

## $O(4)$ -invariant formulation of the nodal liquid

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We consider the  $O(4)$  symmetric point in the phase diagram of an electron system in which there is a transition between  $d_{x^2-y^2}$  density-wave order and  $d_{x^2-y^2}$  superconductivity. If the pseudospin  $SU(2) \subset O(4)$  symmetry is disordered by quantum fluctuations, the nodal liquid can result. In this context, we (1) construct a pseudospin  $\sigma$  model, (2) discuss its topological excitations, (3) point out the possibility of a *pseudospin-Peierls* state, and (4) propose a phase diagram for the underdoped cuprate superconductors.

*Introduction.* Competing interactions and fluctuations have led to a cornucopia of interesting phenomena in the cuprate superconductors. Unfortunately, these phenomena have not led to the unambiguous determination of the phase diagram of these materials, possibly because some of the phases realized in these materials are characterized by particularly subtle forms of order. This dilemma is rather acute on the underdoped side of the phase diagram, where it is still not clear if the pseudogap can be ascribed to a new phase of matter, a nearby critical point, or a crossover. Since a better understanding of proposed exotic phases and the transitions between them may mitigate this difficulty, we study the transition between the  $d_{x^2-y^2}$  superconducting state of the cuprates and a putative  $d_{x^2-y^2}$  density-wave state (also known as the staggered flux state;<sup>1</sup> see Ref. 2) which is a semimetallic state with charge-carrying excitations at the nodes of the gap. We ask if the pseudogap—which appears to have  $d_{x^2-y^2}$  symmetry—could be due to the proximity of the experimental system to this transition. The resulting phase diagram automatically includes the nodal liquid state,<sup>3-5</sup> a state with spin-charge separation. We discuss the possible relevance of this theoretical cuprate phase diagram to the experimental one.

*$O(4)$  formulation of  $d_{x^2-y^2}$  ordered states at half-filling.* In Ref. 2, we adapted Yang's pseudospin  $SU(2)$  symmetry<sup>6-8</sup> to a critical point between a  $d_{x^2-y^2}$  density-wave state and a  $d_{x^2-y^2}$  superconductor. The original pseudospin  $SU(2)$  was germane to the transition between a charge-density wave and an  $s$ -wave superconductor; Zhang's closely related  $SO(5)$ ,<sup>9</sup> was germane to the transition between an antiferromagnet and a  $d$ -wave superconductor.

We first consider a transition at half-filling between a singlet commensurate  $d_{x^2-y^2}$  density wave and a  $d_{x^2-y^2}$  superconductor. We combine the order parameters into

$$\Phi_{\underline{i}}(q)f(k) = \begin{pmatrix} \sqrt{2} \operatorname{Re}\{\langle \psi_{\uparrow}^{\dagger}(k+q/2)\psi_{\downarrow}^{\dagger}(-k+q/2) \rangle\} \\ \sqrt{2} \operatorname{Im}\{\langle \psi_{\uparrow}^{\dagger}(k+q/2)\psi_{\downarrow}^{\dagger}(-k+q/2) \rangle\} \\ i\langle \psi^{\alpha\dagger}(k+Q+q/2)\psi_{\alpha}(k-q/2) \rangle \end{pmatrix}, \quad (1)$$

where  $f(k) = \cos k_x a - \cos k_y a$ . Although we focus initially on half-filling, both order parameters make perfect sense away from half-filling as well. Following Yang,<sup>6</sup> we introduce the pseudospin  $SU(2)$  generators  $O^3, O^+, O^- = (O^+)^{\dagger}$

$$O^3 = \int_{\text{RBZ}} \frac{d^2k}{(2\pi)^2} [\psi^{\alpha\dagger}(k)\psi_{\alpha}(k) + k \rightarrow k+Q],$$

$$O^+ = \int_{\text{RBZ}} \frac{d^2k}{(2\pi)^2} i\psi_{\uparrow}^{\dagger}(k)\psi_{\downarrow}^{\dagger}(-k+Q). \quad (2)$$

The order parameters form a triplet under this  $SU(2)$ . We will use underlined lowercase Roman letters such as  $i = 1, 2, 3$  to denote pseudospin triplet indices and uppercase  $\bar{R}\bar{O}\bar{M}\bar{A}\bar{N}$  letters to denote pseudospin doublet indices  $A = 1, 2$ . Lowercase Roman indices  $a = 1, 2, 3$  will be vector indices (i.e., real spin triplet indices) and Greek letters  $\alpha = 1, 2$  will be used for real spin  $SU(2)$  spinor indices. Pauli matrices  $\tau^i$  will be used for pseudospin, while  $\sigma^a$  will be reserved for spin. The integrals are over the reduced Brillouin zone (RBZ).

