Nonlocal effects in magnetization of high-κ superconductors

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The reversible magnetization $M(T,H)$ in high quality single crystals of $Lu(Ni_{1-x}Co_x)_2B_2C$ with $x=0-0.09$ is measured in a broad T,H domain and interpreted within London theory corrected for nonlocality of the current – vector-potential relation. Profound deviations in the data from the standard London result, $M \propto \ln(H_{c2}/H)$ in intermediate fields $H_{c1} \ll H \ll H_{c2}$, are seen in clean samples at low T's. Unlike strongly anisotropic high-T_c compounds, for nearly isotropic borocarbides this behavior cannot be attributed to fluctuations of weakly interacting pancake vortices. We show that the nonlocal London model describes qualitatively and consistently the whole set of $M(T,H)$ data and, in particular, its temperature and the mean-free path dependence. The scaling field H_0, which arises in $M=M(H/H_0)$ due to nonlocality, is found to be nearly proportional to the the field H_2 at which the vortex lattice undergoes the symmetry change (the ''square-to-hex'' transition).

I. INTRODUCTION AND THEORY

Magnetic properties of high-κ superconductors are commonly treated within the standard London approach. This is possible because the core contribution to the total energy is small relative to the magnetic and kinetic parts constituting the London energy. Within this scheme[1]

$$M=-M_0 \ln\frac{\eta H_{c2}}{B}, \quad M_0=\frac{\phi_0}{32\pi^2\lambda^2}. \tag{1}$$

Comparison with data shows that close to T_c, $\eta \approx 1.2-1.5$.[2,3] The constant η accommodates a number of inherent uncertainties of the London approach, the question discussed originally by de Gennes' group[1] and in some detail in Ref. 4. First, in deriving Eq. (1) the energy F of the vortex lattice is expressed as a logarithmically divergent sum over the reciprocal lattice **G**; the latter is replaced with an integral from $G_{min}\sim 2\pi/a$ with the intervortex spacing $a\sim\sqrt{\phi_0/B}$ to $G_{max}\sim 2\pi/\xi$ where ξ is an *effective* core size. The divergence and the cutoff at $G\sim 1/\xi$ are inherent shortcomings of the London approach which breaks down at distances $\sim\xi$. This procedure yields $\tilde{F}=F-B^2/8\pi=M_0 B\ln(\eta' H_{c2}/B)$, where η' absorbs uncertainties in both the lower and upper integration limits. Second, the core correction $M_0 B\eta_c$ should be added with an uncertain factor η_c. Since $M=-\partial\tilde{F}/\partial B$, one obtains Eq. (1) with $\eta=\eta'\exp(\eta_c-1)$. One of the major consequences of Eq. (1) is the field-independent slope

$$\frac{\partial M}{\partial \ln B}=M_0, \tag{2}$$

which does not contain the uncertainties mentioned above.

The question of validity of Eqs. (1) and (2) at low temperatures arose since they offer a simple method to extract the penetration depth λ and, in particular, to obtain a rough estimate of H_{c2} from the magnetization data at low temperatures (where H_{c2} may not be readily accessible).

While at high temperatures, the analysis of data for Tl-2223 and Hg-1201 based on Eq. (1) (corrected for vortex fluctuations) produced a well-behaved $H_{c2}\propto(T_c-T)$,[5] the method failed when applied to low T's. Extensive magnetization data on Bi-2212 by Cho *et al.*[6] analyzed with the help of either Eq. (1) or the variational GL approach of Hao-Clem[7] have generated a nearly constant $H_{c2}(T)$ between 35 and 70 K, whereas the standard Helfand-Werthamer estimate predicts a reduction by a factor of 3 or 5.[8] Similar results were reported by other groups, see, e.g., Refs. 9 and 10.

This difficulty motivated Kogan and Gurevich to review the microscopic derivation of the London equations and to obtain corrections due to the basic nonlocality of the relation between current density and the vector potential.[4] We outline relevant features of this work below and develop it further paying special attention to the impurity dependence at low temperatures.

