Identifying the pairing symmetry in the Sr_2RuO_4 superconductor

Matthias J. Graf and A. V. Balatsky

Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

We have analyzed heat capacity and thermal conductivity measurements of Sr_2RuO_4 in the normal and superconducting state and come to the conclusion that an order parameter with nodal lines on the Fermi surface is required to account for the observed low-temperature behavior. A gapped order parameter is inconsistent with the reported thermodynamic and transport data. Guided by a strongly peaked dynamical susceptibility along the diagonals of the Brillouin zone in neutron-scattering data, we suggest a spin-fluctuation mechanism that would favor the pairing state with the gap maxima along the zone diagonals (such as for a d_{xy} gap). The most plausible candidates are an odd parity, spin-triplet, f-wave pairing state, or an even parity, spin-singlet, d-wave state. Based on our analysis of possible pairing functions we propose measurements of the ultrasound attenuation and thermal conductivity in the magnetic field to further constrain the list of possible pairing states.

I. INTRODUCTION

The search for the superconducting pairing symmetry in the layered perovskite material Sr_2RuO_4 (SrRuO), and its attempted theoretical predictions, show remarkable parallels to the heavy-fermion superconductor UPt_3. In both systems, early specific-heat measurements showed a large residual value of C/T at low temperatures and were interpreted in terms of a superconducting phase with a nonunitary p-wave order parameter.[1–5] The observation of a strong T_c suppression with nonmagnetic impurities was an additional indication of a superconducting phase with an unconventional order parameter.[6–9] However, newer measurements on high-quality single crystals have shown that the most likely pairing state in UPt_3 is an f-wave state, or more precisely a spin-triplet state whose orbital basis function belongs to the E_{2u} representation of the hexagonal crystallographic point group (D_{6h}).[10,11] The experience with UPt_3 suggests that the early identification of the pairing state, based on low-quality, inhomogeneous samples, is at best inconclusive (for a review on UPt_3 see, for example, Refs. 12,13). However, with improving sample quality it becomes feasible to identify the pairing state by studying transport properties.

Here we analyze new heat capacity measurements on high-quality single crystals of SrRuO, as well as thermal conductivity data on dirty samples with a strong T_c suppression, to show that the proposed p-wave model,[14–16] $\Delta(\mathbf{p}_f) \sim (p_x+ip_y)\hat{\mathbf{z}}$, is inconsistent with the available data. Our conclusion is that the pairing state in SrRuO, most likely, has lines of nodes with gap nodes given by the d_{xy} gap function. This can occur in either an f-wave state, i.e., a spin-triplet pairing state belonging to the E_u representation of the tetragonal crystallographic point group (D_{4h}) or in a d_{xy} singlet state. We argue that the f-wave nodal state is consistent with measurements of the heat capacity,[2] thermal conductivity,[17,18] penetration depth,[19] Andreev reflection,[19a] NMR,[20] Knight shift,[21] and μSR experiments.[22]

Recent band-structure calculations by Mazin and Singh[23] indicate that there is an increase in the spin susceptibility $\chi(\mathbf{q},\omega)$ at four points in the Brillouin zone at approximately $\mathbf{q}_0 \approx (\pm 2\pi/3, \pm 2\pi/3)$ that occur due to strong nesting effects of quasi-one-dimensional bands (ξ and ζ). Nesting effects among these bands lead to the increased interaction between particles on the Fermi surface near $\mathbf{q}_0$, see Fig. 1. In recent neutron-scattering experiments[24] the predicted four incommensurate peaks near $\mathbf{q}_0$ were indeed observed thus supporting that nesting effects near these points are important.[23]

In this paper we propose (1) to identify the regions at the Fermi surface near $\mathbf{q}_0$ with the ones that develop the largest gap. We use the neutron-scattering data as an indication that near the nesting regions the particle-particle (or particle-hole) interactions are dominant and that these are the regions that would benefit the most from opening a superconducting gap. (2) We suggest that regardless of the singlet or triplet
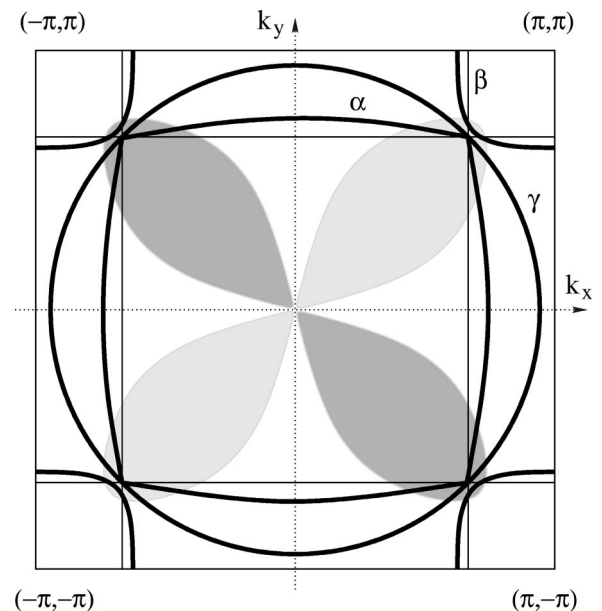

