Electrodynamics of metal-dielectric composites and electromagnetic crystals

Andrey K. Sarychev
Center for Applied Problems of Electrodynamics, 127412, Moscow, Russia

R. C. McPhedran
School of Physics, University of Sydney, NSW 2006, Australia

Vladimir M. Shalaev
Department of Physics, New Mexico State University, Las Cruces, New Mexico 88003
(Received 7 June 1999; revised manuscript received 16 May 2000)

A theory that takes into account effects of retardation is developed for calculating the effective dielectric constant and magnetic permeability of metal-dielectric composites and photonic crystals containing a metallic component. The effective parameters depend, in general, on the local microgeometry of composites and electromagnetic crystals. For example, in metal-wire crystals the effective dielectric constant becomes negative at frequencies below the cutoff frequency, which, in the case of a strong skin effect, is determined by the crystal structure only. It is also shown that the effective high-frequency magnetic permeability, which originates from the eddy currents in metal grains or wires, does not vanish, even in systems with no intrinsic magnetism. In metal-dielectric composites, it is shown that the effective dielectric constant has a positive maximum and a negative minimum in the vicinity of the percolation threshold. A new class of photonic crystals is proposed, where both dielectric permittivity and magnetic permeability are negative in optical spectral range.

I. INTRODUCTION

In this paper we consider electromagnetic properties of metal-dielectric composites irradiated by a high-frequency electromagnetic field under conditions when the skin effect in metal grains is strong. Two different classes of metal-dielectric systems are analyzed, artificial periodic electromagnetic crystals[1,2] and random percolation composites.[3–7]

The electromagnetic crystals are three-dimensional periodic structures of metal inclusions in a dielectric host. They are similar to the well-known photonic crystals composed of periodic structures of dielectric particles. Since metals have nonzero losses at the optical frequencies, most studies on photonic crystals are focused on dielectric structures. Nevertheless, metallic $3d$ structures can find applications in the microwave range and, under some conditions, in the optical spectral range as well. Below the interaction of a simple-cubic metal lattice with an electromagnetic field is considered.

Metal-dielectric percolation composites and $3d$ electromagnetic crystals are quite different objects at the first glance. In the present paper it is shown, however, that these objects can be described in common terms of the effective dielectric constant and magnetic permeability, provided that the wavelength of an incident wave is much larger than an intrinsic spatial scale of the system.

The wavelength inside a metal component can be very small. The most interesting effects are expected in the limit of a strong skin effect. Our consideration goes beyond the quasistatic approximation employed in most papers on metal-dielectric composites.[3–7] It is important that our approach for calculating the effective dielectric constant and magnetic permeability is essentially the same for composites and electromagnetic crystals. Moreover, results for the effective parameters are also, to large extent, similar. We believe that the method developed in this paper can be applied to arbitrary metal-dielectric systems, regular and random.

In electromagnetic crystals and composites, the propagation of electromagnetic waves with wavelength λ smaller than a spatial scale of an inhomogeneity (correlation length ξ) may be accompanied by strong scattering or diffraction. On the other hand, it seems plausible that the wave propagation for $\lambda \gg \xi$ can be described by Maxwell's equations with the effective dielectric constant ε_e and effective magnetic permeability μ_e. Below we calculate and discuss the effective parameters ε_e and μ_e, in the case where the skin effect is strong. In order to calculate the effective parameters, the approach suggested in Refs. 8–10 is further developed and applied for both metal-dielectric composites and electromagnetic crystals.

We consider an optically thin sample of a composite or electromagnetic crystal of the size $\mathcal{L} \ll \lambda/\sqrt{|\varepsilon_e \mu_e|}$; the system is homogeneous from the macroscopic point of view, i.e., $\mathcal{L} \gg \xi$. The sample is placed inside a resonator, where electromagnetic standing waves are excited. We assume that the volume of the resonator is much larger than the volume of the sample, $v \sim \mathcal{L}^3$. Since the system is described in terms of the effective parameters ε_e and μ_e, the macroscopic Maxwell equations, which are averaged over a spatial scale much larger than the correlation length ξ, can be written in the following form:

$$\mathrm{curl}\,\mathbf{E} = ik\mu\mathbf{H}, \quad \mathrm{curl}\,\mathbf{H} = -ik\varepsilon\mathbf{E}, \tag{1}$$

where $k = \omega/c$ is the wave vector, and $\mathbf{E}$ and $\mathbf{H}$ are monochromatic fields with the usual $\exp(-i\omega t)$ time dependence. The dielectric constant ε and magnetic permeability μ are equal to the effective parameters, $\varepsilon = \varepsilon_e$ and $\mu = \mu_e$, inside

the sample. Outside the metal-dielectric system, $\varepsilon=\mu=1$. It is supposed that the frequency ω equals one of the resonance frequencies of the resonator. The fields $\mathbf{E}_1$ and $\mathbf{H}_1$ in the empty resonator satisfy the Maxwell equations

$$\operatorname{curl}\mathbf{E}_1=ik_1\mathbf{H}_1,\quad \operatorname{curl}\mathbf{H}_1=-ik_1\mathbf{E}_1, \tag{2}$$

where $k_1=\omega_1/c$, and ω_1 is the resonance frequency of the empty resonator that corresponds to the same mode as ω. The shift of the resonance frequency $\Delta\omega=\omega_1-\omega$ is small, $|\Delta\omega|/\omega\ll 1$, since the volume of the resonator is much larger than the volume v of the composite. We multiply the first of Eqs. (1) by $\mathbf{H}_1^*$, and the second one by $-\mathbf{E}_1^*$. After complex conjugation of Eq. (2), we multiply the first of Eqs. (2) by $\mathbf{H}$ and the second by $-\mathbf{E}$. After this, all the equations are summed together, resulting in the following:

$$\mathbf{H}_1^*\operatorname{curl}\ \mathbf{E}-\mathbf{E}\operatorname{curl}\mathbf{H}_1^*+\mathbf{H}\operatorname{curl}\mathbf{E}_1^*-\mathbf{E}_1^*\operatorname{curl}\mathbf{H}$$
$$=i(k\varepsilon-k_1)\mathbf{E}\mathbf{E}_1^*+i(k\mu-k_1)\mathbf{H}\mathbf{H}_1^*. \tag{3}$$

