Relevance of the scheme of regularization of the density-of-state fluctuation contribution in an arbitrary magnetic field

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The problem of the regularization of the fluctuations induced renormalization of the one-electron density of states (DOS) has been recently clarified. So far, only the weak-field limit was considered because of the field dependence of the cutoff in the DOS expression. Recently, theoretical results [A. I Buzdin and A. A. Varlamov, Phys. Rev. B **58**, 14 195 (1998)] have proposed a scheme of regularization of the problem allowing numerical calculations for an arbitrary magnetic field. By comparing magnetoconductivity measurements under a strong magnetic field and the theoretical predictions, the robustness of the expression for the DOS contribution obtained as a convergent series independent of cutoff is shown. Such an analysis allows us to calculate the zero-field contribution of the fluctuations to the measured resistivity. The great influence of this effect on both the magnitude and the temperature dependence of the resistivity above T_c is demonstrated.

Among the various scenarios proposed to elucidate the unusual behaviors of *c* -axis temperature dependence, optical conductivity, *c*-axis magnetoresistance, nuclear magnetic resonance (NMR) spin-relaxation rate near T_c, tunneling conductance and any other normal-state anomalies of HTSC's in the metallic part of the phase diagram, interpretation in terms of a pseudogap originating from the fluctuations induced renormalization of the one-electron density of states at the Fermi level found excellent agreement with experimental results.[1] Other proposals have been made to explain the occurrence of a pseudogap, featuring spin charge separation, charge confinement, and a van Hove scenario.[2–6] However, since there exists no theoretical formalism for the latters, a quantitative analysis cannot be achieved. Such a formalism is available for the theory of the fluctuation renormalization of the density of states (DOS). This has been initially proposed by Dorin *et al.*[7] to describe experimental results concerning magnetotransport measurements. In particular, the prediction of a sign change in the *c*-axis magnetoresistivity (MR) is very interesting and matches perfectly experimental results in numerous papers.[8–14] Nevertheless, as emphasized recently by Buzdin and Varlamov,[15] the field dependence of the cutoff parameter in the DOS contribution to conductivity prevents any numerical analysis for an arbitrary magnetic field. Thus to make this model convincing and to perform numerical calculation, Dorin *et al.*[7] had to distinguish asymptotic regions of relevant field strength: the weak-field region ($h \ll \epsilon$) and the strong-field one ($\epsilon \ll h$), where ϵ and h are the reduced temperature and field, respectively. Buzdin and Varlamov[15] have proposed a scheme of regularization of the DOS contribution in order to obtain expressions valid for any field strengths. It is shown in this latter paper that the usual weak-field asymptotic expression derived from the this new scheme is similar to the result initially proposed by Dorin. As pointed out in the following, the proposed expressions are very suitable for numerical calculations in the case of *c*-axis MR for anisotropic superconductors.

Underdoped Bi-2212 single crystals are good candidates for studying the change of sign in the MR because of the strong semiconducting behavior of the out-of-plane resistivity and the large magnitude of the observed negative MR.[14] Measurements achieved under strong magnetic fields allow us to reach the domain of validity of the high-field asymptotic and more generally, to show the robustness of the DOS contribution expression obtained as a convergent series independent of cutoff. In this paper, the prediction of the theory for fluctuations contribution to magnetoconductivity is numerically calculated considering previously published results and is compared with high-field experimental data (27 T). The derived parameters are then used to evaluate the zero-field contribution of the fluctuations to resistivity. Thus it is possible to determine the ''true'' normal state resistivity just above the peak after substraction from experimental data. The results obtained in this last section emphasize the great role played by the superconducting fluctuations in the temperature dependence of the out-of-plane resistivity well above the critical temperature.

