

Half-quantum vortex and \hat{d} -soliton in Sr_2RuO_4

Hae-Young Kee,¹ Yong Baek Kim,² and Kazumi Maki³

¹*Department of Physics, University of California, Los Angeles, California 90095*

²*Department of Physics, The Ohio State University, Columbus, Ohio 43210*

³*Department of Physics, University of Southern California, Los Angeles, California 90089*

(Received 30 May 2000)

Assuming that the superconductivity in Sr_2RuO_4 is described by a planar p -wave order parameter, we consider possible topological defects in Sr_2RuO_4 . In particular, it is shown that both the \hat{d} -soliton and half-quantum vortex can be created in the presence of the magnetic field parallel to the a - b plane. We discuss how one can detect the \hat{d} -soliton and half-quantum vortex experimentally.

It has been suggested that the unconventional superconducting state of Sr_2RuO_4 is described by the planar spin-triplet p -wave order parameter with broken time-reversal symmetry in analogy to the ^3He A phase. As known from the example of superfluid ^3He , one of the hallmarks of the triplet superconductivity is the presence of a manifold of topological defects.^{1,2} Thus, we expect that the creation and detection of topological defects in Sr_2RuO_4 (or spin-triplet superconductor) will provide further insights about the nature of the unconventional superconducting state of Sr_2RuO_4 . In this sense, the study of the topological defects in the planar p -wave superconducting state with broken time-reversal symmetry is of great interest. Moreover, half-quantum vortices can exist in different condensed matter systems, such as the Bose condensates with hyperfine spin $F = 1$ and the mixture of two Bose condensates.³

It has been proposed^{4,5} that the superconducting order parameter in Sr_2RuO_4 is described by the planar p -wave form written as

$$\Delta_{\alpha\beta}(\vec{k}) = \vec{d}(\vec{k}) \cdot (\vec{\sigma}i\sigma_2)_{\alpha\beta} \quad (1)$$

with

$$\vec{d}(\vec{k}) = \Delta \hat{d}(\hat{k}_1 \pm i\hat{k}_2), \quad (2)$$

where σ_μ ($\mu=1,2,3$) are Pauli matrices and α, β represent spin \uparrow or \downarrow . Here \hat{k}_j ($j=1,2$) represent the projection of the unit wave vector \hat{k} along two perpendicular directions \hat{e}_1 and \hat{e}_2 in two-dimensional space. This order parameter describes the Cooper pair state with the zero-spin projection on \hat{d} and with the unique projection of the pair orbital angular momentum given by $\hat{l} = \hat{e}_1 \times \hat{e}_2$. In Sr_2RuO_4 , due to the spin-orbit coupling, \hat{d} is forced to be parallel to $\pm \hat{c}$ and \vec{k} is the quasiparticle momentum in the a - b plane.

Indeed the spontaneous magnetization seen by muon spin-relaxation experiment⁶ and flat Knight shift seen by NMR (Ref. 7) seem to be consistent with this picture. On the other hand, the origin of the spontaneous magnetization seen by muon spin-relaxation experiment is somewhat mysterious since we do not expect such a magnetization in a homogeneous system.

It is important to notice that the superconducting ground state described by the order parameter of Eq. (2) is doubly degenerate. We can designate these two ground states by the angular momentum $l_z = \pm 1$, where \hat{l} is parallel to the c axis. Sigrist and Agterberg⁸ proposed recently that there will be in general a domain wall between $l_z = 1$ and $l_z = -1$ states, which we shall call \hat{l} -soliton^{9,10} in analogy to the case of superfluid ^3He -A. It is likely that such a soliton is magnetically active, so it may be an origin of the spontaneous magnetization seen in muon spin-relaxation experiment.⁶ In particular, in a magnetic field $\mathbf{H} \parallel \hat{c}$, only one of these degenerate states is favored.^{11,12} Therefore, it is possible to control \hat{l} -solitons by a magnetic field parallel to the c axis.⁸ They also proposed that these \hat{l} -solitons would provide very efficient barriers for the vortex motion and this effect is possibly related to the pinning of vortices observed in Sr_2RuO_4 below $T = 30$ mK.¹³