Employing the ''nonlocal London'' approach in a manner similar to the derivation of Eq. (1) one obtains[4]

$$-\frac{M}{M_0}=\ln\left(\frac{H_0}{B}+1\right)+\frac{B}{H_0+B}+\zeta(T), \tag{3}$$

$$\zeta=\eta_c-1-\ln\left(\frac{H_0}{\eta' H_{c2}}+1\right), \tag{4}$$

where we have deliberately separated the field-independent quantity ζ which slowly decreases with temperature. In any situation when $H_0 \gg H_{c2}$ (and therefore $H_0 \gg B$), this result reduces to the standard Eq. (1); it is shown in the following that this happens as $T\to T_c$ and with increasing scattering at all T's. The field scale

$$H_0=\frac{\eta^*\phi_0}{4\pi^2\mathcal{R}^2}; \quad \mathcal{R}^2=\frac{\pi^2}{20}\xi_0^2\gamma(T,l), \tag{5}$$

where $\mathcal{R}$ is the ''nonlocality range''[11] and ξ_0 is the BCS zero-T coherence length. The prefactor η^* depends on the vortex lattice structure and on the choice of the lower limit G_{min} in the integral over the reciprocal space. Physically, H_0 is related to the nonlocality range similar to the way in which

H_{c1} and H_{c2} are related to λ and ξ. It is worth mentioning that as derived the nonlocal corrections to London equations make sense only for materials with large κ. For $\kappa \sim 1$, there is no field range where intervortex spacing exceeds substantially the core size $\xi(T)$, in other words, there is no domain where the corrections due to nonlocality can be separated from those due to spatial variations of the order parameter.

The temperature and mean-free path dependent quantity $\gamma(T,l)$ is defined as

$$\gamma = \Delta^2(0) \sum \beta^{-2}\beta_1^{-3} / \sum \beta^{-2}\beta_1^{-1}, \tag{6}$$

where the sum is over the Matsubara frequencies $\hbar\omega = \pi T(2n+1)$, $\beta^2 = \Delta^2 + \hbar^2\omega^2$, $\beta_1 = \beta + \hbar/2\tau$, and τ is the scattering time by nonmagnetic impurities. This quantity has been calculated numerically in Ref. 4 for the two-dimensional (2D) case (proper for the strongly anisotropic layered compounds). For the 3D situation, this was done in Ref.11; in the clean limit $\gamma = 2/3$ at $T=0$ and drops to ≈ 0.30 at T_c. Electron scattering suppresses γ; in the dirty limit $\gamma \to l^2/\xi_0^2 \to 0$, i.e., nonlocal effects vanish. Thus in the clean case $\mathcal{R} \sim \xi_0$, and decreases slowly with temperature remaining finite even at T_c. Consequently, the field H_0 is of the order of $H_{c2}(0)$ at low T's and *increases* slowly with T reaching at T_c a value greater than $H_0(0)$. In the dirty limit, $\mathcal{R} \sim l$, so that $H_0 \gg H_{c2}$ even at $T=0$ and so it remains at all T's.

At $T=0$, the sums in Eq. (6) can be replaced with integrals according to $2\pi T\Sigma \to \int_0^\infty d(\hbar\omega)$. It is readily shown that

$$\gamma(0) = \frac{I''(\alpha)}{2I(\alpha)}, \quad I = \int_0^\infty \frac{dx}{(1+x^2)(\sqrt{1+x^2}+\alpha)}, \tag{7}$$

where $\alpha = \hbar v/2\Delta l = \pi\xi_0/2l$. The integral is

$$I = \frac{\pi}{2\alpha} - \frac{2}{\alpha\sqrt{1-\alpha^2}} \tan^{-1}\sqrt{\frac{1-\alpha}{1+\alpha}}, \quad \alpha < 1, \tag{8}$$

$$I = \frac{\pi}{2\alpha} - \frac{1}{\alpha\sqrt{\alpha^2-1}} \ln\frac{\alpha+1+\sqrt{\alpha^2-1}}{\alpha+1-\sqrt{\alpha^2-1}}, \quad \alpha > 1. \tag{9}$$

We will not write down cumbersome expressions for $\gamma(0,l)$. Instead, we provide a simple polynomial approximation in the interval $0 \leqslant \alpha \leqslant 10$:

$$\gamma^{-1}(0,l) \approx 1.5(1 + 2.034\alpha + 0.717\alpha^2) \tag{10}$$

with a better that 1% accuracy, see Fig. 1.

