}$	$(3\epsilon-2\tilde{\epsilon})(p-4)/12$	$-(3\epsilon-2\tilde{\epsilon})(p-4)/12$

be reached. Depending on the number p of order-parameter components, the remaining fixed points (Nos. 10, 12, 14, and 16) are either also unphysical or they are unstable. Consequently, for $p<4$ and to one-loop order, there is no stable fixed point. Renormalization-group trajectories, which in the conventional theory would go to the dirty Heisenberg fixed point, show runaway flow to large disorder strength. This runaway flow could either indicate a unconventional phase transition, e.g., an infinite-disorder critical point as in the one-dimensional random Ising model,[8] or a percolative rather than a homogeneous transition, or even a destruction of the phase transition. Within the present approach, we cannot decide between these alternatives.

The influence of the rare regions on the stability of the fixed points in our model is similar to that in the isotropic case.[12] For $p<4$, the conventional fixed point is destroyed in both models. For $p>4$, the conventional fixed point is stable. In our model, this is the dirty cubic fixed point, while in the isotropic case this stable fixed point is the dirty Heisenberg fixed point.

C. The fluctuation-driven first-order transition

In addition to the continuous phase transitions associated with the critical points discussed above, there is also the possibility for a first-order transition in the model considered here. Let us first discuss the mechanism for a clean system and discuss the effects of disorder and rare regions later.

According to a mean-field stability analysis of the effective action (2.10) with $\Delta=\bar{w}=0$, the inequalities $u+\lambda>0$ (for $u>0$) and $u+\lambda/p>0$ (for $u<0$) have to be fulfilled for the theory to be stable. Now consider a bare theory with $u<0,\lambda>0$ or $u>0,\lambda<0$, but still fulfilling the above stability conditions. In these cases, the flow equations (3.1) can lead the renormalization-group trajectories to the mean-field unstable region. This indicates a fluctuation-driven first-order transition.[19,20] It was later shown[21,22] that the fluctuation-driven first order in this model survives the presence of quenched disorder, at least within the conventional theory. Let us now consider the influence of the rare regions. As already mentioned, the rare region coefficient $\bar{w}$ does not couple into the flow equations for u and λ, but only into the flow equation for Δ. Thus a renormalization-group trajectory going to the mean-field unstable region within the conventional theory will generically also do so in the presence of rare regions, the only modification being a different disorder value at the stability boundary.

Therefore, we conclude that the fluctuation-driven first-order transition also occurs when taking the rare regions into account. However, since the rare regions modify the flow of the disorder strength Δ, the boundaries of the first-order region may change.

IV. SUMMARY AND CONCLUSIONS

We have investigated the influence of rare regions on the quantum phase transition of a disordered itinerant antiferromagnet with cubic anisotropy. In this final section we want to summarize the results and discuss the potential and limitations of our approach.

Our method consists of two main parts: First, we consider a particular realization of the disorder potential. We identify the inhomogenous saddle-point solutions of the field theory for this disorder realization. Physically, the inhomogeneous saddle points describe the formation of local magnetic moments on the rare regions. The interaction between the local moments and the order-parameter fluctuations generates a new term in the effective field theory, which has the form of static annealed disorder. This first part of our method is nonperturbative with respect to the rare regions.

Once we have identified the rare regions and their coupling to the order-parameter fluctuations, we perform the average over all possible disorder realizations. Next, in order to study the critical properties of the model in question, we perform a momentum-shell renormalization procedure. In order to control the perturbation theory, we implement a double expansion in terms of $(4-\epsilon)$ spatial dimensions and ϵ_τ imaginary time dimensions. This part of our procedure is perturbative, and hence, it is clear that it will be useful to describe fixed points at which the renormalized disorder is zero or finite. Our method cannot describe infinite-disorder fixed points as those found in random Ising systems.[8,9] However, *a priori*, one does not know whether a given transition is described by a fixed point with finite or infinite disorder.

