Acoustoelectric effect in a finite-length ballistic quantum channel

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The dc current induced by a coherent surface acoustic wave (SAW) of wave vector q in a ballistic channel of length L is calculated. The current contains two contributions, even and odd in q. The even current exists only in a asymmetric channel, when the electron reflection coefficients r_1 and r_2, at both channel ends, are different. The direction of the even current does not depend on the direction of the SAW propagation, but is reversed upon interchanging r_1 and r_2. The direction of the odd current is correlated with the direction of the SAW propagation, but is insensitive to the interchange of r_1 and r_2. It is shown that both contributions to the current are nonzero only when the electron reflection coefficients at the channel ends are energy dependent. The current exhibits geometric oscillations as a function of qL. These oscillations are the hallmark of the coherence of the SAW and are completely washed out when the current is induced by a flux of noncoherent phonons. The results are compared with those obtained previously by different methods and under different assumptions.

I. INTRODUCTION

The acoustoelectric effect is the generation of a dc electric current (the so-called *acoustoelectric current*) in a nonbiased device by a coherent acoustic wave or a flux of phonons. There has been recently a growing interest in observing this effect in mesoscopic structures. In particular, the acoustoelectric current due to a surface acoustic wave (SAW) was investigated experimentally in a point contact (PC) defined in a GaAs/AlGaAs heterostructure by a split gate.[1–3] In mesoscopic structures one can expect to observe effects related to ballistic transport, when the length of the PC channel L is shorter than the electron mean-free path l.

The theoretical considerations of the acoustoelectric effect can be divided into two groups. The first one, based on a classical approach, uses the Boltzmann equation for the electrons, with the acoustic wave considered either as a classical coherent force,[1,4] or as a flux of noncoherent quasimonochromatic phonons.[5,6] The classical approach for the description of electrons is valid for not very low temperatures, when the temperature smearing destroys the interference of the electron waves. For a ballistic PC the relevant interference is due to reflection from the channel ends, and one can use the Boltzmann equation for the electrons when $T \gg v/L$, where v is the relevant electron velocity. (For brevity here, as well as in the following expressions, we put $\hbar = 1$.) For lower temperatures the quantum approach has to be used.[7,8]

The situation considered in Refs. 1 and 4 does not correspond to a ballistic electron propagation, since the channel was assumed to be infinitely long, which means that $qL, kL \gg 1$ and $L \gg l$, where q is the wave vector of the SAW and k is the relevant electron momentum. Nevertheless, it has been conjectured that those results can be carried over to the ballistic PC by replacing the mean-free path l by the channel length L. As we show this is not totally correct. A ballistic situation was considered in Refs. 5 and 6, where the SAW was represented by a phonon flux, instead of a classical force. It is known (see, e.g. Ref. 9), that for an infinite channel both representations of the acoustical wave are equivalent for the derivation of the acoustoelectric current. But, as follows from our results, the SAW representation by a phonon flux is not always adequate for a ballistic channel of a finite length.

In what follows we consider the classical approach for a ballistic channel of a finite length, representing the SAW by a classical force, and allowing for electron reflections from the channel ends. As has been observed in Ref. 1 and explained in Refs. 1 and 5, the acoustoelectric current is high at the thresholds of the channel openings, i.e., at the steps of the PC quantized conductance (giant acoustoelectric current oscillations). In this situation the current is due to ''resonant'' electrons, whose velocities v are of order of the SAW velocity s. However, these slow electrons have short mean-free paths and their propagation in most PC's is not ballistic. We will calculate the acoustoelectric current that corresponds to the plateaus of the quantized conductance, i.e., far from the channel opening threshold, where $v \gg s$ and a ballistic electron propagation is more realistic. This current appears to be larger when the PC is not symmetric, i.e., when the reflection coefficients from both channel ends are different.

