Effect of surface defects on the first field for vortex entry in type-II superconductors

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(Received 7 April 2000)

The effect of arbitrary-size surface defects on conditions for vortex entry in bulk type-II superconductors is studied by numerically solving the time-dependent Ginzburg-Landau equations. It is found out that the field $H_{\rm en}$ for a vortex entry in a sample decreases with a larger size of defects, yet, beginning from a certain size, it practically stops varying and sets at a minimum possible value $H_{\rm en}^{\rm min}$ (which is larger than the first critical field H_{c1}). An interpolation expression is found to describe field $H_{\rm en}^{\rm min}$ as a function of Ginzburg-Landau parameter κ, which fits the numerical results with a high degree of accuracy.

The effect of a surface (edge, geometrical) barrier on magnetic characteristics of type-II superconductors has been an issue of considerable interest recently.[1–6] A closely related problem is the onset of a magnetic flux (vortices) in superconducting samples. The influence of an ideal (defect-free) surface on the conditions for the first vortex entry was studied in detail in a number of works (see, for example, Refs. 7–9) for bulk superconductors. It was shown that vortices start penetrating in bulk type-II superconductors at some critical field H_s (nearly the value of thermodynamical field H_c) which is larger than H_{c1}, i.e., the first critical field. It is obvious that surface defects will decrease field $H_{\rm en}$ for the first vortex entry in a superconductor: $H_{\rm en}<H_s$. In Refs. 10–13 the influence of small-size[10,11] ($\ll\lambda$ which is the London penetration depth) and large-size[12,13] ($\gg\lambda$) surface defects on the value of $H_{\rm en}$ for bulk type-II superconductors was studied within the London model. In Ref. 12 (see also Ref. 13) an analytical expression $H_{\rm en}^{\rm min}=H_c\sqrt{\pi/\kappa}$ was derived in the limit $\kappa\gg1$ for a minimum possible field $H_{\rm en}^{\rm min}$ (feasible for thin deep $\gg\lambda$ cracks). The effect of surface roughness on the conditions providing stability of a vortex-free superflow of liquid helium ^{4}He was considered in Ref. 14. It was established that surface defects are responsible for a decrease in the critical rate of a vortex-free superflow, q_c, nearly $\sim(l/\xi)^n$-fold (where l is the defect length, ξ is the coherence length, index $n>0$ depends on the defect width w).

The objective of this work was to study the effect of rectangular-shaped surface defects on the first field for vortices entry in bulk type-II superconductors, based on numerical solution of the time-dependent Ginzburg-Landau equations[15]

$$\frac{\partial\Psi}{\partial t}=-\frac{1}{C}[(-\mathrm{i}\nabla-\mathbf{A})^2\Psi+\Psi(|\Psi|^2-1)]+\chi, \qquad (1)$$

$$\frac{\partial\mathbf{A}}{\partial t}=\mathrm{Re}[\Psi^*(-\mathrm{i}\nabla-\mathbf{A})\Psi]-\kappa^2\ \mathrm{rot\,rot}\,\mathbf{A}. \qquad (2)$$

The length here is scaled in units $\xi(T)$, the time is in units $\tau=4\pi\sigma_n\lambda^2(T)/c^2$, the vector potential $\mathbf{A}$ is in units $\Phi_0/(2\pi\xi)$, where $\Phi_0=ch/2e$ is a magnetic flux quantum, σ_n the normal-state conductivity of a superconductor, C the coefficient of proportionality,[16,17] χ is the dimensionless random ''force'' simulating fluctuations of the order parameter,[18] and Re stands for ''real part of.'' The magnetic field is measured in units $H_{c2}=\Phi_0/2\pi\xi^2$ which is the second critical field.