FIG. 1. Fermi surfaces in the Brillouin zone after Mazin and Singh (Ref. 23). The plotted order parameter (proportional to d_{xy}) opens a gap along $(\pm\pi,\pm\pi)$ where the incommensurate peaks of the spin susceptibility are observed. The corresponding quasi-one-dimensional model bands ξ at $(k_x,\pm 2\pi/3)$ and ζ at $(\pm 2\pi/3, k_y)$ are shown as thin lines.

nature of the pairing in SrRuO the gap function should be proportional to a d_{xy} harmonics. Such an order parameter would lead to *lines of nodes* along the k_z axis in the gap and to power-law behavior in the thermodynamic and transport properties. Line nodes on the Fermi surface lead in clean superconductors, and for scattering in the Born limit, to the well-known temperature dependences[25–28] of the specific heat $C \sim T^2$, the nuclear spin-relaxation rate $1/T_1 \sim T^3$, the deviation of the penetration depth from its zero-temperature value $\triangle\lambda \sim T$, the thermal conductivity $\kappa \sim T$, and the longitudinal sound attenuation $\alpha_L \sim$ const., as well as for the transverse attenuation $\alpha_T \sim T^2$. (3) Based on the proposed line nodes in the gap we make predictions for ultrasound attenuation and thermal conductivity measurements that can further distinguish between the remaining possible basis functions. We propose complimentary longitudinal and transverse attenuation measurements that can help to locate the location of the nodal lines of the order parameter on the Fermi surface. Another crucial experiment is the thermal conductivity with an in-plane magnetic field. We expect the *four-fold* modulation of the thermal conductivity $\kappa(\theta, H)$ as a function of the angle between the nodes of the gap [along the (1,0) and (0,1) direction] and the field directions. Thermal conductivity measures the unpaired quasiparticle heat transport and is therefore sensitive to the angular (field) dependence of the quasiparticle scattering rate, which ''knows'' about the angular dependence of the gap. We use the analogy with the suggested *d*-wave paring state in high-T_c superconductors where this *fourfold* modulation has been observed.[29–32]

II. MODEL

The gap function for even parity (spin-singlet) or odd parity (spin-triplet) representations is described by an order parameter of the form

$$\Delta_{\alpha\beta}(\mathbf{p}_f) = \Delta(\mathbf{p}_f)(i\sigma_y)_{\alpha\beta}, \quad \text{(singlet)} \qquad (1)$$

$$\Delta_{\alpha\beta}(\mathbf{p}_f) = \mathbf{\Delta}(\mathbf{p}_f) \cdot (i\boldsymbol{\sigma}\sigma_y)_{\alpha\beta}, \quad \text{(triplet)} \qquad (2)$$

with σ_α being Pauli matrices. Since nonunitary states,[5,4] i.e., $\mathbf{\Delta} \times \mathbf{\Delta}^* \neq 0$, have been ruled out by the very small residual value of the specific heat C/T at zero temperature,[33] we restrict our study of spin-triplet states to unitary order parameters that factorize into a single spin vector and an orbital amplitude, i.e., $\mathbf{\Delta}(\mathbf{p}_f) = \mathbf{d}\,\Delta(\mathbf{p}_f)$, where $\mathbf{d}$ is a real unit vector in spin space and $\Delta(\mathbf{p}_f)$ is an odd-parity orbital function. The vector $\mathbf{d}$ defines the axis along which the Cooper pairs have zero spin projection, e.g., if $\mathbf{d} \| \hat{\mathbf{z}}$, then $\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow} = 0$ and $\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow} = \Delta(\mathbf{p}_f)$.