This yields

$$H_0(0,l) \approx \frac{15\eta^*\phi_0}{2\pi^4\xi_0^2}\left(1 + 3.194\frac{\xi_0}{l} + 1.126\frac{\xi_0^2}{l^2}\right). \tag{11}$$

As expected, H_0 increases with ''impurity parameter'' ξ_0/l. The increase of H_0 is faster than that of H_{c2}: the latter can be estimated using $H_{c2} \approx H_c\kappa \propto 1/\chi(\alpha)$ (see, e.g., Ref. 12) where the Gor'kov function

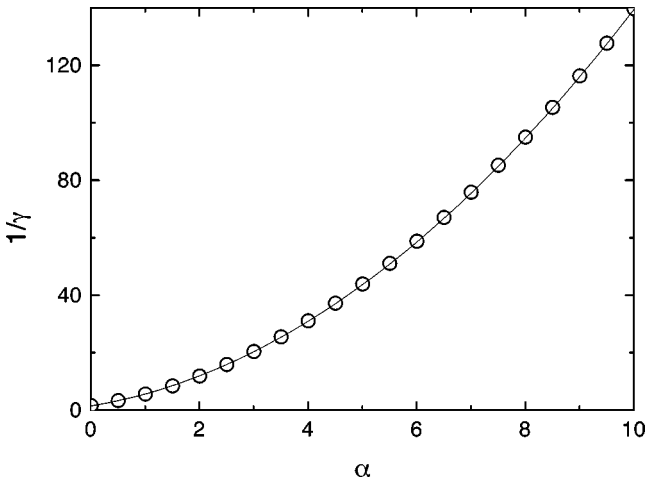

FIG. 1. The dots are calculated using Eqs. (7), (8), and (9), whereas the solid line shows the polynomial of Eq. (10).

$$\chi = \frac{8}{7\zeta(3)} \sum \frac{\Delta^3}{\beta^2\beta_1} \approx \frac{1}{1+0.9\alpha}. \tag{12}$$

Coming back to Eq. (3), we observe that the field B enters M in the combination B/H_0 instead of the standard London or GL ratio B/H_{c2}. The field H_0 of a given sample increases with T; at a given T, H_0 of a set of samples increases fast with shorter mean-free path l. We also note that instead of the standard slope (2) we have now

$$\frac{\partial M}{\partial \ln B} = \frac{M_0}{(1+B/H_0)^2}, \tag{13}$$

i.e., the slope $\partial M/\partial \ln B$ *decreases* with B.

It is worth recalling that in the linear domain near $H_{c2}(T)$ where $4\pi M \approx (B - H_{c2})/2\kappa_2^2$ for large κ's (see, e.g., Ref. 12), we have

$$\frac{\partial M}{\partial \ln B} = B\frac{\partial M}{\partial B} \approx \frac{B}{8\pi\kappa_2^2}, \tag{14}$$

i.e., here the slope $\partial M/\partial \ln B$ *increases* with B [$\kappa_2(T) = \kappa$ at $T = T_c$ and increases with lowering the temperature, see, e.g., Ref. 12].

As we have mentioned, interpretation of the magnetization data for strongly anisotropic compounds (Bi-2212, Tl-2212, Hg-1201) on the basis of Eq. (3) was reasonably successful.[4] Nevertheless, the evidence in favor of nonlocality as the reason for deviation in $M(T,H)$ from the standard London behavior was incomplete. The point was that in strongly anisotropic layered compounds, the thermal fluctuations in vortex positions dominate magnetization at high temperatures. Moreover, it has been argued by Bulaevskii *et al.* that the quantum fluctuations of weakly interacting 2D *pancake* vortices at low temperatures may well account for the observed deviations in slopes $\partial M/\partial \ln B$ from the London prediction (2).[13]

Convincing indications that the nonlocal corrections to London indeed describe the vortex physics in high-κ materials came after the problem of vortex lattices in borocarbides has been addressed experimentally and theoretically.