II. THE BOLTZMANN EQUATION

We consider a PC that is shaped by a split gate as a relatively long and uniform channel, which opens to (nonbiased) terminals, see Fig. 1. In the channel along the x direction the electronic states are quantized in the transverse direction y and as a result the electron energy is $E_n + \epsilon_k$, where n labels the transverse modes and k is the electron momentum along the channel. E_n is the threshold energy for the nth mode and $\epsilon_k = k^2/2m$ is the longitudinal electron energy. A mode n contributes to the current if $E_n < E_F$, where E_F is the

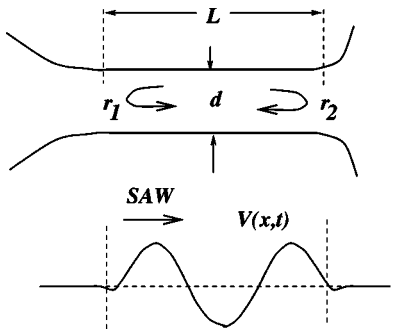

FIG. 1. Upper panel—a sketch of the system under consideration. An asymmetric channel of length L and width d is defined in a 2DEG by a split gate. $r_{1,2}$ are the electron reflection coefficients from the channel's edges. Lower panel—a snapshot of the profile of the potential created by the SAW. Outside the channel the field is screened by the wide leads.

Fermi energy in the device terminals. We assume that only the lowest transverse mode $n=0$ is relevant, namely, that E_F is above the first threshold (the PC pinch-off), but below the next one. The electrons of the relevant mode are described by the distribution function $f(x,k,t)$, which satisfies the Boltzmann equation. In the absence of scattering this equation is

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}-\frac{\partial V}{\partial x}\frac{\partial f}{\partial k}=0, \tag{1}$$

where $v=\partial\epsilon_k/\partial k$ is the electron velocity and V is the potential created by the SAW,

$$V(x,t)=V_\omega \exp(iqx-i\omega t)+\text{c.c.}, \tag{2}$$

with $\omega=sq$. We assume the SAW potential to be totally screened in the terminals and not screened inside the channel. (For a more detailed consideration of the potential and its screening see Refs. 10 and 11.) The boundary conditions at the edges of the channel at $x=0,L$ are

$$f(k)=\begin{cases} t_1 f_0(k)+r_1 f(-k) & (x=0,\ k>0),\\ t_2 f_0(k)+r_2 f(-k) & (x=L,\ k<0).\end{cases} \tag{3}$$

Here $t_{1,2}$ and $r_{1,2}$ are the transmission and the reflection coefficients for electrons approaching the ends of the channel, which satisfy the normalization $t_{1,2}+r_{1,2}=1$ and depend on the electron energy ϵ_k. In Eq. (3),

$$f_0(k)=f_T(\epsilon_k)=\left[\exp\left(\frac{\epsilon_k+E_0-E_F}{T}\right)+1\right]^{-1} \tag{4}$$

is the equilibrium electron Fermi distribution in the absence of the SAW. The terms with f_0 represent electrons penetrating from the terminals into the channel, and the terms with $r_{1,2}$ describe electrons backscattered from the terminals.

For a weak SAW, Eq. (1) can be solved by expanding the distribution function in powers of the SAW potential V,

$$f(x,k,t)=f_T(\epsilon_k)+f_1(x,k,t)+f_2(x,k,t)+\cdots. \tag{5}$$

The linear response f_1 contains Fourier components with frequencies $\pm\omega$, while f_2 contains the frequencies 0 and $\pm 2\omega$. The dc part of f_2 is the one yielding the dc acoustoelectric current.

Expanding the Boltzmann equation (1) and the boundary conditions (3) we find the equations for $f_{1,2}$,

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)f_1=-\frac{\partial V}{\partial x}\left(-\frac{\partial f_T}{\partial \epsilon}\right)v, \tag{6}$$

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)f_2=\frac{\partial V}{\partial x}\frac{\partial f_1}{\partial k}, \tag{7}$$

and the boundary conditions

$$f_{1,2}(k)=\begin{cases} r_1 f_{1,2}(-k) & (x=0,\ k>0),\\ r_2 f_{1,2}(-k) & (x=L,\ k<0).\end{cases} \tag{8}$$