Consider a bulk superconductor $(-\infty<y<\infty,-\infty<z<\infty)$ of width D in an external magnetic field $\mathbf{H}=(0,0,H)$ (see Fig. 1). Further we will focus on the influence produced by a chain of identical surface defects (measuring $l\times w$) with a period $S\gg\lambda$ on the first field for flux penetration. Therefore, we can choose a length S of this superconductor, which has a surface defect, and set periodic boundary conditions $\Psi(-S/2)=\Psi(S/2)$, $\mathbf{A}(-S/2)=\mathbf{A}(S/2)$ along the y direction. Since the selected period S is large and the width of the defects under study are small, $w<2\lambda$, it may be assumed that here we study the influence of one isolated defect on the value of field $H_{\rm en}$ (as numerical calculations show, the value of $H_{\rm en}$ reached for an isolated defect is less than the one corresponding to the chain of the identical surface defects[19]). The boundary conditions for Eqs. (1) and (2) at the superconductor-vacuum interface have a standard form: $\nabla\times\mathbf{A}|_z=H$ and $(-i\nabla-\mathbf{A})\Psi|_n=0$. Calculations within the London model show that the current density tends to infinity when approaching the interior corners of the defect (points A,B in Fig. 1) (see, for example, Refs. 12 and 13). As a

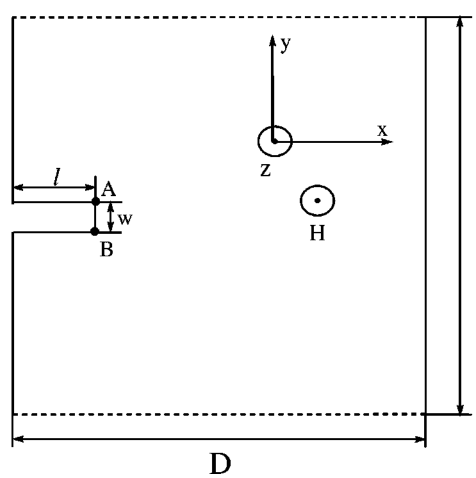

FIG. 1. The bulk superconductor $(-\infty<y<\infty,-\infty<z<\infty)$ with surface defect (chain of defects—see text) in an external magnetic field $\mathbf{H}=(0,0,H)$.

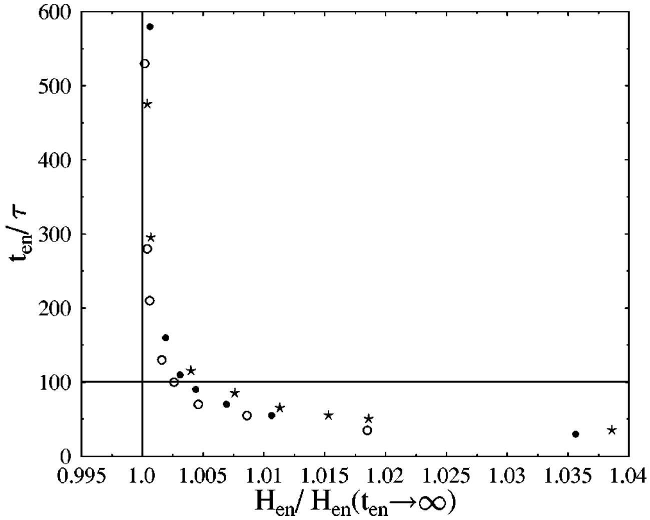

FIG. 2. Vortex entry time t_{en} as a function of an applied magnetic field H. Circles are for $\kappa=7$, dots for $\kappa=5$, and stars for $\kappa=2$. The results were obtained for a defect with size: $l=2\lambda$; $w=\xi$.

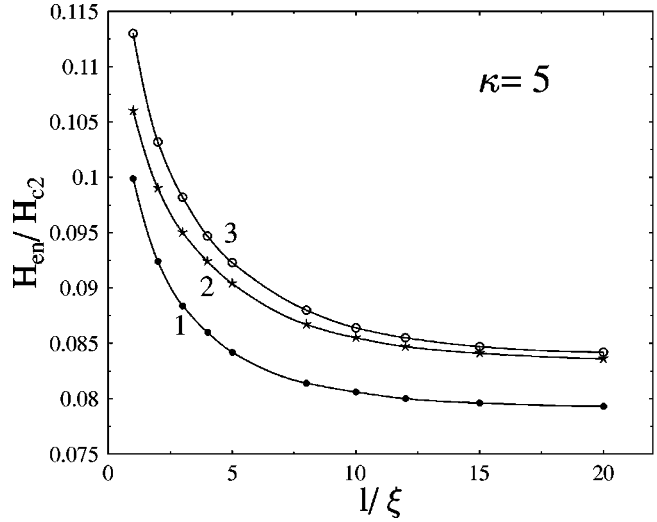

FIG. 3. Dependence of field H_{en} on a defect length at $w=\text{const}$ for type-II bulk superconductor ($\kappa=5$), obtained from the results of numerical calculation of Eqs. (1) and (2). Curve 1 (dots) is for a defect with $w=1\xi$, curve 2 (stars) is for $w=2\xi$, and curve 3 (circles) is for $w=3\xi$.

result, the order parameter in these points within the Ginzburg-Landau model will be zero. This circumstance has been accounted for in our numerical calculations by setting $\Psi=0$ in the interior defect corners.

