Localization length and impurity dielectric susceptibility in the critical regime of the metal-insulator transition in homogeneously doped *p*-type Ge

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(Received 10 April 2000)

We have determined the localization length ξ and the impurity dielectric susceptibility χ_{imp} as a function of Ga acceptor concentrations (N) in nominally uncompensated ^{70}Ge:Ga just below the critical concentration (N_c) for the metal-insulator transition. Both ξ and χ_{imp} diverge at N_c according to the functions $\xi \propto (1 - N/N_c)^{-\nu}$ and $\chi_{\text{imp}} \propto (N_c/N - 1)^{-\zeta}$, respectively, with $\nu = 1.2 \pm 0.3$ and $\zeta = 2.3 \pm 0.6$ for $0.99N_c < N < N_c$. Outside of this region ($N < 0.99N_c$), the values of the exponents drop to $\nu = 0.33 \pm 0.03$ and $\zeta = 0.62 \pm 0.05$. The effect of the small amount of compensating dopants that are present in our nominally uncompensated samples, may be responsible for the change of the critical exponents at $N \approx 0.99N_c$.

The metal-insulator transition (MIT) in doped semiconductors is a unique quantum phase transition in the sense that both disorder and electron-electron interaction play a key role.[1] Important information about the MIT is provided by the values of the critical exponents for the zero-temperature conductivity, correlation length, localization length, and impurity dielectric susceptibility. From a theoretical point of view, the correlation length in the metallic phase and the localization length in the insulating phase diverge at the critical point with the same exponent ν, i.e., they are proportional to $|N/N_c - 1|^{-\nu}$ in the critical regime of the MIT. (N is the dopant concentration and N_c is the critical concentration for the MIT.) Since direct experimental determination of ν is extremely difficult, researchers have usually determined, instead of ν, the value of μ defined by $\sigma(0) \propto (N/N_c - 1)^\mu$ where $\sigma(0)$ is the conductivity extrapolated to $T=0$.[2,3] It is also possible to evaluate μ from finite-temperature scaling of the form $\sigma(N,T) \propto T^x f(|N/N_c - 1|/T^y)$ where x/y is equivalent to μ.[4-6] Values of ν are then obtained assuming $\nu = \mu$ for three-dimensional systems.[7]

In this work we have determined directly the localization length ξ and the impurity dielectric susceptibility χ_{imp} in neutron-transmutation-doped (NTD), nominally uncompensated ^{70}Ge:Ga just below N_c. The application of NTD to isotopically enriched ^{70}Ge leads to unsurpassed doping homogeneity and precisely controlled doping concentration. As a result, we have been able to approach the transition as close as $0.999N_c$ from the insulating side and $1.0004N_c$ from the metallic side.[3] In zero magnetic field, the low-temperature resistivity of the samples is described by variable-range hopping (VRH) conduction within the Coulomb gap.[8] Magnetic field and temperature dependence of the resistivity are subsequently measured in order to determine directly ξ and χ_{imp} in the context of the VRH theory.[8]

This kind of determination of ξ and χ_{imp} was performed for compensated Ge:As by Ionov *et al.*[9] They found $\xi \propto (1 - N/N_c)^{-\nu}$ and $\chi_{\text{imp}} \propto (N_c/N - 1)^{-\zeta}$ with $\nu = 0.60 \pm 0.04$ and $\zeta = 1.38 \pm 0.07$, respectively, for $N < 0.96N_c$. The signifi-

cance of their result is the experimental verification of the relation $2\nu \approx \zeta$ that had been predicted by scaling theories.[10,11] However, the critical exponents of compensated samples are known to be different from those of nominally uncompensated samples. Therefore, the present work which probes ξ and χ_{imp} in nominally uncompensated samples is relevant for the fundamental understanding of the MIT. The previous effort to measure χ_{imp} has also contributed.[12-14] Hess *et al.* found $\zeta = 1.15 \pm 0.15$ in nominally uncompensated Si:P.[13] Since $\mu \approx 0.5$ was determined for the same series of Si:P samples, $2\mu \approx \zeta$ was again valid. Katsumoto has found $\zeta \approx 2$ and $\mu \approx 1$ for compensated Al$_x$Ga$_{1-x}$As:Si, i.e., again, $2\mu \approx \zeta$ applies.[14] Thus, in these cases the conclusion $2\nu \approx \zeta$ was reached indirectly, *by assuming $\mu = \nu$.* The work reported here, on the other hand, determines ν directly, i.e., we do not have to rely on the assumption $\mu = \nu$ in order to study the behavior of ξ near N_c.