Expressing the linear response in Eq. (6) as

$$f_1(x,k,t)=e^{-i\omega t}f_\omega(k,x)+\text{c.c.}, \tag{9}$$

we find the equation for the linear-response amplitude,

$$\left(-i\omega+v\frac{\partial}{\partial x}\right)f_\omega(k,x)=-iqV_\omega e^{iqx}\left(-\frac{\partial f_T}{\partial \epsilon}\right)v. \tag{10}$$

Introducing Eq. (9) into Eq. (7) and averaging the latter over time, we find the equation for the dc contribution of f_2,

$$v\frac{\partial \overline{f_2}}{\partial x}=2q\frac{\partial}{\partial k}\mathrm{Im}[V_\omega^* e^{-iqx}f_\omega]. \tag{11}$$

Integrating the last equation over k and noting that the right-hand side is a full derivative with respect to k, one can see that the time-averaged acoustoelectric current

$$\overline{j}=e\int_{-\infty}^{+\infty}\frac{dk}{2\pi}v\overline{f_2} \tag{12}$$

is constant along the channel.

Solving Eq. (10) one finds

$$f_\omega(k,x)=-iqLV_\omega\left(-\frac{\partial f_T}{\partial \epsilon}\right)F(k,x) \tag{13}$$

with

$$F(k,x)=e^{ipx}\left[A(k)+\frac{e^{i(q-p)x}-1}{i(q-p)L}\right], \tag{14}$$

where $p\equiv\omega/v$. The integration constant $A(k)$ is determined by the boundary conditions (8) for f_1 giving, for $k>0$,

$$A(k)=\frac{e^{2ipL}r_1r_2\phi_k-r_1\phi_{-k}}{1-e^{2ipL}r_1r_2},\quad A(-k)=\frac{A(k)}{r_1} \tag{15}$$

with

$$\phi_k=\frac{e^{i(q-p)L}-1}{i(q-p)L}. \tag{16}$$

Introducing Eq. (13) into Eq. (11) we find

$$v\frac{\partial \overline{f_2}}{\partial x}=-q^2|V_\omega|^2 L\,\mathrm{Re}\left\{e^{-iqx}\frac{\partial}{\partial k}\left[\left(-\frac{\partial f_T}{\partial \epsilon}\right)F(k,x)\right]\right\}. \tag{17}$$

Solving Eq. (17) we have

$$v\overline{f_2(k,x)}=-(qL)^2|V_\omega|^2\left\{\frac{\partial}{\partial k}\left[\left(-\frac{\partial f_T}{\partial \epsilon}\right)G(k,x)\right]+B(k)\right\}, \tag{18}$$

where

$$G(k,x)=2\int_0^x \frac{dx}{L}\mathrm{Re}[e^{-iqx}F(k,x)]. \tag{19}$$

The integration constant $B(k)$ is determined by the boundary conditions (8) for f_2, to which enters $\overline{G}(k)\equiv G(k,L)$. Using Eq. (15) one finds after lengthy but straightforward calculations, for $k>0$,

$$\overline{G}(k)=\frac{1-r_1r_2}{|1-r_1r_2e^{2ipL}|^2}[(1+r_1r_2)|\phi_k|^2-2r_1C_k\cos pL],$$

$$\overline{G}(-k)=-\frac{1-r_1r_2}{|1-r_1r_2e^{2ipL}|^2}[(1+r_1r_2)|\phi_{-k}|^2-2r_2C_k\cos pL], \tag{20}$$

where

$$C_k=2(\cos pL-\cos qL)/(q^2-p^2)L^2. \tag{21}$$

Using Eq. (18) for $x=0,L$ in the boundary conditions (8) for f_2 we find for $k>0$,

$$B(k)=\frac{r_1(r_2\psi_k+\psi_{-k})}{1-r_1r_2},\quad B(-k)=-\frac{B(k)}{r_1}, \tag{22}$$

where

$$\psi_k=\frac{\partial}{\partial k}\left[\left(-\frac{\partial f_T}{\partial \epsilon}\right)\overline{G}(k)\right]. \tag{23}$$