Whether or not spin-orbit coupling is weak or strong in Sr_2RuO_4 has important ramifications for both spin and orbital components of the order parameter that are allowed by symmetry. While spin-orbit coupling is believed to be strong in the heavy-fermion system UPt_3 there are no experimental indications that this is likewise true for Sr_2RuO_4. In the meantime we will use the classification of basis functions in terms of irreducible representations of the tetragonal point group (D_{4h}) listed in Table I, implying that spin-orbit coupling is strong. Since the band-structure calculations[23] and de Haas–van Alphen measurements[34] show very little dispersion along k_z, we will consider only two-dimensional (2D)

TABLE I. 2D polynomial basis functions for the irreducible representations of D_{4h} of several pairing models [after Yip and Garg (Ref. 37)]. Notice that $B_g \times E_u = E_u$. The commonly proposed $p_x + ip_y$ state belongs to the two-dimensional E_u representation. We present *both* singlet, d_{xy}, and triplet states, $B_{2g} \otimes E_u$, which have lines of nodes, as plausible candidates for Sr_2RuO_4. For simplicity we list only the nodal angles on the dominating γ and α Fermi sheets.

#	Γ	$\Delta(\mathbf{p}_f)$	Nodes
1	B_{2g}	$p_x p_y$	$\phi = 0, \pi/2, \pi, 3\pi/2$
2	E_u	$(p_x + ip_y)$	No
3	$B_{2g} \otimes E_u$	$p_x p_y(p_x + ip_y)$	$\phi = 0, \pi/2, \pi, 3\pi/2$

basis functions on a more or less cylindrical Fermi surface. A similar list of possible basis functions was recently compiled by Hasegawa and co-workers[35] for further investigations. The listed *hybrid* state (3) of the direct product $B_{2g} \otimes E_u = E_u$ is a nontrivial realization of the E_u representation (also referred to as *f*-wave state). So far Knight shift data with an in-plane magnetic field $\mathbf{H} \| [100]$ show no change below T_c and have been interpreted in terms of spin-triplet pairing with the spin vector $\mathbf{d}$ locked to the crystal $\mathbf{c}$ axis.[21] On the other hand, muon spin rotation (μSR) experiments observed a spontaneous internal magnetic field on entering the superconducting state,[22] consistent with a time-reversal symmetry-breaking state belonging to the two-dimensional E_u representation.

At this place a caveat is warranted because neither Knight-shift data at high fields and for a single field orientation, nor μSR measurements in impure samples provide a clear-cut identification for spin-triplet pairing or broken time-reversal symmetry states. For example, in UPt_3 early μSR measurements indicated broken time-reversal symmetry in the superconducting phase (probably due to impurities), while newer measurements on very clean samples fail to detect any effect at all.[36] What makes the interpretation of the Knight-shift data in SrRuO for magnetic fields parallel to the planes even more complicated, is, that (1) the experiment was not performed in the low field limit, but rather deep in the mixed phase, $H \sim H_{c2}/2$, where contributions from the vortices may be important, and (2) nonlocal and surface effects may be relevant due to a small Ginzburg-Landau parameter for in-plane currents, $\kappa_\| = \lambda_\| / \xi_\| \sim 2.6$.

III. THERMODYNAMIC AND TRANSPORT PROPERTIES

We calculate the specific heat and thermal conductivity for the order-parameter models listed in Table I and fit the results to existing experiments. This way, we can determine the model parameters and make predictions for sound attenuation measurements. It is important to point out that none of the here analyzed transport experiments can distinguish between a spin-singlet and a spin triplet order parameter. Thus we obtain identical results for the states 1 and 3.

The specific heat, $C = T dS/dT$, can easily be obtained from the entropy,[38,28]

$$S = 4 \int_0^\infty d\epsilon\, N(\epsilon) \left(\frac{\epsilon}{T} f(\epsilon) - \ln(1 - f(\epsilon)) \right), \qquad (3)$$

by numerical differentiation. Here $f(\epsilon)=1/[1+\exp(\epsilon/T)]$ is the Fermi-Dirac function and $N(\epsilon)=-(N_f/\pi)\mathrm{Im}\int d\mathbf{p}_f g^R(\mathbf{p}_f,\epsilon)$ is the density of states per spin with N_f being the normal-state density of states at the Fermi surface.