The purpose of this paper is to propose an alternative model for the appearance of the spontaneous magnetization and another mechanism of the pinning of vortices: \hat{d} -soliton and half-quantum vortex. The \hat{d} -soliton is a domain wall between $\hat{d} \parallel \hat{c}$ and $\hat{d} \parallel -\hat{c}$ as in superfluid ^3He -A. We believe that \hat{d} is parallel and antiparallel to \hat{c} , because they are forced to be parallel to the angular momentum \hat{l} (or $-\hat{l}$) due to the spin-orbit coupling characterized by an energy scale Ω_d .^{14,15} Therefore, if we use Ω_d as a parameter, we can calculate the energy and shape of the \hat{d} -soliton provided that $\Omega_d \ll \Delta(T)$, where $\Delta(T)$ is the superconducting gap. Unfortunately we do not know the precise value of Ω_d , but it may be about $\frac{1}{10}\Delta(T)$. In this picture, moving the \hat{d} -soliton generates the local magnetization which can result in the spontaneous magnetization seen in muon spin-relaxation experiment. One can also generate a large number of \hat{d} -solitons by applying a burst of high-frequency microwaves with frequency $\sim \Omega_d$ sent parallel to the a - b plane.⁹

As in superfluid ^3He -A, each \hat{d} -soliton is terminated by a pair of half-quantum vortices.¹⁶ We find that these pairs of half-quantum vortices are more stable than the usual single-quantum vortex in the superconducting state in the presence of the magnetic field parallel to the a - b plane. This means

that the usual single-quantum vortex would split into a pair of half-quantum vortices connected by the \hat{d} -soliton.^{2,17} In this case, these objects would provide an extremely efficient pinning mechanism of vortices in Sr_2RuO_4 . Also the half-quantum vortices should exhibit a clear electron-spin resonance signature. Further we believe that these objects are visible by the scanning tunneling microscopy (STM) imaging and by micromagnetometry developed by Kirtley *et al.*¹⁸ used in high- T_c cuprate compounds.

Free energy of the conventional single vortex when the magnetic field is parallel to the a - b plane. Let us assume that the magnetic field is parallel to the a axis. Then the free energy of the conventional vortex with the flux quantum $\phi_0 = hc/2e$ is obtained within the London approximations as

$$f_v = \left(\frac{\phi_0}{4\pi\lambda} \right)^2 \ln \left(\frac{\lambda}{\xi} \right), \quad (3)$$

where ξ is the coherence length and λ is the magnetic penetration depth. The magnetic penetration depth λ is related to the superfluid density $\rho_s(T)$ by $\lambda^{-2} = (4\pi e^2/mc^2)\rho_s(T)$. When the magnetic field is parallel to the a axis, in an anisotropic system like Sr_2RuO_4 , λ and ξ should be reinterpreted as $\lambda = \sqrt{\lambda_b\lambda_c}$ and $\xi = \sqrt{\xi_b\xi_c}$. Here $\lambda_{b,c}$ and $\xi_{b,c}$ are the magnetic penetration depth and coherence length in the b and c directions, respectively.

\hat{d} -soliton and a pair of half-quantum vortices. There exists huge anisotropy in the in-plane and out-of-plane transport properties in Sr_2RuO_4 . Thus Sr_2RuO_4 may be regarded as an effectively two-dimensional system. The large anisotropy or the effective two-dimensionality of the system forces the angular momentum of the Cooper pair to be parallel or antiparallel to the c axis. In the p -wave superconducting state described by the order parameter described by Eqs. (1) and (2), the \hat{d} vector is oriented along $\pm \hat{l}$ in the presence of the spin-orbit coupling. Here we consider the case that the angular momentum \hat{l} is uniform in the entire system. We can assume, without loss of generality, that $\hat{l} \parallel \hat{c}$. We are interested in the deformation of the \hat{d} configuration from the uniform case; for example, $\hat{d} \parallel \hat{c}$. Any deviation from the uniform state would cost the energy associated with the spin-orbit coupling characterized by an energy scale Ω_d .^{14,15} However, we will show that the so-called \hat{d} -soliton (a particular form of the \hat{d} configuration) with a pair of half-quantum vortices can have lower energy than the conventional single vortex. Thus it is easier to excite a pair of half-quantum vortices with a \hat{d} -soliton compared to single conventional vortex. In particular, a magnetic field parallel to the a - b plane generates very likely pairs of half-quantum vortices rather than usual vortices when the former are stable.