III. THE ACOUSTOELECTRIC CURRENT

To obtain a symmetric expression for the current it is convenient to calculate it as the average of its values at $x=0$ and $x=L$, which is

$$\overline{j}=-\frac{e}{4\pi}|V_\omega|^2(qL)^2\int_0^\infty dk\left\{\frac{(1-r_1)(1-r_2)}{1-r_1r_2}\times\frac{\partial}{\partial k}\left[\left(-\frac{\partial f_T}{\partial \epsilon}\right)[\overline{G}(k)+\overline{G}(-k)]\right]+\frac{r_1-r_2}{1-r_1r_2}\frac{\partial}{\partial k}\left[\left(-\frac{\partial f_T}{\partial \epsilon}\right)[\overline{G}(k)-\overline{G}(-k)]\right]\right\}. \tag{24}$$

One can calculate the integral by parts, using that $\overline{G}(\pm k)$ is zero at $k=0$ and $(-\partial f_T/\partial\epsilon)$ is zero at $k=\infty$. Strictly speaking, for electrons with $k=0$, the inequality $L\gg l$ required for the channel to be ballistic may be not fulfilled. In this case the above solution of the Boltzmann equation is not valid. However, in any case the contribution from $k=0$ is exponentially small due to the factor $(-\partial f_T/\partial\epsilon)$, for $E_F-E_0\gg T$. After integration by parts, one can see that the acoustoelectric current vanishes for an open channel ($r_1=r_2=0$) or when the electron reflection coefficients from the channel ends are energy independent. This is a specific property of a ballistic channel.

One may separate the acoustoelectric current given by Eq. (24) into two contributions, the *even* current $\overline{j_e}$, and the *odd* current $\overline{j_o}$. The even current does not change its sign upon the replacement $q\to -q$, but changes the sign upon interchanging r_1 and r_2. The odd current reverses its sign upon the replacement $q\to -q$, but is symmetric with respect to r_1 and r_2. The correlation between the change of the propagation direction of the SAW and the interchange of the channel ends follows from obvious symmetry considerations. Note that only the odd current exists in a symmetric PC, as well as in any homogeneous medium.

Simpler expressions for the acoustoelectric current can be given far from the threshold. The scattering time for electrons in a high-quality two-dimensional electron gas (2DEG) can be taken to be $\tau=30$ ps, which corresponds, for a Fermi velocity $v_F=3\times 10^7$ cm/s to a mean-free path $l=10$ μm. This means that channels with $L\lesssim 3$ μm are ballistic far from the threshold. The situation near the threshold where $v\simeq s$ is less clear. Assuming that τ is velocity independent and using $s=3\times 10^5$ cm/s, we estimate the mean-free path as $l=0.1$ μm, which means that for resonant electrons most channel-shaped PC's with $L\simeq 1$ μm are nonballistic. However, the specific velocity dependence of the relaxation rate in quasi-one-dimensional channels is not known.

For estimates one can use L between 1 μm and 10 μm, $\omega/2\pi$ between 100 MHz and 1 GHz, and temperature $T=1$ K. The change of pL in $\overline{G}(\pm k)$ within the thermal smearing defined by $(-\partial f_T/\partial\epsilon)$ is $(T/\epsilon_F)(s/v_F)qL$. This is small even for the highest frequencies and the longest channels available at present time. Hence one can replace the integration over k by taking the integrand at $k=k_F$. Then $pL\simeq\omega L/v_F$ and $p/q\simeq s/v_F$ are also small.

Based on these estimates we can simplify the expressions for the even and odd currents using $pL\ll 1$ and $p\ll q$. The results are

$$\overline{j_e}=(e|V_\omega|^2/\pi)\beta[1-\cos qL], \tag{25}$$

$$\overline{j_o}=\mathrm{sign}\,q\;(e|V_\omega|^2/\pi)\alpha(s/v_F)\times[2(1-\cos qL)-qL\sin qL], \tag{26}$$

where

$$\beta=\left[\frac{(1-r_1)(1-r_2)}{1-r_1r_2}\right]^2\frac{\partial}{\partial\epsilon}\frac{r_1-r_2}{(1-r_1)(1-r_2)},$$

$$\alpha=\frac{1+r_1r_2}{1-r_1r_2}\frac{\partial}{\partial\epsilon}\frac{(1-r_1)(1-r_2)}{1-r_1r_2}, \tag{27}$$

are calculated at the Fermi energy. The odd current is smaller by a factor s/v_F compared to the even one.

