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Low-energy dynamics of the one-dimensional multichannel Kondo-Heisenberg lattice

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We determine *exactly* the fixed point Hamiltonian of the one-dimensional multichannel Kondo-Heisenberg lattice model for any number of channels $N \geqslant 2$. It is found to belong to a new class of non-Fermi-liquid fixed points, different from the usual Luttinger or Luther-Emery liquids. The fixed point describes an anomalous singlet with nontrivial internal dynamics manifesting itself in unconventional order. We compute the correlation functions of the various conventional and composite order parameters of the system, and find that for $N \leqslant 4$ the composite order parameters induce the dominant instabilities.

The Kondo lattice is one of the most challenging problems in contemporary theoretical condensed matter physics. If the single impurity Kondo problem, a local moment antiferromagnetically coupled to conduction electrons, is by now well understood thanks to a variety of theoretical techniques,[1] the problem of a regular three-dimensional array of local moments in a metal (the Kondo lattice) still defies theoretical analysis. The difficulty stems from the fact that there are two competing effects, the tendency of the local moments to form singlets with the conduction electrons (or more complicated states[2] if more than one band is available) and the tendency of the moments to order due to the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction mediated by the conduction electrons. Besides its intrinsic theoretical interest for the general theory of strongly correlated fermions, the Kondo lattice model is believed to capture the non-Fermi liquid physics of a class of rare-earth or actinide compounds[3] in which *f*-shell local moments couple to the electron bands. In this context,[4] it has been suggested that the two-channel Kondo model could be relevant in Ce^{3+} or U^{4+} based alloys such as UBe_{13} and $CeCu_2Si_2$. This hypothesis is supported by experiments on diluted alloys that show a two-channel single impurity behavior in the specific heat or magnetic susceptibility. A full understanding of the multichannel Kondo lattice would also be of great significance for experiment.

Many problems of strongly correlated fermions can be treated very effectively in one dimension thanks to powerful analytical[5] and numerical[6] techniques available there. In particular, the full phase diagram of the one-dimensional single channel Kondo lattice has been determined.[7] An insulating state with a spin gap obtains at half-filling. Away from half-filling a small Kondo coupling produces a paramagnetic metal and a large one leads to ferromagnetism. Adding an antiferromagnetic Heisenberg interaction between the local moments (Kondo-Heisenberg lattice) produces a spin-gapped metal[8] with nonconventional pairing fluctuations.[9]

In this Communication, we investigate the ground state, excitations, and dominant instabilities of the multichannel one-dimensional Kondo-Heisenberg lattice model for incommensurate fillings and zero temperature. We find that screening of the local moments occurs via the ''chiral stabilization'' mechanism,[12] resulting in a ground state that is a chiral non-Fermi liquid we call *coset singlet*. We show that the dominant instability for a low number of channels $N \leqslant 4$ is of the nonconventional pairing type as in the single channel case,[9] and that for a larger number of channels, one recovers conventional pairing instabilities as the dominant ones. Fermi liquid physics is regained in the limit of a very large number of channels. We also discuss the effect of channel anisotropy.

The Hamiltonian of the one-dimensional multichannel Kondo-Heisenberg lattice is:

$$H = -t \sum_{i,n,\sigma} (c^\dagger_{i,n,\sigma} c_{i+1,n,\sigma} + c^\dagger_{i+1,n,\sigma} c_{i,n,\sigma}) + \lambda_H \sum_i \vec{S}_i \vec{S}_{i+1} + \lambda_K \sum_{i,n} \vec{S}_i \cdot c^\dagger_{i,n,\alpha} \vec{\sigma}_{\alpha,\beta} c_{i,n,\beta}, \quad (1)$$

where $\sigma = \uparrow,\downarrow$ is the spin of an electron, $n = 1, \ldots, N$ is the channel index, and $\vec{S}_i$ is a localized spin 1/2. t is the bandwidth, $\lambda_H > 0$ the Heisenberg coupling of the localized spins, and $\lambda_K > 0$ the Kondo coupling.