Equations (1) and (2) were solved numerically by the method described in detail in Ref. 18. Specifically, we used a new variable $U_j=\exp(-i\int A_j dx_j)$ ($x_j=x,y$), which allows to satisfy the condition of gauge invariance for Eqs. (1) and (2) on the computational mesh.

The superconductor parameters were chosen as follows: width $D=10\lambda$, period $S=10\lambda$. The value of the constant C, coordinate $(\Delta x,\Delta y)$ and time (Δt) step was chosen so as to minimize the count time; it turned out that the obtained results were practically independent of a value of C (we chose $C=0.5$), Δx (if $\Delta x=\Delta y\leqslant 1.0$) and Δt [if $\Delta t\leqslant 1/(2\kappa\Delta x)^2$]. Note that the values for H_{c1} and H_s obtained in this work agreed to within the numerical accuracy ($\sim 2\%$) with the results reported in Ref. 20 and Ref. 9 for fields H_{c1} and H_s, respectively.

Let us define the value of the field H_{en}. By H_{en} here we denote the value of a magnetic field, at which the time t_{en} of the first vortex entry in a surface-defect superconductor does not exceed 100τ (the magnetic field instantly is increased from zero to H_{en}). In Fig. 2 the vortex entry time is shown versus the value of the applied magnetic field (for three different values of Ginzburg-Landau parameter $\kappa=2,5,7$). The mistake inherent in such a finding of the H_{en} value is, as can be easily seen from Fig. 2, not more than 1%.

Let us first consider the value of field H_{en} as a function of length and width of defect for a superconductor with $\kappa=5$. In Fig. 3 it is clearly seen that starting from the length $l>10\xi$, the field H_{en} reaches its minimum and practically does not vary with a further increase in the defect length. Defect width w determines the minimum possible value of field H_{en}: the narrower the defect, the lower is field H_{en}; the most sudden change in its value being in the range $0<w<2\xi$ (see Fig. 4). It should be noted that in real samples, as a defect width is decreasing to dimensions of the order of coherence length ξ, an increasing influence is produced on the situation by the Josephson currents flowing through a defect. This factor is responsible for a lower current density near a interior defect corner and, hence, a higher H_{en}. Therefore, a change in the value of H_{en} in the range of small widths $w\sim\xi$ of a defect should be less abrupt, than that shown in Fig. 4 (because we neglected the effect of the Josephson currents in our calculations).

The dependences $H_{en}(l)$ and $H_{en}(w)$ were analyzed for other values of the Ginzburg-Landau parameter ($\kappa=2,3,7,10$), which yielded the following results (similar to the above): (i) at $l\geqslant 2\lambda$ the field H_{en} practically stops to vary and reaches its minimum value (for the given κ and defect width w); (ii) the most sharp change (decrease) in the value of H_{en} occurs in the range of defect widths $0<w<2\xi$.

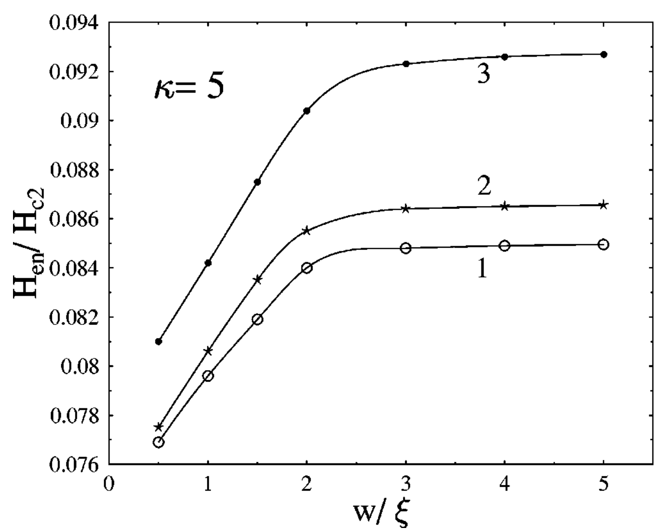

FIG. 4. Dependence of field H_{en} on a defect width at $l=\text{const}$ for a type-II bulk superconductor ($\kappa=5$), obtained from the results of numerical calculation of Eqs. (1) and (2). Curve 1 (circles) is for a defect with $l=15\xi$, curve 2 (stars) is for $l=10\xi$, and curve 3 (dots) is for $l=5\xi$.