All of the ^{70}Ge:Ga samples used in this study were prepared by NTD of isotopically enriched ^{70}Ge single crystals. We use the NTD process since it is known to produce the most homogeneous dopant distribution.[2] Details of the sample preparation and characterization are described elsewhere.[3] In this study, we determined the low-temperature (0.05–0.5 K) resistivity of nine samples in weak magnetic fields (<0.4 T) applied in the direction perpendicular to the current flow.

The electrical conduction of doped semiconductors on the insulating side of the MIT is often dominated by VRH at low temperatures. The temperature dependence of the resistivity $\rho(T)$ for VRH is written in the form of

$$\rho(T) = \rho_0(T) \exp[(T_0/T)^p], \qquad (1)$$

where $p = 1/2$ for the excitation within a parabolic-shaped energy gap (the Coulomb gap),[8] and $p = 1/4$ for a constant density of states around the Fermi level.[15] In our earlier work,[3] we reported that $p = 1/2$ for $N < 0.991N_c$ ($N_c = 1.860 \times 10^{17}$ cm^{-3}) and that p decreases rapidly as N approaches N_c from $0.991N_c$ and becomes even smaller than

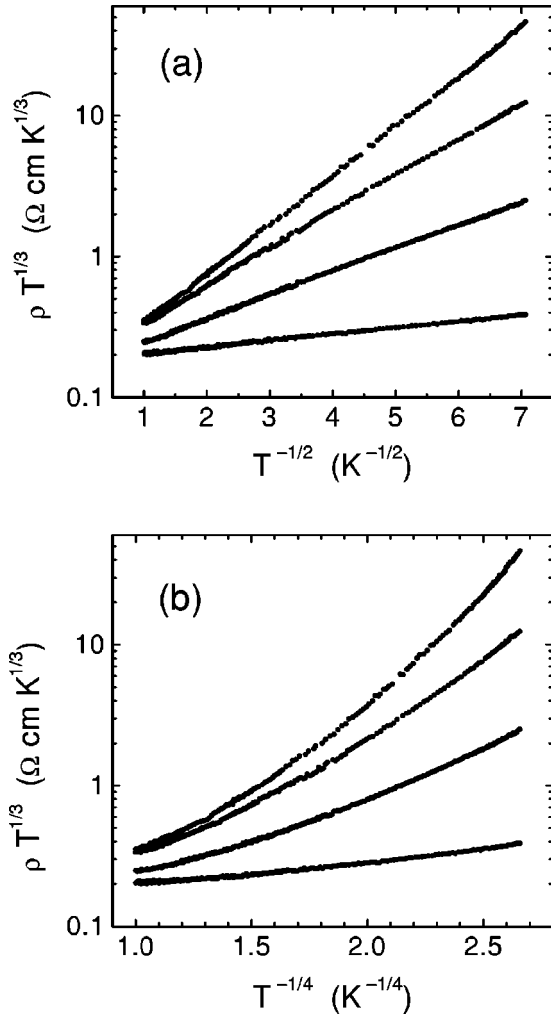FIG. 1. Resistivity multiplied by $T^{1/3}$ vs (a) $T^{-1/2}$ and (b) $T^{-1/4}$ for ^{70}Ge:Ga. From top to bottom in units of 10^{17} cm^{-3}, the Ga concentrations are 1.848, 1.850, 1.853, and 1.856.

1/4 when we neglect the temperature variation of $\rho_0(T)$. However, the variation contributes greatly to the temperature dependence of $\rho(T)$ near N_c because the factor T_0/T in the exponential terms become very small, i.e., the temperature dependencies of $\rho_0(T)$ and that of the exponential term become comparable. Theoretically, $\rho_0(T)$ is expected to vary as $\rho_0 \propto T^{-r}$ but the value of r including the sign has not been derived yet for VRH with both $p = 1/2$ (Ref. 16) and $p = 1/4$ (Ref. 17).

