Ferromagnetic film on a superconducting substrate

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We study the equilibrium domain structure and magnetic flux around a ferromagnetic film with perpendicular magnetization M_0 on a superconducting (SC) substrate. At $4\pi M_0 < H_{c1}$ the SC is in the Meissner state and the equilibrium domain width in the film, l, scales as $(l/4\pi\lambda_L) = (l_N/4\pi\lambda_L)^{2/3}$ with the domain width on a normal (nonsuperconducting) substrate, $l_N/4\pi\lambda_L \gg 1$; λ_L being the London penetration length.

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The interaction between magnetism and superconductivity has been intensively studied in the past, see, e.g., Ref. 1. The discovery of high-temperature superconductors and advances in manufacturing of nanoscale multilayered systems have added new dimensions to these studies. In this paper we investigate equilibrium magnetic and superconducting phases in a system consisting of a ferromagnetic (FM) film with perpendicular magnetic anisotropy[2] on the surface of a superconductor (SC). The needs of magneto-optic technology have produced a large variety of magnetic films with perpendicular magnetic anisotropy. Some are synthesized on metallic substrates (e.g., Nb) and are well characterized at room temperature, including their domain structure.[3–5] We will show that, as the temperature of such a system is lowered, its magnetic state must be affected, in a nontrivial manner, by the superconducting transition.

The system under consideration is shown in Fig. 1. We are assuming no exchange of electrons between the FM film and the superconductor. This will be true either when the ferromagnet is an insulator or when it is separated from the superconductor by a thin insulating buffer layer. The systems with the exchange of electrons between the FM and SC layers have been discussed in Refs. 6,7. (This situation corresponds to very thin layers with parallel magnetization, which do not interact via Meissner effect.) In our case the FM film and the superconductor are coupled only by the magnetic field. In this case the superconductivity makes a profound effect on the domain structure in the FM layer, which must be easy to detect in experiment. The physics behind this effect is explained below. Consider a FM film of thickness d_M, with perpendicular magnetic anisotropy. In the absence of the superconductor adjacent to the film, its domain structure is determined by the balance of the energy of the magnetic field surrounding the film and the energy of domain walls. The positive energy of the magnetic field favors small domains, so that the field does not spread too far from the film. On the contrary, the positive energy of domain walls favors less walls, that is, large domains. The minimization of the total magnetic energy gives a well-known result[8] for the equilibrium domain width $l \propto \sqrt{\delta d_M}$, with δ being the domain wall thickness. Domains typically observed in magneto-optic films have thickness of a few microns. In the presence of a superconductor adjacent to the FM film, the balance of the magnetic energy changes drastically. This is because the magnetic field must be either expelled from the superconductor due to the Meissner effect or it should penetrate into the superconductor in the form of vortices. The Meissner state should always be the case when $4\pi M_0 < H_{c1}$, where M_0 is the magnetization. At $4\pi M_0 > H_{c1}$ the equilibrium energy of the Meissner state should be compared with the energy of the vortex state. Such a study goes beyond the framework of this paper and will be done elsewhere.[10] In the Meissner case, the superconductor favors FM domains of width below the London penetration depth λ_L. If the room temperature domains are significantly greater then λ_L, the effect of the SC phase transition on the domain structure will be dramatic. As we shall see, the new equilibrium will be achieved at $l \propto (\delta d_M \lambda_L)^{1/3}$. Consequently, on lowering the temperature below the SC critical temperature, the domains in the FM film can shrink by an appreciable factor.

We are assuming the stripe domain structure in the FM film. The width of the FM domain l is presumed large compared with the domain wall thickness δ. The latter is the smallest length in our consideration. Two other characteristic lengths are the thickness of the FM film d_M and the London penetration depth λ_L of the SC. In the case of $l < \lambda_L$ the magnetic flux penetrates into the SC as it would penetrate into a normal nonmagnetic metal, making superconductivity irrelevant. The case of interest is, therefore, $l > \lambda_L$. We shall begin with the study of the Meissner state, that is, the state where equilibrium vortices are absent.