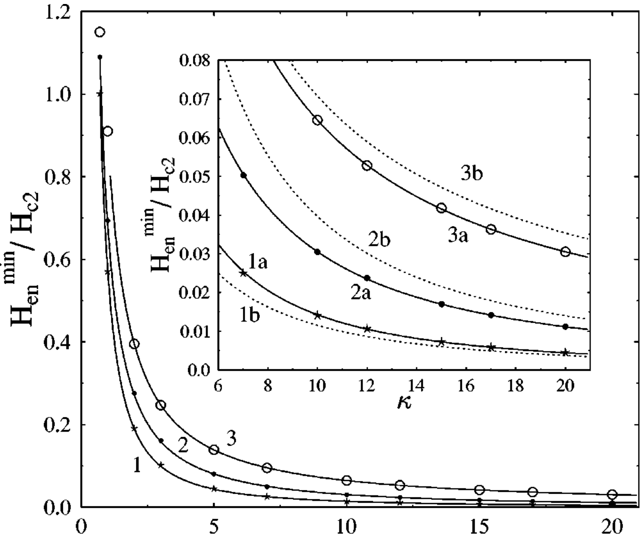

FIG. 5. Fields H_s (circles), H_{en}^{min} (dots), and H_{c1} (stars) versus the value of the Ginzburg-Landau parameter κ. All fields have been found by numerical solution of Eqs. (1) and (2). Curve 1 corresponds to expression (4), curve 2 is for Eq. (3), and curve 3 for the dependence $H_s(\kappa)$ taken from Ref. 9. In the inset are the same quantities (curves 3a, 2a, and 1a, respectively). Curve 3b is for thermodynamical field $H_c = 1/\sqrt{2}\kappa$, curve 2b is for the dependence $H_{en}^{min} = \sqrt{\pi/2}/\kappa^{3/2}$ obtained in Ref. 12, and curve 1b corresponds to the ''classic'' expression for the first critical field $H_{c1} = \ln(\kappa)/2\kappa^2$.

Note that unlike the results in Ref. 14, we have shown field H_{en} to have a minimum possible value (for a given defect width) different from zero, whereas in Ref. 14 the quantity q_c characterizing stability of a vortex-free superflow in ^{4}He proved to be proportional to $(\xi/l)^n (n>0)$ and for an infinite-length defect it reduces to zero. It can be shown that for type-II superconductors having large values of κ and surface defects meeting the condition $w < l \ll \lambda$ the field H_{en} will also be proportional to $(\xi/l)^n$ (where, same as in the liquid ^{4}He case, the power index $n>0$). Such similarity between superconductors and ^{4}He can be accounted for by the fact that formally it may be assumed that $\lambda = \infty$ in ^{4}He.

The obtained data led us to a conclusion that the entry field $H_{en}(l=2\lambda, w=1\xi)$ slightly (by about a few percent for $\kappa \gg 1$) exceeds the minimum attainable field H_{en}^{min} for vortices entry in a superconductor with surface defects for a given value of κ. Based on this conclusion we plotted a dependence of field $H_{en}^{min} \simeq H_{en}(l=2\lambda, w=1\xi)$ on the Ginzburg-Landau parameter κ, as shown in Fig. 5 (for $\kappa \leqslant 1$ the length of a defect was 3λ, the width was 0.5ξ). In the same figure we provide the dependences $H_s(\kappa)$ and $H_{c1}(\kappa)$. It is easily seen that in the entire range of values in question: $1/\sqrt{2} \leqslant \kappa \leqslant 20$ the field H_{en}^{min} exceeds H_{c1}. Note that at $\kappa = 1/\sqrt{2}$ also the first field for vortex entry, even in the presence of surface defects, is larger than the first critical field H_{c1} which in this case is equal to field H_{c2}. Figure 6 shows the dependence H_s/H_{en}^{min} and H_{en}^{min}/H_{c1} on the Ginzburg-Landau parameter. One can see that both these quantities are, essentially, the monotonically increasing functions of parameter κ.