Recently, we have shown that the temperature dependence of the conductivity of the same series of ^{70}Ge:Ga samples within $\pm 0.3\%$ of N_c is proportional to $T^{1/3}$ at 0.02–1 K.[3] Since both the $T^{1/3}$ dependence of the conductivity and the Efros-Shklovskii VRH are results of the electron-electron interaction in disordered systems, they can be expressed, in principle, in a unified form. Moreover, the electronic transport in barely metallic samples and that in barely insulating samples should be essentially the same at high temperatures so long as the inelastic scattering length and the thermal diffusion length are smaller than, or at most comparable to the correlation length or the localization length. So, the temperature dependence of conductivity at high temperatures should be the same on both sides of the
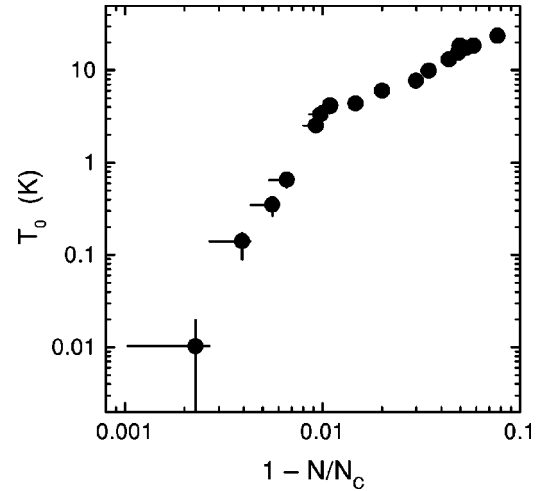

FIG. 2. T_0 determined by $\rho(T) \propto T^{-1/3} \exp[(T_0/T)^{1/2}]$ as a function of the dimensionless concentration $1 - N/N_c$.

transition. Such behavior is confirmed experimentally in the present system,[3] i.e., the conductivity of samples very close to N_c shows a $T^{1/3}$ dependence at $T \approx 0.5$ K, irrespective of the phase (metal or insulator) to which they belong at $T = 0$. Based on this consideration we fix $r = 1/3$. Figure 1 shows ρT^r with $r = 1/3$ for four samples ($N/N_c = 0.993$, 0.994, 0.996, and 0.998) as a function of (a) $T^{-1/2}$ and (b) $T^{-1/4}$. All the data points lie on straight lines with $p = 1/2$ in Fig. 1(a) while they curve upward with $p = 1/4$ in Fig. 1(b). This dependence is maintained even when we change the values of r between 1/2 and 1/4. Thus we conclude that the resistivity of all samples for N up to $0.998N_c$ is described by the VRH theory where the excitation occurs within the Coulomb gap, i.e., Eq. (1) with $p = 1/2$.

Based on these findings, we evaluate T_0 in Eq. (1) with $p = 1/2$ and $r = 1/3$, and show it as a function of $1 - N/N_c$ in Fig. 2. The vertical and horizontal error bars have been estimated based on the values of T_0 obtained with $r = 1/2$ and $r = 1/4$, and the values of $1 - N/(1.858 \times 10^{17}$ cm$^{-3})$ and $1 - N/(1.861 \times 10^{17}$ cm$^{-3})$, where 1.858×10^{17} cm^{-3} is the highest concentration in the insulating phase and 1.861×10^{17} cm^{-3} is the lowest in the metallic phase, respectively. According to theory,[8] T_0 in Eq. (1) is given by

$$k_B T_0 \approx 2.8 e^2/4\pi\epsilon_0\epsilon(N)\xi(N) \qquad (2)$$

in SI units, where $\epsilon(N)$ is the dielectric constant. Here, we should note that the condition $T < T_0$ is needed for the theory to be valid, i.e., T_0 has to be evaluated only from the data obtained at temperatures low enough to satisfy the condition. This requirement is fulfilled in Fig. 2 for all the samples except for the one with $N = 0.998N_c$. Concerning this latter sample, we will include it for the determination of ξ and χ_{imp} (Fig. 5) but not for the calculation of the critical exponents.

Our next step is to separate T_0 into ϵ and ξ. For $\xi/\lambda \ll 1$, the magnetoresistance is expressed as

$$\ln[\rho(B,T)/\rho(0,T)] \approx 0.0015(\xi/\lambda)^4, \qquad (3)$$

where $\lambda \equiv \sqrt{\hbar/eB}$ is the magnetic length in SI units.[8] According to Eq. (3), the magnetic-field variation of $\ln\rho$ at $T = $ const. is proportional to B^2, i.e., $\ln\rho(B,T) = \ln\rho(0,T)$
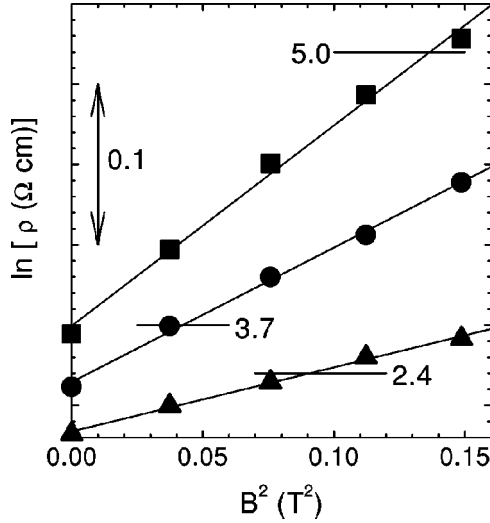
FIG. 3. Logarithm of resistivity vs B^2 at constant temperatures for the sample having $N = 1.840 \times 10^{17}$ cm^{-3}. From top to bottom the temperatures are 0.095 K, 0.135 K, and 0.215 K, respectively. The solid lines represent the best fits.