To study the low-energy physics of the model we follow the standard strategy:[11] keep only linear electron modes around $\pm k_F$ captured in terms of the left and right moving fields $\psi_{L,R,\sigma,n}(x)$, as well as the low-lying spin-lattice modes around π/a. The low-energy physics is then described by a continuum Hamiltonian expressed using non-Abelian bosonization[10,11] as a quadratic form of various currents: the $SU(2)_1$ (left and right) spin currents $\vec{\sigma}_{L,R}$ of the local moments, the $SU(2)_N$ (left and right) spin current $\vec{S}_{L,R}(x)$ of the electrons and the electron charge and channel currents which decouple from the spin currents in the continuum. The notation $SU(M)_k$ indicates that the currents (generically denoted $J^a_{L,R}$) satisfy a Kac-Moody (KM) algebra: $[J^a_L(x), J^b_L(x')] = \delta(x-x') f^{abc} J^c_L(x) + (k/2\pi)\delta'(x-x')$, where f_{abc} are the structure constants of the SU(M) Lie algebra. A similar relation holds for the currents J_R, while the J_R and J_L currents commute. The KM algebra generates a conformal field theory (CFT) known in Lagrangian form as the Wess-Zumino-Novikov-Witten[10] (WZNW) model. All the physical operators of the original lattice theory (1) can be expressed in terms of the primary or descendant fields of this CFT, allowing the calculation of the asymptotic behavior of their correlators. This is at the heart of all applications of non-Abelian bosonization to condensed matter physics.

Consider the Hamiltonian with $\lambda_K=0$. The electrons are free, and in the basis described above, charge, spin, and channel excitations propagate independently of each other, and decouple from the local-moment excitations. Turning on a small antiferromagnetic Kondo coupling $0<\lambda_K\ll t,\lambda_H$ at incommensurate filling, the charge and flavor excitations remain decoupled from the spin excitations. We can thus restrict ourselves to the latter, described by the Hamiltonian:

$$H=\int\left[\frac{2\pi v}{N+2}(\vec{S}_L\cdot\vec{S}_L+\vec{S}_R\cdot\vec{S}_R)+\frac{2\pi v_s}{3}(\vec{\sigma}_L\cdot\vec{\sigma}_L+\vec{\sigma}_R\cdot\vec{\sigma}_R)+\lambda_K^f(\vec{S}_L\cdot\vec{\sigma}_L+\vec{S}_R\cdot\vec{\sigma}_R)+\lambda_K^b(\vec{S}_L\cdot\vec{\sigma}_R+\vec{S}_R\cdot\vec{\sigma}_L)\right], \tag{2}$$

where we dropped irrelevant terms such as $\vec{\sigma}_L\cdot\vec{\sigma}_R$ The coupling λ_K^f describes the forward scattering, λ_K^b the backward scattering, and $\lambda_K^f=\lambda_K^b=\lambda_K$ to begin with. Under renormalization-group (RG) transformations, λ_K^f does not flow near the weak coupling fixed point and merely renormalizes the velocity. On the other hand, λ_K^b is relevant and drives the system to a new strong coupling fixed point. We will thus take $\lambda_K^f=0$ and $v_s=v$ in Eq. (2) and determine the fixed point. We will later prove that the neglected terms are irrelevant near the strong coupling fixed point.

Under these circumstances the spin sector is described by $H=H_1+H_2$, where

$$H_1=\int dx\left[\frac{2\pi v}{N+2}\vec{S}_R\cdot\vec{S}_R+\frac{2\pi v}{3}\vec{\sigma}_L\cdot\vec{\sigma}_L+\lambda_K\vec{S}_R\cdot\vec{\sigma}_L\right],$$

$$H_2=\int dx\left[\frac{2\pi v}{N+2}\vec{S}_L\cdot\vec{S}_L+\frac{2\pi v}{3}\vec{\sigma}_R\cdot\vec{\sigma}_R+\lambda_K\vec{S}_L\cdot\vec{\sigma}_R\right].$$

In H_1 (H_2), the left (right) branch of the $\mathrm{SU}(2)_N$ WZNW model is coupled to the right (left) branch of the $\mathrm{SU}(2)_1$ WZNW model, leading to a chiral asymmetry of H_1 (H_2). We readily identify the strong coupling fixed point of Eq. (3) exactly via ‘‘chiral stabilization.’’[12] The chiral asymmetry in H_1 or H_2 is invariant under the RG flow and characterizes the fixed point. We find this way that the theory (3) flows under RG to a fixed point theory that is the product of a coset theory[13] by a WZNW theory:

$$H^*=\frac{\mathrm{SU}(2)_1\times\mathrm{SU}(2)_{N-1}}{\mathrm{SU}(2)_N}\otimes\mathrm{SU}(2)_{N-1}, \tag{3}$$

where the coset theory, $\mathrm{SU}(2)_1\times\mathrm{SU}(2)_{N-1}/\mathrm{SU}(2)_N$ describes the local moment spin sector, and the $\mathrm{SU}(2)_{N-1}$ WZNW theory describes the electron spin sector. Note that at the fixed point, the left and right components H_1 and H_2 are recombined, and chiral symmetry is globally preserved.