We consider the \hat{d} -soliton that is a topological planar defect in the \hat{d} configuration. The orientation of \hat{d} changes by π across the planar defect while \hat{d} vectors at far distances are still along the c axis. Typical configurations of the \hat{d} -soliton in the y - z plane can be found in Figs. 1 and 2, which we will explain later. We take y and z as the coordinates along the b axis and c axis, respectively. Now let us attach a pair of

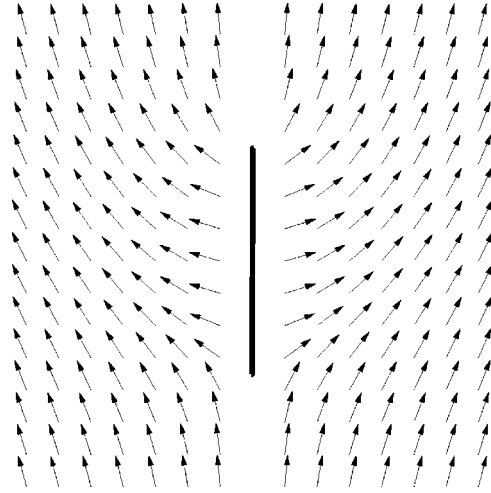


FIG. 1. The spatial configuration of the d vector in the b - c plane given by Eq. (6). The thick line denotes the domain wall with the length R , which is parallel to the c axis.

half-quantum vortices to the end points of the \hat{d} -soliton of length R in the y - z plane. In the case of an isolated half-quantum vortex, we have a $e^{i\pi} = -1$ factor in the order parameter due to phase winding around the half-quantum vortex. Therefore, an isolated half-quantum vortex cannot occur. On the other hand, if the half-quantum vortex is attached to the end points of the \hat{d} -soliton, the disgyration in \hat{d} at the same point compensates the phase π so that there is no net change in the overall phase of the order parameter.

In order to show that a pair of half-quantum vortices with the \hat{d} -soliton is a lower energy excitation compared to a single conventional vortex, we have to compare the free energies of the two cases. The free energy required to create the \hat{d} -soliton is obtained from

$$f_d = \frac{1}{2} \chi_N C^2 \int d^3r \left[\sum_{ij} |\partial_i \hat{d}_j|^2 + \xi_d^{-2} (1 - d_z^2) \right], \quad (4)$$

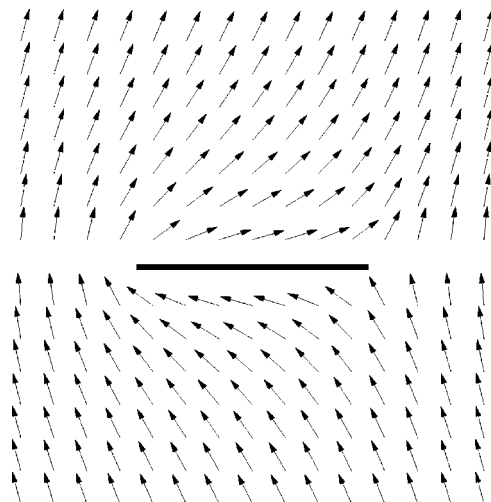


FIG. 2. The spatial configuration of the d vector in the b - c plane given by Eq. (7). The thick line denotes the domain wall with the length R , which is parallel to the b axis.

where χ_N is the spin susceptibility, $\xi_d(T) = C(T)/\Omega_d(T)$ where $C(T)$ is the spin-wave velocity, and $\Omega_d(T)$ is the longitudinal spin-resonance frequency.¹⁴

On the other hand, the \hat{d} vector of the \hat{d} -soliton can be parametrized by the following expression:

$$\hat{d} = \cos \psi \hat{z} + \sin \psi \hat{y}, \quad (5)$$

where

$$\psi(y, z) = \frac{1}{2} \left(\arctan \frac{z+R/2}{y} - \arctan \frac{z-R/2}{y} \right), \quad (6)$$

where we put two half-quantum vortices at $(y, z) = (0, R/2)$ and $(0, -R/2)$.