The left hand side of Eq. (3) can be written as $\operatorname{div}([\mathbf{H}_1^*\times\mathbf{E}]+[\mathbf{E}_1^*\times\mathbf{H}])$. Provided that Eq. (3) is integrated over the volume of the resonator, the integral of the left-hand side transforms into a surface integral that vanishes since the tangential components of the electric field vanish on the walls of the resonator. (For simplicity, it is assumed that the resonator is made of a perfect conductor.) The integral in the right-hand side of Eq. (3) can be rearranged as follows:

$$\int\ [(\varepsilon-1)\mathbf{E}\cdot\mathbf{E}_1^*+(\mu-1)\mathbf{H}\cdot\mathbf{H}_1^*]d\mathbf{r}$$
$$=\frac{\Delta\omega}{\omega}\int(\mathbf{E}\cdot\mathbf{E}_1^*+\mathbf{H}\cdot\mathbf{H}_1^*)d\mathbf{r}, \tag{4}$$

where the terms on the left of Eq. (4) are nonzero in the composite only. Therefore, the integral on the left of Eq. (4) is over the volume v of the composite, where $\varepsilon=\varepsilon_e$ and $\mu=\mu_e$. Since the volume v is much smaller than the volume of the resonator the main contribution in the right-hand side of Eq. (4) comes from distances much larger than the size $\mathcal{L}$ of the composite. At these distances the perturbation due to the composite can be neglected so that $\mathbf{E}\approx\mathbf{E}_1$ and $\mathbf{H}\approx\mathbf{H}_1$. Thus the frequency shift according to Eq. (4) is estimated as

$$\frac{\Delta\omega}{\omega}=\frac{\int_v[(\varepsilon_e-1)\mathbf{E}_1^*(\mathbf{r}_0)\cdot\mathbf{E}_0(\mathbf{r})+(\mu_e-1)\mathbf{H}_1^*(\mathbf{r}_0)\cdot\mathbf{H}_0(\mathbf{r})]d\mathbf{r}}{\int(|\mathbf{E}_1|^2+|\mathbf{H}_1|^2)d\mathbf{r}}, \tag{5}$$

where $\mathbf{r}_0$ defines the coordinates of the composite, $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{H}_0(\mathbf{r})$ are the macroscopic fields in the composite, and $\mathbf{E}_1(\mathbf{r}_0)$ and $\mathbf{H}_1(\mathbf{r}_0)$ are the fields at the position $\mathbf{r}_0$ in the empty resonator. Because $\mathcal{L}\ll\lambda$, we neglect in Eq. (5) the variation of the fields $\mathbf{E}_1$ and $\mathbf{H}_1$ within the volume v. The field distribution $\mathbf{E}_1(\mathbf{r})$ and $\mathbf{H}_1(\mathbf{r})$ in the empty resonator can be calculated for an arbitrary resonator. The field distributions $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{H}_0(\mathbf{r})$ in the composite are unambiguously determined by the effective parameters ε_e and μ_e and by the shape of the composite. Thus, by measuring the shift $\Delta\omega$ at different positions of the sample in the resonator one can determine the effective parameters ε_e and μ_e.

We suppose now that the composite is placed at the maximum of the electric field in the resonator. The magnetic field $\mathbf{H}_1(\mathbf{r}_0)$ at the maximum of the electric field is zero, $\mathbf{H}_1(\mathbf{r}_0)=0$; moreover, since the composite is optically thin, i.e., its size $\mathcal{L}\ll\lambda/\sqrt{|\varepsilon_e\mu_e|}$, the magnetic field $\mathbf{H}_0$ inside the composite is also nearly zero. Therefore, Eq. (4) takes the form

$$\frac{\Delta\omega_E}{\omega}=\frac{(\varepsilon_e-1)\int_v\mathbf{E}_1^*(\mathbf{r}_0)\cdot\mathbf{E}_0(\mathbf{r})d\mathbf{r}}{2\int|\mathbf{E}_1|^2d\mathbf{r}}, \tag{6}$$

where in the denominator we used the equality $\int|\mathbf{E}_1|^2d\mathbf{r}=\int|\mathbf{H}_1|^2d\mathbf{r}$ that holds for the fields in a resonator (see, e.g., Ref. 11, Sec. 90). Now we assume, for simplicity, that the composite has a spherical shape. Then the macroscopic field inside the composite, $\mathbf{E}_0$, is uniform and equal to (Ref. 11, Sec. 8)

$$\mathbf{E}_0=\frac{3}{\varepsilon_e+2}\mathbf{E}_1(\mathbf{r}_0), \tag{7}$$

so that the frequency shift

$$\frac{\Delta\omega_E}{v\omega}=\frac{(\varepsilon_e-1)\mathbf{E}_0\cdot\mathbf{E}_1^*(\mathbf{r}_0)}{2\int|\mathbf{E}_1|^2d\mathbf{r}}=\frac{3}{2}\frac{\varepsilon_e-1}{\varepsilon_e+2}\frac{|\mathbf{E}_1(\mathbf{r}_0)|^2}{\int|\mathbf{E}_1|^2d\mathbf{r}}, \tag{8}$$

can be unambiguously related to the effective dielectric constant ε_e. If the composite is placed now at the maximum of the magnetic field $\mathbf{H}_1(\mathbf{r}_0)$ the corresponding shift in the resonance frequency is given by

$$\frac{\Delta\omega_H}{v\omega}=\frac{(\mu_e-1)\mathbf{H}_0\cdot\mathbf{H}_1^*(\mathbf{r}_0)}{2\int|\mathbf{H}_1|^2d\mathbf{r}}=\frac{3}{2}\frac{\mu_e-1}{\mu_e+2}\frac{|\mathbf{H}_1(\mathbf{r}_0)|^2}{\int|\mathbf{H}_1|^2d\mathbf{r}}, \tag{9}$$

and it can be related to the effective magnetic permeability μ_e. Below, it is shown how to obtain the self-consistent equations for parameters ε_e, μ_e, and the macroscopic fields $\mathbf{E}_0$ and $\mathbf{H}_0$ in electromagnetic crystals and percolation composites.

The rest of the paper is organized as follows. In Sec. II we consider the effective parameters of $3d$ metal-dielectric electromagnetic crystals. A simple-cubic lattice of spheres and wires is studied. Explicit formulas are derived for the effective dielectric constant and magnetic permeability. It is shown that the effective dielectric constant of a wire crystal is negative for frequencies smaller than the ''plasma frequency'' defined below. Section III considers percolation metal-dielectric composites. In this section we show that the critical behavior of the dielectric constant near the percolation threshold is rather different from the known results of the standard percolation theory. Concluding remarks are presented in Sec. IV.

II. ELECTROMAGNETIC CRYSTALS

In this section two basic types of electromagnetic crystals are considered: a simple-cubic lattice of unconnected metal spheres and a three-dimensional conducting wire-mesh configured into a cubic lattice.