Applying our method to the disordered itinerant antiferromagnet with cubic anisotropy, we have found that for order-parameter dimension $p>4$ the rare regions do not change the critical behavior, which is characterized by the dirty cubic fixed point. In contrast, for $p<4$, the rare-region term renders the conventional critical fixed point unstable. The renormalization-group trajectories show runaway flow to large disorder. Within our approach, we cannot determine the ultimate fate of the transition. It could be an infinite-disorder critical point as in the random quantum Ising systems; however, there are other possibilities, e.g., an inhomogeneous transition, a first-order transition, or even the complete destruction of the phase transition. We have also found that the fluctuation-driven first-order transition occurring in this model in certain parameter regions remains qualitatively unchanged by the local moments, while the precise position of the first-order region in parameter space will change.

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[1] A.B. Harris, J. Phys. C **7**, 1671 (1974); J. Chayes, L. Chayes, D.S. Fisher, and T. Spencer, Phys. Rev. Lett. **57**, 2999 (1986).

[2] R.B. Griffiths, Phys. Rev. Lett. **23**, 17 (1969).

[3] B.M. McCoy and T.T. Wu, Phys. Rev. **176**, 631 (1968); **188**, 982 (1969); B.M. McCoy, *ibid.* **188**, 1014 (1969); See also R. Shankar and G. Murthy, Phys. Rev. B **36**, 536 (1987).

[4] Viktor Dotsenko, A.B. Harris, D. Sherrington, and R.B. Stinchcombe, J. Phys. A **28**, 3093 (1995); Viktor Dotsenko and D.E. Feldman, *ibid.* **28**, 5183 (1995).

[5] For a pedagogical discussion, see G. Grinstein, in *Fundamental Problems in Statistical Mechanics VI*, edited by E.G.D. Cohen, (Elsevier, New York 1985), p. 147, and references therein.

[6] J. Villain, J. Phys. (Paris) **46**, 1843 (1985); D.S. Fisher, Phys. Rev. Lett. **56**, 416 (1986); A.E. Nash, A.R. King, and V. Jaccarino, Phys. Rev. B **43**, 1272 (1991).

[7] For recent reviews of quantum phase transitions, see, e.g., S.L. Sondhi, S.M. Girvin, J.P. Carini, and D. Shahar, Rev. Mod. Phys. **69**, 315 (1997); D. Belitz and T.R. Kirkpatrick, cond-mat/9811058 (unpublished); T. Vojta, Ann. Phys. (Leipzig) **9**, 403 (2000).

[8] D.S. Fisher, Phys. Rev. B **51**, 6411 (1995).

[9] O. Motronich, S.-C. Mau, D.A. Huse, and D.S. Fisher, Phys. Rev. B **61**, 1160 (2000).

[10] H. Rieger and F. Iglói, Europhys. Lett. **39**, 135 (1997); J. Kisker and A.P. Young, Phys. Rev. B **58**, 14 397 (1998).

[11] C. Pich, A.P. Young, H. Rieger, and N. Kawashima, Phys. Rev. Lett. **81**, 5916 (1998).

[12] R. Narayanan, T. Vojta, D. Belitz, and T.R. Kirkpatrick, Phys. Rev. Lett. **82**, 5132 (1999); Phys. Rev. B **60**, 10 150 (1999).

[13] Y. Yamazaki, A. Holz, M. Ochiai, and Y. Fukuda, Phys. Rev. B **33**, 3460 (1986).

[14] J. Hertz, Phys. Rev. B **14**, 1165 (1976).

[15] T.R. Kirkpatrick and D. Belitz, Phys. Rev. Lett. **76**, 2571 (1996); **78**, 1197 (1997).

[16] S.F. Edwards and P.W. Anderson, J. Phys. F: Met. Phys. **5**, 965 (1975).

[17] S.N. Dorogovtsev, Phys. Lett. **76A**, 169 (1980); D. Boyanovsky and J.L. Cardy, Phys. Rev. B **26**, 154 (1982).

[18] In Ref. 12, the temperature factor was found to be necessary to give sensible flow equations. Its relation to the Boltzman factor of the static saddle points was realized only later.

[19] J. Rudnick, Phys. Rev. B **18**, 1406 (1978).

[20] H. Jacobsen and D.J. Amit, Ann. Phys. (N. Y.) **133**, 57 (1981).

[21] Y. Yamazaki, M. Ochiai, A. Holz, and Y. Fukuda, Phys. Rev. B **33**, 3474 (1986).

[22] J. Cardy, J. Phys. A **29**, 1897 (1996).