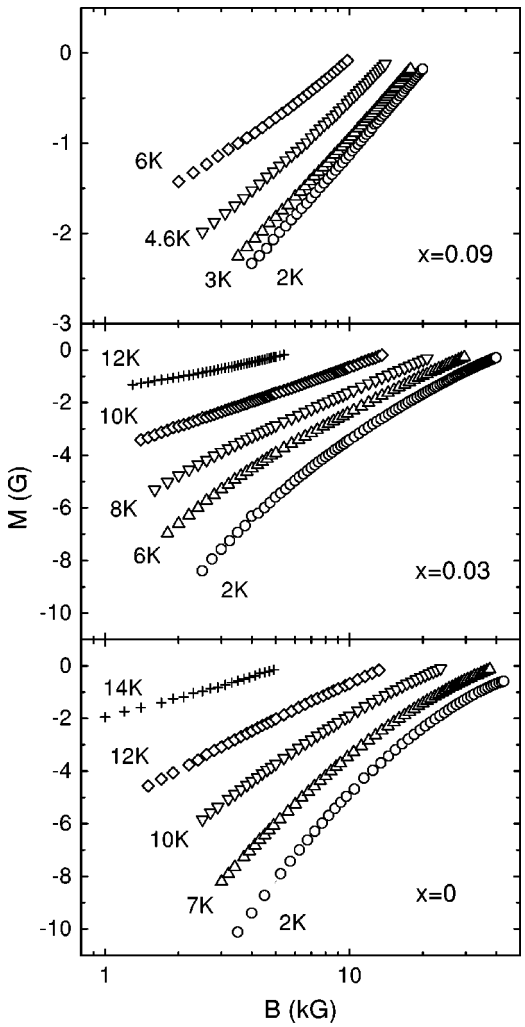

FIG. 2. Magnetization versus field for three samples of $Lu(Ni_{1-x}Co_x)_2B_2C$ with $x=0,0.03$ and 0.09.

The small-angle-neutron scattering experiments (SANS) on large high quality single crystals revealed that in increasing fields, the flux-line lattice may undergo structural transition (from triangular to square arrangement).[14] These results were confirmed and extended at Argonne,[15] Oak Ridge,[16] and Grenoble.[17] By and large the theoretical predictions based on the nonlocal London description were confirmed.[18–20]

Further support to the idea of nonlocality as responsible for the ''hex-to-square'' transition came in recent SANS experiments done on a series of $Lu(Ni_{1-x}Co_x)_2B_2C$ crystals prepared in Ames for the purpose of studying the mean-free path dependence of this transition.[21,22] The results confirmed the theoretical prediction for the transition field to increase with decreasing mean-free path l.

II. EXPERIMENT

As far as magnetization of borocarbides is concerned, Song *et al.*[23] applied the nonlocal analysis to the data on YNi_2B_2C. Here we report the magnetization data on the same $Lu(Ni_{1-x}Co_x)_2B_2C$ samples which were used for the SANS experiment.[21] Single crystals of this compound were grown from the $(Ni_{1-x}Co_x)_2B$ flux, in a manner similar to the growth of other borocarbide crystals, for details see Ref. 22 and references therein. Magnetization measurements were performed on a Quantum Design MPMS-5 superconducting quantum interference device (SQUID) magnetometer with the applied magnetic field parallel to the c axis of the crystals. For each temperature, $M(H)$ data were taken in increasing and decreasing field to ensure that only the reversible part of the data enters our analysis.

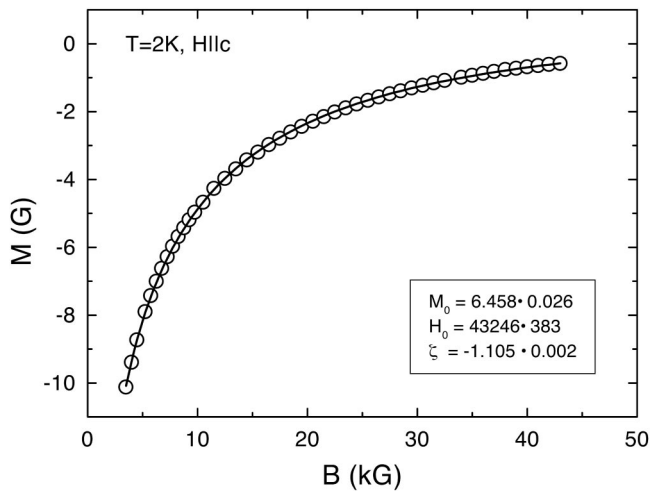

FIG. 3. A typical example of the fitting procedure. Open dots are the magnetization isotherm for $LuNi_2B_2C$ at 2 K. The solid line is calculated with the help of Eq. (3) with fitting parameters shown in the inset.

Figure 2 shows the reversible part of the $M(B)$ data on a semilog scale. On the bottom panel for the Co-free sample, one clearly sees deviations from the standard London logarithmic behavior; the deviations decrease with increasing temperature. On the middle panel ($x=0.03$) one sees that the standard London behavior occupies a broader part of the phase diagram and wins completely in the crystal with $x=0.09$, the top panel.