For short channels with $qL\ll 1$ one finds $\overline{j_e}\sim(qL)^2$ and $\overline{j_o}\sim(qL)^4$, while for $qL\gtrsim 1$ both contributions to the acoustoelectric current show geometric oscillations as a function of the product qL (qL oscillations), similar to the result of the quantum consideration.[8] These oscillations are due to the coherence of the SAW and result from the compensation of the forces acting on the electrons in different half-waves of the SAW. No qL oscillations are predicted when the acoustic wave is described as a noncoherent flux of phonons,[6] in which case $\overline{j}$ is simply proportional to L.

Finally, let us specify the expression for a symmetric channel with $r_1=r_2=r$, which is convenient for comparison with the results obtained by previous approaches. After integration by parts, Eq. (24) reduces to

$$\overline{j}=\frac{e}{4\pi}|V_\omega|^2(qL)^2\int_0^\infty dk\left(-\frac{\partial f_T}{\partial\epsilon}\right)(|\phi_k|^2-|\phi_{-k}|^2)\times\frac{1-r^4}{|1-r^2e^{2ipL}|^2}\frac{\partial}{\partial k}\frac{1-r}{1+r}. \tag{28}$$

Disregarding the fact that for low-energy electrons realistic channels may be nonballistic, we can use Eq. (28) to compare the values of the acoustoelectric current at the conductance plateaus, i.e., between the thresholds for $\epsilon\simeq\Delta$ (where Δ is a typical distance between the thresholds), and at the conductance steps, i.e., near the thresholds for $\epsilon\simeq ms^2$. To estimate the reflection coefficient and its derivative we use the results in the Appendix, assuming that the channel opening are not adiabatic. At the plateau we find the following order of magnitude estimates: $r\simeq1$ and $\partial r/\partial\epsilon\simeq\Delta^{-1}$, while at the step we use Eq. (A5). Replacing the Fermi function derivative by a delta function and assuming $qL\simeq1$, we obtain at the plateau

$$\overline{j}\simeq(e|V_\omega|^2/\Delta)\ (s/v_F) \tag{29}$$

(with $mv_F^2\simeq\Delta$) and at the step

$$\overline{j}\simeq(e|V_\omega|^2/\Delta). \tag{30}$$

The current at the plateau is smaller by the factor $(ms^2/\Delta)^{1/2}$ compared to that at the step.

IV. COMPARISON WITH PREVIOUS APPROACHES

The aim of this section is a more detailed comparison of our results with the results obtained in Refs. 4 and 6. For simplicity we confine to a symmetric channel. The result for a (nonballistic) infinite channel obtained in Ref. 4 can be expressed as follows:

$$\overline{j}=\frac{2e|V_\omega|^2}{\pi s}\int_0^\infty\frac{dv\ (v/s)^2}{(1-v^2/s^2)^2+(\omega\tau)^{-2}}\left(-\frac{\partial f_T}{\partial\epsilon}\right). \tag{31}$$

The integrand is an overlap of two peaks, centered at $v=v_F$ and $v=s$. Hence the current is maximal when $v_F=s$, i.e., near the channel opening threshold. When the scattering is weak, $\omega\tau\to\infty$, only resonant electrons with $v=s$ contribute to the current, which diverges as $\omega\tau$.