What is the physics around the fixed point? The coset theory describes a spin singlet which the local moments form with the electrons. It is a different type of a singlet, a *coset singlet*: a fraction $6/(N+1)(N+2)$ of the local moment ‘‘modes’’ are paired with the same number of electron spin ‘‘modes.’’ Thus the system loses twice this amount of degrees of freedom as seen in the total specific heat (including channel and charge degrees of freedom):

$$C_{\mathrm{total}}=\frac{\pi}{6}\left(2N+1-\frac{12}{(N+1)(N+2)}\right)T. \tag{4}$$

The susceptibility is given by

$$\chi=\frac{1}{2\pi v}(N-1) \tag{5}$$

and the Wilson ratio: $R_w=\{2N+1-[12/(N+1)(N+2)]\}/(N-1)$. The coset singlet still retains degrees of freedom whose number is given by the central charge of the theory $c=1-6/(N+1)(N+2)$. This fraction decreases with the number of channels N since it is ‘‘easier’’ to form the singlet when N increases. This also shows up in the effective coupling of the electrons to the local moments which decreases with the number of channels, $\lambda_K^*\sim 1/N$.

Consider the two-channel case.[14] For $N=2$, local moments are described by a $SU(2)_1\times SU(2)_1/SU(2)_2=$Ising theory, or equivalently by a Majorana fermion. Such a Majorana fermion picture is very appealing since it is well known[15] that the single impurity two channel Kondo model will be described at the fixed point by a local Majorana fermion degree of freedom. These Majorana fermions form a band when coupled with each other, thus suppressing the single impurity residual entropy at $T=0$.

Having obtained the low energy theory (3) describing the spin sector, we proceed to express the original operators in terms of the operators of the fixed point theory. This will enable us to check that the operators we discarded in Eq. (2) are indeed irrelevant at the strong coupling fixed point, as well as determine the dominant instability of the multichannel Kondo-Heisenberg lattice. The needed identification of operators as well as the calculation of the scaling dimensions was done in Ref. 12. Let us briefly summarize the method. To obtain the conformal weight of a given operator, we first decompose it into a product of operators belonging to each of the two decoupled chiral theories. Then, for each chiral theory, we decompose operators of $\mathrm{SU}(2)_N\otimes\mathrm{SU}(2)_1$ on operators of $\mathrm{SU}(2)_{N-1}$ in an expansion formally similar to the Clebsch-Gordan expansion, the role of the Clebsch-Gordan coefficients being played by operators of the coset theory.[12] The operator with the lowest scaling dimension in this expansion is then retained as the fixed point form of the original operator.

The results are summarized in Table I for the theory described by H_1. The conformal weights of the theory described by H_2 are obtained by interchanging L and R. These conformal weights are such that the operators $\vec{S}_L(x)\cdot\vec{\sigma}_L(x)$, $\vec{S}_R(x)\cdot\vec{\sigma}_R(x)$, and $(v_s-v)(\vec{\sigma}_R\cdot\vec{\sigma}_R+\vec{\sigma}_L\cdot\vec{\sigma}_L)$ that we have previously discarded are indeed irrelevant (marginally irrelevant for $N=2$). This proves the self-consistency of our treatment.

The fixed point is a non-Fermi liquid. The Green's function of the right moving fermions is given by

$$\langle T\psi_R(x,t)\psi_R^\dagger(0,0)\rangle\sim\frac{1}{(x-vt)^{1+(1/2)\delta_N}(x+vt)^{(1/2)\delta_N}},$$

TABLE I. The conformal weights of the operators in the theory described by Eq. (3). Here, $\delta_N=3/(N+1)(N+2)$. For $N=2$, the conformal weights of $\vec{\sigma}_L$ and $\vec{\sigma}_R$ are, respectively, (0,1) and (1,0).