The free energy functional for the magnetic field $\mathbf{B} = [B_x(x,z), 0, B_z(x,z)]$ is

$$\mathcal{F}(\mathbf{B},\mathbf{M}) = \mathcal{F}_S(\mathbf{B},\mathbf{M}) + \mathcal{F}_M(\mathbf{B},\mathbf{M}), \tag{1}$$

where

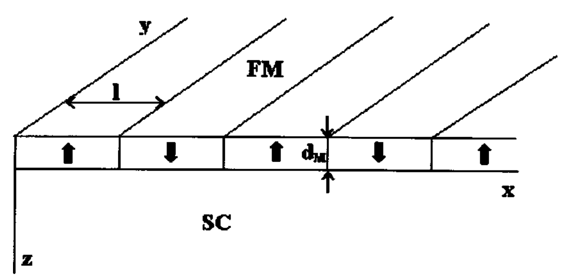

FIG. 1. FM film with stripe domains on a SC substrate.

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$$\mathcal{F}_S(\mathbf{B}) = \frac{1}{8\pi}\int dV[\lambda_L^2(\nabla\times\mathbf{B})^2+\mathbf{B}^2],$$

$$\mathcal{F}_M(\mathbf{B},\mathbf{M}) = \int dV\left[\frac{\mathbf{B}^2}{8\pi}-\mathbf{B}\cdot\mathbf{M}\right]+\mathcal{F}_D. \tag{2}$$

Here $\mathcal{F}_S$ is the free energy due to the magnetic field in the superconductor, $\mathcal{F}_M$ is the free energy of the magnetic film and the empty space above the film, $\mathbf{M}(x)$ is the magnetization inside the magnetic film, and $\mathcal{F}_D$ is the energy of domain walls. At $l \gg \delta$, a good approximation for $M(x)$ (see Fig. 1) is the steplike function along the X axis, $M(x) = \pm M_0$ inside the domains. Its Fourier expansion is

$$M(x) = \frac{4M_0}{\pi}\sum_{k=0}^{\infty}\frac{\sin[(2k+1)Qx]}{2k+1}, \tag{3}$$

where $Q=2\pi/l$. For this domain structure $\mathcal{F}_D(l)=\sigma d_M/l$. Here $\sigma=\sqrt{2}\beta M_0^2\delta/\pi$ is the energy of the unit area of the domain wall and βM_0^2 is the energy density of the perpendicular magnetic anisotropy.

The equilibrium distribution of the magnetic field should be obtained by the minimization of $\mathcal{F}(\mathbf{B},\mathbf{M})$ at given configurations of magnetic domains $\mathbf{M}(x)$. Introducing $\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}$, one obtains in terms of $\mathbf{H}$:

$$\mathcal{F}_M(\mathbf{H},\mathbf{M}) = \int dV\left[\frac{\mathbf{H}^2}{8\pi}-2\pi M^2\right]+\mathcal{F}_D,$$

$$\mathcal{F}_S(\mathbf{H}) = \mathcal{F}_S(\mathbf{B}=\mathbf{H}). \tag{4}$$

The field $\mathbf{H}$ is induced by alternating magnetic charges, $\nabla\cdot\mathbf{M}$, on the two surfaces of the magnetic film.[9] With account of the Maxwell equation, $\nabla\cdot\mathbf{B}=0$, it satisfies

$$\nabla\cdot\mathbf{H} = -4\pi\nabla\cdot\mathbf{M} = -4\pi[\delta(z)-\delta(z+d_M)]M(x),$$

$$\nabla\times\mathbf{H}=0 \tag{5}$$

outside the superconductor, that is, inside the magnetic film, in the buffer layer, and in the empty space above the film. Here $\delta(z)$ is the delta function, $z=0$ and $z=-d_M$ are coordinates of the film surfaces. Inside the superconductor $\mathbf{H}$ satisfies the London equation

$$\nabla^2\mathbf{H}-\lambda_L^{-2}\mathbf{H}=0, \tag{6}$$

and the boundary condition that $\mathbf{H}$ is continuous across the interface between the buffer layer and the superconductor. Equation (6) is valid if the magnetic field changes on the scale greater than the correlation length ξ. In our case the smallest relevant scale of spatial variations of the field is the width of FM domains l. We shall assume that $l \gg \xi$, which is relevant to most situations of practical interest.