It is worth noting that for $x=0.09$, the isotherms $M(\ln B)$ at high temperatures even acquire a positive curvature, in other words, the slope $\partial M/\partial(\ln B)$ increases with B thus signaling the crossover from the London to the linear regime near H_{c2}. We estimate from the plot $\partial M/\partial B \approx 1.1\times 10^{-4}$ at the high field end of the 6-K isotherm. Using the slope (14) we obtain $\kappa_2 \approx 20$. This is an upper bound on κ because $\kappa_2(T)$ being equal to κ at $T_c \approx 9.5$ K increases with T.

In Fig. 3 we give a typical example of the fitting procedure. The circles are the data and the solid line is the fit to $M(B)$ given in Eq. (3). We treat M_0, H_0 and ζ as fitting parameters shown in the inset. The quality of the fit is usually high, but deteriorates at high T's and for large x's where the data approach the standard London limit. As is seen from Eqs. (3) and (4), when $H_0 \gg H_{c2}$ the parameter H_0 drops from the sum of two logarithmic terms. In the numerical procedure, this translates in exceedingly shallow minima with respect to the fitting parameter H_0 and causes numerical errors to increase. Value of M_0 obtained from the fit yields the penetration depth $\lambda \approx 1000$ Å at $T=2$ K.

Figure 4 summarizes the temperature behavior of the parameters M_0 and H_0 for samples with $x=0,0.015,0.03$. We see that $M_0 \propto 1/\lambda^2$ decreases with T—as it should—and extrapolates to zero near the corresponding T_c shown by black

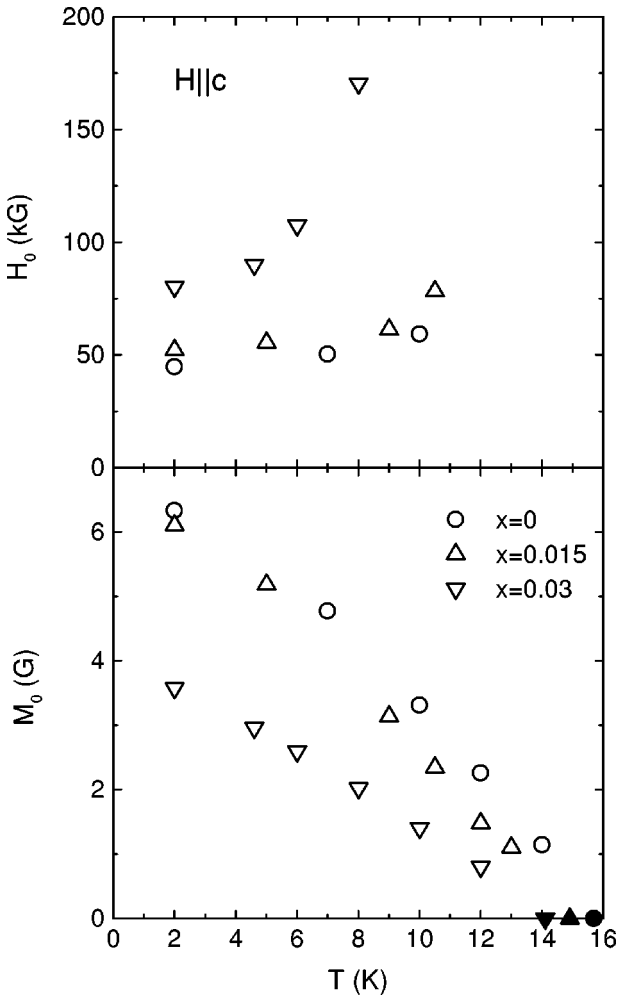

FIG. 4. Parameters M_0 and H_0 obtained with the help of Eq. (3) to fit the data for $Lu(Ni_{1-x}Co_x)_2B_2C$ with $x=0$, 0.015, and 0.03. At the lower panel for M_0, two last points at the high-temperature side are obtained using the standard London Eq. (1). The black symbols at the T axis show the corresponding T_c's.

symbols at the T axis. It is tempting to interpret the nearly linear T dependence of λ^{-2} at low temperatures as evidence for an unconventional order parameter symmetry. We are, however, reluctant to claim that λ^{-2}, which is obtained from a three-parameter fit of relatively featureless data, see Fig. 3, is accurate enough to support such a claim without falling into a danger of overinterpretation.