In the past, a similar form of ψ was also discussed in a different context in regard to ^3He .¹⁶ As one can see, there is a discontinuity in ψ across the line defined by $-R/2 < z < R/2$ and $y = 0$. The spatial configuration of the corresponding \hat{d} around a pair of half-quantum vortices is shown in Fig. 1. As one can see from the figure, the planar defect is parallel to the \hat{z} direction or c axis. One can also consider the planar defect lying along the \hat{y} axis given by

$$\psi = \frac{1}{2} \left(\arctan \frac{y+R/2}{z} - \arctan \frac{y-R/2}{z} \right), \quad (7)$$

where two half-quantum vortices are located at $(y, z) = (R/2, 0)$ and $(-R/2, 0)$. The configuration of the \hat{d} vector using the above ψ is shown in Fig. 2. One can easily see that the free energies, f_d , associated with two possible \hat{d} configurations are the same.

The total free energy of the \hat{d} -soliton and a pair of half-quantum vortices is given by

$$\begin{aligned} f_{pair} &= \frac{1}{2} \chi_N C^2 \int dy dz \left[K (\nabla \Phi)^2 + \sum_{ij} |\partial_i \hat{d}_j|^2 + \xi_d^{-2} \sin^2 \psi \right] \\ &= \frac{1}{2} \chi_N C^2 \left(\pi K \ln \frac{\lambda}{R} + I_1 + I_2 \right), \end{aligned} \quad (8)$$

where Φ represents the phase of the order parameter which couples to the external electromagnetic field. The parameter K is defined by

$$K = \frac{\rho_s}{\rho_{sp}} = \frac{1 + 1/3F_1}{1 + 1/3F_1^a} \frac{1 + 1/3F_1^a(1 - \rho_s^0)}{1 + 1/3F_1(1 - \rho_s^0)}, \quad (9)$$

where ρ_s and ρ_{sp} are the superfluid density and the spin superfluid density respectively. F_1 and F_1^a are the Landau parameters and $\rho_s^0 \equiv [1 - Y(T)]$ and $Y(T)$ is the Yosida function] is the superfluid density without the Fermi-liquid correction.¹ Notice that $K(T_c) = 1$ at $T = T_c$ and $K(0) = (1 + 1/3F_1)/(1 + 1/3F_1^a)$ at $T = 0$. The temperature dependence of the parameter K is shown in Fig. 3 assuming that $F_1 = 9$ and $F_1^a = 0$. This choice of the parameters will be explained later.

The first term in the first and second lines of Eq. (8) is the contribution from two half-quantum vortices and represents the fact that these half-quantum vortices repel each other. $I_{1,2}$

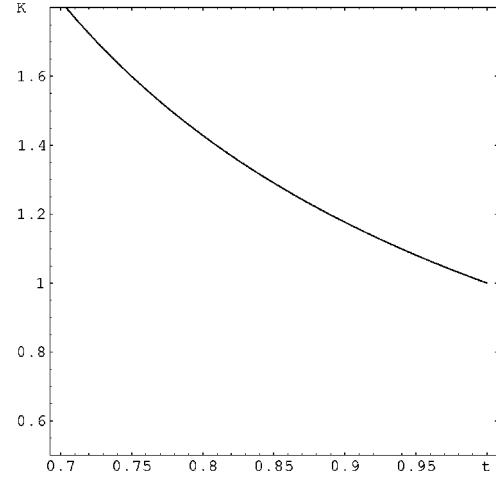


FIG. 3. The parameter K as a function of the reduced temperature $t = T/T_c$.

are the contributions from the second and third terms in the first line of Eq. (8). These contributions come from the declination of the \hat{d} -vector.