Solving the above equations we obtain that, due to the domain structure, $\mathbf{H}(x,z)$ decays exponentially away from the surfaces. Taking into account that $d_M \gg l$ and neglecting exponentially small terms of order $\propto \exp[-d_M(4\pi^2 l^{-2}+\lambda_L^{-2})^{1/2}]$, we get

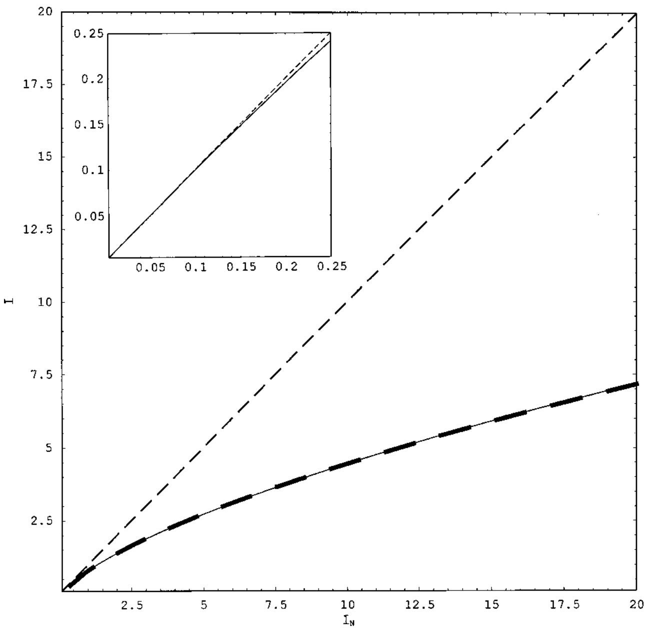

FIG. 2. Solid line: Exact dependence of $\overline{l}$ (normalized equilibrium domain width on the SC substrate) on $\overline{l_N}$ (normalized domain width on the normal substrate) obtained by the minimization of the total free energy, Eq. (13). Thick dashed line: Approximation of $\overline{l}(\overline{l_N})$ by Eq. (14). Thin dashed line: Normal substrate. The inset shows the dependence of $\overline{l}$ on $\overline{l_N}$ at small $\overline{l_N}$.

$$\mathbf{H}(x,z) = \sum_q \mathbf{H}_q \exp(-q_z|\tilde{z}|+iqx) \tag{7}$$

for the magnetic field inside the superconductor, the film, and in the empty space. Here $\tilde{z}$ is the distance along the z axis from the nearest film surface. This gives the Fourier components

$$H_{z,q} = \sum_{k=0}^{\infty}\frac{4M_0}{2k+1}\delta[q-Q(2k+1)],$$

$$H_{z,-q} = -\sum_{k=0}^{\infty}\frac{4M_0}{2k+1}\delta[q+Q(2k+1)], \tag{8}$$

and $H_{x,q}=-iq_zH_{z,q}/q$ with $q_z^2=q^2+\lambda_L^{-2}$ inside the superconductor and $q_z=q$ elsewhere. Substituting this equilibrium magnetic field $\mathbf{H}$ at a given l into the free energy functional, Eq. (4), we obtain the following expressions for $\mathcal{F}_S(l)$ and $\mathcal{F}_M(l)$ per unit area:

$$\mathcal{F}_S(l) = \frac{4M_0^2}{\pi Q^2\lambda_L}\sum_{k=0}^{\infty}\frac{[1+(2k+1)^2Q^2\lambda_L{}^2]^{1/2}}{(2k+1)^4}, \tag{9}$$

$$\mathcal{F}_M(l) = 3\mathcal{F}_S(l,\lambda_L^{-1}=0)+\mathcal{F}_D(l). \tag{10}$$