Since Eq. (31) makes sense only when $qL\gg1$, consider the limit $L\to\infty$ in Eq. (28). Using the representation of a delta function in this limit we have

$$|\phi_{\pm k}|^2=\frac{2[1-\cos(q\pm p)L]}{(q\pm p)^2L^2}\to\frac{(2/L)^2}{(q\pm p)^2+(2/L)^2}. \tag{32}$$

For $qL\gg1$ the term with $|\phi_{-k}|^2$ does not contribute to the integral in Eq. (28) and we obtain from this equation

$$\overline{j}=\frac{2e|V_\omega|^2}{\pi s}\int_0^\infty\frac{dv\ (v/s)^2R\{r\}}{(1-v^2/s^2)^2+(4v/\omega L)^2}\left(-\frac{\partial f_T}{\partial\epsilon}\right);$$

$$R\{r\}=\frac{2(1-r^4)}{|1-r^2\exp(2i\omega L/v)|^2}\,ms\,\frac{\partial}{\partial k}\frac{1-r}{1+r}. \tag{33}$$

Comparing Eq. (33) and Eq. (31) one can see that in the resonance factor, indeed, as conjectured, the scattering time τ is replaced (up to a numerical factor) by the lifetime L/v of the electron in the open channel. However the result given by Eq. (31) misses the crucial factor $R\{r\}$, which combines two features: (i) When there is no reflection, or for an energy-independent reflection, the acoustoelectric current vanishes. (ii) qL oscillations resulting from the factor $|1-r^2\exp(2i\omega L/v)|^{-2}$. Within the resonance factor width the exponent oscillates once, and hence some residual qL oscillations are present in the current given by Eq. (33).

Next we compare our results with those obtained in Ref. 6, where the acoustoelectric current is due to a monochromatic flux of phonons with frequency $\omega=sq$ and the electron-phonon interaction is presented as an electron-phonon collision term in the Boltzmann equation for the electrons. The comparison can be accomplished only up to some q-dependent factors: Ref. 6 considers bulk phonons, for which, as shown in Ref. 12, the electron-phonon interaction matrix elements have a different q dependence, compared to surface phonons.

The properties of the acoustoelectric current obtained in Ref. 6 are dominated by the energy-momentum conservation in the electron-phonon collision term. As a result only electrons with $k_\pm=ms\pm q/2$ contribute to the current: an electron with k_- can absorb a phonon from the flux, being excited into k_+, while an electron with k_+ can omit a phonon to the flux, being de-excited into k_-. It is now obvious that the current is large only near the threshold of channel opening, when v_F is of order s (for $\hbar\omega\lesssim ms^2$, which is usually the case). Most importantly, this collision picture is valid only when the phonon energy is larger than the energy uncertainty of the electron state due to the finite escape time from the channel, i.e., when $\omega\gg v/L$. For resonant electrons with $v\simeq s$ it means $\omega\gg s/L$ or, equivalently, $qL\gg1$. This latter condition, together with $kL\gg1$, allows the description of the electron-phonon interaction as a *local* scattering term in the Boltzmann equation, as used in Ref. 6.

Comparing our results with those of Ref. 6 one has to bear in mind that the description of the acoustic wave as a classical force in the Boltzmann equation for electrons is valid only when $q\ll k$ and $\hbar\omega\ll\epsilon_k$. The latter condition allows us to neglect the discreteness of quantum transitions for phonon emission or absorption. For resonant electrons with $v\simeq s$ both these conditions hold as long as $\hbar\omega\ll ms^2$ or, equivalently $q\ll ms$.

The flux of phonons can be considered as noncoherent when the channel is longer than the phonon coherence length

$\Lambda = s/\delta\omega$, where $\delta\omega$ (smaller than the central frequency ω) is the width of the phonon distribution or the SAW spectral width. The condition $\Lambda \ll L$ can be fulfilled only when $qL \gg 1$ and we may begin the comparison using our result in the form of Eq. (33). To imitate the noncoherence of the phonons we average this equation over ω with a Lorentzian form factor having $\delta\omega$ as half-width at half-height. This corresponds to a phonon field which can be obtained, for example, by selecting from a thermal phonon distribution those phonons, which have frequencies within $\delta\omega$ and wave vectors along the channel direction. In the time representation it corresponds a quasimonochromatic wave with fluctuating amplitude and phase. The only factor to be averaged is $|1 - r^2 \exp(2i\omega L/v)|^{-2}$, since the change of the width in the resonance factor is small. The result of the averaging is

$$\frac{1}{1-r^4}\,\frac{1+\eta\cos\varphi}{1+2\eta\cos\varphi+\eta^2}, \tag{34}$$

where $\varphi = 2\omega L/v$ (with the central frequency ω) and $\eta = r^2\exp(-2\delta\omega L/v)$ (with $v \simeq s$). For strong decoherence, $\Lambda \ll L$, one finds η to be exponentially small, and the qL oscillations are totally suppressed.