There is a small but important difference between our pseudospin  $SU(2)$  and Yang's:<sup>6</sup> the factors of  $i$  in  $O^{\pm}$ . They are necessary since a commensurate  $d_{x^2-y^2}$  density wave breaks  $T$ , while a superconductor does not; hence, our pseudospin  $SU(2)$  does not commute with  $T$ . Pseudospin  $SU(2)$ , spin  $SU(2)$ , and time reversal combine to form the symmetry group  $O(4)$ .

The electron fields transform as a doublet under both  $SU(2)$ 's. We will group them into  $\Psi_{A\alpha}$ :

$$\begin{pmatrix} \Psi_{1\alpha} \\ \Psi_{2\alpha} \end{pmatrix} = \begin{pmatrix} \psi_{\alpha}(k) \\ i\epsilon_{\alpha\beta}\psi^{\beta\dagger}(-k+Q) \end{pmatrix}. \quad (3)$$

Near the transition between a  $d_{x^2-y^2}$  density wave and a  $d_{x^2-y^2}$  superconductor, we can focus on the low-energy degrees of freedom: the order parameters and the nodal fermionic excitations. We can write down an  $O(4)$ -invariant action for this transition:

$$S_{\text{eff}} = \int d\tau \frac{d^2k}{(2\pi)^2} \Psi^{A\alpha\dagger} [\partial_{\tau} - \epsilon(k)] \Psi_{A\alpha}$$

$$+ ig \int d\tau \frac{d^2k}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Phi_{\underline{i}}(q)f(k) \left[ \epsilon^{\alpha\beta} \Psi_{C\alpha} \left( k + \frac{q}{2} \right) \right.$$

$$\left. \times \epsilon^{CA} \tau_A^{iB} \Psi_{B\beta} \left( -k + \frac{q}{2} \right) + \epsilon_{\alpha\beta} \Psi^{A\alpha\dagger} \left( k + \frac{q}{2} \right) \right]$$

$$\begin{aligned} & \times \tau_A^{iB} \epsilon^{BC} \Psi^{B\beta\ddagger} \left( -k + \frac{q}{2} \right) \Big] + \int d\tau d^2x \left( (\partial_\mu \Phi_i^-)^2 \right. \\ & \left. + \frac{1}{2} r \Phi_i^- \Phi_i^- + \frac{1}{4!} u (\Phi_i^- \Phi_i^-)^2 \right). \end{aligned} \quad (4)$$

‘‘Microscopic’’ models with this symmetry were constructed in Ref. 2. In this  $O(4)$ -symmetric action, we have, by a rescaling, set the  $\Phi_i$  velocities,  $v_i$ , and stiffnesses,  $\rho_i$ , to 1. This cannot be done in the asymmetric case,  $\rho_1 = \rho_2 \equiv \rho_s \neq \rho_{DW} = \rho_3$ . In general, symmetry-breaking terms will be present, but they can scale to zero at a critical point, thereby dynamically restoring the symmetry, as we discuss later. Hence, we focus on the symmetric case.

When  $\Phi_i$  is ordered, the fermionic spectrum is  $E(k) = \sqrt{\epsilon^2(k) + g^2 \Phi_i f^2(k)}$ . In the following, we ignore the fermionic excitations which are not associated with the nodes of the  $d_{x^2-y^2}$  order parameter. We linearize  $\epsilon(k)$  about the Fermi surface and  $f(k)$  about the nodes. If we rotate our axes so that the  $k_x$  axis is perpendicular to the Fermi surface at one antipodal pair of nodes, then we can write  $\epsilon(k) \approx v_F k_x$  and  $g|\Phi_i|f(k) \approx v_\Delta k_y$ . As in Ref. 3, we will have to introduce an additional index  $a=1,2$  for the two sets of antipodal nodes which differ by the replacement  $k_x \leftrightarrow k_y$ . In order to avoid unnecessary clutter, this index will be suppressed.