The crystals of $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+\delta}$ used in this study were grown by a self-flux method which has been described elsewhere.[16,17] The structural investigations undertaken to check the quality of crystals, in particular their actual cationic compositions are reported in an earlier paper;[18] it is demonstrated that substitutions of low concentrations of Y^{3+} on the Ca^{2+} site lead to a set of samples with different doping states. The actual cations contents were checked by energy dispersive spectroscopy (EDS) x-ray spectroscopy with a Kevex analyzer mounted on a 200-kV electron microscope following the procedure described in Ref. 18 for each batch. In this work, we investigate charge transport in $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+\delta}$ with $x_{EDS}=0.36$ which corresponds to a heavily underdoped state.[18] The crystal, which had typical dimensions $1.0\times1.0\times0.01$ mm^3 was contacted in the direct ''cross'' configuration.[19] One can directly derive a good estimate of ρ_c from such a cross measurement providing that the sample is small enough to ensure that the current and voltage contacts are close together and close to the edges of the crystal. Several comparative studies with more sophisticated methods,[20,21] such as the measurement of a ''bottom'' voltage, have shown that the direct combination of R_{cross} with the dimensions of the crystals yields reliable values of ρ_c.[21,22] Gold wires were attached to the ''evaporated-silver stripes'' with silver paint. The samples were then annealed in air at 400 °C for 10 min. The c-axis resistance was measured as a function of temperature at a series of fixed fields ($B\|c\|I$) in the 20-MW resistive magnet of the Grenoble High Magnetic Field Laboratory (France). The temperature was controlled by an Oxford instrument cryostat with an ITC5 controller using a Cernox sensor. The sample resistance was measured by a four points AC-technique (3 mA at $f=17$ Hz).

As it was shown in the framework of a Gaussian model developed by Ioffe *et al.*[23] and Dorin *et al.*,[7] c-axis conductivity fluctuations in high-temperature superconductivity is comprised of four terms. The direct contribution, initially proposed by Aslamasov and Larkin (AL),[24] is due to the acceleration in an electric field of short-lived Cooper pairs in thermal nonequilibrium. On the basis of various conditions, alternative expressions of the AL contribution have been derived.[25,26] The DOS contribution arises from corrections to the normal quasiparticle density of states owing to fluctuations of normal quasiparticles into the superconducting state. This contribution, negative in sign, causes a decrease of resistivity in a magnetic field. It is possible to observe a negative fluctuation induced c-axis magnetoresistance in the temperature region where the DOS contribution exceeds the positive AL one. The regular and anomalous Maki-Thompson (MT) contributions respectively result from the scattering of the normal-state particles and the superconducting pairs.[7,26] These MT contributions are usually small and in the following, the main issue is to understand the relevancy of this contribution to describe the experimental results since they are not considered in many papers.[27,28] In this paper, the interactions of the magnetic field with electron spins leading to a Zeeman effect are neglected. Finally, the fluctuation magnetoconductivity is given by

$$\Delta\sigma_c=\Delta\sigma_c^{AL}+\Delta\sigma_c^{DOS}+\Delta\sigma_c^{MT(reg)}+\Delta\sigma_c^{MT(an)}, \quad (1)$$

where $\Delta\sigma_c^{AL}=\sigma_c^{AL}(B,T)-\sigma_c^{AL}(0,T)$, etc.

However, the full expressions of $\Delta\sigma_c^{MT(reg)}$ and $\Delta\sigma_c^{DOS}$ contributions given by Dorin *et al.*[7] cannot directly be used for comparison with experiments. The field-dependent cutoff for those contributions is unknown and can only be roughly approximated. Up to now, this lack in the theory has imposed experimentalists to lie in the weak-field regime where explicit formalism could be obtained. As mentioned above, Buzdin and Varlamov[15] have proposed interesting expressions that can be used in an arbitrary magnetic field and which are suitable for numerical calculations. For completeness, these expressions with the field-independent cutoff are given:

$$\Delta\sigma_c^{DOS}=\sigma_c^{DOS}(B,T)-\sigma_c^{DOS}(0,T)=\frac{e^2 s r\kappa}{8\eta}\times h\sum_{n=0}^{\infty}\left\{\frac{1}{h}\ln\frac{\sqrt{\epsilon+2nh+2h}+\sqrt{\epsilon+r+2nh+2h}}{\sqrt{\epsilon+2nh}+\sqrt{\epsilon+r+2nh}}-\frac{1}{\sqrt{\epsilon+2nh+h}\sqrt{\epsilon+r+2nh+h}}\right\}, \quad (2)$$

$$\Delta\sigma_c^{MT(reg)}=\sigma_c^{MT(reg)}(B,T)-\sigma_c^{MT(reg)}(0,T)=\frac{e^2 s\tilde{\kappa}}{4\eta}h\sum_{n=0}^{\infty}\left\{\frac{\epsilon+(2n+1)h+r/2}{\sqrt{\epsilon+(2n+1)h}\sqrt{\epsilon+r+(2n+1)h}}-\frac{1}{2h}\left[\sqrt{\epsilon+(2n+1)h}\sqrt{\epsilon+r+(2n+1)h}-\sqrt{\epsilon+2nh}\sqrt{\epsilon+r+2nh}\right]\right\}. \quad (3)$$