A. Simple-cubic lattice of metal spheres

Here, the local electromagnetic fields and effective parameters are considered for a system of metal spheres of radius a that are embedded in a dielectric host (a vacuum, for example) at the sites of the cubic lattice with the period $\xi > 2a$. The wavelength λ of an incident electromagnetic wave is assumed to be much larger than the lattice period ξ. We suppose that a sample of the electromagnetic crystal with the size $\xi \ll \mathcal{L} \ll \lambda$ is placed at the maximum of the electric field in a resonator, as discussed above. We consider the electric field distribution in a lattice cell of the crystal. It is convenient to assume that the cell is centered at the metal sphere. The electric field outside the sphere can be expanded in the multipole series and for simplicity, the dipole approximation can be used (see discussion in Refs. 12 and 13), which holds in the limit $\lambda \gg \xi \gg a$. (We note, however, that after a proper renormalization, the dipole approximation can also be applied to touching metal spheres, with a high accuracy.[7]) In the dipole approximation, the outside field $\mathbf{E}_{out}$ has only two components, namely, the constant and dipole fields:

$$\mathbf{E}_{out}(\mathbf{r}) = \mathbf{E}_1 + A_E a^3 \boldsymbol{\nabla}\left(\frac{\mathbf{E}_1 \cdot \mathbf{r}}{r^3}\right), \tag{10}$$

where $\mathbf{E}_1$ is some electric field aligned with the electric field at the maximum and A_E is a coefficient, which is determined below. Provided that the external field $\mathbf{E}_{out}$ is specified, the electric field $\mathbf{E}_{in}$ inside the metal grain can be found unambiguously by solving the Maxwell equations together with the boundary conditions $\mathbf{E}_{in} \times \mathbf{n} = \mathbf{E}_{out} \times \mathbf{n}$ and $\varepsilon_m \mathbf{E}_{in} \cdot \mathbf{n} = \varepsilon_d \mathbf{E}_{out} \cdot \mathbf{n}$ imposed at the metal surface ($\mathbf{n} = \mathbf{r}/r$ is the normal unit vector directed away from the metal sphere). Thus we obtain the following equation (see Ref. 11, Sec. 59, and Refs. 9 and 10) for the electric field inside the metal grain:

$$\mathbf{E}_{in}(\mathbf{r}) = \mathbf{E}_{in,0} + 4\pi \mathbf{L}(\mathbf{r}), \quad \mathbf{E}_{in,0} = \frac{3\varepsilon_d}{2\varepsilon_d + \tilde{\varepsilon}_m}\mathbf{E}_1, \tag{11}$$

where the uniform field $\mathbf{E}_{in,0}$ is determined by the renormalized metal permittivity

$$\tilde{\varepsilon}_m = \varepsilon_m \frac{2F(k_m a)}{1 - F(k_m a)}, \quad F(x) = \frac{1}{x^2} - \frac{\cot(x)}{x}, \tag{12}$$

where $k_m = k\sqrt{\varepsilon_m \mu_m}$ is the wave vector inside the metal. The skin (penetration) depth δ is equal to $\delta = 1/\mathrm{Im}\, k_m$. The local electric field $\mathbf{L}$ in Eq. (11) is determined by the following equation:

$$\mathrm{curl}\, \mathbf{L}(\mathbf{r}) = \frac{1}{4\pi}\mathrm{curl}\, \mathbf{E}_{in,m}(\mathbf{r}) = \frac{ik}{4\pi}\mathbf{B}_E, \tag{13}$$

where the circular magnetic induction

$$\mathbf{B}_E = -3iE_0 \frac{ak\varepsilon_m \varepsilon_d \sin(k_m r) F(k_m r)}{(2\varepsilon_d + \tilde{\varepsilon}_m)\sin(k_m a)(F(k_m a) - 1)}\left(\frac{y}{r}, -\frac{x}{r}, 0\right) \tag{14}$$

is written in the Cartesian coordinate system with the z axis directed along the field $\mathbf{E}_1$. The solenoidal magnetic induction $\mathbf{B}_E$ is generated in a metal particle by the electric current. Therefore, the ''inside'' electric field consists of the uniform curl-free part $\mathbf{E}_{in,0}$ and the rotational part $\mathbf{L}(\mathbf{r})$ that depends on the position inside the metal sphere.

The prefactor A_E in Eq. (10) can also be found by matching the fields $\mathbf{E}_{in}$ and $\mathbf{E}_{out}$ at the surface of the metal grain; this gives $A_E = (\varepsilon_d - \tilde{\varepsilon}_m)/(2\varepsilon_d + \tilde{\varepsilon}_m)$. Therefore, the electric field outside the metal grain coincides with the dipole field from a particle with the polarizability $\tilde{\alpha} = a^3 A_E$. The shift in the resonance frequency $\Delta\omega$ of the resonator is due to the change in the fields when the crystal is placed inside the resonator. In accordance with this, the shift in the resonance frequency $\Delta\omega_E$ is due to the electric fields scattered by metal particles, i.e., due to the dipole fields. The effective dielectric constant ε_e is unambiguously related to the frequency shift $\Delta\omega_E$ by Eqs. (6) and (8). Thus, ε_e is determined by the dipoles induced in metal particles by the electrical field and is given by the well-known Clausius-Mossotti equation for a cubic array of dipoles (which can be obtained using the Lorentz-Lorenz consideration of the local fields[14]):

$$\frac{\varepsilon_e - \varepsilon_d}{\varepsilon_e + 2\varepsilon_d} = p\frac{\varepsilon_d - \tilde{\varepsilon}_m}{2\varepsilon_d + \tilde{\varepsilon}_m}, \tag{15}$$

where $p = (4\pi/3)a^3\xi^{-3}$ is the volume concentration of metal grains. We would like to stress that this result does not depend explicitly on the field distribution inside a metal grain. All the retardation effects are reduced to a renormalization of the metal dielectric constant $\tilde{\varepsilon}_m$. In this sense, Eq. (15) is exact; at least, it becomes exact in the dilute limit $a/\xi \to 0$, when the multipoles higher than the dipole can be neglected for the outside field [see Eq. (10)]. In the dense limit of $\xi \to 2a$, all multipoles, in general, should be taken into account and the field $\mathbf{E}_{out}$ may be written as $\mathbf{E}_{out} = \mathbf{E}_1 + A_E\mathbf{E}^{(1)} + \Sigma_{l=2} A_l \mathbf{E}^{(l)}$, where $\mathbf{E}^{(l)}$ is the multipole of lth order. The coefficients A_l completely determine the scattered field and thus the effective dielectric constant $\varepsilon_e(A_E, A_2, \ldots, A_l)$. The skin (retardation) effect leads to a renormalization of the coefficients A_l though the functional

dependence $\varepsilon_e(A_E, A_2, \ldots, A_l)$ remains the same. For the sake of simplicity, we restrict our consideration in this paper to the standard dipole approximations, i.e., to Eq. (10). As mentioned, this approximation can further be improved, if needed, by a proper renormalization (which is based on replacing the real touching spheres by some ''effective'' spheres, geometrically intersecting each other[7]). We also note that very efficient numerical methods have been recently developed in Refs. 15–18; these methods can also be used for calculating the local fields in the close-packing limit.