It is convenient to adopt a nonlinear  $\sigma$ -model approach and assume that the magnitude of  $\Phi_i$  is fixed,  $\Phi_i^2 = a^2$ . Following Refs. 10 and 11, we employ a  $CP^1$  representation of the nonlinear  $\sigma$  model:

$$\Phi_i^a = z^{A\dagger} \tau_A^{iB} z_B \quad (5)$$

with  $|z_1|^2 + |z_2|^2 = a^2$  and rotate the pseudospins of the fermions to the local direction of the order parameter:

$$\Psi_A = U_A^B \chi_B, \quad (6)$$

where

$$U = \frac{1}{a} \begin{pmatrix} z_1 & -z_2^* \\ z_2 & z_1^* \end{pmatrix}. \quad (7)$$

The latter change of variables is a direct  $SU(2)$  analog of the original  $U(1)$  nodal liquid construction.<sup>3</sup> As in that case, it is double valued, so we must introduce a Chern-Simons term as in Ref. 4 which couples the  $\chi$ 's to the topological current  $j_\mu = \epsilon_{\mu\nu\lambda} \epsilon_{ijk} \Phi^i \partial^\nu \Phi^j \partial^\lambda \Phi^k$ . This term is only important at the phase transitions since the topological current vanishes in the ordered phases since the pseudospins are aligned and in the disordered phases since it is odd under the  $Z_2$  symmetry  $\Phi^3 \rightarrow -\Phi^3$ . We suppress this term below.

In terms of  $z_A, \chi_A$ , the effective action takes the form

$$\begin{aligned} S_{\text{eff}} = & \int d\tau d^2x \chi^{A\alpha\dagger} (\partial_\tau + \alpha_\tau \tau_-^3 - v_F i \partial_x - v_F \alpha_x \tau_-^3) \chi_{A\alpha} \\ & + i \int d\tau \frac{d^2k}{(2\pi)^2} [\epsilon^{\alpha\beta} \chi_{C\alpha} \epsilon^{CA} \tau_A^{3B} v_\Delta i \partial_y \chi_{B\beta} \\ & + \epsilon_{\alpha\beta} \chi^{A\alpha\dagger} \tau_A^{3B} \epsilon^{BC} v_\Delta i \partial_y \chi^{B\beta\dagger}] \end{aligned}$$

$$\begin{aligned} & + \int d\tau \frac{d^2k}{(2\pi)^2} \chi^{\alpha\dagger} [U^\dagger (\partial_\tau - A_\tau \tau_-^3) U - \alpha_\tau \tau_-^3 \\ & - v_F U^\dagger (i \partial_x - A_x \tau_-^3) U + v_F \alpha_x \tau_-^3] \chi_\alpha \\ & + \int d\tau d^2x [(i \partial_\mu - \alpha_\mu - A_\mu \tau_-^3) z]^2 + \lambda (z^\dagger z - a)]. \end{aligned} \quad (8)$$

The  $U(1)$  gauge field  $\alpha_\mu$  is a Lagrange multiplier which removes the redundant phase variable in the parametrization of  $CP^1$  by  $z_A$ . A coupling between  $\alpha_\mu$  and  $\chi_A$  has been added to the first term and subtracted from the  $U^\dagger \partial U$  terms so as to make the latter invariant under the gauge transformation  $z_A \rightarrow e^{i\theta} z_A$ .  $\lambda$  is a Lagrange multiplier which fixes  $\Phi_i^2 = a^2$ . We have introduced the external electromagnetic field,  $A_\mu$ , in order to keep track of the charge quantum numbers of the fields. When  $a$  is large,  $\Phi^i = z^{A\dagger} \tau_A^{iB} z_B$  condenses and the system is in one of the  $d_{x^2-y^2}$  ordered states. When  $a$  is small,  $\Phi^i$  is disordered. There is a critical point at  $a = a_c$ .