In these formulas, e is the electron charge, s is the lattice period along the c axis, and

$$\eta=-\frac{v_F^2\tau^2}{2}\left[\Psi\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B T\tau}\right)-\Psi\left(\frac{1}{2}\right)-\frac{\hbar}{4\pi k_B T\tau}\Psi'\left(\frac{1}{2}\right)\right], \quad (4)$$

where v_F is the Fermi velocity parallel to the layers, τ is the quasiparticle scattering time, $\Psi(x)$ and $\Psi'(x)$ are the digamma function and its derivative. $r=4\eta J^2k_B^2/v_F^2\hbar^2$ is the usual anisotropy parameter characterizing the dimensionality of the fluctuations and J is an effective interlayer energy in Kelvin. $\beta=2\eta eB/\hbar$ and $\epsilon=\ln(T/T_c)$. The constant κ and $\tilde{\kappa}$ are ruled by the impurity concentration and are function of τT:

$$\kappa=\frac{-\Psi'\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B T\tau}\right)+\frac{\hbar}{2\pi k_B T\tau}\Psi''\left(\frac{1}{2}\right)}{\pi^2\left[\Psi\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B T\tau}\right)-\Psi\left(\frac{1}{2}\right)-\frac{\hbar}{4\pi k_B T\tau}\Psi'\left(\frac{1}{2}\right)\right]}, \quad (5)$$

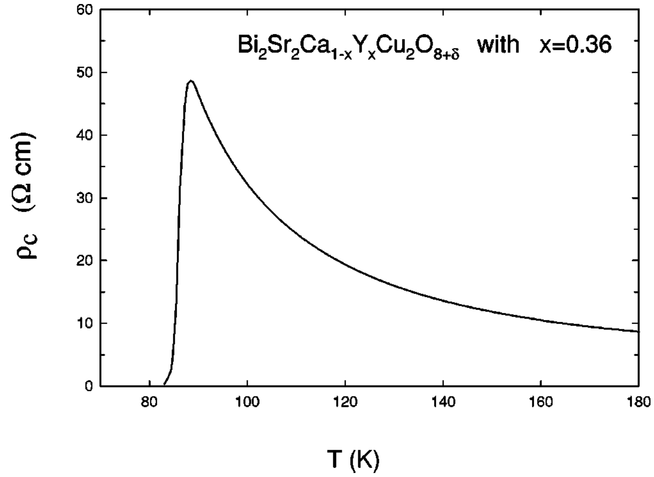

FIG. 1. Zero-field out-of-plane resistivity ρ_c of an underdoped Bi-2212 single crystal versus temperature.

$$\tilde{\kappa}=\frac{-\Psi'\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B T\tau}\right)+\Psi'\left(\frac{1}{2}\right)+\frac{\hbar}{4\pi k_B T\tau}\Psi''\left(\frac{1}{2}\right)}{\pi^2\left[\Psi\left(\frac{1}{2}+\frac{\hbar}{4\pi k_B T\tau}\right)-\Psi\left(\frac{1}{2}\right)-\frac{\hbar}{4\pi k_B T\tau}\Psi'\left(\frac{1}{2}\right)\right]},\tag{6}$$

where τ_Φ is the pair breaking life time. The asymptotics for all regions of field can be found in Ref. 15. For numerical calculations, we took $s=c/2=1.5$ nm; T_c is determined from the midpoint of the zero-field resistive transition (89 K) (see Fig. 1) and we assumed τ and $\tau_\Phi \propto 1/T$.

The experimental magnetoconductivity (MC) is defined as follows: $\Delta\sigma=\sigma(B,T)-\sigma(0,T)=1/\rho(B,T)-1/\rho(0,T)$. In order to intend connection with the magnetoresistivity, we have plotted in the following the temperature dependence of $-\Delta\sigma$.