For the dependences $H_{en}^{min}(\kappa)$ and $H_{c1}(\kappa)$ we derived interpolation expressions that satisfy the numerical results with a high degree of accuracy (the error is not more than 2%)

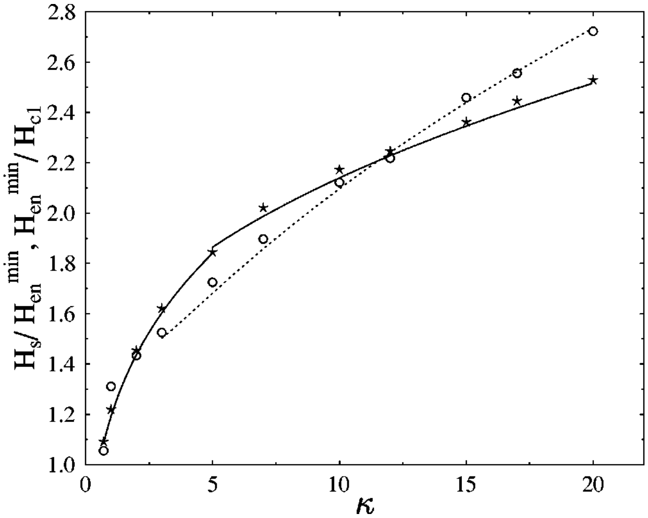

FIG. 6. The ratios H_s/H_{en}^{min} (circles) and H_{en}^{min}/H_{c1} (stars) as functions of the Ginzburg-Landau parameter. The solid line was obtained from expressions (3) and (4) and dotted line from Eq. (3) and dependence $H_s(\kappa) = \sqrt{5/2}(1+\sqrt{1/2\kappa})/3\kappa$ [which is valid for $\kappa \geqslant 1.1$ (Ref. 9)]. The small discontinuity of the solid curve is connected with the mistake of approximate expressions (3) and (4), which are discontinuous functions at $\kappa = 5$. The solid curve jump at $\kappa = 5$ is less than the accuracy (2%) of these expressions.

$$H_{en}^{min} \simeq \begin{cases} \dfrac{1}{(4\kappa/3)^{4/3}} & 1/\sqrt{2} \leqslant \kappa \leqslant 5, \\[2ex] \dfrac{1.03}{\kappa^{3/2}}(1-0.63/\kappa) & 5 \leqslant \kappa \leqslant 20, \end{cases} \tag{3}$$

$$H_{c1} \simeq \begin{cases} \dfrac{1}{(\sqrt{2}\kappa)^{8/5}} & 1/\sqrt{2} \leqslant \kappa \leqslant 5, \\[2ex] \dfrac{\ln(\kappa)+0.55}{2\kappa^2} & 5 \leqslant \kappa \leqslant 20. \end{cases} \tag{4}$$

When seeking an approximation expression (3) we assumed that in the limit of large values of κ it should approach the asymptotics obtained in Refs. 12 and 13, $H_{en}^{min} \sim 1/\kappa^{3/2}$. Similarly, field H_{c1} was sought for on the assumption that in the limit $\kappa \gg 1$ expression (4) should coincide with the ''classic'' expression $H_{c1} = \ln(\kappa)/2\kappa^2$ (see, for example, Ref. 21). In the limit of small κ the best dependences (in terms of agreement with the numerical results and simplicity of expression) were Eqs. (3) and (4) (at $1/\sqrt{2} \leqslant \kappa \leqslant 5$) obtained for the fields H_{en}^{min} and H_{c1}, respectively. Note that even for $\kappa = 100$ the difference of the above interpolation expression for field H_{c1} from dependence $H_{c1} = \ln(\kappa)/2\kappa^2$ makes to a 13%.