$+ C(T)B^2$, and the slope $C(T)$ is proportional to $T^{-3/2}$. In order to demonstrate that these relations hold for our samples, we show for the $N = 0.989 N_c$ sample $\ln \rho(B,T)$ vs B^2 in Fig. 3 and $C(T)$ determined by least-square fitting of $\partial \ln \rho / \partial B^2$ vs $T^{-3/2}$ in Fig. 4. Since Eq. (3) is equivalent to

$$\gamma \equiv C(T)/T^{-3/2} \approx 0.0015 (e/\hbar)^2 \xi^4 T_0^{3/2}, \qquad (4)$$

ξ is given by

$$\xi \approx 5.1 (\hbar/e)^{1/2} \gamma^{1/4} T_0^{-3/8}. \qquad (5)$$

In this way we have determined γ for nine samples. The inset of Fig. 5 shows γ as a function of T_0. The value of γ is almost independent of T_0, and if one assumes $\gamma \propto T_0^\delta$, one obtains a small value of $\delta = 0.094 \pm 0.005$ from least-square fitting. Figure 5 shows ξ and $\chi_{\rm imp} = \epsilon - \epsilon_h$ determined from Eqs. (2) and (5). Here, ϵ_h is the dielectric constant of the host Ge, and hence, $\chi_{\rm imp}$ is the dielectric susceptibility of the Ga
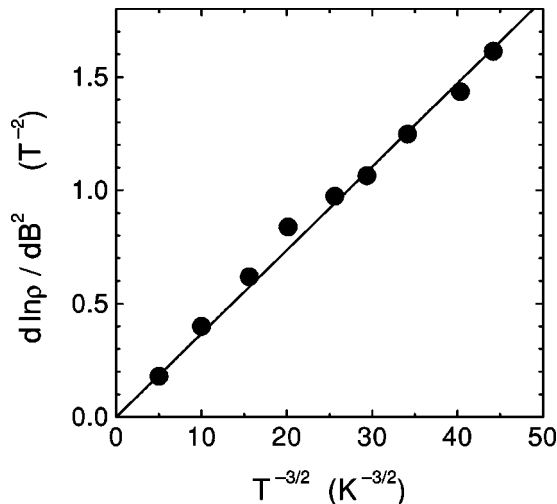

FIG. 4. Slope $d \ln \rho / dB^2$ vs $T^{-3/2}$ for the sample having $N = 1.840 \times 10^{17}$ cm^{-3}. The solid line represents the best fit.
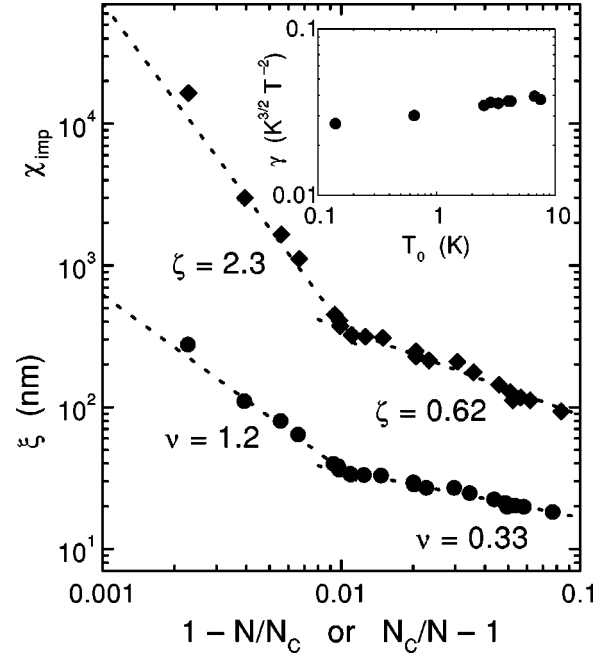