Above T_c the free energy of the system as a function of l is given by

$$\mathcal{F}_N(l) = 4\mathcal{F}_S(l,\lambda_L^{-1}=0)+\mathcal{F}_D. \tag{11}$$

The minimization of Eq. (11) gives the well known result for the equilibrium width of the domains when the superconductor is in the normal state[8]

$$l_N=\left[\frac{\sqrt{2}\pi}{7\zeta(3)}\right]^{1/2}(\beta\delta d_M)^{1/2}. \tag{12}$$

For the superconducting state of the substrate it is convenient to introduce $\overline{l}=l/4\pi\lambda_L$ and $\overline{l_N}=l_N/4\pi\lambda_L$. Then

$$\mathcal{F}(\overline{l})=\frac{8M_0^2\lambda_L}{\pi}\sum_{k=0}^{\infty}\frac{\overline{l}}{(2k+1)^3}\times\left\{3+\left[1+\frac{4\overline{l}^2}{(2k+1)^2}\right]^{1/2}+\frac{4\overline{l}_N^2}{\overline{l}^2}\right\}. \tag{13}$$

The minimization of $\mathcal{F}$ with respect to $\overline{l}$ produces the dependence of $\overline{l}$ on $\overline{l}_N$ shown in Fig. 2. At $\overline{l}_N \ll 1$ the field penetrates into the SC the same way as it penetrates into the normal metal and $\overline{l}\approx\overline{l}_N$ (see inset to Fig. 2). It is easy to obtain from Eq. (13) that the assymptotic dependence of $\overline{l}$ on $\overline{l}_N$ in the limit of large $\overline{l}_N$ is $\overline{l}=\overline{l}_N^{2/3}$. As is demonstrated in Fig. 2, a rather accurate approximation at $\overline{l}_N>1$ is

$$\overline{l}=\overline{l}_N^{2/3}-0.2. \tag{14}$$

We, therefore, conclude that the SC phase transition in the substrate can result in a significant shrinkage of the equilibrium domain width in the FM film if the substrate is in the Meissner state.

The effects described above fall within common experimental range of parameters. They will be noticeable if the room-temperature domains in the FM film are wider than $l_N\sim 0.5$ micron for, e.g., the Nb substrate ($\lambda_L\sim 40$ nm) or wider than $l_N\sim 1.6$ micron for a high-temperature SC ($\lambda_L \sim 130$ nm). Because equilibrium l_N depends on the thickness of the FM film d_M the above condition on l_N translates, through Eq. (12), into the lower bound on d_M. For, e.g., a TbFe film[3] ($\beta\sim 10^2$ and $\delta\sim 15$ nm) the equilibrium domain width should decrease below T_c in films of thickness greater than 0.3 micron on a Nb substrate or in films thicker than 3 micron on a high-T_c substrate. In a real FM film the stripe domains are curved due to thermal fluctuations[11] and due to the pinning of domain walls. This, however, should not affect our conclusions as long as the corresponding radius of curvature of the domains is large compared with other characteristic lengths. Since we are interested in the equilibrium magnetic structure due to the FM-SC interactions, it is important to acknowledge that strong pinning of domain walls by the imperfections may prevent the system from reaching that equilibrium. Possible ways to study the equilibrium magnetic structures include choosing systems with low coercivity (that is, weak pinning of domain walls), or low Curie temperature (below the critical temperature of the SC), or rotating the system in a slowly decaying magnetic field. It should be also possible to extract changes in the magnetic equilibrium from the study of the magnetic hysteresis in the FM film above and below T_c. The existing large variety of magnetic materials and superconductors should allow experiments in all interesting ranges of temperature and coercivity.

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