Giant thermo-emf in multiterminal superconductor/normal-metal mesoscopic structures

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We considered a mesoscopic superconductor/normal-metal (S/N) structure in which the N reservoirs are maintained at different temperatures. It is shown that in the absence of current between the N reservoirs a voltage difference V_T arises between the superconducting and normal conductors. The voltage V_T oscillates with increasing phase difference φ between the superconductors, and its magnitude does not depend on the small parameter (T/ϵ_F).

It is well known that if the terminals of a normal conductor are maintained at different temperatures, then in the absence of a current a thermoelectric voltage (V_{emf}) appears between the terminals. The magnitude of V_{emf} is equal to $c_1(T/\epsilon_F)\delta T/e$, where c_1 is a factor of the order 1, ϵ_F is the Fermi energy, and δT is the temperature difference (see, for example, Ref. 1).

In this paper we analyze the thermoelectric effect in the mesoscopic structure shown in Fig. 1. We show that when a temperature difference exists between the normal (N) reservoirs, a voltage between normal and superconducting circuits V_T appears. Unlike in the normal case, the magnitude of this voltage does not depend on a small parameter (T/ϵ_F), and also this voltage oscillates as the phase difference φ between the superconductors varies. We assume that the superconductors are connected via a superconducting loop and the phase difference between them φ is controlled by an applied magnetic field. There is no current between the N reservoirs and the temperatures of the reservoirs are different: $T(\pm L) = T_0 \pm \delta T$. We will calculate the electric potential in the N film and, in particular, the potential V_T in the N reservoirs. Since we set the potential in the superconductors equal to zero, the potential V_T is the voltage difference between the N reservoirs and superconductors which arises in the presence of the temperature difference δT.

In order to find the potential V_T, we need to determine the distribution functions $f_\pm$ and the condensate wave functions $\hat{F}^{R(A)}$ induced in the N film. The distribution functions $f_\pm$ are related to the ordinary distribution functions for electrons $n_\uparrow$ and holes $p_\downarrow$: $f_{+\uparrow} = f_{+\downarrow} = 1 - (n_\uparrow + p_\downarrow)$ and $f_{-\uparrow} = f_{-\downarrow} = -(n_\uparrow - p_\downarrow)$ (we assume that there is no spin-dependent interaction in the system). The function f_+ determines the condensate current and the function f_- determines the quasiparticle current and electrical potential (see, for example, Ref. 2, where f_+ and f_- are denoted by f and f_1, respectively, and Ref. 3 where the application of the Green's function technique to the study of transport in S/N mesoscopic structures is discussed). These functions satisfy the kinetic equation (see Ref. 4)

$$L\partial_x[M_\pm \partial_x f_\pm(x) + J_S f_\mp(x) \pm J_{an}\partial_x f_\mp(x)]$$