FIG. 5. Localization length ξ vs $1 - N/N_c$ (lower data set) and the dielectric susceptibility $\chi_{\rm imp}$ arising from the impurities vs $N_c/N - 1$ (upper data set). The inset shows the coefficient γ defined by Eq. (4) as a function of T_0.

acceptors. We should note that both ξ and $\chi_{\rm imp}$ are sufficiently larger than the Bohr radius (8 nm for Ge) and $\epsilon_h = 15.4$ (Ref. 12), respectively. According to the theories of the MIT, both ξ and $\chi_{\rm imp}$ diverge at N_c as $\xi(N) \propto (1 - N/N_c)^{-\nu}$ and $\chi_{\rm imp}(N) \propto (N_c/N - 1)^{-\zeta}$, respectively. We find, however, both ξ and $\chi_{\rm imp}$ do not show such simple dependencies on N in the range shown in Fig. 5, and that there is a sharp change of both dependencies at $N \approx 0.99 N_c$. On both sides of the change in slope, the concentration dependence of ξ and $\chi_{\rm imp}$ are expressed well by the scaling formula as shown in Fig. 5. Theoretically, the quantities should show the critical behavior when N is very close to N_c. So $\nu = 1.2 \pm 0.3$ and $\zeta = 2.3 \pm 0.6$ may be concluded from the data in $0.99 < N/N_c$. However, the other region ($0.9 < N/N_c < 0.99$), where we obtain $\nu = 0.33 \pm 0.03$ and $\zeta = 0.62 \pm 0.05$, is also very close to N_c in a conventional experimental sense.

As a possible origin for the change in slope, we refer to the effect of compensation. Although our samples are nominally uncompensated, doping compensation of less than 0.1% may present due to residual isotopes that become n-type impurities after NTD. In addition to the doping compensation, the effect known as ''self compensation'' may play an important role near N_c.[18] It is empirically known that the doping compensation affects the value of the critical exponents. Rentzsch et al. studied VRH conduction of n-type Ge in the concentration range of $0.2 < N/N_c < 0.91$, and showed that T_0 vanishes as $T_0 \propto (1 - N/N_c)^\alpha$ with $\alpha \approx 3$ for $K = 38\%$ and 54%, where K is the compensation ratio.[19] Since $\alpha \approx \nu + \zeta$ [Eq. (2)], we find for our ^{70}Ge:Ga samples $\alpha = 3.5 \pm 0.8$ for $0.99 < N/N_c < 1$ and $\alpha = 0.95 \pm 0.08$ for $0.9 < N/N_c < 0.99$. Interestingly, $\alpha = 3.5 \pm 0.8$ agrees with $\alpha \approx 3$ found for compensated samples. Moreover, we have recently proposed the possibility that the conductivity critical

exponent $\mu \approx 1$ in the same ^{70}Ge:Ga only within the very vicinity of N_c (up to about $+0.1\%$ of N_c).[6] An exponent of $\mu = 0.50 \pm 0.04$, on the other hand, holds for a wide region of N up to $1.4 N_c$.[3] Again, $\mu \approx 1$ near N_c may be viewed as the effect of compensation. Therefore, it may be possible that the region of N around N_c where $\nu \approx 1$ and $\mu \approx 1$ changes its width as a function of the doping compensation. In the limit of zero compensation, the part which is characterized by $\nu \approx 1$ and $\mu \approx 1$ vanishes, i.e., we propose $\nu = 0.33 \pm 0.03$, $\zeta = 0.62 \pm 0.05$, and $\mu = 0.50 \pm 0.04$ for truly uncompensated systems and that Wegner's scaling law of $\nu = \mu$ is not satisfied. In compensated systems, on the other hand, Wegner's law may hold as it does in the very vicinity of N_c. The experiment on compensated $Al_xGa_{1-x}As$:Si that showed $\zeta \approx 2$ and $\mu \approx 1$ (Ref. 14) is also consistent with the law. However, our preceding discussion needs to be proven in the future by experiments in samples with precisely and systematically controlled compensation ratios.

In summary, we have determined directly the localization length and the dielectric susceptibility arising from the impurities in nominally uncompensated NTD ^{70}Ge:Ga samples near the critical point for the MIT. While the relation $2\nu \approx \zeta$ predicted by scaling theory[10,11] holds for $0.9 < N/N_c < 1$, the critical exponents for localization length and impurity susceptibility change at $N/N_c \approx 0.99$. The small amount of doping compensation that is unavoidably present in our samples may be responsible for such a change in the exponents.

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