In the limit of Born (weak) or unitarity (strong) impurity scattering the in-plane thermal conductivity of unitary spin-triplet superconductors is given by[39,40]

$$\kappa_{ii}=-\frac{N_f v_f^2}{8\pi^3 T^2}\int d\epsilon\,\epsilon^2\,\mathrm{sech}^2\frac{\epsilon}{2T}\int d\mathbf{p}_f\hat{\mathbf{v}}_{fi}^2\mathcal{K}(\mathbf{p}_f,\epsilon),\quad (4)$$

$$\mathcal{K}(\mathbf{p}_f,\epsilon)=\frac{1}{\mathrm{Re}\,C^R}[g^R(g^R)^*-\mathbf{f}^R\cdot(\mathbf{f}^R)^*+\pi^2],\qquad (5)$$

with the unit vector of the Fermi velocity, $\hat{\mathbf{v}}_{fi}$, and $C^R=-1/\pi\sqrt{|\boldsymbol{\Delta}|^2-(\widetilde{\epsilon}^R)^2}$. The quasiclassical equilibrium Green functions are $g^R=\widetilde{\epsilon}^R/C^R$ and $\mathbf{f}^R=-\boldsymbol{\Delta}/C^R$. Within the t-matrix approximation for isotropic scattering the impurity renormalized quasiparticle energy is $\widetilde{\epsilon}^R=\epsilon-\sigma_{imp}^R(\epsilon)$. For weak scattering $\sigma_{imp}^R(\epsilon)=(\Gamma/\pi)\int d\mathbf{p}_f g^R$, and for strong scattering $\sigma_{imp}^R(\epsilon)=-\Gamma/(\pi\int d\mathbf{p}_f g^R)$, with the normal-state scattering rate $\Gamma=\hbar/2\tau$.

In the hydrodynamic regime, $\omega\tau\ll 1$, and long-wavelength limit, $ql\ll 1$, the absorption of ultrasound of polarization $\boldsymbol{\varepsilon}$ propagating along direction $\mathbf{q}$ is related to the viscosity by[26–28]

$$\alpha=\frac{\omega^2}{\varrho c_s^3}\eta_{ij,kl}\hat{\boldsymbol{\varepsilon}}_i\hat{\mathbf{q}}_j\hat{\boldsymbol{\varepsilon}}_k\hat{\mathbf{q}}_l,\qquad (6)$$

with the speed of sound c_s, the mass density ϱ, and the viscosity tensor evaluated at $\omega\to 0$,

$$\eta_{ij,kl}=-\frac{N_f v_f^2 p_f^2}{8\pi^3 T}\int d\epsilon\,\mathrm{sech}^2\frac{\epsilon}{2T}\int d\mathbf{p}_f\pi_{ij}\pi_{kl}\mathcal{K}(\mathbf{p}_f,\epsilon),$$
$$(7)$$

where $\pi_{ij}=\hat{\mathbf{v}}_{fi}\hat{\mathbf{p}}_{fj}-\frac{1}{2}\delta_{ij}$.

Here we confine our discussion to order parameters with vanishing averages, $\int d\mathbf{p}_f\boldsymbol{\Delta}(\mathbf{p}_f)=0$, which satisfy the gap equation for triplet (singlet) pairing interactions,

$$\boldsymbol{\Delta}(\mathbf{p}_f)=\int\frac{d\epsilon}{2\pi}\tanh\frac{\epsilon}{2T}\int d\mathbf{p}_f' V(\mathbf{p}_f,\mathbf{p}_f')\mathrm{Im}\,\mathbf{f}^R(\mathbf{p}_f',\epsilon).\quad (8)$$

Note that for spin-singlet pairing all vector functions get replaced by the corresponding scalar functions. In the weak-coupling spin-fluctuation model the pairing interaction is written as

$$V(\mathbf{p}_f,\mathbf{p}_f')\sim V^*(\mathbf{p}_f,\mathbf{p}_f')\chi(\mathbf{p}_f-\mathbf{p}_f'),\qquad (9)$$

$$\chi(\mathbf{q})=\chi_0/[1+\xi^2(\mathbf{q}-\mathbf{q}_0)^2].\qquad (10)$$

The detailed form of the effective pairing interaction $V^*(\mathbf{p}_f,\mathbf{p}_f')$ depends on the form of the spin singlet or spin triplet pairing interaction. χ_0 is the static spin susceptibility, ξ is the antiferromagnetic correlation length, and the incommensurate wave vectors are $\mathbf{q}_0\approx(\pm 2\pi/3,\pm 2\pi/3)$. The spin-fluctuation scenario proposed here is similar to the one

studied by many authors in the context of the heavy-fermion systems,[41] the high-T_c cuprates,[42] the quasi-two-dimensional organic superconductors,[43] and even SrRuO.[23,44] In contrast to the microscopic model calculations in Refs. 23,44, we propose the existence of either an attractive triplet f-wave or singlet d-wave pairing channel in order to describe the power-laws observed in thermodynamic and transport coefficients. Our approach is guided by the neutron-scattering data of the spin susceptibility that can lead to a gap function that is gapped along the $(\pm\pi,\pm\pi)$ directions and has nodes along $(\pi,0)$ and $(0,\pi)$. The immediate consequence of the proposed state is that the superconducting gap on the hole-like β band develops nodes at $(\pm 2\pi/3,\pm\pi)$ and $(\pm\pi,\pm 2\pi/3)$.