$$= r[A_\pm \delta(x-L_1) + \bar{A}_\pm \delta(x+L_1)], \quad (1)$$

where all the coefficients are expressed in terms of the retarded (advanced) Green's functions: $\hat{G}^{R(A)} = G^{R(A)}\hat{\sigma}_z + \hat{F}^{R(A)}$; $M_\pm = [1 - G^R G^A \mp (\hat{F}^R \hat{F}^A)_1]/2$; $J_{an} = (\hat{F}^R \hat{F}^A)_z/2$, $J_s = (1/2)(\hat{F}^R \partial_x \hat{F}^R - \hat{F}^A \partial_x \hat{F}^A)_z$, $A_\pm = (\nu\nu_S + g_{1\mp})(f_\pm - f_{S\pm}) - (g_{z\pm}f_{S\mp} + g_{z\mp}f_\mp)$; $g_{1\pm} = (1/4)[(\hat{F}^R \pm \hat{F}^A)(\hat{F}^R_S \pm \hat{F}^R_S)]_1$; $g_{z\pm} = (1/4)[(\hat{F}^R \mp \hat{F}^A)(\hat{F}^R_S \pm \hat{F}^R_S)]_z$. The parameter $r = R/R_b$ is the ratio of the resistance of the N wire R and S/N interface resistance R_b; the functions $\bar{A}_-$ and $\bar{A}_+$ coincide with A_-, A_+ if we make a substitution $\varphi \rightarrow -\varphi$. We introduced above the following notations $(\hat{F}^R \hat{F}^A)_1 = \text{Tr}(\hat{F}^R \hat{F}^A)$, $(\hat{F}^R \hat{F}^A)_z = \text{Tr}(\hat{\sigma}_z \hat{F}^R \hat{F}^A)$, etc.; ν, ν_S are the density of states in the N film at $x = L_1$ and in the superconductors. The functions $f_{S\pm}$ are the distribution functions in the superconductors which are assumed to have equilibrium forms. This means that $f_{S+} \equiv f_{eq} = \tanh(\epsilon\beta_0)$ and $f_{S-} = 0$, because we set the potential of the superconductors equal to zero (no branch imbalance in the superconductors). At the reservoirs the distribution functions $f_\pm$ obey the boundary conditions: $f_\pm(\pm L) = F_{V\pm}(\pm L)$, where $F_{V\pm}(L) = (1/2)\{\tanh[\beta_0(\epsilon + eV_T)] \pm \tanh[\beta_0(\epsilon - eV_T)]\}, \beta_0 = (2T_0)^{-1}$.
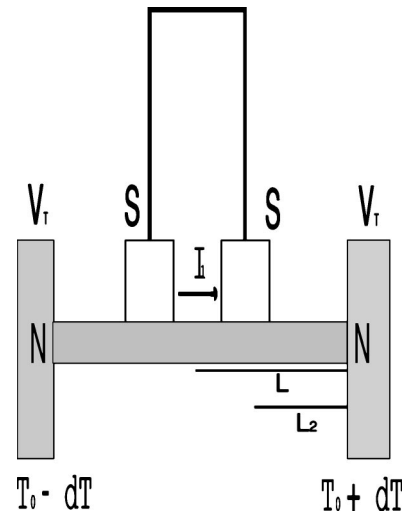

FIG. 1. Schematic view of the four-terminal S/N/S structure under consideration. The electric potential of the superconductors is zero. The N reservoirs are disconnected from the external circuit.

In order to clarify the physical meaning of different terms in Eq. (1), consider the equation for f_-. The first term in this equation is the partial quasiparticle current (the quasiparticle current at a given energy) in the N film. The second term is the condensate current and the third term is also the condensate current which appears under nonequilibrium conditions. The factors A_- and $\bar{A}_-$ are the partial currents through the S/N interfaces. The first term in A_- is the partial quasiparticle current above (the term $\nu\nu_S f_-$) and below (the term $g_{1\mp} f_-$) the gap Δ, and the last term is the condensate current. The factors A_+ and $\bar{A}_+$ are equal to zero below the gap (complete Andreev reflection).

In this paper we consider the case when the S/N interface resistance is greater than or approximately equal to the resistance of the N wire ($r < 1$). However r should not be too small because in Eq. (1) we neglected the inelastic collision integral. This implies that the condition

$$(\epsilon\tau_\epsilon)^{-1} \ll r \lesssim 1 \qquad (2)$$

must be fulfilled; here τ_ϵ is the energy relaxation time, $\epsilon \approx \min\{T, \epsilon_L\}$, and $\epsilon_L = D/L^2$ is the Thouless energy. We consider the most interesting case of low temperatures ($T \ll \Delta$) when for characteristic energies $\epsilon \ll \Delta$ the functions A_+ and $\bar{A}_+$ equal zero as in this case $\nu_S \approx 0$ and $\hat{F}^R \cong \hat{F}^A$. The first integration of Eq. (1) yields

$$M_+ \partial_x f_+(x) + J_S f_-(x) + J_{an}\partial_x f_-(x) = J_+, \qquad (3)$$

where J_+ is a constant of integration. In the limit of the small parameter r, all the corrections related to the proximity effect are small (the functions $\hat{F}^{R(A)}$ are proportional to r). Therefore in the main approximation in the parameter r, we have for the distribution function $f_+(x)$

$$f_+(x) \cong \delta\beta\epsilon(x/L)\cosh^{-2}(\beta_0\epsilon) + f_{eq}. \qquad (4)$$

