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Role of in-plane dissipation in dynamics of a Josephson vortex lattice in high-temperature superconductors

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We calculate the flux-flow resistivity of the Josephson vortex lattice in a layered superconductor taking into account both the interplane and in-plane dissipation channels. We consider the limiting cases of small fields (isolated vortices) and high fields (overlapping vortices). In the case of the dominating in-plane dissipation, typical for high-temperature superconductors, the field dependence of flux-flow resistivity is characterized by *three* distinct regions. As usual, at low fields the flux-flow resistivity grows linearly with field. When the Josephson vortices start to overlap the flux-flow resistivity crosses over to the regime of *quadratic* field dependence. Finally, at very high fields the flux-flow resistivity saturates at the *c*-axis quasiparticle resistivity. The intermediate quadratic regime indicates the dominant role of the in-plane dissipation mechanism. The shape of the field dependence of the flux-flow resistivity can be used to extract both components of the quasiparticle conductivity.

A stack of low dissipative Josephson junctions formed by the atomic layers of high-temperature superconductors[1] represents a nonlinear system with unique dynamic properties. The magnetic field applied along the layers creates the lattice of Josephson vortices (JV's).[2] Transport properties of this state are determined by dynamics of the Josephson lattice. Two distinct regimes exist depending on the strength of the applied magnetic field. At low fields the Josephson vortices are isolated and form a triangular lattice, strongly stretched along the direction of layers. An isolated JV is characterized by the nonlinear core, the region within which the phase difference between the two central layers sweeps from 0 to 2π. The core size is given by the Josephson length, γs, where γ is the anisotropy of the London penetration depth and s is the interlayer spacing. This regime of a dilute lattice is characterized by the linear field dependence of flux-flow resistivity $\rho_{ff}\propto B$. The linear flux-flow branch in $Bi_2Sr_2CaCu_2O_8$ (BSCCO) at small fields has been observed experimentally.[3,4]

When a magnetic field exceeds the crossover field, $B_{cr}=\Phi_0/\pi\gamma s^2$, the cores of JV's start to overlap and a dense Josephson lattice is formed, in which JV's fill all layers.[5] In this regime the linear field dependence of the flux-flow resistivity breaks down. Further field behavior depends on the mechanism of dissipation.

Moving Josephson vortices generate both in-plane and interplane electric fields, which induce dissipative quasiparticle currents. Usually only dissipation due to the tunneling of quasiparticles between the layers is taken into account in calculation of the viscosity of JV's.[6] However in the high-T_c superconductors the in-plane quasiparticle conductivity σ_{ab} is strongly enhanced in the superconducting state as compared to the normal conductivity due to reduction of phase space for scattering,[7,8] while the *c*-axis component σ_c rapidly decreases with temperature in the superconducting state.[9] Below the transition temperature the anisotropy of dissipation σ_{ab}/σ_c becomes larger than the superconducting anisotropy γ^2. This leads to the dominating role of the in-plane dissipation in the dynamics of the Josephson lattice.

In this paper we calculate the field dependence of the flux-flow resistivity $\rho_{ff}(B)$ taking into account both the in-plane and interplane dissipation channels. We separately consider the regimes of small and high fields. The flux-flow resistivity at high fields, taking into account the in-plane dissipation, has been studied before in Ref. 10. For the case of purely *c*-axis dissipation linear growth of the flux-flow resistivity saturates at the *c*-axis quasiparticle resistivity ρ_c when the magnetic field exceeds the crossover field B_{cr}. Dominating in-plane dissipation leads to qualitative change of the shape of $\rho_{ff}(B)$. In this case ρ_{ff} also increases linearly at small fields. The slope of this dependence is mainly determined by σ_{ab}, and at $B\approx B_{cr}$ the resistivity ρ_{ff} is still much smaller than ρ_c. At $B\approx B_{cr}$ the field dependence of ρ_{ff} crosses over to even faster, *quadratic*, dependence. Only at a significantly higher field, $B\approx\sqrt{\sigma_{ab}/(\gamma^2\sigma_c)}B_{cr}$, ρ_{ff} reaches ρ_c. Therefore the field dependence of ρ_{ff} can be used to extract both components of the quasiparticle conductivity.