After this averaging, having in mind that for the comparison we are interested in the case $qL \gg 1$, the resonance factor in Eq. (33) can be replaced by a delta function, giving

$$\overline{j} = e|V_\omega|^2 qL\left(-\frac{\partial f_T}{\partial \epsilon}\right)_{v=s}\left[\frac{1}{2}ms\frac{\partial}{\partial k}\frac{1-r}{1+r}\right]_{v=s}. \tag{35}$$

This result has to be compared with Eq. (27) of Ref. 6, which corresponds to phonon momenta q below the Čerenkov threshold $q = 2ms$. The current given by Eq. (15) of Ref. 6, for phonon momenta q above the threshold, cannot be compared with Eq. (35), since our approach, as mentioned above, overlaps with the approach used in Ref. 6 only when $q \ll ms$.

In Eq. (27) of Ref. 6 one finds the difference of the Fermi distributions at $k = ms \pm q/2$, which for $q \ll ms$ reproduces the derivative of the equilibrium distribution in Eq. (35). The acoustoelectric current is proportional to L in both cases. However, there is a difference regarding the role of reflection from the channel ends.

According to Ref. 6, above the Čerenkov threshold the acoustoelectric current is nonzero even for an open channel $r = 0$. As already mentioned, this does not contradict Eq. (35). Below the Čerenkov threshold, the current is nonzero only in the case of an energy dependent reflection, which agrees with Eq. (35). Nevertheless, the details of the derivative function in this equation and in Eq. (27) of Ref. 6 are different.

The physical origin of the discrepancy is as follows. In the Boltzmann equation used in Ref. 6 the electron-phonon collision term is local in space and is separated from the boundary conditions, which are also local. The locality of the collision term is determined by the largest of the two wavelengths, $2\pi/k$ and $2\pi/q$, while the locality of the boundary condition is determined by $2\pi/k$. They can be separated only when they are of the same order, i.e., when $q \simeq k$. However, when $q \ll k$, the electron-phonon scattering is nonlocal near the boundary. Formally, the right-hand side of Eq. (4) in Ref. 6 for the nonequilibrium part of the electron distribution function contains no information about the boundary conditions, while the right-hand side of Eq. (17) depends on the boundary conditions, because the linear response f_ω is sensitive to them.

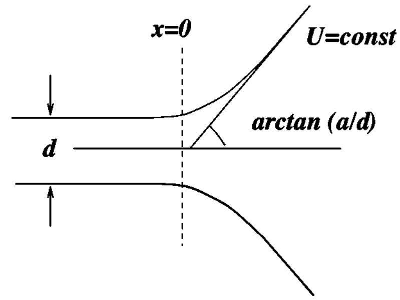

FIG. 2. Opening of a semi-infinite channel of width d into a horn with the opening angle defined by the ratio a/d. Shown are the equipotential lines.

As a result we have to conclude, that the acoustoelectric currents produced in a ballistic channel by a coherent SAW or a flux of noncoherent quasimonochromatic phonons are different. Even after destroying the coherence of the SAW the results are different. *A priori* there are no reasons why both representations of the SAW have to give the same results.

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APPENDIX: REFLECTION AT THE END OF A CHANNEL

To have an explicit expression for the reflection coefficient r entering the boundary condition for the electron distribution function, we calculate the reflection coefficient from the end of a channel. For that we use the following model for the confining potential, see Fig. 2.

$$U(x,y) = \frac{1}{2md^2}\begin{cases}[-(x/a)^2 + (y/d)^2] & (x>0)\\ (y/d)^2 & (x<0).\end{cases} \tag{A1}$$

This potential describes a semi-infinite waveguide at $x<0$ of width d connected to a horn with an opening angle d/a at $x>0$. The variables x and y can be separated and the solution of the wave equation is of the form $\Phi_n(y)\psi(x)$, see Refs. 8 and 13. Here Φ_n is a normalized harmonic-oscillator wave function with energy $E_n = \Delta(n+1/2)$, where $\Delta = 1/md^2$, and $n = 0,1,2,\ldots$ labels the modes in the waveguide and in the horn. There is no channel mixing upon wave transmission and reflection from the waveguide to the horn and vice versa.