In obtaining Eq. (4) we have taken into account that in the N reservoirs the function $f_+(\pm L)$ has an equilibrium form with different temperatures: $f_+(\pm L) = f_{eq} \pm \delta\beta\epsilon\cosh^{-2}(\beta_0\epsilon)$, where $\delta\beta/\beta \equiv -\delta T/T, \beta_0 = (2T_0)^{-1}$. Equation (4) implies that the temperature gradient leads to a flow of nonequilibrium electrons and holes in the N wire. In order to find the distribution function $f_-(x)$, we integrate Eq. (1)

$$M_- \partial_x f_-(x) + J_S f_+(x) - J_{an}\partial_x f_+(x)$$
$$= J_1\Theta(L_1 - |x|) + J_{2+,2-}\Theta(|x| - L_1). \qquad (5)$$

The constants J_1 and $J_{2+,2-}$ are the partial currents in regions $(-L_1, +L_1)$, (L_1, L), and $(-L, -L_1)$ respectively. Outside the interval $(-L_1, +L_1)$ the "supercurrent" J_S is zero (this follows directly from the expressions for J_S and for $\hat{F}^R$). As there is no current between the N reservoirs, the integrals over energy ϵ from $J_{2+,2-}$ should be equal to zero. In the absence of the temperature gradient we obtain from Eq. (5) $f_- = 0$ and $J_S f_{eq} = J_{1eq} = -r(g_{z-} + g_{z+})f_{eq}$. According to Eq. (1) the constants J_1 and $J_{2+,2-}$ are related to each other (Kirchoff's law)

$$J_{2+,2-} - \delta J_1 = \pm r[g_{1+}f_- - g_{z+}\delta f_+]_{(\pm L_1)}, \qquad (6)$$

where $\delta J_1 = J_1 - J_S$ and $\delta f_+ = f_+ - f_{eq}$. Integrating Eq. (5) outside the interval $(-L_1, +L_1)$ and taking into account the boundary condition for $f_-(x)$, we obtain $J_{2+,2-} = \pm[f_-(\pm L) - f_-(\pm L_1)]/L_2$. Using this expression and Eq. (6), one can show that in the main approximation in r the function $f_-(x)$ is almost constant along the N wire and equal to $f_-(x) \approx f_-(0) \approx f_-(\pm L) = eV_T\beta\cosh^{-2}(\beta_0\epsilon)$. Integrating Eq. (6) over energies we readily find eV_T

$$eV_T = \frac{L_1}{L}\frac{\delta T}{T_0}\int_0^\infty d\epsilon\, g_{z+}\epsilon$$

$$\times \cosh^{-2}(\beta_0\epsilon) \bigg/ \int_0^\infty d\epsilon\, g_{1+}\cosh^{-2}(\beta_0\epsilon). \qquad (7)$$

The potential V_T equals approximately the voltage difference between the N reservoirs and the superconducting loop. It is worth noting that V_T, determined by Eq. (7), does not depend on the small parameter r because both functions g_{z+} and g_{1+} are proportinal to r [however, one should have in mind that according to the condition (2) this parameter must not be too small]. The integrand in Eq. (7) can be calculated if the function $\hat{F}^R$ is known from an approximate or numerical solution of the Usadel equation. In the limit considered, of small r, the retarded (advanced) Green's functions are readily found from the linearized Usadel equation. In this case we find

$$g_{z+} = r\,\mathrm{Re}\left(F_S^R\frac{\sinh^2\theta_2}{\theta\sinh(2\theta)}\right)\sin\varphi;$$

$$g_{1+} = r\,\mathrm{Im}\left\{F_S^R[\sinh(\theta_2 + 2\theta_1) + \cos\varphi\sinh\theta_2]\frac{\sinh\theta_2}{\theta\sinh(2\theta)}\right\}, \qquad (8)$$

where $\theta = \theta_1 + i\theta_2$, $\theta_{1(2)} = k_\epsilon L_{1(2)}$, $k_\epsilon = \sqrt{-2i\epsilon/\epsilon_L}$, $L_2 = L - L_1$, and $F_S^R = \Delta/\sqrt{(\epsilon + i\Gamma)^2 - \Delta^2}$ is the retarded Green's function in the superconductor. One can see that the ratio in Eq. (7) indeed does not depend on r. Numerical analysis of the Usadel equation shows that at a characteristic energy $\epsilon \cong \epsilon_L$ the difference between the linearized and exact numerical solutions of the Usadel equation is less than 10% even for $r \approx 1$.