Dynamics of the moving Josephson lattice is governed by the coupled Sine-Gordon equations for the interlayer phase differences.[11] The equations taking into account in-plane dissipation have been derived in Refs. 10 and 12. Consider a layered superconductor in a magnetic field applied along the layers (y direction) and carrying transport current along the c axis (z direction). We express fields and currents in terms of the gauge invariant phase difference between the layers $\theta_n=\phi_{n+1}-\phi_n-(2\pi s/\Phi_0)A_z$ and the in-plane superconducting momentum $p_n=\nabla_x\phi_n-(2\pi/\Phi_0)A_x$. The local magnetic field B_n between the layers n and $n+1$ can be expressed as

$$B_n(x)=\frac{\Phi_0}{2\pi s}\left(\frac{\partial\theta_n}{\partial x}-p_{n+1}+p_n\right). \tag{1}$$

The components of the electric field can be approximately represented as

$$E_x\approx\frac{\Phi_0}{2\pi c}\frac{\partial p_n}{\partial t};\quad E_z\approx\frac{\Phi_0}{2\pi cs}\frac{\partial\theta_n}{\partial t}. \tag{2}$$

These expressions are valid assuming fast equilibration inside the layers. More general situations have been considered in Refs. 10,13–15. The components of electric current, j_x and j_z, consist of the quasiparticle and superconducting contributions

$$j_x=\sigma_{ab}\frac{\Phi_0}{2\pi c}\frac{\partial p_n}{\partial t}+\frac{c\Phi_0}{8\pi^2\lambda_{ab}^2}p_n, \tag{3}$$

$$j_z=\sigma_c\frac{\Phi_0}{2\pi cs}\frac{\partial\theta_n}{\partial t}+j_J\sin\theta_n, \tag{4}$$

where σ_{ab} and σ_c are the components of the quasiparticle conductivity, λ_{ab} and λ_c are the components of the London penetration depth, and $j_J=c\Phi_0/(8\pi^2 s\lambda_c^2)$ is the Josephson current density. Note that the in-plane current is actually concentrated inside the superconducting layers and physically meaningful quantities are the two-dimensional current densities J_{xn} in the layers. To be precise, the bulk current in Eq. (3) is defined at discrete points $z_n=ns$ as $j_x(z_n)\equiv J_{xn}/s$. Using the above relations we rewrite the z and x components of the Maxwell equation

$$\frac{4\pi}{c}\mathbf{j}+\frac{\partial\mathbf{D}}{\partial t}=\nabla\times\mathbf{B}$$

as

$$\frac{2\sigma_c\Phi_0}{c^2 s}\frac{\partial\theta_n}{\partial t}+\frac{4\pi}{c}j_J\sin\theta_n+\frac{\varepsilon_c\Phi_0}{2\pi c^2 s}\frac{\partial^2\theta_n}{\partial t^2}=\frac{\partial B_n}{\partial x}, \tag{5}$$

$$\frac{2\sigma_{ab}\Phi_0}{c^2}\frac{\partial p_n}{\partial t}+\frac{\Phi_0}{2\pi\lambda_{ab}^2}p_n=-\frac{B_n-B_{n-1}}{s}. \tag{6}$$

In the second equation we replaced $\partial B/\partial z$ by the discrete derivative $(B_n-B_{n-1})/s$. We also neglected the in-plane displacement current $\partial D_x/dt$, because typical frequencies involved in Josephson dynamics are much smaller than the in-plane plasma frequency c/λ_{ab}. Equations (1), (5), and (6) give closed system which describes dynamics of the phases $\theta_n(x,t)$, fields $B_n(x,t)$, and momenta $p_n(x,t)$. The moving lattice generates both in-plane and c-axis electric fields [Eq. (2)] leading to dissipation. The rate of energy dissipation W per unit volume is given by

$$W=\left(\frac{\Phi_0}{2\pi c}\right)^2\left[\frac{\sigma_c}{s^2}\left\langle\left(\frac{\partial\theta_n}{\partial t}\right)^2\right\rangle+\sigma_{ab}\left\langle\left(\frac{\partial p_n}{\partial t}\right)^2\right\rangle\right]. \tag{7}$$

For the steady state motion with small velocity v the phase differences vary in space and time as $\theta_n(x,t)=\theta_n^{(0)}(x-vt)$, where $\theta_n^{(0)}(x)$ is the static phase distribution, and Eq. (7) can be rewritten as

$$W=\eta_J v^2,$$

where

$$\eta_J=\left(\frac{\Phi_0}{2\pi c}\right)^2\left[\frac{\sigma_c}{s^2}\left\langle\left(\frac{\partial\theta_n^{(0)}}{\partial x}\right)^2\right\rangle+\sigma_{ab}\left\langle\left(\frac{\partial p_n^{(0)}}{\partial x}\right)^2\right\rangle\right] \tag{8}$$

is the linear viscosity coefficient of the lattice per unit volume and $\langle\cdots\rangle$ means averaging with respect to x and n. The flux-flow resistivity ρ_{Jff} is connected with η_J by relation $\rho_{Jff}=B^2/(c^2\eta_J)$.