Operator	Conformal weights at the fixed point
$\psi_{R,n,\sigma}$	$(\frac{1}{2}+\delta_N/4,\delta_N/4)$
$\psi_{L,n,\sigma}$	$(\delta_N/4,\frac{1}{2}+\delta_N/4)$
$\tilde{g}_{L,\beta}$	$[3/4(N+1),\frac{1}{4}+3/4(N+1)]$
$\tilde{g}_{R,\beta}$	$[\frac{1}{4}+3/4(N+1),3/4(N+1)]$
$\vec{\sigma}_L(x)$	$[2/(N+1),1+2/(N+1)],N\geqslant 3$
$\vec{\sigma}_R(x)$	$[1+2/(N+1),2/(N+1)],N\geqslant 3$
$\vec{S}_R(x)$	(1,0)
$\vec{S}_L(x)$	(0,1)
$O_{\mathrm{CDW}}=\psi^\dagger_{L,n,\sigma}\psi_{R,n,\sigma}$	$[(1+\delta_N)/2,(1+\delta_N)/2]$
$\vec{O}_{\mathrm{SDW}}=\psi^\dagger_{L,n,\sigma}\vec{\sigma}_{\sigma,\sigma'}\psi_{R,n,\sigma'}$	$[(1+\delta_N)/2,(1+\delta_N)/2]$
$O_{\mathrm{SS}}=-\imath\psi_{L,n,\sigma}\sigma^y_{\sigma,\sigma'}\psi_{R,n,\sigma'}$	$[(1+\delta_N)/2,(1+\delta_N)/2]$
$\vec{O}_{\mathrm{TS}}=-\imath\psi_{L,n,\sigma}(\vec{\sigma}\sigma_y)_{\sigma,\sigma'}\psi_{R,n,\sigma'}$	$[(1+\delta_N)/2,(1+\delta_N)/2]$
$O_{\text{c-SP}}=\vec{n}\cdot\vec{O}_{TS}$	$[\frac{3}{4}-3/2(N+2),\frac{3}{4}-3/2(N+2)]$
$O_{\text{c-CDW}}=\vec{n}\cdot\vec{O}_{\mathrm{SDW}}$	$[\frac{3}{4}-3/2(N+2),\frac{3}{4}-3/2(N+2)]$

where $\delta_N=3/(N+1)(N+2)$. Here we have taken, for simplicity, charge, spin, and channel velocities to be equal, and combined exponents from all sectors, neglecting the nonuniversal Luttinger interaction in the charge sector (it can be easily taken into account[12]). Further, if a contribution in the channel sector is generated it is ferromagnetic and flows to zero. $G_L(x,t)$ is obtained by replacing $x\pm vt$ by $x\mp vt$. We note that a weak singularity appears at the Fermi level k_F, and there is no large Fermi surface. Also note that all dimensions tend to their Fermi liquid values in the limit of a large number of channels.

We now examine the possible order parameters of the system. Beginning with the localized moments, we have $\vec{S}_i=\vec{\sigma}_L(x)+\vec{\sigma}_R(x)+e^{\imath\pi x/a}\vec{n}(x)$, where $\vec{n}(x)$, the staggered magnetization, is given by $\vec{n}(x)=\frac{1}{2}\sum_{\alpha,\beta}\tilde{g}^\dagger_{R,\alpha}\vec{\sigma}_{\alpha,\beta}\tilde{g}_{L,\beta}$, where $\tilde{g}_{R,\alpha}$, $(\tilde{g}_{L,\alpha})$ are the right (left) WZNW fields. We find

$$\langle\vec{n}(x)\cdot\vec{n}^\dagger(x')\rangle\sim\frac{1}{|x-x'|^{1+6/(N+1)}}. \tag{6}$$

More order parameters are available in the electron sector. The order parameters for charge density wave (CDW), spin density wave (SDW), singlet (SS), and triplet (TS) superconducting are defined in the table and their dimension is given, from which it follows that they are degenerate and fall with a power of $2+2\delta_N$. As the fluctuations of these order parameters are weaker than in the one-dimensional metal, we are led to investigate the possibility of dominant fluctuations associated with a nonconventional order parameter[9] odd-frequency singlet pairing[16] (c-SP) and composite charge-density wave order (c-CDW): $O_{\text{c-SP}}=\vec{n}\cdot\vec{O}_{TS}$ and $O_{\text{c-CDW}}=\vec{n}\cdot\vec{O}_{\mathrm{SDW}}$. The composite operators have momentum $\pm\pi/a$ and $2k_F\pm\pi/a$, respectively, and their correlations decay with the power of $3-6/(N+2)$. The presence of gapless excitations at $2k_F\pm\pi/a$ is compatible with the results of Yamanaka *et al.*[17]

