

Experimental critical current patterns in Josephson junction ladders

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We present an experimental and theoretical study of the magnetic-field dependence of the critical current of Josephson junction ladders. At variance with the well-known case of a one-dimensional (1D) parallel array of Josephson junctions the magnetic-field patterns display a single minimum even for very low values of the self-inductance parameter β_L . Experiments performed changing both the geometrical value of the inductance and the critical current of the junctions show a good agreement with numerical simulations. We argue that the observed magnetic-field patterns are due to a peculiar mapping between the isotropic Josephson ladder and the 1D parallel array with the self-inductance parameter $\beta_L^{\text{eff}} = \beta_L + 2$.

I. INTRODUCTION

A Josephson junction ladder is an array of coupled superconducting loops containing small Josephson junctions as shown schematically in Fig. 1(a). In the past years, ladders of Josephson junctions have attracted considerable interest for a number of reasons. On one hand, more complex systems such as two-dimensional arrays of Josephson junctions can be viewed as elementary ladders coupled to each other.^{1,2} On the other hand, the ladders are an ideal model system for the investigations of discrete nonlinear entities,³⁻⁶ such as breathers⁷⁻¹⁰ and vortex propagation.¹¹

Superconducting (static) properties of Josephson ladders are also of a great interest due to a peculiar shielding of an external magnetic field¹² and the presence of different metastable states^{11,13} in these structures. Although the numerical and theoretical studies of such systems have been very advanced, a systematic experimental verification of as simple facts as the magnetic pattern (that does not involve any in-

vestigation of the dynamics) has not yet been carried out. In fact, the magnetic patterns have been investigated in Refs. 11 and 14; Grimaldi *et al.*¹⁴ have performed numerical simulations on these systems. Their findings are that the behavior of ladders is quite different from that of one-dimensional (1D) parallel arrays. In contrast to a ladder, a 1D parallel array contains Josephson junctions only in the direction of the bias current I_B but not transverse to it. A 1D parallel array of Josephson junctions placed in magnetic field shows a pattern of critical current with as many minima as the number of loops (for relatively low β_L , as defined below).^{15,16} In Josephson ladders with junctions on the horizontal branches not only does the number of minima in the pattern not correspond to the number of loops even for extremely low β_L , but also the pattern dependence on the parameter β_L is different: the critical current I_C never reduces to zero for fully frustrated arrays (i.e., when there is a half flux quantum in each cell). Baharona *et al.*¹¹ have shown that one can analytically estimate the depinning current of fluxons trapped into the ladder in the limit of zero inductance. They have also computed the onset of instability in the case of no fluxons, thus retrieving analytically the numerical result of Ref. 14 for very low inductance. Moreover, the authors of Ref. 11 have estimated that the critical current of a ladder with a fluxon trapped in each second cell is higher than the depinning from the empty ground state for a moderately high magnetic field.

The aim of this paper is to present experiments performed on isotropic Josephson ladders with various values of the self-inductance parameter β_L . We call as isotropic a ladder consisting of identical junctions. We also make an analysis of the model to explain the observed dependence of the pattern upon β_L . The work is organized as follows. In Sec. II we describe a model for the Josephson ladders, in Sec. III we show the experimental findings and make the comparison with the numerical predictions. Finally, Sec. IV contains a discussion of our results and Sec. V is the conclusion.

II. MODEL

To derive the equations for the ladder, we start from the fluxoid quantization over a cell:

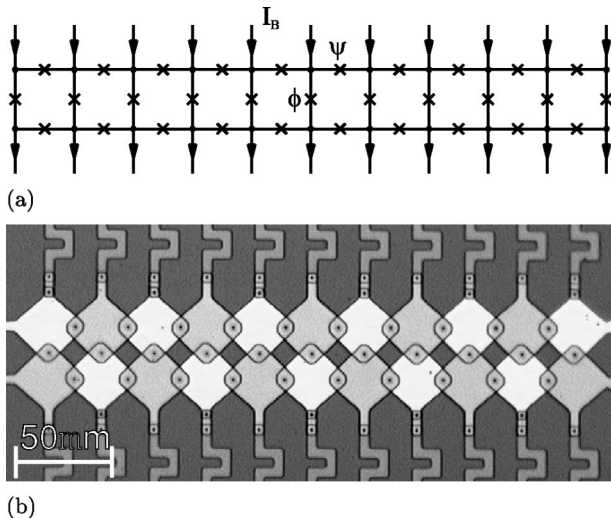


FIG. 1. (a) The electrical scheme of a Josephson junction ladder; crosses ($\times$) indicate Josephson junctions. (b) Optical image of one of the studied samples.

$$\sum_{cell} \varphi = -\frac{2\pi}{\Phi_0} [\Phi^{ext} + \Phi^{ind}], \quad (1)$$

where φ are the phases of all the junctions in the cell, Φ_0 is the flux quantum, and Φ^{ext} and Φ^{ind} are the applied and induced flux, respectively. To evaluate Φ^{ind} we retain only self-inductance terms, i.e., we assume that $\Phi^{ind} = LI^s$, I^s being the screening current circulating in the elementary cell and L is the self-inductance of the cell. More correctly, Φ^{ind} should include the field generated by all currents, including the bias current. Such effects can be included using, for instance, the approach of Ref. 15. However, the influence of mutual inductances on the dynamic properties of parallel arrays has been studied in Ref. 18 and it was found that the effect is rather small. It is because in 1D arrays as well as in ladders the fields generated by not nearest-neighbor currents are mostly outside the cells. Thus we take into account only self-inductances in our numerics. For the junctions we assume the resistive and capacitive shunted junction (RCSJ) model and suppose that all junctions are identical (isotropic ladder). With these ingredients it is possible to derive the following set of equations for the gauge invariant phase difference across the vertical (ϕ_i) and horizontal (ψ_i) junctions:⁶

$$\begin{aligned} \ddot{\phi}_i + \alpha \dot{\phi}_i + \sin \phi_i &= \frac{1}{\beta_L} [\phi_{i-1} - 2\phi_i + \phi_{i+1} + 2(\psi_i - \psi_{i-1})] \\ &+ \gamma, \quad i=2, \dots, N, \\ \ddot{\psi}_i + \alpha \dot{\psi}_i + \sin \psi_i &= \frac{1}{\eta\beta_L} [\phi_i - \phi_{i+1} - 2\psi_i - 2\pi f], \\ &i=1, \dots, N. \end{aligned} \quad (2)$$

The boundary conditions are

$$\begin{aligned} \ddot{\phi}_1 + \alpha \dot{\phi}_1 + \sin \phi_1 &= \frac{1}{\beta_L} [\phi_2 - \phi_1 + 2\psi_1 + 2\pi f] + \gamma, \\ \ddot{\phi}_{N+1} + \alpha \dot{\phi}_{N+1} + \sin \phi_{N+1} &= \frac{1}{\beta_L} [\phi_N - \phi_{N+1} - 2\psi_N - 2\pi f] \\ &+ \gamma. \end{aligned} \quad (3)$$

Here, $\gamma = I_B / i_c^{ver}$ is the bias current normalized to the single vertical junction critical current, $\beta_L = 2\pi L i_c^{ver} / \Phi_0$ is the self-inductance parameter (the self-inductance L of a square cell with the side a can be estimated¹⁷ as $L = 1.25\mu_0 a$, where μ_0 is the magnetic permeability), the ratio $\eta = i_c^{hor} / i_c^{ver}$ between the horizontal and the vertical junction critical currents is the anisotropy parameter, α is the normalized dissipation parameter, $f = \Phi^{ext} / \Phi_0$ is the normalized external flux which is called frustration, and N is the number of loops. For the static case, the parallel arrays considered in Refs. 4, 5, and 15 correspond to the limit $\eta \rightarrow \infty$. In deriving Eqs. (2)–(3) we take advantage of the fact that, due to the symmetry of the system, the current flowing in the top and bottom horizontal junctions of the same cell [see Fig. 1(a)] differs only in direction but not in amplitude, and therefore we can write the equation for just one of them.¹ Finally, we want to stress that in this work we are interested only in the transition point