Using the form of $\psi(y, z)$ discussed above, Eq. (6), I_1 and I_2 can be obtained as follows:

$$\begin{aligned} I_1 &= \frac{1}{4} \int dy dz \frac{R^2}{[y^2 + (z+R/2)^2][y^2 + (z-R/2)^2]} = \pi \ln \frac{R}{\xi}, \\ I_2 &= \frac{1}{2\xi_d^2} \int dy dz \left(1 - \frac{y^2 + z^2 - R^2/4}{\sqrt{(y^2 + z^2 - R^2/4)^2 + y^2 R^2}} \right) \\ &= \pi \left(\frac{R}{2\xi_d} \right)^2 \ln \frac{4\xi_d}{R}, \end{aligned} \quad (10)$$

where ξ_d is the length scale associated with the spin-orbit coupling defined by $\xi_d(T) = C(T)/\Omega_d(T)$.

By minimizing f_{pair} with respect to R , we obtain the optimal R_0 for the lowest free energy configuration of a pair of half-quantum vortices and the \hat{d} -soliton. The optimal R_0 is given by

$$R_0^2 = \frac{(K-1)2\xi_d^2}{\ln(4\xi_d/\sqrt{e}R_0)} > 0. \quad (11)$$

Here we have assumed that $\xi_d > \sqrt{e}R_0/4$. Notice that the half-quantum vortices with a \hat{d} -soliton is possible only when $K > 1$ in order to have $R_0 > 0$. Although we have no information about F_1^a , it is most likely that $F_1^a \sim 0$. The ratio between the effective mass and the bare mass, m^*/m , is about 4, which means that $F_1 \sim 9$. Therefore, $K > 1$ in the superconducting state, as one can see from Eq. (9). Thus this condition is always satisfied below T_c . However, the existence of the solution for R_0 depends on the value of K . We find that the solution exists only if $1 < K \leq 1.5$. For example, for $K = 1.5$, $\xi_d/R = 0.85$. Since the parameter K depends on temperature as shown in Fig. 3, we find that a pair of half-quantum vortices with the \hat{d} -soliton exist only for $0.78 \leq T/T_c < 1$.

Now the free energy of a pair of half-quantum vortices and the \hat{d} -soliton at the optimal R_0 can be obtained as

$$f_{pair} = \frac{1}{2} \pi \chi_N C^2 \left[K \ln \frac{\lambda}{\xi} + \frac{(K-1)}{2} \ln \frac{\Lambda \xi^2}{2(K-1) \xi_d^2} + \frac{K-1}{2} \right], \quad (12)$$

where $\Lambda = \ln(4 \xi_d / \sqrt{e} R_0)$. In order to examine the stability of the half-quantum vortices, we have to compare f_{pair} and the free energy of single vortex, f_v . The difference is given by

$$f_v - f_{pair} = \frac{1}{2} \pi \chi_N C^2 \times \left[\ln \frac{\lambda}{\xi} + \frac{(K-1)}{2} \ln \frac{2(K-1) \xi_d^2}{\Lambda \lambda^2} - \frac{(K-1)}{2} \right]. \quad (13)$$

If $f_v - f_{pair} > 0$ for some values of $K > 1$, a pair of half-quantum vortices are more stable than the conventional single vortex. This condition can be rewritten as

$$\frac{\lambda}{\xi} \left(\frac{\xi_d}{\lambda} \right)^{K-1} > \frac{e^{(K-1)/2} \Lambda^{(K-1)/2}}{2^{(K-1)/2} (K-1)^{(K-1)/2}}. \quad (14)$$

Recalling that the solution of Eq. (11) exists if $1 < K \leq 1.5$, one can investigate the stability condition given by Eq. (14). One can see from Eqs. (11) and (14) that, as long as $K > 1$, a pair of half-quantum vortices can be stabilized over a single vortex under certain conditions for the ratio between ξ_d and λ . For example, for $K = 1.1$ and $K = 1.5$, the conditions for the stability of a pair of half-quantum vortices over a single vortex are given by

$$\frac{\xi_d}{\lambda} > 10^{-11} \quad \text{and} \quad \frac{\xi_d}{\lambda} > 0.0094, \quad (15)$$

respectively. One can see that these conditions are easily satisfied. Here we use $\lambda/\xi = \sqrt{(\lambda_b \lambda_c)/(\xi_b \xi_c)} = 12.186$,

which is appropriate for Sr_2RuO_4 . One can also see that the stability of a pair of half-quantum vortices with the \hat{d} -soliton is determined by the value of K which depends on temperatures as shown in Fig. 3.