It follows from Eqs. (7) and (8) that the voltage V_T caused by the temperature gradient is zero when the phase difference between the superconductors is zero and oscillates with increasing φ. One can easily estimate the order of magnitude of V_T. We find

$$eV_T = \delta T(L_1/L)\sin\varphi\begin{cases}(T/\epsilon_L)C_1(\varphi), & T \ll \epsilon_L \\ (\epsilon_L/T)^{3/2}C_2(\varphi), & T \gg \epsilon_L.\end{cases} \qquad (9)$$

Here $C_{1,2}(\varphi)$ are periodic functions of the phase difference φ of order 1 and are not zero when $\varphi = 0$.

If we define the thermo-emf V_T as the voltage between the N reservoirs and the superconducting circuit, we can state that this thermo-emf is much larger than the thermo-emf between the N reservoirs in the absence of superconductors because V_T does not contain the small parameter (T/ϵ_F) as is the case in a normal system (see, for example, Ref. 1). In addition, V_T oscillates with an applied magnetic field H (φ is
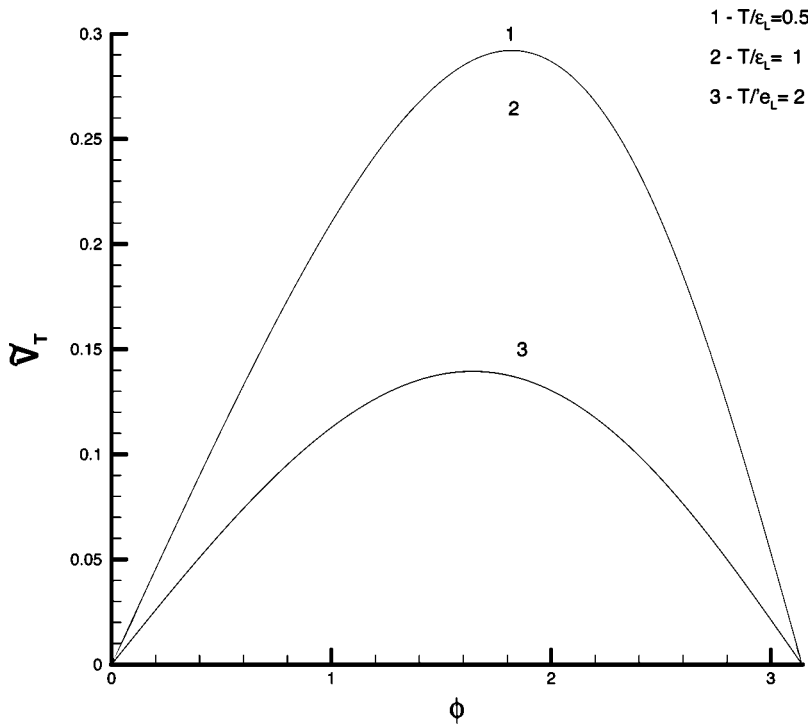
FIG. 2. The dependence of the normalized thermoeletrical voltage $\tilde{V}_T = eV_T/(\delta T L_1/L)$ on the phase difference φ for various $\beta = T/\epsilon_L$ (the parameters are $\Delta/\epsilon_L = 10, L_1/L = 0.5, r = 0.3$).