In the quasistatic case, when the skin effect is negligible, Eq. (15) coincides with the well-known Maxwell-Garnett formula for the effective parameters.[19] It has been shown that the Maxwell-Garnett approximation, which emerges from the dipole approximation, gives very accurate results for the effective properties of various metal-dielectric periodic composites, even at large filling factors $p \lesssim 0.5$.[12,13,20]

Now we consider constitutive equations for electromagnetic crystals. The average electric field $\langle \mathbf{E} \rangle$ can be found from Eq. (11) as

$$\langle \mathbf{E} \rangle = \mathbf{E}_0 + 4\pi \langle \mathbf{L} \rangle, \tag{16}$$

where

$$\mathbf{E}_0 = \left[p \frac{3\varepsilon_d}{2\varepsilon_d + \tilde{\varepsilon}_m} + (1-p) \right] \mathbf{E}_1, \tag{17}$$

is the average curl-free local electric field. The average solenoidal part of the local electric field, $\langle \mathbf{L} \rangle$, can be represented as the average moment of the eddy magnetic induction,

$$\langle \mathbf{L} \rangle = \frac{ik}{8\pi\xi^3} \int \mathbf{r} \times \mathbf{B}_E d\mathbf{r} = \frac{1}{8\pi\xi^3} \int [\mathbf{r} \times \operatorname{curl} \mathbf{E}] d\mathbf{r}, \tag{18}$$

with the induction $\mathbf{B}_E$ given by Eq. (14). Recall now that the moment $\mathbf{L}$ is nonzero within the grain volume $v_0 = (4\pi/3) a^3$ only. The average electric displacement is defined as $\langle \mathbf{D} \rangle = \xi^{-3} \int \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}) d\mathbf{r}$, where the local permittivity $\varepsilon(\mathbf{r})$ takes values ε_m and ε_d inside and outside the metal grain. By substituting in this formula Eqs. (10) and (11), we find

$$\langle \mathbf{D} \rangle = \left[p \frac{3\varepsilon_d \tilde{\varepsilon}_m}{2\varepsilon_d + \tilde{\varepsilon}_m} + (1-p)\varepsilon_d \right] \mathbf{E}_1, \tag{19}$$

where $\tilde{\varepsilon}_m$ is defined in Eq. (12). It follows from Eqs. (15) and (17) that Eq. (19) can be rewritten as

$$\langle \mathbf{D} \rangle = \varepsilon_e \mathbf{E}_0. \tag{20}$$

Therefore, the average electric displacement is proportional to the irrotational part of the local field averaged over the system and the coefficient is exactly equal to the effective dielectric constant. Equation (20) replaces the usual constitutive equation $\langle \mathbf{D} \rangle = \varepsilon_e \langle \mathbf{E} \rangle$, which is valid in the quasistatic case only.

We consider now magnetic properties of electromagnetic crystals by following the same procedure as above. Using the dipole approximation for the magnetic field distribution in the lattice cell, we obtain the effective magnetic permeability

$$\mu_e = \frac{(1+2p)\tilde{\mu}_m + 2(1-p)}{(1-p)\tilde{\mu}_m + (p+2)}, \quad \tilde{\mu}_m = \mu_m \frac{2F(k_m a)}{1 - F(k_m a)}, \tag{21}$$

where the function F is defined in Eq. (12). Note that the renormalized magnetic permeability $\tilde{\mu}_m$ is not equal to unity, even if the metal is nonmagnetic, i.e., even if there is no ''seed'' magnetic permeability, $\mu_m = 1$.

The constitutive equation for magnetic properties is similar to its ''electric'' counterpart given by Eq. (20):

$$\langle \mathbf{B} \rangle = \langle \mu \mathbf{H} \rangle = \mu_e \mathbf{H}_0, \tag{22}$$

where $\mathbf{H}_0$ is the average, curl-free part of the magnetic field

$$\mathbf{H}_0 = \langle \mathbf{H} \rangle - 4\pi \langle \mathbf{M} \rangle, \tag{23}$$

and the magnetic moment is defined by

$$\operatorname{curl} \mathbf{M} = \frac{1}{4\pi} \operatorname{curl} \mathbf{H}_{in,m} \equiv -\frac{ik}{4\pi} \mathbf{D}_H,$$

$$\langle \mathbf{M} \rangle = \frac{1}{8\pi\xi^3} \int [\mathbf{r} \times \operatorname{curl} \mathbf{H}] d\mathbf{r}. \tag{24}$$

Equation (22) replaces the usual ''magnetic'' constitutive equation $\langle \mathbf{B} \rangle = \mu_e \langle \mathbf{H} \rangle$, which is valid in the quasistatic case only.

We proceed now with a derivation of the equations for the macroscopic electromagnetism in electromagnetic crystals. In the considerations above an optically thin sample of a crystal was placed at the maximum of either electric or magnetic field. In this way, the effective parameters ε_e and μ_e were determined. We consider the propagation of an electromagnetic wave in bulk crystals. As known, to obtain the macroscopic equations, the ''microscopic'' Maxwell equations should be averaged over the scale $\mathcal{L}$, which is much larger than the lattice constant ξ but smaller than the wavelength λ, $\xi \ll \mathcal{L} \ll \lambda$. The macroscopic electric $\mathbf{E}_0$ and magnetic $\mathbf{H}_0$ fields are defined by Eqs. (16) and (23); both fields are nonzero, in general. The electric field $\mathbf{E}_0$ excites the average electric displacement given by Eq. (20); the magnetic field $\mathbf{H}_0$ also excites the Foucault currents. By summing the electric displacement $\mathbf{D}_H$ from Eq. (24) and the average displacement from Eq. (20), we find the full electric displacement,

$$\langle \mathbf{D} \rangle_f = \varepsilon_e \mathbf{E}_0 + i \frac{4\pi}{k} \langle \operatorname{curl} \mathbf{M} \rangle. \tag{25}$$

Similarly, the average full magnetic induction $\langle \mathbf{B} \rangle_f$ is as follows:

$$\langle \mathbf{B} \rangle_f = \mu_e \mathbf{H}_0 - i \frac{4\pi}{k} \langle \operatorname{curl} \mathbf{L} \rangle, \tag{26}$$

where the vector $\mathbf{L}$ is given by Eq. (13). Now we average the Maxwell equations over the macroscopic volume $v \sim \mathcal{L}^3$ centered at the point $\mathbf{r}$. Thus we obtain

$$\langle \mathrm{curl}\,\mathbf{E}(\mathbf{r})\rangle = ik\langle \mathbf{B}(\mathbf{r})\rangle_f = ik\mu_e \mathbf{H}_0(\mathbf{r}) + 4\pi\langle \mathrm{curl}\,\mathbf{L}(\mathbf{r})\rangle, \tag{27}$$

$$\langle \mathrm{curl}\,\mathbf{H}(\mathbf{r})\rangle = -ik\langle \mathbf{D}(\mathbf{r})\rangle_f = -ik\varepsilon_e \mathbf{E}_0(\mathbf{r}) + 4\pi\langle \mathrm{curl}\,\mathbf{M}(\mathbf{r})\rangle, \tag{28}$$

where Eqs. (25) and (26) were used. The order of the curl operation and the volume averaging in Eqs. (27) and (28) can be interchanged (see, e.g., Ref. 21, Chap. 6, Sec. 6.6). Then, the Maxwell equations (27) and (28) become

$$\mathrm{curl}\,\mathbf{E}_0(\mathbf{r}) = ik\mu_e \mathbf{H}_0(\mathbf{r}), \tag{29}$$

$$\mathrm{curl}\,\mathbf{H}_0(\mathbf{r}) = -ik\varepsilon_e \mathbf{E}_0(\mathbf{r}), \tag{30}$$

i.e., they acquire the form typical for the macroscopic electromagnetism, describing propagation of electromagnetic waves in continuous media. Thus Eqs. (29) and (30) coincide with Eqs. (1) conjectured for a metal-dielectric system placed into a resonator.