Consider the regime of small fields, $B\ll B_{cr}=\Phi_0/(\pi\gamma s^2)$. In this regime the JV's are isolated and dissipation is concentrated in the vicinity of nonlinear vortex cores. In this case η_J is proportional to the field $\eta_J=B\eta_{Jv}/\Phi_0$, where η_{Jv} is the viscosity coefficient of an individual JV per unit length. The viscosity coefficient due to the c-axis dissipation has been calculated by Clem and Coffey.[6] Similar problems of viscous friction have been studied for an Abrikosov vortex (see, e.g., Ref. 16) and for a Josephson vortex in a single junction.[17]

In the vicinity of the vortex cores one can neglect screening effects and express the phase differences $\theta_n(x)$ and momenta $p_n(x)$ via the in-plane phases $\phi_n(x)$, $\theta_n^{(0)}\approx\phi_{n+1}-\phi_n$, $p_n^{(0)}\approx\nabla_x\phi_n$ (we are using the gauge div$\mathbf{A}=0$). Numerically accurate phase distribution $\phi_n(x)$ in the vicinity of the vortex core was obtained in Ref. 18,

$$\phi_n(u)\approx\arctan\frac{n-1/2}{u}+\frac{0.35(n-1/2)u}{[(n-1/2)^2+u^2+0.38]^2}+\frac{8.81(n-1/2)u(u^2-(n-1/2)^2+2.77)}{[(n-1/2)^2+u^2+2.02]^4} \tag{9}$$

with $u\equiv x/\gamma s$.[19] We can represent now the viscosity coefficient η_{Jv} as

$$\eta_{Jv}=\frac{1}{\gamma s^2}\left(\frac{\Phi_0}{2\pi c}\right)^2\left[C_c\sigma_c+C_{ab}\frac{\sigma_{ab}}{\gamma^2}\right],$$

with

$$C_c=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}du\left(\frac{\partial(\phi_{n+1}-\phi_n)}{\partial u}\right)^2,$$

$$C_{ab}=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}du\left(\frac{\partial^2\phi_n}{\partial u^2}\right)^2.$$

Using the phase distribution [Eq. (9)] we compute $C_c\approx 9.0$ and $C_{ab}\approx 2.4$. Finally, we obtain the following result for the flux-flow resistivity at small fields $\rho_{Jff}=\Phi_0 B/(c^2\eta_{Jv})$:

$$\rho_{Jff}\approx\frac{4.4\gamma s^2 B}{\Phi_0(\sigma_c+0.27\sigma_{ab}/\gamma^2)},\quad \text{at } B<B_{cr}. \tag{10}$$

The case of dominating c-axis dissipation ($\sigma_{ab}\ll\gamma^2\sigma_c$) has been considered by Coffey and Clem.[6] They obtain the coefficient 2.8 instead of 4.4 using the approximate phase distribution.

Now we consider the regime of high fields, $B>B_{cr}$. In this regime we can obtain a simple analytical result using expansion with respect to the Josephson current.[10] For the

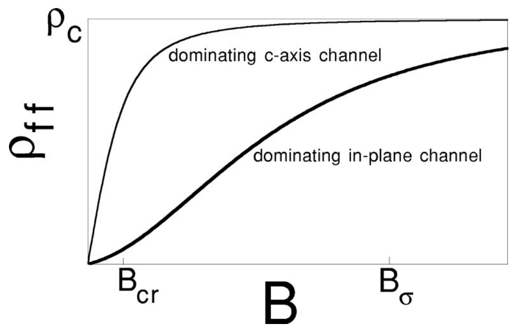

FIG. 1. Schematic field dependence of the flux-flow resistivity for the cases of dominating c-axis dissipation channel ($\sigma_{ab}/\sigma_c \ll \gamma^2$) and dominating in-plane dissipation channel ($\sigma_{ab}/\sigma_c \gg \gamma^2$). For strong in-plane dissipation the dependence $\rho_{ff}(B)$ has pronounced upward curvature at $B \gtrsim B_{cr}$ and approaches ρ_c at the field B_σ much larger than the crossover field B_{cr}.