IV. RESULTS AND DISCUSSIONS

In our analysis of the thermodynamic and transport properties we make the simplifying assumption that all three Fermi surfaces (α,β,γ) simultaneously go superconducting and can be described by one effective, cylindrical band. At the present time we cannot rule out any admixture of the p-wave state 2 to the f-wave state 3, since both gap functions belong to the same two-dimensional representation E_u. However, from a detailed analysis of the calculated heat capacity we find, rather conservatively, that the admixture of a nodeless p-wave state has to be less than 20% to be consistent with the experimental $C(T)$.[45] Thus, we neglect the possibility of a p-wave admixture to the f-wave gap function in the remainder of this work. Impurity calculations for the p-wave state 2 also were performed by Maki and Puchkaryov,[46] who reported reasonably good agreement between experiment and the calculated p-wave order parameter. Very recently, Dahm, Won, and Maki[47] discarded the nodeless p-wave state and argued in favor of f-wave pairing.

In the temperature range $T^*\ll T\ll T_c$, where T^* is the characteristic temperature of the impurity band width, and in the clean limit, $\Gamma\ll\Delta_0$, the evaluation of the entropy and transport coefficients simplifies significantly. In the presence of line nodes on the Fermi surface the density of states is $N(\epsilon)\sim(\epsilon/\Delta_0)N_f$. Similarly, we obtain for the Fermi surface averaged integrand $\int d\mathbf{p}_f\mathcal{K}(\mathbf{p}_f,\epsilon)\sim-\epsilon\tau(\epsilon)/\Delta_0$, because of $\mathcal{K}\approx\pi^4/(\mathrm{Re}\,\widetilde{\epsilon}^R\,\mathrm{Im}\,\sigma_{imp}^R)\mathrm{Im}\,C^R$, with $\widetilde{\epsilon}^R\approx\epsilon+i0^+$. Thus, the transport coefficients show the usual power laws of clean superconductors when using the approximate relations for the scattering self-energies, $1/2\tau(\epsilon)=(-\pi)^{-1}\mathrm{Im}\,\sigma_{imp}^R\sim\Gamma\epsilon/\Delta_0$ in the Born limit, or $1/2\tau(\epsilon)\sim\Gamma\Delta_0/\epsilon$ for unitarity scattering.

A. Specific heat

The states 1 and 3 with line nodes yield $C\sim N_f T^2/\mu\Delta_0$, in excellent agreement with experiments, while the gapped state 2 disagrees with the data. The proposed multiband order-parameter model by Agterberg and co-workers,[15] which assumes that only one band (γ) out of three possible bands goes superconducting at T_c, fails to describe the low-T dependence (see Fig. 2). Our result for the p-wave state 2 is in agreement with calculations of the heat capacity by Agterberg.[48] In the multiband model the density of states
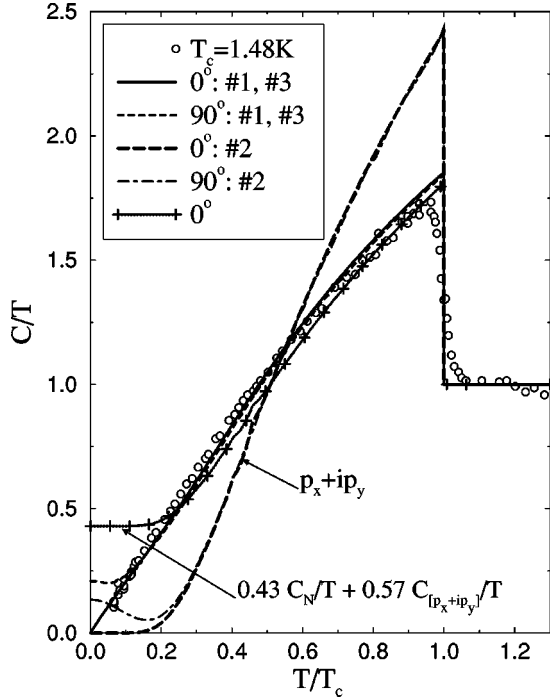
FIG. 2. The specific heat normalized at T_c for pairing states with line nodes (1 or 3). A scattering phase shift of $\delta_0 = 0°$ (Born) or $\delta_0 = 90°$ (resonant), a scattering rate $\Gamma/\pi T_{c0} = 0.01$, and a nodal parameter $\mu = 1.5$, were assumed. For comparison the p-wave state 2 for Born (long-dash) and resonant (dot-dash) scattering and the multiband state by Agterberg in the Born limit (cross-dot) are shown. The data are from Ref. 33.