B. Wire mesh electromagnetic crystal

We consider now electromagnetic properties of a $3d$ metal wire mesh configurated in a simple-cubic lattice with period ξ. The lattice axes coincide with (x,y,z) coordinates. The average electric field is aligned with the z axis. We suppose that the radius of the wires $a \ll \xi$ and neglect interaction of the electric field with the y and z wires. Then, the local electric field in the cell can be found in the dipole approximation as follows:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1 J_0(k_m r), \quad r < a$$

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1\left[J_0(k_m a) - k_m a J_1(k_m a)\ln\left(\frac{r}{a}\right)\right], \quad r > a \tag{31}$$

where $\mathbf{E}_1 = (0,0,E_1)$ is an electric field, and J_n is the Bessel function of the nth order. It follows from Eqs. (16), (18), and (20) that the effective dielectric constant ε is given by

$$\varepsilon_e = \frac{4\varepsilon_d\left(1-\frac{p}{3}\right) + \varepsilon_m\left\{\frac{8}{3}p + (ak)^2\varepsilon_d\left[6 - \frac{2}{3}p - \pi + 2\ln\left(\frac{2a^2}{\xi^2}\right)\right]\right\}}{4 + k_m a\frac{J_1(k_m a)}{J_0(k_m a)}\left[4 - \pi + 2\ln\left(\frac{2a^2}{\xi^2}\right)\right]}, \tag{32}$$

where $p \approx 3(a/\xi)^2$ is the metal concentration.

As seen in Figs. 1 and 2, with the skin effect increasing (i.e., with decreasing the field penetration length) the effective dielectric constant becomes negative. The electric field of an incident wave excites not only a current in metal wires but also a magnetic induction $\mathbf{B}_E$ that ''wraps'' the wires. Thus, the energy of an incident wave reversibly converts into the energy of the circular magnetic field concentrated around the wires. This field, in turn, generates an electric field, which is phase-shifted by π with respect to the external field. When this secondary field is larger than the primary electric field, the average electric field is opposite to the external field, so that the effective dielectric constant is negative. Negative values of the effective dielectric constant in metal-dielectric composites formed by conducting sticks were predicted in Refs. 10 and 22 and obtained in experiments.[23,24]

For a strong skin effect ($a\,\mathrm{Im}\,k_m \to +\infty$) and thin cylinders, with the radius a so small that $\ln(\xi/a) \gg 1$, Eq. (32) simplifies to the expression

$$\varepsilon_e = \varepsilon_d - \frac{2p}{3(ak)^2\ln\left(\frac{\xi}{a}\right)} \equiv \varepsilon_d - \frac{\omega_p^2}{\omega^2}, \quad \omega_p^2 = \frac{2\pi c^2}{3\xi^2\ln\left(\frac{\xi}{a}\right)}, \tag{33}$$

where ε_e is a real quantity. Thus, for ω smaller than the ''plasma'' frequency ω_p, an electromagnetic wave exponentially decays in the crystal, without losses. Equation (33) and the ''plasma'' frequency ω_p were first suggested by Pendry *et al.*[2]

III. METAL-DIELECTRIC PERCOLATION COMPOSITES

The problem of the effective conductivity σ_e and dielectric constant ε_e in metal-dielectric composites have attracted much attention in the last three decades.[3–7] An example of such a system is a composite material consisting of a disordered mixture of metallic and insulating particles. A reduction in the volume concentration p of the metallic (conducting) component reduces the static conductivity of the composite so that it vanishes at some critical concentration p_c known as the percolation threshold. That is, the metal-dielectric composite undergoes a metal-insulator transition at the percolation threshold. The inhomogeneity scale ξ corresponding to the transition is known as the percolation correlation length.

Electric and transport properties of a composite exhibit a number of unusual features near the percolation threshold p_c. In the static limit the effective conductivity vanishes as $(p-p_c)^t$, where t is a critical exponent; the effective dielectric constant ε_e diverges at p_c as $\varepsilon_e \sim |p-p_c|^{-s}$, where s is another critical exponent. Here we consider how the effective dielectric constant ε_e depends on the skin effect in metal grains. As above, it is assumed that a typical size a of the composite grains is much smaller than the wavelength, $a \ll \lambda$.

To find the effective dielectric constant ε_e we suppose that an optically thin sample of a composite is placed at the maximum of the electric field in the resonator, as discussed in Sec. I. The change in the field when the composite is placed inside the resonator is determined by a superposition of the fields scattered from individual metal and dielectric

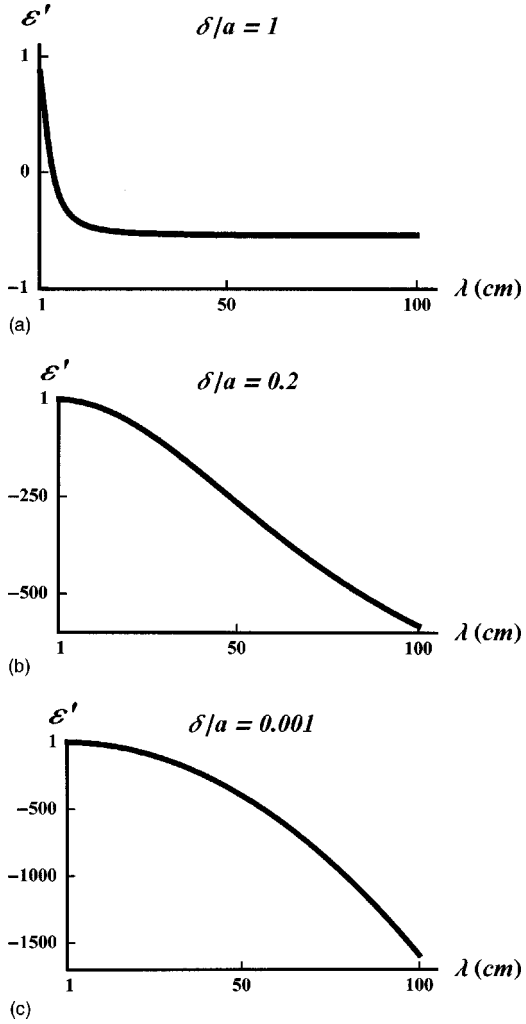

FIG. 1. Real part of the effective dielectric constant $\varepsilon_e'(\lambda)$ in the simple-cubic lattice of metal wires. The period of the lattice $L=1$ cm, the diameter of the wire $2a=1$ mm. (a)–(c) correspond to the skin effect of different strengths. The skin depth δ is fixed at wavelength $\lambda=1$ cm.