proportional to H) allowing one to detect small temperature gradients. We stress once again that the thermo-emf analyzed in this work arises not between the normal reservoirs, as takes place in the case of the ordinary thermoelectric effect, but between the superconducting and normal circuits. In Fig. 2 we plot the dependence V_N on φ for various $\beta^{-1} = (2T)$. In Fig. 3 the temperature dependence V_T at $\varphi = \pi/2$ is presented. We see that this dependence is nonmonotonic with a maximum at $T \approx \epsilon_L$ (reentrant behavior). We note that the influence of the proximity effect on the ordinary thermoelectric effect was studied theoretically in Ref. 5. We ignore this effect regarding the thermoelectric current $\alpha\nabla T$ as negli-

gible, compared to the effect under consideration. In Ref. 6 the thermoelectric voltage was measured in complicated S/N structures which differ from the simple structure considered by us. The reentrant behavior of V_T and its oscillations on φ were observed in this work. It is possible that the observed effects are related to those considered in this paper. It is worthwhile noting that the influence of the ordinary thermoelectric current $\alpha\nabla T$ on the Josephson effect was studied long ago.[7]

The physical explanation of the effect is the following. The temperature gradient creates a deviation of the distrubtion function $\delta f_+ = -(\delta n + \delta p)$ from the equilibrium form.
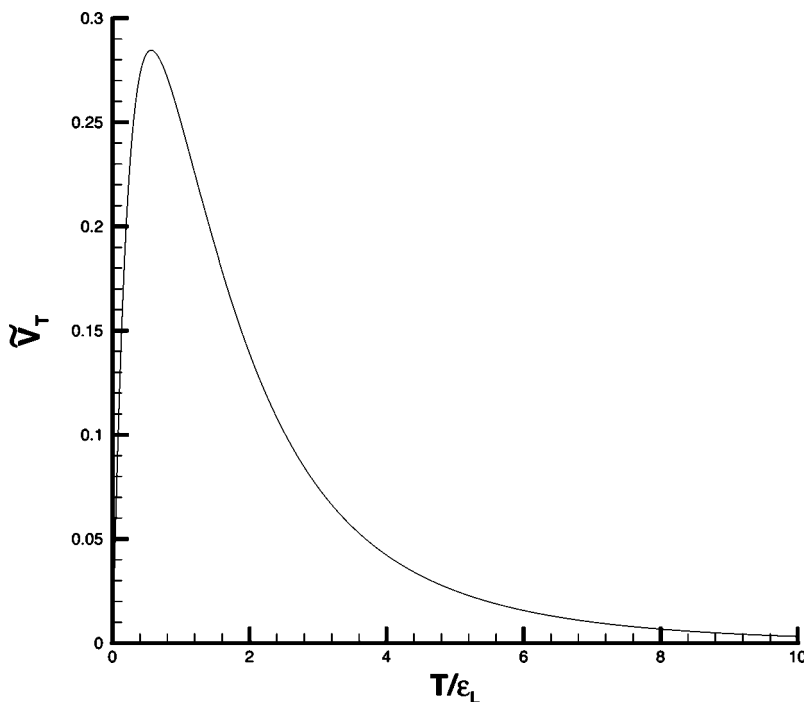

FIG. 3. The temperature dependence of the normalized voltage $\tilde{V}_T$ at $\varphi = \pi/2$, for the same parameters as in Fig. 2.

On the other hand, the superconductors do not affect this function because complete Andreev reflection conserves the total number of excess electrons and holes. The function δf_+ changes the condensate current flowing across the S/N interface. If δf_+ is created by an electrical current flowing between the N reserviors, it has the same sign at $x = \pm L$ and leads to a change in the Josephson current. In the case considered here the function δf_+ has different signs at these points and leads to a variation of the condesate current of the same sign at different S/N interfaces. Therefore, the potential V_T arises in the N wire producing a subgap current $rg_{1+}f_-(\pm L_1)$ which compensates the current δJ_S.

In summary, we have calculated the voltage V_T between the superconducting and normal-metal curcuits in a S/N mesoscopic structure where the normal-metal reservoirs are maintained at different temperatures. This voltage arises due to a branch imbalance in the N film and oscillates with varying phase difference. Its magnitude does not contain the small parameter (T/ϵ_F) which is present in normal systems and is of the order $\delta T(L_1/L)/e$.

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