TABLE II. T_c suppression due to the pair-breaking effects of nonmagnetic impurities after Abrikosov and Gorkov. (Ref. 52).

$\Gamma/\pi T_{c0}$	0.0	10^{-3}	10^{-2}	0.10	0.20	0.25
$T_c(\Gamma)/T_{c0}$	1.0	0.998	0.98	0.74	0.44	0.25
$T_c(\Gamma)$ [K]	1.51	1.507	1.48	1.12	0.67	0.38

the multicomponent superconducting order parameter in UPt$_3$ uniaxial strain (pressure) in the plane would lift the degeneracy of the two-component order parameter.[49] As a consequence the transition temperature will split into two. This is a crucial test of the multicomponent nature of the order parameter.[50,14] Along the same line of arguments, a magnetic field in the plane should also split T_c, as was pointed out in Ref. 51.

B. Thermal conductivity

The in-plane thermal conductivity is isotropic for all order-parameter models listed in Table I, assuming a cylindrical Fermi surface. In the clean limit, $T^* \ll T \ll T_c$, and neglecting logarithmic corrections, $\kappa_{ii} \sim T$ for weak scattering and $\sim T^3$ for strong scattering. In the dirty limit, $T \ll T^* \ll T_c$, the thermal conductivity is linear in temperature, $\kappa_{ii} \sim T$, and independent of the scattering strength. Unfortunately the samples studied by Suderow et al.[17] exhibit a very strong T_c suppression. The reported resistive transitions for samples 2 and 4, $T_c^\varrho(2) \approx 0.81$ K and $T_c^\varrho(4) \approx 0.58$ K, occurred significantly above the bulk superconducting transitions identified by the thermal conductivity, $T_c^\kappa(2) \approx 0.60$ K and $T_c^\kappa(4) \approx 0.47$ K. Not only does this suggest that the samples are in the dirty limit but also that they are considerably inhomogeneous. Thus the standard scattering t-matrix analysis in terms of pointlike defects in the dilute limit will most likely fail to give a quantitative description. Nevertheless, combining the facts of the T_c suppression and that the ratios of the residual resistivities and the normal-state thermal conductivities are related to the scattering rates, $\Gamma(4)/\Gamma(2) \sim \varrho_0(4)/\varrho_0(2) \sim \kappa_N(2)/\kappa_N(4) \approx 1.25$,[17] we find that the normal-state scattering rates are approximately given by $\Gamma(2)/\pi T_{c0} \approx 0.20$ and $\Gamma(4)/\pi T_{c0} \approx 0.25$ (see Table II for the corresponding T_c suppression).

In Figs. 3 and 4 we show the best fits of κ_{xx} for samples 2 and 4 measured by Suderow et al.[17] Although we cannot obtain a quantitatively good fit for any of the pairing models, we are able to ascribe the large residual value of κ/T to impurity scattering (see Fig. 3) without having to invoke a multiband order-parameter model (see Fig. 4). A surprising result of these fits is that, generally, we find better agreement between theory and experiment for weak impurity scattering in the Born limit. Very recently, Tanatar et al.[18] reported measurements of κ on cleaner crystals ($T_c \approx 1.4$ K) that are in good quantitative agreement with the gapless states 1 or 3 and impurity scattering in the unitarity limit.[45]