In Fig. 2, the experimental MC is reported for the underdoped single crystal of Bi-2212 under a magnetic field of 27 T. It should be pointed out that, in a previously published paper,[13] the same analysis in the weak-field regime was successfully applied using the same values of parameters at different magnetic fields. For our field strength and within the range of temperature investigated, we are not in the so-called weak-field limit ($h \ll \epsilon$). So far, most of the analyses performed in the papers dealing with this subject have only considered the weak-field expressions since the work of Buzdin and Varlamov was not available yet. In the present study, the experimental data are compared to the total MC predicted by those latter authors. In a recent paper[14] dealing with the doping dependence of the description in terms of superconducting fluctuations, we have observed a weak doping dependence of the values derived for τ and τ_Φ. Moreover, the doping state of the sample investigated in the present paper is rather close to the $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+\delta}$ with $x_{EDS}=0.21$ for which the values of J and v_F were found to be 28 K and 4.35×10^6 cm/s, respectively.[14] As a starting point, the values of the parameters obtained for this latter compound have to be considered. Then, comparing experimental data at 27 T with the numerical calculation [Eq. (1)], the values were slightly modified until obtaining the result shown in Fig. 2 where the four terms are considered (AL, anomalous, and regular MT and DOS). The best agreement exhibited in Fig. 2 is obtained using the following parameters: $J=30$ K, $\tau(100\ \text{K})=1.07\times10^{-14}$, $\tau_\Phi(100\ \text{K})\approx1.23\times10^{-13}$ s, and $v_F=4.35\times10^6$ cm/s. Those values are in excellent agreement with previous results derived through the same analysis for underdoped Bi-2212 single crystals in the weak-field limit.[14] Let us now focus on the respective magnitude of the various contributions involved in the overall result (Fig 3). The term in Eq. (3), referred to as the regular MT contribution, gives a negative correction, as does the DOS one [Eq. (2)], while the anomalous MT term is positive. Neither the anomalous nor the regular MT contributions are large enough to dominate the AL and DOS ones, respectively, in the studied range of temperature. It can also be observed that those corrections enter as a very small part in the total MC when one compares to the AL (in the vincinity of T_c) and DOS contributions. In Fig. 3, the prediction for the high-field asymptotic is calculated using the values given above for J ,

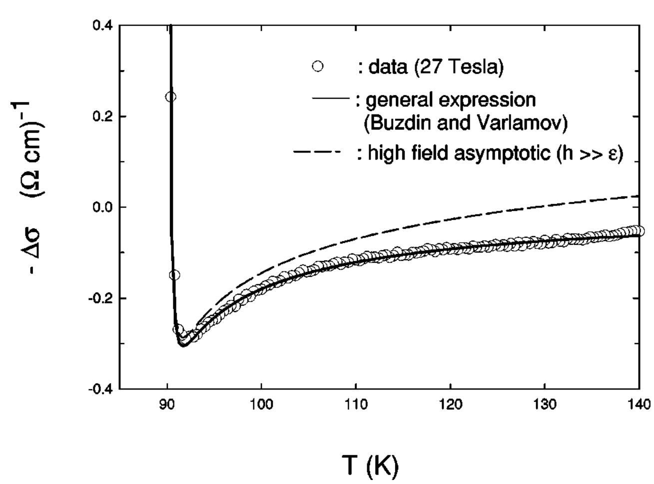

FIG. 2. Magnetoconductivity versus temperature at 27 T for an underdoped Bi-2212 single crystal. The solid line represents the theoretical calculation with parameters given in the text. The symbols are the experimental magnetoconductivity $\Delta\sigma_c$ $(B\|c\|I)$.

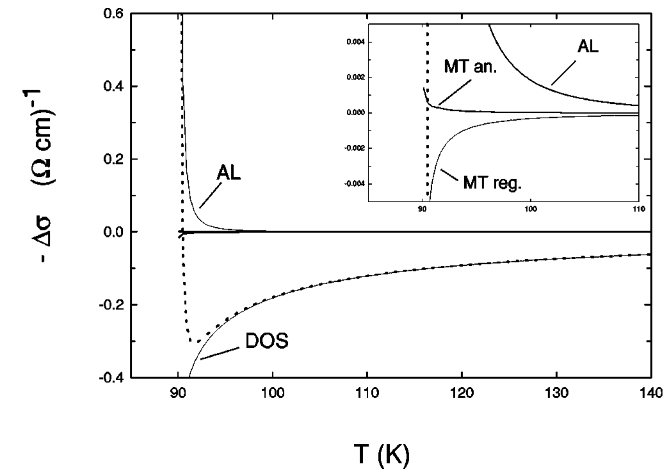

FIG. 3. Decomposition of the calculation of total theoretical magnetoconductivity for an underdoped Bi-2212 single crystal at 27 T with parameters given in the text. The inset shows the MT regular (reg.) and MT anomalous (an.) contributions which are too small to be presented in the same scale as the AL and DOS contributions.

τ, τ_Φ, and v_F. We observe that there exists a good agreement of the theory with experimental data only in the vincinity of T_c where the condition $h \gg \epsilon$ is fulfilled. As the temperature is increased, one comes out of the range of validity of the asymptotic and, as expected, the agreement fails.