To calculate the reflection amplitude R for the nth mode for an electron energy E we choose the wave function in the waveguide at $x<0$ as

$$\psi(x) = A(e^{ikx} + Re^{-ikx}), \quad k = [2m(E-E_n)]^{1/2} \tag{A2}$$

and in the horn at $x>0$ as

$$\psi(x)=B\mathrm{E}(-\varepsilon,\xi),\quad \xi=(2/dL)^{1/2}x,\quad \varepsilon=(E-E_n)/\delta, \tag{A3}$$

where E is the complex Weber (parabolic cylinder) function as defined in Ref. 14 and $\delta=1/mad$. (We use the notation E instead of E to avoid confusion with the energy.) $\varepsilon=0$ corresponds to the threshold of the nth mode opening. As defined, $\psi(x)$ at $x>0$ has only a wave propagating to $x=+\infty$. It is convenient to use the representation, see Ref. 14,

$$\mathrm{E}(-\varepsilon,\xi)=\sigma(\varepsilon)^{-1/2}\mathrm{W}(-\varepsilon,\xi)+i\sigma(\varepsilon)^{1/2}\mathrm{W}(-\varepsilon,-\xi),$$

where W are the real Weber functions and

$$\sigma(\varepsilon)=(1+e^{-2\pi\varepsilon})^{1/2}-e^{-\pi\varepsilon}.$$

Matching the logarithmic derivative at $x=0$ one finds

$$\begin{aligned}\frac{1-R}{1+R}&=-i\varepsilon^{-1/2}\frac{1-i\sigma(\varepsilon)}{1+i\sigma(\varepsilon)}\frac{\mathrm{W}'(-\varepsilon,0)}{\mathrm{W}(-\varepsilon,0)}\\&=i\left(\frac{2}{\varepsilon}\right)^{1/2}\frac{1-i\sigma(\varepsilon)}{1+i\sigma(\varepsilon)}\left|\frac{\Gamma(3/4-i\varepsilon/2)}{\Gamma(1/4-i\varepsilon/2)}\right|,\end{aligned} \tag{A4}$$

where the prime denotes the derivative of $\mathrm{W}(-\varepsilon,\xi)$ with respect to ξ, and we expressed W and W′ at $\xi=0$ in terms of the Γ function, see Ref. 14. The reflection coefficient entering the boundary conditions for the Boltzmann equation is $r=|R|^2$.

Close to the mode opening threshold one can put $\varepsilon=0$ in σ and Γ. Far from the threshold one can use the asymptotic for $\Gamma(x+iy)$ at $y\to\infty$, see Ref. 14. This gives

$$r=\begin{cases}1-c\varepsilon^{1/2} & (\varepsilon\ll 1),\\ (1/4)e^{-2\pi\varepsilon} & (\varepsilon\gg 1).\end{cases} \tag{A5}$$

Here $c=2\Gamma(1/4)/\Gamma(3/4)\approx 5.92$.

Consider an electron with energy E at the midpoint between the thresholds E_0 and E_1, i.e., $E-E_0=\Delta/2$. For $a\gg d$ the waveguide opening is adiabatic. In this case $\delta\ll\Delta$, and for the chosen energy E we have $\varepsilon\gg 1$ and r is exponentially small. However for a nonadiabatic opening, when $a\simeq d$, for the same energy E we find $\delta\simeq\Delta$. As a result we have $\varepsilon\simeq 1$ and $r\simeq 1$.

Using these results we can estimate the derivatives α and β, which determine the even and odd currents [Eqs. (25) and (26)]. For channel openings that are not specially designed, $a\simeq d$, and a typical Fermi energy in the midpoint between the thresholds E_0 and E_1 (at the center of quantized conductance plateau) will yield $\alpha,\beta\simeq 1$.

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