*The nodal liquid revisited.* In the ordered phases,  $U$  is a constant, so the  $U^\dagger \partial U$  terms in Eq. (8) can be dropped; the nodal quasiparticles are coupled to the external electromagnetic field. Note that the  $d_{x^2-y^2}$  density wave is an ordered state in this formalism, unlike in Refs. 3–5, where it is a disordered state. In the disordered phases, the  $z_A$  sector of the theory develops a gap. Hence, the fourth and fifth lines of Eq. (8) can be dropped at low energies. To analyze these phases further, we introduce a dual representation for  $z_A$ , following Refs. 5 and 12. The effective action now takes the form

$$\begin{aligned} S_{\text{eff}} = & S_F[\chi_A, \alpha_\mu] + \sum_A S_{GL} \left[ \Phi^A, \frac{1}{2} (a_\mu^+ \pm a_\mu^-) \right] \\ & + \int d\tau d^2x (\alpha_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda^+ + A_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda^-), \end{aligned} \quad (9)$$

where  $S_F[\chi_A, \alpha_\mu]$  is the first three lines of Eq. (8),  $\Phi^A$  annihilates a vortex in  $z_A$ , and

$$\mathcal{L}_{GL}(\Phi, a_\mu) = \frac{1}{2} |(i \partial_\mu - a_\mu) \Phi|^2 + V(\Phi) + \frac{1}{2} (f_{\mu\nu})^2 \quad (10)$$

and  $J_\mu^\pm = \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda^\pm$  are the  $z_A$  number and pseudospin 3 currents. When the  $Z_2$  symmetry  $\Phi^A \rightarrow -\Phi^A$  is unbroken, we can rewrite the effective action in terms of the fields  $\Phi^+ = \Phi^1 \Phi^2$ ,  $\Phi^- = \Phi^1 \Phi^2^\dagger$ . We now have

$$\begin{aligned} S_{\text{eff}} = & S_F[\chi_A, \alpha_\mu] + S_{GL}[\Phi^+, a_\mu^+] + S_{GL}[\Phi^-, a_\mu^-] \\ & + \int d\tau d^2x (\alpha_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda^+ + A_\mu \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda^-). \end{aligned} \quad (11)$$

Integrating out  $\alpha_\mu$ , we can solve the resulting constraint to express  $a_\mu^+$  in terms of  $\chi_A$ :  $J_0^+ = \chi^\dagger \tau^3 \chi$ ,  $J_x^+ = v_F \chi^\dagger \tau^3 \chi$ .

Now suppose that the system becomes disordered as a result of the condensation of  $\Phi^-$ . By the Anderson-Higgs mechanism,  $a_\mu^-$  acquires a gap. Integrating out  $a_\mu^-$ , we find no coupling of  $A_\mu$  to the remaining degrees of freedom:  $\chi_A$  is a

neutral spin-1/2 fermion. The change of variables [Eq. (6)] has effectively “bleached” the fermions by using the order parameter to screen their pseudospin (including their charge). This state is none other than the nodal liquid.

*Pseudospin-Peierls order.* If the system is, instead, disordered by the condensation of  $\Phi^{1,2}$ , then  $J_\mu^\pm$  must vanish at low energies. The only allowed excitations at low energies are those combinations of  $\chi_A$ 's which are invariant under  $\tau_A^{3B}$  rotations, i.e., neutral excitations. At finite energy, there are also solitonic excitations which carry one quantum of  $(a_\mu^+ \pm a_\mu^-)/2$  flux, i.e., charge  $e$  and spin 1/2. According to the analogy between the pseudospin SU(2) physics of our system and the spin SU(2) physics of a quantum antiferromagnet, we might, in this disordered phase, expect the pseudospin analog of spin-Peierls order, *pseudospin Peierls* order,

$$\langle \vec{\Phi}(k+K) \cdot \vec{\Phi}(k) - \vec{\Phi} \times \partial_\tau \vec{\Phi}(k+K) \cdot \vec{\Phi} \times \partial_\tau \vec{\Phi}(k) \rangle = \sin k_x a \quad (12)$$

with  $K = (\pi/a, 0)$  or  $(0, \pi/a)$ , as a result of Berry phases<sup>16</sup> which we have neglected in Eq. (8).