The field H_0 increases with T as expected according to Eq. (5). Numerical estimates of $\gamma(T)$ show that even in the clean limit, the relative increase of H_0 should not exceed $[H_0(T_c)-H_0(0)]/H_0(0)\sim 1$. As is seen on the upper panel, this upper bound is compatible with $H_0(T)$ for $x=0$ and $x=0.015$, but is clearly violated for $x=0.03$. The exaggerated increase of H_0 at high-T edge is related to the numerical instability discussed above: as is seen at Fig. 2, at $T=8$ K $M(B)$ is very close to London behavior $M\propto \ln(H_{c2}/B)$.

Parameters H_0 needed to fit the data to Eq. (3) at the lowest temperature of the experiment (2 K) are collected in Table I. We also show T_c determined from the magnetization data, the residual resistivities ρ_0, and the specific-heat coefficient γ_e which are needed for estimates of l and ξ_0.

The undoped $LuNi_2B_2C$ is known to have a sharp peak of the density of states $N(0)$ close to the Fermi level.[24] To have realistic estimates of quantities depending on $N(0)$, we have measured the electronic specific-heat coefficient γ_e for the Co-doped crystals, in which the superconductivity has been suppressed by application of high magnetic field up to 9 T [γ_e should not be confused with the function $\gamma(T,l)$ of Eqs. (5) and (6)]. As is shown in Fig. 5, the data are well approximated by

$$\gamma_e=(19.7-33.6x-75.7x^2)\frac{\mathrm{mJ}}{\mathrm{mol\,K^2}}, \tag{15}$$

where x is the Co content. The suppression of γ_e and a corresponding reduction in $N(0)$ are in qualitative agreement with the suppression of T_c.

We observe that $\gamma_e(x=3\%)$ differs from $\gamma_e(x=0)$ by a mere 5%, so that we do not expect much of a change in v_F for the samples studied here. This, in turn, suggests that $\xi_0\propto v_F/T_c$ and the mean free path $l=3/2\rho_0e^2N(0)v_F$ should roughly scale as $1/T_c$ and $1/\rho_0$, respectively. Then, the ratios ξ_0/l which enter the expression (11) for H_0 are related to each other:

$$\frac{\xi_0(x)}{l(x)}\Big/\frac{\xi_0(0)}{l(0)}\approx\frac{T_c(0)}{T_c(x)}\frac{\rho(x)}{\rho(0)}. \tag{16}$$

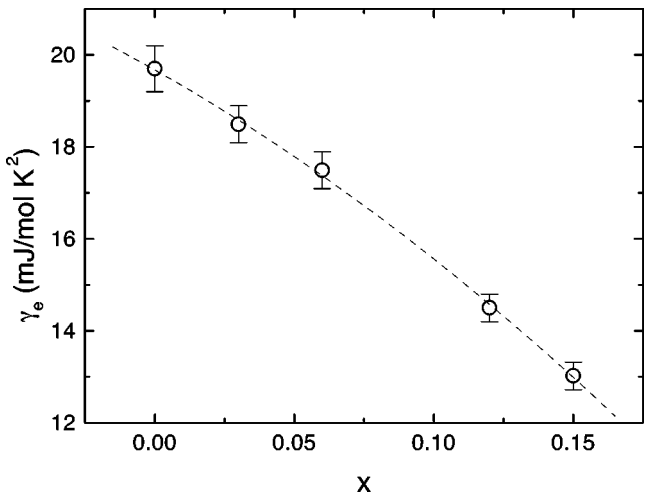

TABLE I. H_0 is obtained fitting the $M(H)$ data at 2 K to Eq. (3); ρ_0 is given in $\mu\Omega$ cm and γ_e in mJ/K^2 mol. The line ''64% Y'' is for $Y_{0.64}Lu_{0.36}Ni_2B_2C$; H_0 is obtained in this work, H_2 by SANS (Ref. 27). ''100% Y'' stands for YNi_2B_2C (Refs. 23, 17, and 30).

x% Co	T_c, K	ρ_0	γ_e	H_0, T	H_2, kG	H_0/H_2
0.	16.0	1.5	19.7	4.3	2 - 2.5	17 - 22
1.5	15.0	4.2	19.2	5.2	3	17
3.0	14.2	5.7	18.6	8.0	4.9	16
64% Y	15.0	4.4	19.8	4.9	3 - 3.5	14 - 16
100% Y	~15	4	19.8	5.2	2 - 2.5	21 - 26

FIG. 5. The specific-heat coefficient γ_e as a function of Co content x for $Lu(Ni_{1-x}Co_x)_2B_2C$. The dashed line is a polynomial fit of the data, Eq. (15).