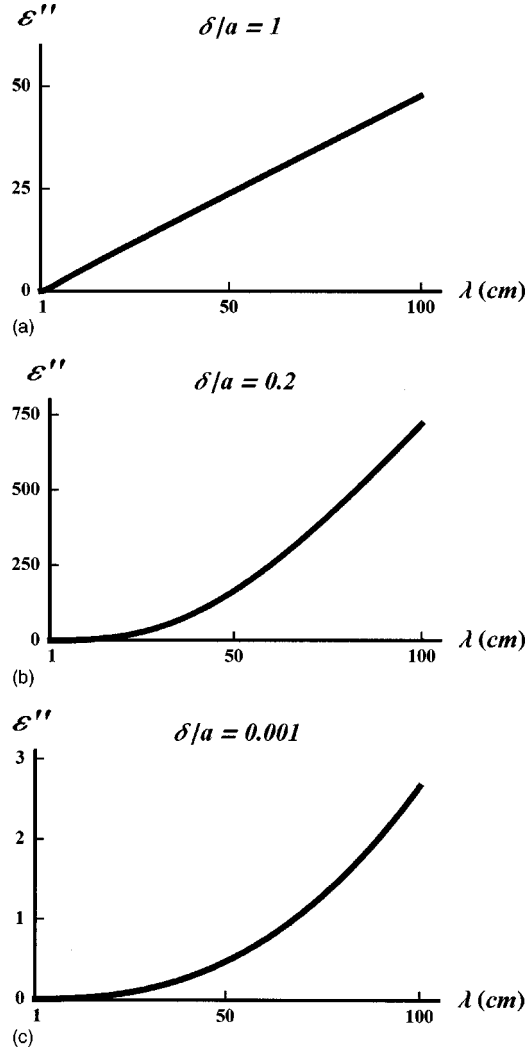

FIG. 2. Imaginary part of the effective dielectric constant $\varepsilon_e''(\lambda)$ in the simple-cubic lattice of metal wires. The period of the lattice $L=1$ cm, the diameter of the wire $2a=1$ mm. (a)–(c) correspond to the skin effect of different strenghts. The skin depth δ is fixed at wavelength $\lambda=1$ cm.

particles that have dielectric constants ε_m and ε_d, respectively. The interaction between the particles can be taken into account in the self-consistent approximation known as the effective medium theory (EMT).[5,25] In this theory, the interaction of a given metal or dielectric particle with the rest of the system is found by replacing the latter by a homogeneous medium with the effective parameters ε_e and μ_e. Assuming that the grains are spherical in shape, the electric fields outside and inside a metal grain are given by Eqs. (10) and (11), respectively, where the dielectric constant of the host is replaced with ε_e. The electric fields outside and inside a dielectric particle are also given by Eqs. (10) and (11), where ε_d is replaced with ε_e and $\tilde{\varepsilon}_m$ is replaced with ε_d. The effective dielectric constant ε_e is determined by Eq. (20) that, in the EMT approximation, acquires the following form:

$$p\frac{\varepsilon_e-\tilde{\varepsilon}_m}{2\varepsilon_e+\tilde{\varepsilon}_m}+(1-p)\frac{\varepsilon_e-\varepsilon_d}{2\varepsilon_e+\varepsilon_d}=0, \tag{34}$$

where the renormalized metal permittivity is given again by Eq. (12).

The effective magnetic permeability μ_e can be found by using arguments similar to those above; then Eq. (22) takes the following form:

$$p\frac{\mu_e-\tilde{\mu}_m}{2\mu_e+\tilde{\mu}_m}+(1-p)\frac{\mu_e-1}{2\mu_e+1}=0. \tag{35}$$

Equations (34) and (35) were first obtained in Refs. 9,10, and 26. They are similar to equations of the standard EMT.[5] Equation (34) predicts the percolation threshold at $p_c=1/3$, which is the same as in the standard EMT. The only difference, a very important one, is that the metal dielectric constant ε_m and magnetic permeability μ_m are replaced with $\tilde{\varepsilon}_m$ and $\tilde{\mu}_m$ [see Eqs. (12) and (21)]. This fact has a substantial effect on the frequency dependence of the effective parameters, as shown below.

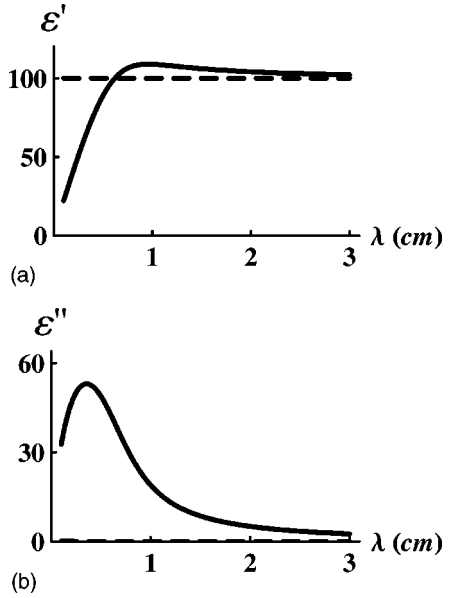

FIG. 3. (a) Real and (b) imaginary parts of the microwave effective dielectric constant $\varepsilon_e(\lambda)$ in the metal-dielectric composite. The metal conductivity $\sigma_m = 10^{17}$ (as in aluminum) and the grain size $a = 100$ μm; the concentration $p = 0.33$ is below the percolation threshold. The solid and dashed lines represent the developed theory and standard EMT, respectively.

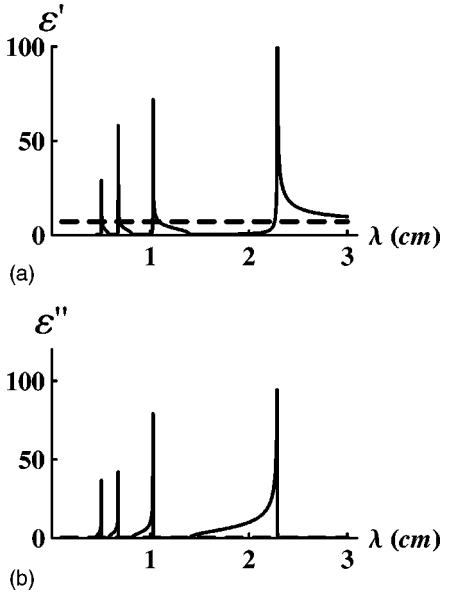

FIG. 4. (a) Real and (b) imaginary parts of the microwave effective dielectric constant $\varepsilon_e(\lambda)$ in the dielectric-dielectric composite. The grain diameter $a = 2$ mm; the first component has the dielectric constant $\varepsilon_1 = 100 + 0.01i$ and volume concentration $p_1 = 0.33$; for the second component, $\varepsilon_2 = 1$ and $p_2 = 1 - p_1$. The solid and dashed lines represent the developed theory and standard EMT, respectively.