*Phase transitions at half-filling.* The transition at half-filling between the  $d_{x^2-y^2}$  density wave and the  $d_{x^2-y^2}$  superconductor is driven by a pseudospin-2 symmetry-breaking field,

$$S_u = u \int d\tau d^2x (\Phi_3^2 - \Phi_1^2 - \Phi_2^2). \quad (13)$$

For  $u < 0$ , the 3 axis is an easy axis and the  $d_{x^2-y^2}$  density-wave state is favored; for  $u > 0$ , the 1-2 plane is an easy plane and the  $d_{x^2-y^2}$  superconducting state is favored. At  $u = 0$ , a first-order *pseudospin-flop* transition occurs, provided  $a > a_c$ . At the bicritical point  $a = a_c$ ,  $u = 0$ , quantum fluctuations destroy order at the  $O(4)$ -symmetric point. This bicritical point and the quantum critical region<sup>13,14</sup> are described by the physics of the critical fluctuations coupled to nodal fermionic excitations. For  $a < a_c$ ,  $u = 0$  the system lies along the  $O(4)$ -symmetric line in the nodal liquid phase. A small increase or decrease of  $u$  will not cause order, and the system will still be in the nodal liquid phase, albeit with lower symmetry,  $U(1) \times Z_2$ . Further increase or decrease of  $u$  will lead to second-order phase transitions at  $u_{cr}^\pm(a)$  into the  $d_{x^2-y^2}$  superconducting and  $d_{x^2-y^2}$  density-wave phases, respectively.

At the second-order transition from the nodal liquid to the  $d_{x^2-y^2}$  density wave, the  $Z_2$  symmetry of translation by one lattice site is broken. At the second-order XY transition from the nodal liquid to the  $d_{x^2-y^2}$  superconductor, electromagnetic U(1) is broken. At the first-order pseudospin-flop transition between the  $d_{x^2-y^2}$  superconductor and the  $d_{x^2-y^2}$  density-wave, U(1) is restored and  $Z_2$  is simultaneously broken. In the formulation discussed here, spin-charge confinement—which, in the language of Refs. 4, 5 and 15 (see also Ref. 17) is due to the absence of vortex pairing—occurs simultaneously with translational symmetry breaking.

*Topological excitations.* We can give a narrative for the destruction of superconductivity in the language of vortex condensation. In the superconducting phase, the pseudospin  $\Phi^i$  lies in the 1-2 plane. In the core of a vortex—a *meron* in

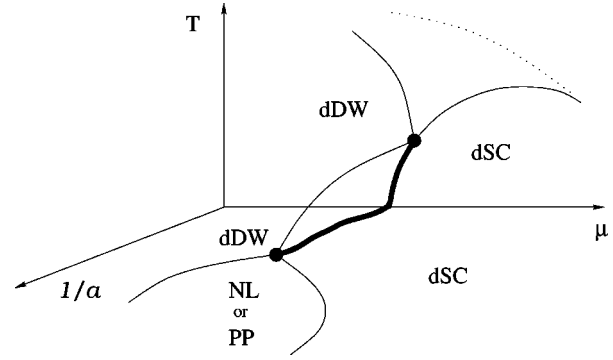


FIG. 1. The proposed phase diagram for the cuprates. The thick lines are first-order phase transitions; the thin one, second-order. The large dots are bicritical points, as is the entire thin line connecting them.

the  $\sigma$  model— $\Phi^i$  must point out of the 1-2 plane. This can be done by pointing along the  $\pm 3$  axis. When  $+3$  merons dominate (in the presence of an infinitesimal  $Z_2$  symmetry-breaking field), the superconductor undergoes a transition to the  $d_{x^2-y^2}$  density-wave state. When there are equal numbers of  $\pm 3$  merons, the superconductor instead undergoes a transition to the disordered state. This condition on the densities of  $\pm 3$  merons is reminiscent of and cognate to the vortex-pairing scenario of Refs. 4 and 5, but is weaker since it allows for the two possibilities discussed earlier. The transition from the  $d_{x^2-y^2}$  density-wave state to the disordered state can be understood in terms of *skyrmion* condensation.