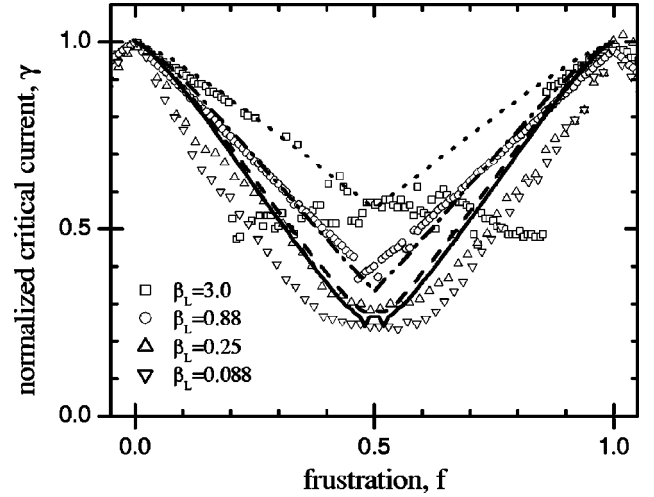


FIG. 2. Experimental (symbols) and numerical (lines) γ vs f patterns of ladders with different cell sizes. Parameters are: $N = 11$; $\beta_L = 3.0$ (squares and dotted line), 0.88 (circles and dashed dotted line), 0.25 (up triangles and dashed line), 0.088 (down triangles and solid line).

from the static to the dynamic solutions (the critical current), therefore the dynamics is nothing but a computational means to find the current point at which the static solution becomes unstable. The value of the dissipation used in the simulations is fictitious, and it has been chosen equal to 1 for computational convenience. The experimentally studied arrays are actually underdamped and therefore have a much smaller damping coefficient, $\alpha \approx 0.005$ – 0.08 , that can be controlled by temperature.

III. EXPERIMENTAL I_C VS f PATTERNS

We present an experimental study of ten-cell Josephson junction ladders. Each elementary cell of the ladders contains four identical small Nb/Al-AIO_x/Nb Josephson tunnel junctions,¹⁹ which have an area of $3 \times 3 \mu\text{m}^2$. To get different values of β_L , we used samples with different critical current density j_c (100 A/cm² or 1000 A/cm²) and also varied the loop size a (2.8 μm or 9.9 μm). An optical image of a typical ladder is shown in Fig. 1(b).

We have measured the ladder critical current I_C versus frustration f for four isotropic ladders ($\eta = 1$) with different β_L . The values selected are $\beta_L = 3, 0.88, 0.25$, and 0.088 , a range where the peculiar behavior of the ladders should be clearly visible. The measurements were performed in a cryoperm shield. The magnetic field H applied perpendicular to the substrate was provided by a coil placed inside the shield. The uniform bias current I_B was injected at every node via on-chip resistors. The voltage across the first vertical junction was measured to define the depinning current of the ladder. Finally, the I_C vs f dependencies were measured using GoldExi software.²⁰

In Fig. 2 we present two measured features of the ladders. In contrast to the case of a 1D parallel array, there are no additional lobes between $f = 0$ and $f = 1$. Also, the critical current I_C remains relatively large, despite a β_L as low as 0.088. Similarly to Ref. 14 we numerically solved Eqs. (2)–(3) by using the same parameters as in the experiment