For the predicted pairing states 1 or 3, we expect to observe a fourfold oscillation of the thermal conductivity when a magnetic field is parallel to the layers and rotated within the layers. However, the amplitude of the oscillations depends on the scattering strength. It is appreciable for strong scattering (unitarity limit) and very small for weak scattering
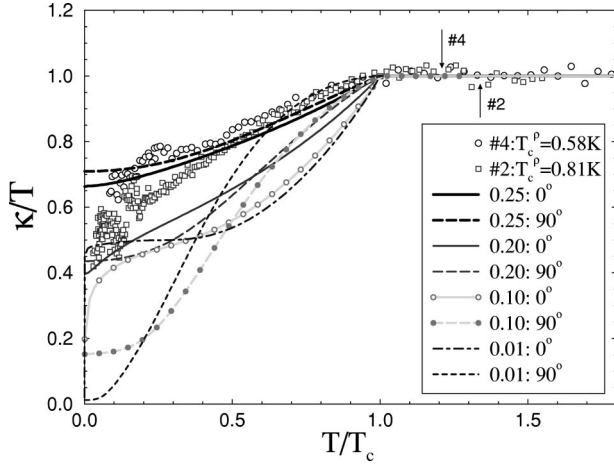
(DOS) of the γ band is weighted with 57% of the total DOS, while the remaining α and β bands account for 43% of the total DOS. It is the γ band on which the p-wave state 2 has been proposed to nucleate. The α and β bands remain normal. Here μ is the slope parameter of the gap function at the nodes, $\mu = |d\Delta(\phi)/\Delta_0 d\phi|_{node}$. In our calculations we have used variational basis functions, $\Delta(\mathbf{p}_f) \rightarrow \Delta(\mathbf{p}_f)\mathcal{F}_{A_{1g}}(\mathbf{p}_f; \mu)$, where the variational function $\mathcal{F}_{A_{1g}}$ belongs to the A_{1g} representation and remains invariant under all group transformations. The slope parameter μ allows us to adjust the opening of the gap function at the nodes, which is otherwise not determined by symmetry. This enables us to quantitatively describe the ground state of the superconducting order parameter as probed by low energetic quasiparticles. An approach that has been quite successful in describing the low energetic quasiparticle excitations in UPt$_3$.[40]

Assuming that pure SrRuO has an optimal transition temperature of $T_{c0} \approx 1.51$ K,[9] we obtain an excellent fit for scattering in the Born limit with a scattering phase shift $\delta_0 \rightarrow 0$ and a scattering rate $\Gamma/\pi T_{c0} = 0.01$. On the other hand, resonant scattering ($\delta_0 \rightarrow \pi/2$) with the same scattering rate gives a residual value of C/T that is too large. If impurity scattering is indeed resonant, then a value of $\Gamma/\pi T_{c0} \lesssim 10^{-3}$ is required to account for the lowest measured values of the specific heat. Furthermore, it would imply that the optimal transition temperature is closer to $T_{c0} \approx 1.48$ K.

The T_c transition of the two components of the triplet p-wave order parameter 2, or of the two components of the f-wave order parameter 3, is doubly degenerate. Similar to

FIG. 3. The thermal conductivity normalized at T_c for the pairing state 1 or 3. The scattering phase shifts are $\delta_0 = 0°, 90°$, the scattering rates are $\Gamma/\pi T_{c0} = 0.01, 0.10, 0.20, 0.25$, and $\mu = 1.5$. The data are from Ref. 17. Note that in these dirty samples the resistive transition occurs at much higher temperatures (see arrows).

(Born limit). So far, no oscillations have been observed.[18,53] Certainly the experimental and theoretical situation remains unresolved and requires more study. Indeed such magnetic oscillations have been reported in the cuprate $YBa_2Cu_3O_7$,[29–32] and are considered as additional proof in support of the $d_{x^2-y^2}$ symmetry of the superconducting state.

C. Sound attenuation

The longitudinal ($\mathbf{q}\|\boldsymbol{\varepsilon}\|[100]$) and transverse ($\mathbf{q}\|[100]$ and $\boldsymbol{\varepsilon}\|[010]$) sound attenuations are identical for the pairing state 2, i.e., $\alpha_{xx}(T)/\alpha_{xx}(T_c) = \alpha_{xy}(T)/\alpha_{xy}(T_c)$. This result also was reported in Ref. 54. Whereas for pairing states 1 and 3 the longitudinal attenuation with $\mathbf{q}\|\boldsymbol{\varepsilon}\|[100]$ is the same as the transverse attenuation rotated by $\pi/4$ with $\mathbf{q}\|[110]$ and $\boldsymbol{\varepsilon}\|[\bar{1}10]$. These relations follow directly from Eq. (7) and are a peculiarity of the 2D Fermi surface and the 2D basis functions of the order parameters. Inspecting the momentum-dependent weighting factors in Eq. (7),
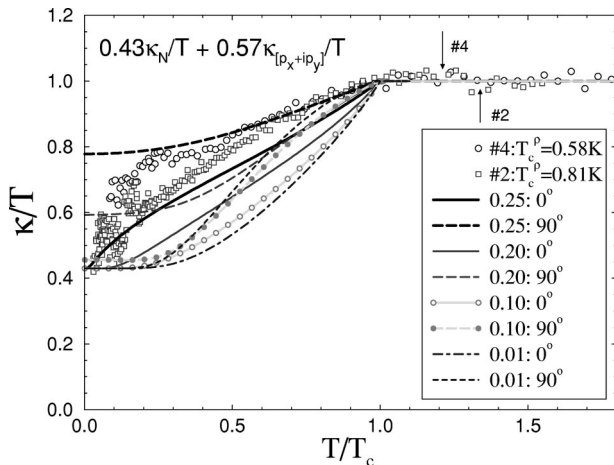