Using T_c's and ρ_0's of Table I we evaluate this ratio for $x=3\%$ as 4.28.

As the next step, we write

$$\frac{H_0(x)}{H_0(0)}=\frac{\xi_0^2(0)}{\xi_0^2(x)}\frac{\gamma(0)}{\gamma(x)}\approx\frac{T_c^2(x)}{T_c^2(0)}\frac{\gamma(0)}{\gamma(x)}, \tag{17}$$

where γ's depend on the ratios ξ_0/l. We now take H_0's and T_c's from Table I for $x=0.03$ and $x=0$, and substitute $(\xi_0/l)_{0.03}=4.28(\xi_0/l)_0$ to obtain Eq. (17) with a single unknown parameter $(\xi_0/l)_0$. The equation is readily solved with the result $(\xi_0/l)_0=0.13$, which corresponds to the Co-free sample being a rather clean superconductor. Equation (11) then yields $\xi_0(0)\approx 70$ Å, where we set the parameter $\eta^*=1$. This estimate is close to that obtained from the SANS data[25] and implies that the zero-temperature upper critical field for the Co-free sample should be about 6 T, which is close to the measured value.[26]

III. DISCUSSION

Thus all qualitative features of $M(H,T)$ predicted by the nonlocal London model are clearly seen in our data: the deviations in the slope $dM/d\ln B$ from the standard London constant at low temperatures in clean samples and its field dependence, the standard London behavior of dirty samples at all temperatures, and the correct qualitative behavior of all fitting parameters. In order to demonstrate that the observed behavior of $M(H,T)$ is generic for high-κ superconductors and is not related to a particular system, we have prepared a crystal with nominally 50 of Lu being substituted with Y (actually $Y_{0.64}Lu_{0.36}Ni_2B_2C$ as determined by microprobe analysis). This crystal turned out to have $T_c=15.0$ K and the residual resistivity $\rho=4.4$ $\mu\Omega$ cm, in other words, its superconducting properties are expected to be similar to those of the Co-doped crystal with $x=1.5\%$, see Table I. Figure 6 shows this remarkable similarity.

As mentioned above, the transition from a square to rhombic cell in the vortex lattice at a field H_2 is another major manifestation of nonlocality. We collect the data on H_2 in Table I for materials with available estimates of H_0 (the data are taken from Ref. 27 for a Co-free $LuNi_2B_2C$ and for $Y_{0.64}Lu_{0.36}Ni_2B_2C$, from Ref. 21 for $Lu(Ni_{1-x}Co_x)_2B_2C$ with $x=1.5\%$ and 3%; other sources are indicated in the table caption). We observe that the field $H_0=\phi_0/4\pi^2\mathcal{R}^2$ extracted from the magnetization and $H_2=\phi_0/a^2$ (a^2 is the intervortex spacing at the transition) are nearly proportional. Given relatively large error bars and uncertainties in the data, the ratio H_0/H_2 is approximately the same (≈ 18) throughout Table I. This means that for the materials of Table I,

$$\frac{H_2}{H_0}=\frac{4\pi^2\mathcal{R}^2}{a^2}\approx\frac{1}{18}, \tag{18}$$

or that at the transition

$$\mathcal{R}\approx 0.04a. \tag{19}$$

In other words, the square-to-rhombus transition happens when the nonlocality range $\mathcal{R}$ reaches a certain fraction of the intervortex distance. This conjecture (reminiscent of the Lindemann criterion for melting) provides a useful relation between two at first sight unrelated phenomena: the macroscopic magnetization (in which the transition at H_2 has not been seen) and the ''microscopic'' structural transition in the vortex lattice. Physically, this criterion can be justified qualitatively by estimating the reduction in the vortex lattice energy due to the nonlocal corrections and equating it to the lattice shear energy.