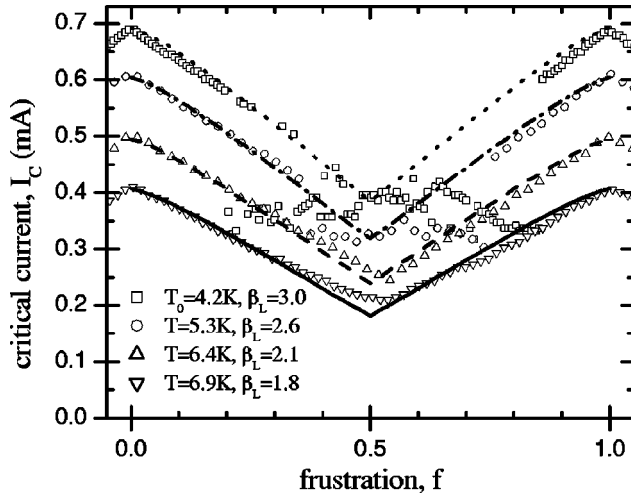


FIG. 3. Experimental (symbols) and numerical (lines) I_C vs f patterns of one of the ladders measured at different temperatures, in order to vary the critical current. The temperature has been derived from the gap voltage. Parameters are: $N=11$; $\beta_L=3.0$ (squares and dotted line), 2.6 (circles and dashed dotted line), 2.1 (up triangles and dashed line), 1.8 (down triangles and solid line).

(except for α , see above). In Fig. 2, we compare the numerical simulations (lines) with the experimental data (symbols), which show good agreement. The β_L used in the simulations was calculated from the critical current of a single junction measured in the experiment. The calculations show some flattening at $f=0.5$ for low β_L , which is also present in the experimental data. We found in simulations that in this region the ladder gets first filled with flux and only subsequently undergoes the depinning. We have observed this behavior only for low inductance, in good agreement with the analytical prediction of Ref. 11 (that neglects inductance terms). In experiments with $\beta_L=3$ and $\beta_L=0.88$ we note the presence of two different states at the same frustration value. We suppose that this is due to distinctly different initial conditions that can be realized in the ladder, while sweeping the bias current I_B through zero. This contradicts the prediction of Ref. 11 where it is stated that the depinning of the whole ladder from a state different from the empty ground state occurs *only* around $f=0.5$. We suppose that this disagreement is due to the neglect of the inductance in calculation (Ref. 11). In our simulations, deviations from the predicted behavior have been occasionally found. Such deviations are ascribed to the contemporary presence of metastable states in the array. When presenting the simulation results we show only the states with the highest critical currents. Other states have been discussed in Ref. 11.

In the ladder with the largest β_L parameter ($\beta_L=3.0$), we have measured the I_C vs f dependence also as a function of the temperature T . This allows to further compare the experimental behavior with the numerical one. At higher temperatures the decreased critical current causes a decrease of β_L . The results of both simulations and experiments are reported in Fig. 3, in physical units to underline the actual change of the critical current. Also in this case the agreement between the model and the experiments is rather good.

IV. DISCUSSION

We have characterized the static properties of Josephson ladders for different values of the self-inductance parameter

β_L . The experimentally observed dependencies of the critical current on frustration are in good agreement with the numerical simulations and show that the behavior of the ladder is clearly different from that of the 1D parallel array (see Fig. 2). As is well-known, in 1D parallel arrays the critical (depinning) current is determined by the parameter β_L , and in the limit of small β_L the minimum frustration-dependent critical current is very small. Instead, the ladder critical current, even in the case of small β_L , never goes to zero. As it was already pointed out in Ref. 14, with respect to 1D parallel arrays, the presence of the horizontal junctions in the ladder leads to an “effective” increased β_L^{eff} , which for small discreteness can be by up to two orders of magnitude larger than the *natural* β_L of the system, calculated (similar to 1D arrays) from the junction critical current and the cell inductance.