In conclusion, the effect of surface defects on the first field for vortex entry was investigated based on numerical solution of the time-dependent Ginzburg-Landau equations. The study was carried out for isotropic bulk type-II superconductors, neglecting the effects arising due to the temperature being nonzero. It is shown that the field H_{en} decreases monotonically with a growing length and a decreasing width of a defect. Physically, variation of the vortex entry field for a superconductor with surface defects, as compared to a defect-free superconductor, is accounted for by the fact that in the former case the current density near the interior corners of defects will largely exceed that on a superconductor surface. Therefore, first vortices will nucleate (due to the depairing current density which sets somewhere in a superconductor in the absence of weak coupling effects) in these places. It was found out that field H_{en} practically reaches a minimum possible value H_{en}^{min} for defects with a length $l \geqslant 2\lambda$ and a width $w \simeq \xi$. It is shown that field H_{en}^{min} lies in the range $H_{c1} < H_{en}^{min} < H_s$, and in the considered range of values for κ it will be larger than field H_{c1}. It should be expected that in superconductors of other geometries (for example, superconducting films in a perpendicular magnetic field) the presence of surface/edge defects would be unable to reduce the field for vortex penetration down to the value H_{c1}. Apparently, a particular geometry would only quantitatively affect the dependence of the ratios H_s/H_{en}^{min} and H_{en}^{min}/H_{c1} on the parameters of superconducting samples.

The author is grateful to Dr. G. M. Maksimova, Dr. I. L. Maksimov, and Dr. A. S. Mel'nikov for helpful discussions. The work was supported in part by the International Centre for Advanced Research (INCAS), Grant No. 99-02-3.

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[1] M. Konczykowski, L.I. Burlachkov, Y. Yeshurun, and F. Holtzberg, Phys. Rev. B **43**, 13 707 (1991); L.I. Burlachkov, Y. Yeshurun, M. Konczykowski, and F. Holtzberg, *ibid.* **45**, 8193 (1992).

[2] E. Zeldov, A.I. Larkin, V.B. Geshkenbein, M. Konczykowski, D. Majer, B. Khaykovich, V.M. Vinokur, and H. Shtrikman, Phys. Rev. Lett. **73**, 1428 (1994).

[3] M. Benkraouda and J.R. Clem, Phys. Rev. B **53**, 5716 (1996).

[4] E.H. Brandt, Phys. Rev. B **59**, 3369 (1999).

[5] F. Mrowka, M. Wurlitzer, P. Esquinazi, E. Zeldov, T. Tamegai, S. Ooi, K. Rogacki, and B. Dabrowski, Phys. Rev. B **60**, 4370 (1999).

[6] V. Jeudy and D. Limagne, Phys. Rev. B **60**, 9720 (1999).

[7] C.P. Bean and J.D. Livingston, Phys. Rev. Lett. **12**, 14 (1964).

[8] P.G. de Gennes, Solid State Commun. **3**, 127 (1965).

[9] H.J. Fink and A.G. Presson, Phys. Rev. **182**, 498 (1969).

[10] L. Burlachkov, M. Konczykowski, Y. Yeshurun, and F. Holtzberg, J. Appl. Phys. **70**, 5759 (1991).

[11] F. Bass, V.D. Freilikher, B.Ya. Shapiro, and M. Shvaster, Physica C **260**, 231 (1996).

[12] A.Yu. Aladyshkin, A.S. Mel'nikov, I.A. Shereshevsky, and I.D. Tokman, cond-mat/9911430 (unpublished).

[13] A. Buzdin and M. Daumens, Physica C **294**, 257 (1998).

[14] P.I. Soininen and N.B. Kopnin, Phys. Rev. B **49**, 12 087 (1994).

[15] A. Schmid, Phys. Kondens. Mater. **5**, 302 (1966); C.R. Hu and P.S. Thompson, Phys. Rev. B **6**, 110 (1972).

[16] L.P. Gor'kov and N.B. Kopnin, Usp. Fiz. Nauk **116**, 413 (1975) [Sov. Phys. Usp. **18**, 496 (1975)].

[17] B.I. Ivlev and N.B. Kopnin, Usp. Fiz. Nauk. **142**, 435 (1984) [Sov. Phys. Usp. **27**, 206 (1984)].

[18] R. Kato, Y. Enomoto, and S. Maekawa, Phys. Rev. B **47**, 8016 (1993).

[19] It is closely connected with the fact that decreasing of distance between defects S leads to decreasing (especially strong at $S \leqslant \lambda$) of concentration of current density line near interior corners of defects.

[20] L. Neumann and L. Tewordt, Z. Phys. **189**, 55 (1966).

[21] P.G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), Chap. 3.