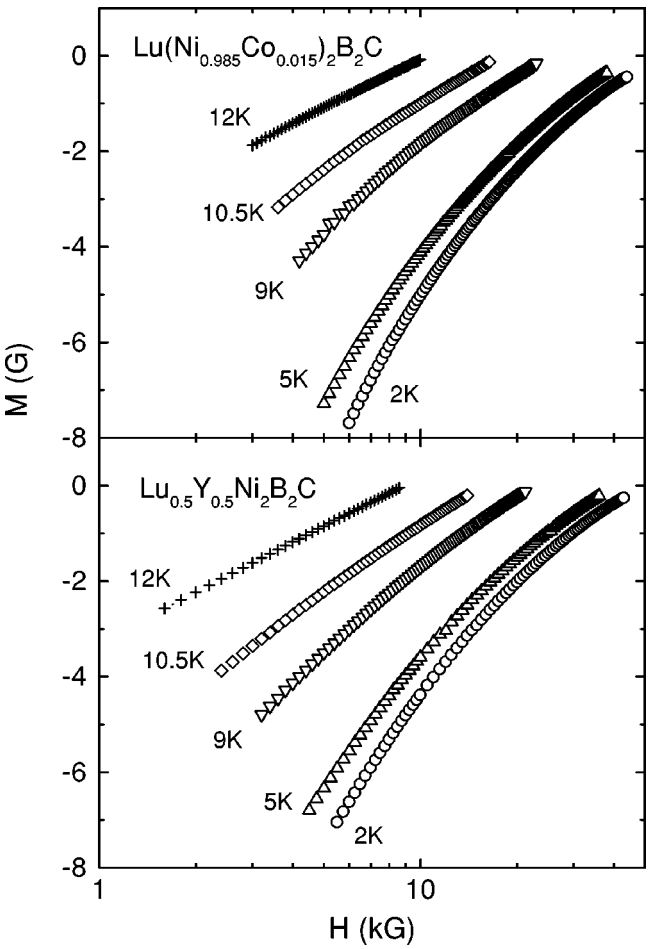

FIG. 6. Magnetization versus field at temperatures indicated for $Lu(Ni_{0.985}Co_{0.015})_2B_2C$ and $Y_{0.64}Lu_{0.36}Ni_2B_2C$.

Finally, we should comment on a sizable difference in estimates of ξ_0/l of this work and of Ref. 22; in the latter these ratios are an order of magnitude larger. In the analysis of this work, we do not use a particular value for the (average) Fermi velocity v_F. With $\xi_0=70$ Å, the free-electron relation $\xi_0=0.57\hbar v_F/\pi T_c$ yields $v_F\approx 0.9\times 10^7$ cm/s for the Co-free sample, whereas the band structure calculations give $\langle v_F^2\rangle^{1/2}$ by a factor of 2 or 3 larger.[20,24] As a consequence, the procedure of Ref. 22 which uses the band structure value for v_F gives higher estimates for ξ_0 and lower for l's. This yields a factor of 7 in the ratios ξ_0/l (although preserves about the same relative increase of the ratios when one goes from the Co-free sample to higher doping levels). It is quite possible that having a complex Fermi surface (made of isolated pockets[24,28]), the borocarbides are in a sense ''unconventional'' even if the superconductivity in them is due to the standard electron-phonon interaction.[29] The estimates based on the free-electron scheme of a metal are nothing but a ''Procrustean'' stretch, so that one should not expect more than a qualitative agreement to be demonstrated.

We conclude with the notion that borocarbides are quite unique as a system for studying the mixed state of type-II superconductors. The materials are almost isotropic as far as superconducting characteristics such as λ and ξ are concerned. They have relatively large GL parameter κ even in clean crystalline samples along with a very weak pinning. These features allow one to study reversible magnetization in a broad range of temperatures and fields. The only other example of such a system is that of high-T_c superconductors, majority of which are strongly anisotropic, in fact, close to being two dimensional. This makes the thermal and quantum fluctuations into a factor to reckon with when considering magnetic properties of the mixed state. Less anisotropic members of the high-T_c family such as YBCO have only a narrow domain of the (H,T) diagram near the transition line where the magnetization is reversible even in the best quality single crystals. On the other hand, the conventional materials like Nb have usually low κ's. Then, it is difficult to study the vortex phase theoretically because nearly at all fields between H_{c1} and H_{c2}, the intervortex spacing is on the order of the coherence length, and one cannot neglect the spatial variations of the order parameter. Impurities, of course, raise the κ, but in conventional materials this is usually accompanied by a considerable pinning and irreversibility.

ACKNOWLEDGMENTS

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