In order to show the particular mapping between Josephson ladders and 1D parallel arrays, we carry out a simple quantitative analysis of the Eqs. (2). Let us consider the static case, when all Josephson junction phases are independent of time and satisfy the system of nonlinear equations:

$$\eta(\sin \psi_{i-1} - \sin \psi_i) = \sin \phi_i - \gamma, \quad i=2, N. \quad (4)$$

By making use of the particular assumption that the horizontal phases ψ_i are small, we can eliminate the phases ψ_i from all equations and write the system of equations for phases ϕ_i in the form

$$\sin \phi_i = \frac{\eta}{\eta\beta_L + 2} [\phi_{i-1} - 2\phi_i + \phi_{i+1}] + \gamma. \quad (5)$$

This system of equations coincides with the one describing the static properties of 1D parallel array (with horizontal junctions replaced by superconducting electrodes). The difference between a ladder and 1D parallel array is that for the ladder we have now to use an effective parameter

$$\beta_L^{\text{eff}} = \beta_L + 2/\eta. \quad (6)$$

The deviation of β_L^{eff} from β_L originates from an additional shielding (and vortex pinning) due to the presence of horizontal junctions, i.e., the horizontal junctions can accommodate part of the phase change. Thus we expect that this deviation disappears in anisotropic ladders when the critical current of horizontal junctions i_c^{hor} is much larger than the critical current of vertical junction i_c^{ver} ($\eta \gg 1$).

To verify the mapping given by Eq. (6), in Fig. 4 we compare the patterns of a 1D parallel array with $\beta_L^{\text{1D}}=2.7$ and an isotropic ladder with $\beta_L=0.88$. For the ladder we expect $\beta_L^{\text{eff}} \approx \beta_L^{\text{1D}}$. The agreement is particularly good at low frustration, but not in the vicinity of $f=0.5$, where the most critical assumption of our theory, i.e. is small values of the horizontal junction phases, breaks down.

We would like to note here that a similar analysis for anisotropic ladders in the limit of small β_L has been carried out in Ref. 12. Moreover, for the case of ladder with three junctions per cell the mapping takes the form: $\beta_L^{\text{eff}} = \beta_L + 1/\eta$. This mapping is in good accord with previously published data on the $I_C(f)$ dependence for ladder with three junctions per cell.²¹ We want to stress here that this mapping is supposed to work only for the static case. In the dynamic

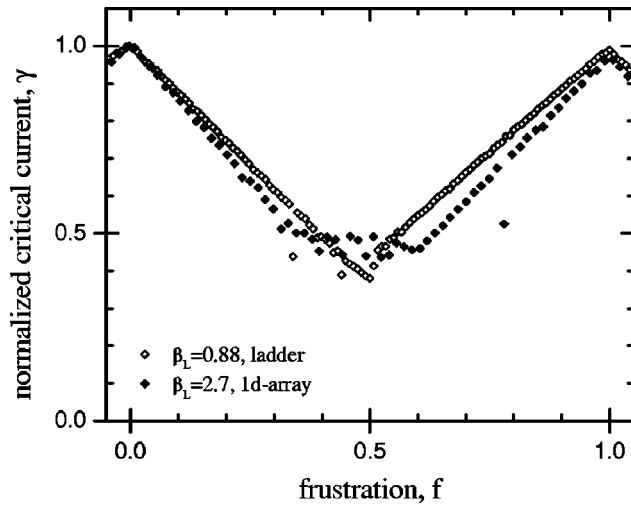


FIG. 4. I_C vs f dependencies of the 1D parallel array with $\beta_L = 2.7$ (solid circles) and the ladder with $\beta_L = 0.88$ (open circles).

state, when the Josephson vortices propagate in the ladder, the phases of the horizontal junctions start to oscillate and Eqs. (4) are not valid anymore, especially in the regime of large vortex velocities. Theoretical and experimental investi-

gation of vortex propagation in Josephson ladders will be reported elsewhere.²²

V. CONCLUSION

We have reported measurements of the critical current in ladders of Josephson junctions. The β_L parameter has been varied by changing both the geometrical inductance and the critical current of the junctions. The results are in good agreement with numerical simulations, and show a behavior clearly distinct from the case of the 1D parallel Josephson junction array without junctions in the horizontal branches. Using a simple quantitative analysis, we have shown that the static properties of 1D parallel arrays and ladders can be mapped by properly scaling the self-inductance parameter β_L . This analysis agrees well with experimental data.

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