static lattice in the zero order with respect to j_J we have $B_n(x)=B$, $p_n(x)=0$, and $\theta_n(x)=2\pi s B x/\Phi_0+\pi n$. The first iteration with respect to $j_J \equiv c\Phi_0/8\pi^2 s \lambda_c^2$ gives

$$B_n(x)=B-\frac{\Phi_0^2}{16\pi^2 s^2 \lambda_c^2 B}\cos\left(\frac{2\pi s B}{\Phi_0}x+\pi n\right), \tag{11}$$

$$p_n(x)=\frac{\Phi_0}{\pi B s^3 \gamma^2}\cos\left(\frac{2\pi s B}{\Phi_0}x+\pi n\right). \tag{12}$$

Substituting expressions for $\theta_n(x)$ and $p_n(x)$ into Eq. (8) we obtain

$$\eta_J=\left(\frac{\Phi_0}{\pi c \gamma s^2}\right)^2\left[\sigma_c\left(\frac{\pi s^2 \gamma B}{\Phi_0}\right)^2+\frac{\sigma_{ab}}{2\gamma^2}\right] \tag{13}$$

and

$$\rho_{Jff}=\frac{B^2}{B^2+B_\sigma^2}\rho_c, \quad B_\sigma=\sqrt{\frac{\sigma_{ab}}{\sigma_c}}\frac{\Phi_0}{\sqrt{2}\pi\gamma^2 s^2} \tag{14}$$

for $B>B_{cr}$.[20] At $B\approx B_{cr}$ this result approximately matches with Eq. (10). Equations (10) and (14) represent the main results of this paper. We see that the shape of $\rho_{Jff}(B)$ is determined by the ratio $\sigma_{ab}/(\gamma^2\sigma_c)$. In the high-temperature superconductors typically $\sigma_{ab}/\sigma_c \approx \gamma^2$ near the transition temperature. However, the ratio σ_{ab}/σ_c rapidly becomes much larger than γ^2 with a temperature decrease because of (i) a significant enhancement of σ_{ab} due to the suppression of in-plane scattering of quasiparticles[8] and (ii) a fast decrease of σ_c.[9] This means that dependence $\rho_c \propto B^2$ holds in a wide field range $B_{cr}<B<B_\sigma$. The field B_{cr} is almost temperature independent and for optimally doped BSCCO ($\gamma \approx 500$) $B_{cr}\approx 0.5$ T. The field B_σ has strong temperature dependence via $\sigma_{ab}(T)$ and $\sigma_c(T)$. Taking typical values for $T\approx 20$ K, $\sigma_c=2\times 10^{-3}(\Omega\ \mathrm{cm})^{-1}$ and $\sigma_{ab}\approx 5\times 10^4(\Omega\ \mathrm{cm})^{-1}$, we obtain estimate $B_\sigma \approx 4$ T. The field dependencies for the cases of dominating c-axis dissipation and dominating in-plane dissipation are sketched in Fig. 1. The shapes of the field dependencies are qualitatively different for these two cases. In particular, they have opposite curvatures at $B \lesssim B_{cr}$. Therefore, the shape of $\rho_{Jff}(B)$ can be used to extract both components of the quasiparticle conductivity. At present, there are no published data for the field dependence of the flux-flow resistivity in a wide field range. Dynamics of the Josephson lattice at high fields has been studied by G. Hechtfischer *et al.*[21] From the I-U curves presented in this paper one can see that the slope dI/dU for the first flux-flow branch indeed has a strong field dependence with upward curvature in the field range $2-3.5$ T.

In conclusion, we calculated the flux-flow resistivity of the Josephson lattice at small and high fields and demonstrated that strong in-plane dissipation qualitatively modifies its field dependence.

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[19] Unfortunately, the formula (B12) in Ref. 18 for phase distribution ϕ_n was written incorrectly. Equation (9) gives correct phase distribution, which was actually used in numerical calculations and is plotted in Fig. 8 of Ref. 18.

[20] Equations (13) and (14) are similar to Eq. (42) from Ref. 10 for the flux-flow conductivity except for the sign in front of the in-plane term.

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