Now let us discuss the relation between \hat{l} - and \hat{d} -solitons. It is difficult to estimate the energy of the \hat{l} -soliton in terms of the texture free energy given by Eq. (4). Precise comparison between the \hat{d} -soliton and the \hat{l} -soliton requires the complete solution of the Euler-Lagrange equations derived from the Ginzburg-Landau functional. This is beyond the scope of the present paper and the subject of future study.

In summary, assuming that the superconducting state of Sr_2RuO_4 is characterized by the spin-triplet order parameter with broken time-reversal symmetry, we investigated the existence of half-quantum vortices and the associated topological defect: the \hat{d} -soliton. We showed that a pair of half-quantum vortices attached to a \hat{d} -soliton can be created in the presence of the magnetic field parallel to the a - b plane. It was found that a pair of half-quantum vortices with a \hat{d} -soliton is more stable than the conventional single vortex for certain temperatures below T_c . As in superfluid $^3\text{He-A}$, the presence of the \hat{d} -soliton may be detected as the deficit in the intensity of electron-spin-resonance signal at $\omega = \Omega_d$.⁹ There should be a clear electron-spin-resonance signature due to the half-quantum vortices. Detection of the half-quantum vortices by STM would also provide a convincing evidence for the spin-triplet pairing state with time-reversal symmetry breaking.

We thank Manfred Sgrist, Ying Liu, and Steven Kivelson for helpful discussions. The work of H.-Y. Kee was conducted under the auspices of the Department of Energy, supported (in part) by funds provided by the University of California for the conduct of discretionary research by Los Alamos National Laboratory. This work was also supported by NSF CAREER award Grant No. DMR-9983783 (Y.B.K.) and the Alfred P. Sloan Foundation (Y.B.K.).

¹D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor & Francis, New York, 1990); A. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).

²G. E. Volovik, *Exotic Properties of Superfluid Helium 3* (World Scientific Pub. Co., Singapore, 1991).

³U. Leonhardt and G. E. Volovik, cond-mat/0003428 (unpublished).

⁴T. M. Rice and M. Sgrist, *J. Phys.: Condens. Matter* **7**, L643 (1995).

⁵M. Sgrist *et al.*, *Physica C* **317-318**, 134 (1999).

⁶G. M. Luke *et al.*, *Nature (London)* **394**, 558 (1998).

⁷K. Ishida *et al.*, *Nature (London)* **396**, 658 (1998).

⁸M. Sgrist and D. F. Agterberg, *Prog. Theor. Phys.* **102**, 965 (1999).

⁹K. Maki, in *Solitons*, edited by S. E. Trullinger, V. E. Zakharov, and V. L. Pokrovskii (North-Holland, Amsterdam, 1986).

¹⁰N. Schopohl and L. Tewordt, *J. Low Temp. Phys.* **41**, 305 (1980).

¹¹D. F. Agterberg, *Phys. Rev. Lett.* **80**, 5184 (1998); *Phys. Rev. B* **58**, 14 484 (1998).

¹²G. F. Wang and K. Maki, *Europhys. Lett.* **45**, 71 (1999).

¹³A. C. Mota, G. Juri, P. Visami, A. Pollini, T. Teruzzi, and K. Aupke, *Physica C* **185-189**, 343 (1999).

¹⁴H. Y. Kee, Y. B. Kim, and K. Maki, *Phys. Rev. B* **61**, 3584 (2000).

¹⁵L. Tewordt, *Phys. Rev. Lett.* **83**, 1007 (1999); *J. Low Temp. Phys.* **117**, 1 (1999).

¹⁶M. M. Salomaa and G. E. Volovik, *Phys. Rev. Lett.* **55**, 1184 (1985).

¹⁷K. Maki, *Phys. Rev. Lett.* **56**, 1312 (1986).

¹⁸J. R. Kirtley *et al.*, *Phys. Rev. Lett.* **81**, 2140 (1998); K. A. Moler *et al.*, *Science* **279**, 1193 (1998).