For example, we consider the limit of low frequencies when the renormalized dielectric function is large, $|\tilde{\varepsilon}_m| \gg 1$. Then it follows from Eq. (34) that the effective dielectric constant

$$\varepsilon_e \sim \varepsilon_d |p - p_c|^{-s} \quad (s = 1) \tag{36}$$

is independent of the frequency. The dispersion of the effective conductivity becomes important in the vicinity of the percolation threshold, where $|p - p_c| \leqslant \sqrt{\varepsilon_d / |\tilde{\varepsilon}_m|}$, as follows from Eq. (34). Therefore, the renormalization of the metal permittivity [see Eq. (12)] results in expansion of the concentration range with a dispersive behavior of the effective dielectric constant, as illustrated in Fig. 3.

In Figs. 4 and 5 we show the effective response of a two-component, dielectric-dielectric composite with $\varepsilon_1 \gg \varepsilon_2 \sim 1$. The effective parameters are obtained from Eqs. (34) and (35), where ε_m and ε_d are replaced with ε_1 and ε_2, respectively ($\mu_m = \mu_d = 1$). The effective dielectric constant $\varepsilon_e(\lambda)$ and the magnetic permeability $\mu_e(\lambda)$ have strong resonances that correspond to the resonant excitation of electric and magnetic dipoles in the ε_1 grains.

The effective medium theory is based on the assumption that the fields are the same in all metal (dielectric) particles. This assumption does not hold when a metal concentration p approaches the percolation threshold p_c since the inhomogeneity scale (percolation correlation length ξ) diverges at p_c as $\xi \sim a|p - p_c|^{-\nu}$, where the critical exponent is given by $\nu \approx 0.88$ for $d = 3$.

Consider, for example, frequencies in the microwave or radio ranges, where for a typical metal conductivity we estimate that $\sigma_m \gg \omega$. Then in the critical concentration region, above the percolation threshold ($p > p_c, \xi \gg a$), the effective conductivity σ_e is determined by the backbone of the infinite cluster of the conducting channels. The backbone can be viewed as the Skal–Shklovskii–de Gennes superlattice, consisting of nodes that are connected via macrobonds.[3] The characteristic geometrical distance in the Skal–Shklovskii–de Gennes superlattice is given by the correlation length ξ. The macrobond connecting two nodes has a complicated, self-similar node-link structure; for a qualitative analysis, however, it is possible to suppose that the con-

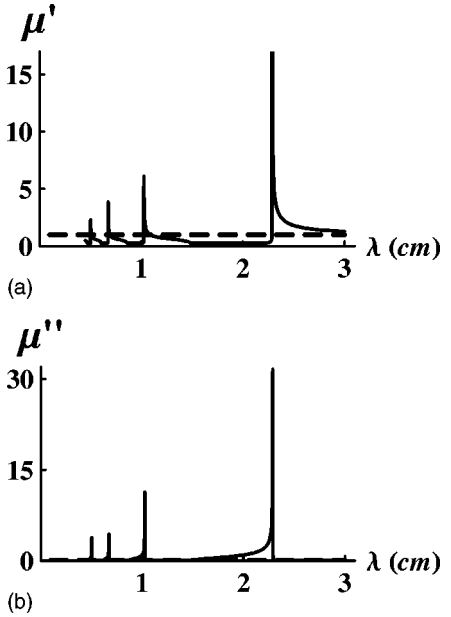

FIG. 5. (a) Real and (b) imaginary parts of the microwave magnetic permeability $\mu_e(\lambda)$ in the same (as in Fig. 4) dielectric-dielectric composite. The solid and dashed lines represent the developed theory and standard EMT, respectively.

ductance of the macrobond is determined by the shortest path between the nodes. The length L_s of the shortest path between two nodes in the superlattice is larger than the geometrical distance ξ between them and it is estimated as $L_s \sim \xi^{\eta}$, where the critical exponent $\eta \simeq 1.2$.[3,27] Since the exponent η is close to unity it is possible, for estimates, to neglect the difference between L_s and ξ. Then, the dielectric properties of the composite are similar to the wire mesh electromagnetic crystal considered in the preceding section.

When the skin effect is negligible the effective dielectric constant is estimated as $\varepsilon_e = i4\pi\sigma_e/\omega \sim i\sigma_m a^2/(\omega\xi^2) \sim i(\sigma_m/\omega)(p-p_c)^t$, where the critical exponent $t = 2\nu \simeq 1.8$ is close to that predicted by recent numerical results $t \approx 2$ (see Ref. 28 and references therein). In the opposite case of a strong skin effect, the effective dielectric constant can be determined from Eq. (33) as follows:

$$\varepsilon_e \simeq \tilde{\varepsilon}_d - \frac{\lambda^2}{2\pi^2\xi^2 \ln\left(\frac{\xi}{a}\right)}, \tag{37}$$

where $\tilde{\varepsilon}_d$ is the renormalized dielectric permittivity.

The dielectric permittivity $\tilde{\varepsilon}_d$ is different from the dielectric constant of the dielectric component that appears in Eq. (33); this is because only a small part of the metal $\sim a^2 L_s/\xi^3 \ll 1$ is concentrated in the conducting channels, whereas most of the metal is ''dissolved'' in the dielectric host in the form of separate grains and finite clusters. The presence of these grains and finite clusters leads to a renormalization of the permittivity of the dielectric medium between the conducting channels. Due to scaling arguments, the renormalized dielectric permittivity $\tilde{\varepsilon}_d$ must be a function of the percolation correlation length ξ only. Therefore, the value of $\tilde{\varepsilon}_d$ is essentially the same, below and above the percolation threshold. Then it is possible to replace $\tilde{\varepsilon}_d$ in Eq. (37) with the effective medium result given by Eq. (36) and obtain the following:

$$\varepsilon_e \sim \frac{\varepsilon_d}{(p-p_c)^s} + \frac{\lambda^2 (p-p_c)^{2\nu}}{2\nu a^2 \pi^2 \ln(p-p_c)}, \quad 0 < p - p_c \ll 1. \tag{38}$$

It follows from this equation that the effective dielectric constant of percolation metal-dielectric composites vanishes above the percolation threshold at a concentration p_1 estimated as

$$p_1 \simeq p_c + \left[\frac{4\pi\nu a^2 \varepsilon_d \ln(\lambda/a)}{\lambda^2 (s+2\nu)}\right]^{1/(s+2\nu)}, \tag{39}$$

where it is still assumed that the wavelength $\lambda \gg a$, so that $|p_1 - p_c| \ll 1$. The estimate (39) is obtained with the so-called ''logarithmic accuracy,'' i.e., it is supposed that $\ln(\lambda/a) \gg 1$. As follows from Eq. (36), the effective dielectric constant ε_e of the composite at $p < p_c$ can be very large and positive, whereas losses are practically zero (since $\varepsilon_d \sim 0$). A small micro-, or even nano-sphere made of such a composite (with the size that can be much less than the wavelength because of the large ε_e) can act as a resonator supporting the magnetic dipole resonance, so that the magnetic permeability is negative at frequencies above the resonance. By combining these composite microspheres with a periodic metal structure, having a negative dielectric constant, one can make electromagnetic crystals, where both magnetic permeability and dielectric permittivity are negative in the visible and near-IR ranges. This opens a fascinating possibility of materials with the negative refrative index in the optical spectral range. These ''left-handed'' materials have very interesting properties with many applications, e.g., they can be used as perfect lenses (see, e.g., Ref. 30).