We observe that for $N\leqslant 5$, the composite order parameters have the most divergent correlations. In this case a large enough fraction of electron spin degrees of freedom is bound to the local moments to suppress the conventional order parameters and enhance the composite ones. This situation is similar to the one-channel one-dimensional Kondo lattice case[9] where composite pairing operators are also dominant. For $N=5$, the two types of order are degenerate *at the fixed point*, and one needs to describe also the approach to the fixed point starting from the bare Hamiltonian to determine the order. For $N\geqslant 6$, the fraction of electron spin degrees of freedom bound to a local moment is insufficient to permit composite order parameters to dominate the conventional ones, thus recovering the Fermi liquid limit.

Thus far, we assumed channel isotropy. In the single impurity problem[2] channel anisotropy is a relevant perturbation, so we have to investigate its effect for a lattice. Assume N_1 channels couple to the local moments with coupling strength λ^1_K, and $N_2=N-N_1$ channels couple with strength λ^2_K. Then, the spin excitations of the N_1 channels are described by a $SU(2)_{N_1}$ KM algebra, whereas those of the N_2 remaining channels are described by a $SU(2)_{N_2}$. For $\lambda^1_K\ll\lambda^2_K$, *coset* screening occurs between the N_1 channels and the local moments. The resulting theory is $SU(2)_1\otimes SU(2)_{N_1-1}/SU(2)_{N_1}\otimes SU(2)_{N_1-1}\times SU(2)_{N_2}$, with central charge $c=2N+1-12/(N_1+1)(N_1+2)$ smaller than $c=2N+1-12/(N+1)(N+2)$, the charge of the symmetric fixed point. It is the stable fixed point having the lower number of degrees of freedom.[18] This implies that channel anisotropy is always relevant. Still, although unstable to anisotropies, it may still determine the behavior over a broad intermediate regime of energies, and perhaps eventually stabilize unconventional order.

What would happen if we relax the condition $\lambda_K\ll\lambda_H,t$? Since in one dimension weak coupling and intermediate coupling are connected this only gives a nonuniversal exponent K_ρ for charge excitations. This adds $K_\rho^{-1}-1$ to the superconducting exponents, $K_\rho-1$ to the density wave exponents, and $(K_\rho+K_\rho^{-1}-2)/4$ to the electron Green's function exponent.

We have derived the fixed point theory describing the one-dimensional multichannel Kondo-Heisenberg lattice. This fixed point is in the class of chiral non-Fermi liquids. We showed that the dominant instability is of the composite pairing type for a number of channels smaller than five and of the conventional pairing type otherwise. Our results were obtained in the limit of weak Kondo coupling, at zero temperature, and away from half-filling. At half-filling, it is known that a spin gap develops only for $N-2S$ integer.[20]

There are many directions for future research: At finite temperature, the behavior we described must disappear above the larger of the Kondo temperatures T^e_K and T^s_K, characterizing the electron and local moment sectors, respectively. Also, the problem of exhaustion,[21] how a large concentration of impurities reduces the Kondo scale, is still unresolved. The conformal field theory approach of our paper is not able to settle this issue. However, the model (2) happens to be Bethe ansatz integrable. This will allow us to settle the issue of the Kondo scales and their dependence on the filling—exhaustion—and discuss the full crossover to the zero temperature behavior. A simple generalization is to replace the spin-1/2 chain by an integrable spin-S chain[19] of local moments, which would lead to a replacement of the $SU(2)_1$ by $SU(2)_{2S}$ Kac-Moody algebra. Another line of future research relates to coupling two or more Kondo chains to investigate the effects of the unstable fluctuations. A last question is whether the picture we have obtained persists for $\lambda_K \gg t, \lambda_H$ or very low density, and if not, what is the strong λ_K regime. Presumably, such a regime would be a ferromagnetic Nagaoka-like state as in the single channel case. Besides the various generalizations of the one-dimensional Kondo-Heisenberg lattice problem, it would be interesting to determine whether the physics of the Kondo-Heisenberg problem persists in the Kondo limit $\lambda_H = 0$. It is known that this is not so in the one-channel case.[7,8] However, since in the multichannel case there is only a partial screening of the electrons by the spins, one may expect that a RKKY interaction could be generated even in the pure Kondo problem, putting it in the universality class of the Kondo-Heisenberg problems.