FIG. 4. The thermal conductivity normalized at T_c for the multi-band model by Agterberg based on the p-wave state (2) and for the same parameters as in Fig. 3.
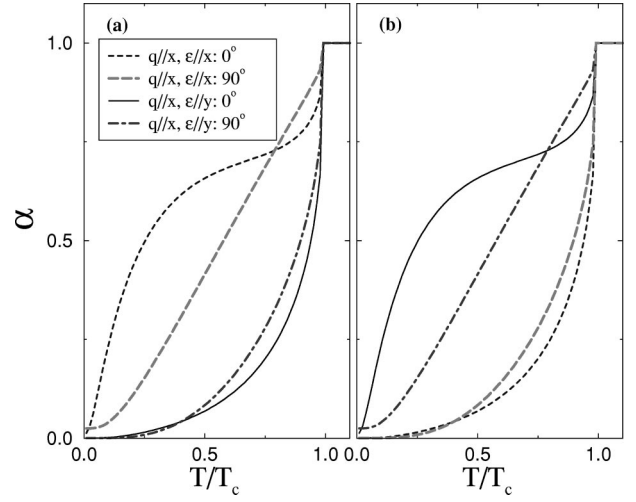

FIG. 5. The longitudinal and transverse sound attenuation normalized at T_c for the states 1 or 3, $\Gamma/\pi T_{c0} = 0.01$, $\delta_0 = 0°$, $90°$, and $\mu = 1.5$. In panel (b) the crystal (or detector) has been rotated by $\pi/4$ around the $\mathbf{c}$ axis relative to the arrangement in panel (a).

$$\pi_{xx}^2 = \frac{1}{4}\cos^2 2\phi, \quad \pi_{xy}^2 = \frac{1}{4}\sin^2 2\phi, \quad (11)$$

it is clear that by rotating the crystal (or the transducer) by $\pi/4$ around the $\mathbf{c}$ axis one simply exchanges these functions, $\pi_{xx}^2 \leftrightarrow \pi_{xy}^2$, and, thus swaps the expressions for the longitudinal and transverse attenuation. Since the integrand $\mathcal{K}(\mathbf{p}_f, \epsilon)$ for the p-wave state (2) is independent of $\mathbf{p}_f$, the longitudinal and transverse attenuations are identical (within an overall scaling factor due to differences in the speed of sound) for arbitrary temperature and impurity concentration. These predictions should be straightforward to check experimentally. In Fig. 5 we show the predicted transverse and longitudinal sound attenuations for the d-wave (1) and f-wave (3) order-parameter models. Our results are similar to the ones discussed by Moreno and Coleman[55] for the case of the $d_{x^2-y^2}$-wave gap function in the high-T_c cuprates.

V. CONCLUSIONS

We have proposed a spin-fluctuation model based on the measured spin susceptibility by neutron scattering that leads to nodes of the gap function on the Fermi surface. We demonstrated that the measured specific heat and thermal conductivity are consistent with a spin-singlet order parameter (d_{xy}-wave symmetry belonging to B_{2g}) or a spin-triplet order parameter (f-wave symmetry belonging to E_u), though inconsistent with a gapped spin-triplet state (p-wave symmetry belonging to E_u). Based on this analysis we proposed sound attenuation measurements and thermal conductivity measurements in a magnetic field to locate the nodes on the Fermi surface, as well as measurements of the specific heat subjected to a uniaxial strain field in the plane in order to split the superconducting transition. It is clear that more experiments are needed to investigate the nodal regions on the Fermi surface and the spin structure of the order parameter.

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