IV. CONCLUSIONS

In this paper, we derived the macroscopic Maxwell equations in metal-dielectric media. In the derivation of the macroscopic equations, the Foucault currents induced by the high-frequency magnetic field in metal component are taken into account as well as the eddy currents of the magnetic induction induced by the high-frequency electric field. The latter has no analogy in the classical electrodynamics, since an electric field does not generate a magnetic induction in atoms and molecules. The theory results in the equations describing metal-dielectric media in terms of the effective dielectric constant ε_e and magnetic permeability μ_e.

In the case of periodic metal-dielectric structures, known as electromagnetic crystals, explicit equations for ε_e and μ_e are obtained. When the skin effect is strong, a cubic lattice of thin conducting wires has a negative dielectric constant and negligible losses (see also Ref. 29). The negative value of the effective dielectric constant is similar to bulk metals in the optical and near-infrared spectral ranges, e.g., surface plasmons can be excited at the boundaries and also at defects inside a crystal. Yet, electromagnetic crystals can operate in this way within any frequency band. Another interesting, high-frequency property of electromagnetic crystals is the effective magnetism that can be observed, even in systems where neither metal nor dielectric possess inherent magnetism.

Electromagnetic crystals and percolation metal-dielectric composites are two extreme cases of fully ordered and fully disordered systems. It is remarkable that in both cases the skin effect can be completely taken into account by appropriate renormalization of the metal permittivity. The renormalization depends on the conductivity and shape of the metal inclusions rather than on the system morphology.

ACKNOWLEDGMENTS

This work was supported in part by an NSF grant (DMR-98101183), a PRF grant (35028-AC5), ARO (DAAG55-98-1-0425), and the AvH Foundation.

[1] D.F. Sievenpiper, M.E. Sickmiller, and E. Yablonovitch, Phys. Rev. Lett. **76**, 2480 (1996).
[2] J.B. Pendry, A.J. Holden, W.J. Stewart, and I. Youngs, Phys. Rev. Lett. **76**, 4773 (1996).
[3] D. Stauffer and A. Aharony, *An Introduction to Percolation Theory*, 2nd ed. (Taylor and Francis, London, 1994).
[4] M. Sahimi, *Applications of Percolation Theory* (Taylor and Francis, London, 1994).
[5] D.J. Bergman and D. Stroud, Solid State Phys. **46**, 14 (1992).
[6] J.P. Clerc, G. Giraud, and J.M. Luck, Adv. Phys. **39**, 191 (1990).
[7] V.M. Shalaev, *Nonlinear Optics of Random Media: Fractal Composites and Metal-Dielectric Films* (Springer Verlag, Berlin, 2000).
[8] A.N. Lagarkov, L.V. Panina, and A.K. Sarychev, Zh. Éksp. Teor. Fiz. **93**, 215 (1987) [Sov. Phys. JETP **66**, 123 (1987)].
[9] A.P. Vinogradov, L.V. Panina, and A.K. Sarychev, Dokl. Akad. Nauk SSSR **306**, 847 (1989).
[10] A.N. Lagarkov, A.K. Sarychev, Y.R. Smychkovich, and A.P. Vinogradov, J. Electromagn. Waves Appl. **6**, 1159 (1992).
[11] L.D. Landau and E.M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, Oxford, 1984).
[12] N.A. Nicorovici, R.C. McPhedran, and L.C. Botten, Phys. Rev. Lett. **75**, 1507 (1995); Phys. Rev. E **52**, 1135 (1995).
[13] R.C. McPhedran, N.A. Nicorovici, and L.C. Botten, J. Electromagn. Waves Appl. **11**, 981 (1997).
[14] See, e.g., N.W. Ashcroft and N.D. Mermin, *Solid State Physics* (Holt, Rinehart and Winston, New York, 1979).
[15] J.A. Porto, F.J. Garcia-Vidal, and J.B. Pendry, Phys. Rev. Lett. **83**, 2845 (1999).
[16] F.J. Garcia de Abajo, Phys. Rev. Lett. **82**, 2776 (1999).
[17] O.J.F. Martin, C. Girard, D.R. Smith, and S. Schultz, Phys. Rev. Lett. **82**, 315 (1999).
[18] O.J.F. Martin, C. Girard, and A. Dereux, Phys. Rev. Lett. **74**, 526 (1995).
[19] J.C. Maxwell Garnett, Philos. Trans. R. Soc. London, Ser. A **203**, 385 (1904).
[20] P. Sheng, *Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena* (Academic, Boston, 1995), Chap. 3.
[21] J.D. Jackson, *Classical Electrodynamics*, 3rd. ed. (John Wiley, New York, 1998).
[22] A.N. Lagarkov and A.K. Sarychev, Phys. Rev. B **53**, 6318 (1996).
[23] A.N. Lagarkov, S.M. Matytsin, K.N. Rozanov, and A.K. Sarychev, Physica A **241**, 58 (1997).
[24] A.N. Lagarkov, S.M. Matytsin, K.N. Rozanov, and A.K. Sarychev, J. Appl. Phys. **84**, 3806 (1998).
[25] D.A.G. Bruggeman, Ann. Phys. (Leipzig) **24**, 636 (1935).
[26] D. Rousselle, A. Berthault, O. Acher, J.P. Bouchaud, and P.G. Zérah, J. Appl. Phys. **74**, 475 (1993).
[27] A.B. Pike and H.E. Stanley, J. Phys. A **13**, 457 (1980); A. Coniglio, Phys. Rev. Lett. **46**, 250 (1981).
[28] L. Tortet, J.R. Gavarri, J. Musso, G. Nihoul, J.P. Clerc, A.N. Lagarkov, and A.K. Sarychev, Phys. Rev. B **58**, 5390 (1998).
[29] W.Y. Zhang, X.Y. Lei, Z.L. Wang, D.G. Zheng, W.Y. Tam, C.T. Chan, and Ping Sheng, Phys. Rev. Lett. **84**, 2853 (2000).
[30] D.R. Smith, Willie J. Padilla, D.C. Vier, S.C. Nemat-Nasser, and Schultz, Phys. Rev. Lett. **84**, 4184 (2000); J. Pendry, Phys. World **13**, 27 (2000).