*Discussion.* Transitions of the type which we have discussed above do not in the cuprates occur at half-filling but—if at all—near  $x_c$ , the doping at which superconductivity first appears. We assume  $u < 0$  to suppress superconductivity at half-filling. In order to move away from half-filling, we vary the chemical potential, which can be done by adding the  $O(4)$ -breaking term:

$$S_\mu = \mu O^3 = \mu \int d\tau d^2x (\epsilon_{3ij} \Phi_i \partial_\tau \Phi_j + \Psi^\dagger \tau^3 \Psi). \quad (14)$$

By increasing  $\mu$ , we can drive the system through a first-order pseudospin-flop transition into the superconducting state. As  $a$  is decreased, a bicritical point will again be reached. The coupling between  $z_A$  and  $\chi_A$  only enters at two loops; at one loop, we can appeal to known results for the pure nonlinear  $\sigma$  model, which indicate that the  $O(4)$  symmetry is dynamically restored at the bicritical point.<sup>18</sup> As a result, the  $O(4)$ -symmetric critical theory<sup>19</sup> discussed above will apply in the low-frequency, long-wavelength limit. A possible phase diagram for the cuprates, based on this scenario, is depicted in Fig. 1. An alternative, not depicted in Fig. 1, can occur if  $\rho_s < \rho_{DW}$ . In this case, there can be a phase with both  $d_{x^2-y^2}$  superconducting and  $d_{x^2-y^2}$  density-wave order, and a tetracritical point,  $T = T_{bc}$ ,  $\mu = \mu_{bc}$ , at which both orders become critical. For  $\mu < \mu_{bc}$ , there will be a regime,  $T_c^{sc} < T < T_c^{dw}$ , above the superconducting transition temperature, which has density-wave order.

The dotted line in Fig. 1 is the pseudogap scale, which we interpret as the scale below which  $\Phi^i$  has fixed magnitude and the nonlinear  $\sigma$  model description is available. Let us consider the physics below this scale. As  $\mu$  is increased,

Fermi pockets open at the nodes of the  $d_{x^2-y^2}$  density wave state. Eventually, the system undergoes a transition from the  $d_{x^2-y^2}$  density-wave to the  $d_{x^2-y^2}$  superconductor. The nature of this transition depends on the value of  $a$  which, ostensibly, varies among the materials in the cuprate family. It may, perhaps, be controlled by chemical substitution or applied pressure. For  $a$  large, the transition will be first order as depicted by the thick line. For  $a$  small, it occurs via two second-order phase transitions; the nodal liquid is sandwiched between these two transitions. Neither the  $d_{x^2-y^2}$  superconductor nor the nodal liquid has Fermi pockets, the latter because the second term in Eq. (14) can be dropped in the disordered phase. Appealing to the phase diagram of the spin-flop transition in magnetically ordered systems, we extend the first-order phase transition to finite temperature, where it meets the second-order  $d_{x^2-y^2}$  density-wave and superconducting ordering transitions.

Whither the antiferromagnet? As Hsu<sup>20</sup> and Gros<sup>21</sup> pointed out, the  $d_{x^2-y^2}$  density-wave state has good short-ranged antiferromagnetic correlations, reflected in its excellent numerical variational energy. Hence, one possibility is that the only additional physics needed to describe the antiferromagnetic state at half-filling is a moderate triplet quasiparticle-quasihole condensate.<sup>20</sup> Similar ideas may apply to the nodal liquid state, making it an equally good platform for the antiferromagnetic state at half-filling. Another possibility is that  $d_{x^2-y^2}$  density-wave order is suppressed as

half-filling is approached, and antiferromagnetism takes over. In neither case will our description of the critical regime be affected.

Our nonlinear  $\sigma$ -model analysis mirrors that of Ref. 9, but is on firmer footing because the  $d_{x^2-y^2}$  density wave—unlike the antiferromagnet—has a nodal fermionic spectrum similar to that of the  $d_{x^2-y^2}$  superconductor into which the pseudospin symmetry rotates it. Fluctuations between the  $d_{x^2-y^2}$  density-wave and superconducting states are also a key feature of the SU(2) mean-field theory of the  $t$ - $J$  model.<sup>22</sup> In fact, a parallel approach to the nodal liquid state was taken in this framework in Ref. 23. However, the SU(2) is local in that approach, which leads to complications arising from the concomitant gauge field. One virtue of the nonlinear  $\sigma$ -model approach is that we can use the physics of quantum antiferromagnets as a guide. In this way, we identified *pseudospin-Peierls* order as a possible alternative to the nodal liquid phase. Another striking upshot of our analysis is the bicritical point at which the  $d_{x^2-y^2}$  density-wave,  $d_{x^2-y^2}$  superconducting, and nodal liquid phases touch. It is possible that it is responsible for recent experimental hints of quantum critical behavior in the cuprates.<sup>24,25</sup>

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