In the case of NMR experiments,[29,30] some papers invoke the extreme sensitivity of the anomalous MT term to the pair breaking process, which is itself related to the symmetry of pairing fluctuations, to discriminate between *s* or *d* wave. It is important to stress that conductivity measurements do not provide a reliable test for possible *s* or *d* pairing since the temperature dependence of the AL and anomalous MT terms are very similar. It is worth mentioning that the τ_Φ parameter only enters in the anomalous MT term which has been shown to be several times smaller than the AL and DOS contributions. Thus the uncertainty for τ_Φ is expected to be large as emphasized in Ref. 13.

At this point of the paper, the proposed scheme of regularization of the density-of-states fluctuations contribution seems to be very suitable for analyzing the MC data in an arbitrary magnetic field. In the next section, we discuss the role of the density-of-states contribution in the temperature dependence of the *c*-axis resistivity. In order to obtain the ''true'' temperature dependence of the normal-state resistivity, the zero-field fluctuations contribution has been calculated on the basis of the expressions proposed by Dorin *et al.*[7] and using the parameters determined in the previous section. In Fig. 4, we report the experimental data and the resistivity obtained after substraction of the fluctuations contribution, referred in the following to as the ''true'' normal-state resistivity, for $T < 130$ K (for $T > 1.25T_c$ the theory is, however, no longer accurate).[10] Such a result exhibits the strong influence of the fluctuations contribution on the magnitude of the measured resistivity on the one hand and on its temperature dependence on the other hand. Many papers have described the peculiar semiconducting behavior of the *c*-axis resistivity only considering some normal-state models.[31–34] However, as emphasized by Balestrino *et al.*[10] and Watanabe *et al.*,[35] attempts to account for the *c*-axis resistivity by means of phenomenological models have failed. For instance, Yan *et al.*[32] have proposed an activation-type phenomenological formula of the type $\rho_c = A + BT + (C/T)\exp(\Delta/T)$ to describe the diverging trend above T_c. Here, Δ is a normal-state gap. Numerical fits were performed on the raw data and on the resistivity corrected from the fluctuations effect. It appears that in both cases, the activation-type component, characterized by the gap Δ, is

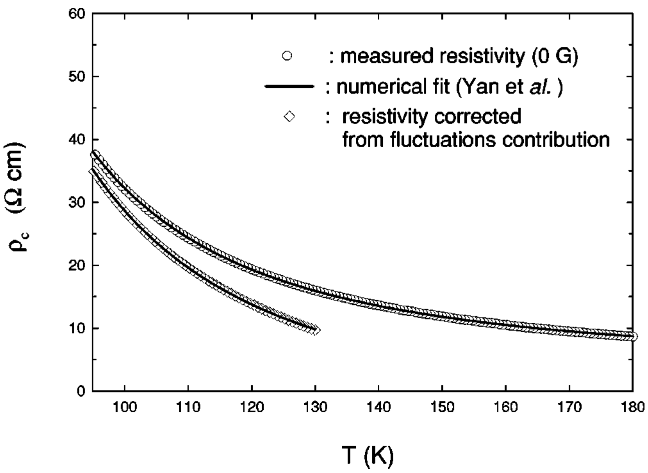

FIG. 4. Possible behavior of the normal-state *c*-axis resistivity just above the peak after subtraction from experimental data of the fluctuations contribution for the parameters given in the text.

suitable for describing the data. However, the derived values for the gap are strongly different. When one applies the phenomenological analysis on the raw data we find $\Delta = 240$ K. This can be compared with values reported by Watanabe *et al.*[35] for underdoped Bi-2212 single crystals. The magnitude of the gap is raised up to 320 K when one considers the true normal-state resistivity. This evidences the major role played by the fluctuations correction in the value of the normal-state gap. When an analysis is performed through a simple normal-state model, it seems essential to take into account the effect of the superconducting fluctuations to correctly describe the resistive behavior of the anisotropic superconductors above T_c.

In conclusion, numerical calculations have been performed on the basis of the field-independent cutoff expressions derived by Buzdin and Varlamov for the DOS and MT regular contributions to conductivity. The predictions have been compared with the experimental magnetoconductivity measured under a magnetic field of 27 T in order to be far outside the weak-field limit. A good agreement is obtained between experimental data and theoretical predictions showing the robustness of the regularization scheme proposed by the authors. Moreover, the parameters obtained through this analysis permit us to evaluate the zero-field contribution of the fluctuations to the resistivity. After substraction of this correction from the experimental data, the behavior of the normal-state transverse resistivity can be obtained and compared with the phenomenological expressions usually proposed to describe it.

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