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[1] P. W. Anderson, J. Phys. C **3**, 2346 (1970); P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B **1**, 4464 (1970); K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975); N. Andrei, K. Furuya, and J. H. Lowenstein, *ibid.* **55**, 331 (1983); A. M. Tsvelik and P. B. Wiegmann, Adv. Phys. **32**, 453 (1983); I. Affleck, Acta Phys. Pol. **26**, 1869 (1995).

[2] P. Nozieres and A. Blandin, J. Phys. (France) **41**, 193 (1980).

[3] M. B. Maple, J. Low Temp. Phys. **99**, 223 (1995).

[4] D. L. Cox and A. Zawadowski, Adv. Phys. **47**, 599 (1998).

[5] H. J. Schulz, in *Mesoscopic Quantum Physics, Les Houches LXI*, edited by E. Akkermans, G. Montambaux, J. L. Pichard, and J. Zinn-Justin (Elsevier, Amsterdam, 1995).

[6] S. R. White, Phys. Rev. Lett. **69**, 2863 (1992).

[7] H. Tsunetsugu, M. Sigrist, and K. Ueda, Rev. Mod. Phys. **69**, 809 (1997); N. Shibata and K. Ueda, J. Phys.: Condens. Matter **11**, R1 (1999).

[8] A. E. Sikkema, I. Affleck, and S. R. White, Phys. Rev. Lett. **79**, 929 (1997).

[9] O. Zachar, S. Kivelson, and V. J. Emery, Phys. Rev. Lett. **77**, 1342 (1996); P. Coleman, A. Georges, and A. Tsvelik, J. Phys.: Condens. Matter **79**, 345 (1997); O. Zachar and A. M. Tsvelik, cond-mat/9909296 (unpublished).

[10] E. Witten, Commun. Math. Phys. **92**, 455 (1984).

[11] I. Affleck, in *Fields, Strings and Critical Phenomena*, edited by E. Brezin and J. Zinn-Justin (Elsevier Science Publishers, Amsterdam, 1988); A. Tsvelik, *Quantum Field Theory in Condensed Matter Physics* (Cambridge University Press, Cambridge, England, 1995).

[12] N. Andrei, M. R. Douglas, and A. Jerez, Phys. Rev. B **58**, 7619 (1998).

[13] P. Goddard, A. Kent, and D. Olive, Commun. Math. Phys. **103**, 105 (1986).

[14] An interesting realization of the same fixed point Hamiltonian in a frustrated three-chain ladder was recently given by P. Azaria, P. Lecheminant, and A. A. Nersesyan, Phys. Rev. B **58**, R8881 (1998). The authors also noticed the relation of their fixed point with the spin sector of the two channel Kondo Heisenberg lattice.

[15] V. J. Emery and S. A. Kivelson, Phys. Rev. B **46**, 10 812 (1992); A. M. Sengupta and A. Georges, *ibid.* **49**, 10 020 (1994); P. Coleman, L. B. Ioffe, and A. M. Tsvelik, *ibid.* **52**, 6611 (1995); A. J. Schofield, *ibid.* **55**, 5627 (1997).

[16] V. L. Berezinskii, Pis'ma Zh. Éksp. Teor. Fiz. **20**, 628 (1974) [JETP Lett. **20**, 287 (1974)]; A. Balatsky and E. Abrahams, Phys. Rev. B **45**, 13 125 (1992).

[17] M. Yamanaka, M. Oshikawa, and I. Affleck, Phys. Rev. Lett. **79**, 1110 (1997).

[18] A. B. Zamolodchikov, Pis'ma Zh. Éksp. Teor. Fiz. **43**, 565 (1986) [JETP Lett. **43**, 730 (1986)].

[19] N. Andrei and E. Orignac, Phys. Rev. B **62**, R763 (2000).

[20] A. M. Tsvelik, Phys. Rev. Lett. **72**, 1048 (1994).

[21] P. Noziéres, Ann. Phys. (Paris) **10**, 19 (1985); P. Noziéres, Eur. Phys. J. B **6**, 447 (1998).