

Contribution to the quasiparticle inelastic lifetime from paramagnons in disordered superconductors

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The paramagnon contribution to the quasiparticle inelastic-scattering rate in disordered superconductors is presented. Using Anderson's exact eigenstate formalism, it is shown that the scattering rate is Stoner enhanced and is further enhanced by the disorder relative to the clean case in a manner similar to the disorder enhancement of the long-range Coulomb contribution. The results are discussed in connection with the possibility of conventional or unconventional superconductivity in the borocarbides. The results are compared to recent tunneling experiments on $\text{LuNi}_2\text{B}_2\text{C}$.

I. INTRODUCTION

In the early 1970s, certain rare-earth ternary compounds were found to display superconductivity while at the same time showed strong tendencies to be magnetic. A large body of theoretical work has been devoted to the interplay of magnetism and superconductivity.¹ Recently, while a great deal of attention has been focused on magnetism and superconductivity in the cuprate high temperature superconductors, there is increasing evidence that there is an interplay of magnetism and superconductivity in the borocarbides² as well as $\text{RuSr}_2\text{GdCu}_2\text{O}_8$.³ The borocarbides $R\text{Ni}_2\text{B}_2\text{C}$ (R =rare earth) can either be magnetic or superconducting depending upon R . Recent NMR measurements have shown the presence of strong antiferromagnetic spin fluctuations in superconducting $\text{YNi}_2\text{B}_2\text{C}$.⁴ Presently it is widely debated whether these materials are conventional superconductors with sharply peaked density of states (DOS) near the Fermi level (similar to some $A-15$'s), or unconventional with ground-state pairing of lower symmetry than the underlying lattice. For instance, in $R=\text{Lu}, \text{Y}$, $R\text{Ni}_2\text{B}_2\text{C}$, scanning tunneling microscopy (STM) has given evidence for conventional BCS behavior, albeit with a substantially smeared DOS, with a smearing parameter $\Gamma/\Delta=0.2$.⁴ Optical conductivity studies also support moderately strong coupled conventional superconductivity with $2\Delta/T_c=3.9-5.2$.⁵ At the same time de Haas-van Alphen,⁶ magnetic field anisotropy,⁷ and electronic Raman scattering⁸ experiments have given evidence for at least very small gaps over a portion of the Fermi surface.

Substitutional or positional disorder has played a crucial role in determining whether a material is conventional (i.e., obeys Anderson's theorem) or not, and recently studies on borocarbides doped with Co have been performed.⁹ Heat capacity and magnetic measurements on $\text{Y}(\text{Ni}_{2-x}\text{Co}_x)\text{B}_2\text{C}$ (Ref. 10) have interpreted the drop in T_c with increasing Co doping as due to the reduction of the DOS at the Fermi level rather than pair breaking by nonmagnetic impurities. On the other hand, Raman measurements on the same systems have shown an increase in spectral weight below the gap edge as Co is doped in, contrary to conventional BCS behavior.⁸

However, it is well known that conventional superconductors that are highly disordered display substantially smeared BCS properties that can mimic unconventional pairing.¹¹ This can result from vanishing of phase coherence

or from the interplay of interactions and disorder. The latter is most manifest in the reduction of quasiparticle (qp) lifetimes. Inelastic collisions broaden qp eigenstates and lead to a smearing of activated or threshold behavior in single- and two-particle correlation functions, measured, e.g., by tunneling, optical conductivity, and electronic Raman scattering. While the present status of the superconducting ground state of the borocarbides remains unclear, it is of interest to inspect whether strong inelastic scattering can modify s -wave properties to the point where the ubiquitous exponential behavior of various thermodynamic and transport quantities is obscured. For instance, the absence of a coherence peak in NMR is usually interpreted as a signal of unconventional electronic pairing. However, it is well known that the coherence peak can be suppressed as a consequence of strong inelastic electron-phonon collisions.¹² While the coherence peak can be fully suppressed only for large electron-phonon couplings in clean superconductors,¹³ it has been shown that the peak can be further suppressed in disordered superconductors and is absent in the region of strong disorder for only moderate couplings.¹⁴

The microscopic interplay of disorder, magnetic fluctuations, and superconductivity is reflected in the behavior of the qp inelastic lifetime. In this paper, we present a calculation for the qp inelastic scattering rate due to spin fluctuations within a formalism developed in previous works.¹⁵ The calculation is undertaken by first obtaining an effective fluctuation propagator in the superconducting state, and then using the exact eigenstate formalism as used in the case of Coulomb scattering with the replacement of the Coulomb propagator and vertex with the derived fluctuation propagators and vertices. It is shown that the rate is qualitatively similar to the rate due to Coulomb interactions with addition of the Stoner enhancement. Finally we discuss our results in terms of STM data on the borocarbide superconductors.

II. CALCULATIONS

The scattering rate from paramagnons in clean superconductors on a lattice is well known for the case of s - or d -wave superconductors.¹⁶ The calculation for the inelastic scattering rate due to paramagnon exchange in disorder metals is also well known.¹⁷ In both cases the results are similar to the scattering rate from long-range Coulomb interactions, with an additional enhancement via the Stoner factor $1/(1-I)$, where $I=UN_F$, U is a phenomenological short-range interaction, and N_F is the DOS per spin at the Fermi level.

For dirty metals and superconductors, the electron-phonon interaction is reduced via ‘‘collision drag’’ relative to the clean case,¹⁸ while the electron-electron interaction is enhanced by disorder due to the breakdown of screening by diffusive electrons.^{19,15} The latter enhancement of the scattering rate at the Fermi surface is $\hat{\rho}^{3/2}(E_F/T)^{1/2}$ compared to that of three-dimensional (3D) clean materials. Here $\hat{\rho}$ is the dimensionless measure of disorder, with $\hat{\rho} = \rho/\rho_M$, where ρ is the extrapolated residual resistivity and ρ_M the Mott number, which in a jellium model is given by $\rho_M = 3\pi^2/e^2k_F$. We use units such that $k_B = \hbar = 1$. However, calculations for the scattering rate calculated for superconductors on a lattice¹⁶ have treated impurities and interactions independently and therefore do not capture the disorder enhancement derived for conventional superconductors. Therefore, in this paper we investigate the interplay of disorder, superconductivity, and magnetism by revisiting the problem of inelastic scattering.

The spin fluctuation propagator is given by the sum of longitudinal $K_{\uparrow\uparrow}$ and transverse $K_{\uparrow\downarrow}$ paramagnons, respectively.²⁰ They can be expressed in terms of the polarization bubble χ as

$$\begin{aligned} K_{\uparrow\uparrow} &= U\chi U + U\chi U\chi K_{\uparrow\uparrow}, \\ K_{\uparrow\downarrow} &= U + U\chi K_{\uparrow\downarrow}. \end{aligned} \quad (1)$$

Solving these equations we obtain the fluctuation propagator $t(\mathbf{q}, \omega)$,

$$t(\mathbf{q}, \omega) = K_{\uparrow\uparrow} + K_{\uparrow\downarrow} = \frac{U^2\chi(\mathbf{q}, \Omega)}{1 - U^2\chi^2(\mathbf{q}, \Omega)} + \frac{U}{1 - U\chi(\mathbf{q}, \Omega)}. \quad (2)$$

However, in the superconducting state one must distinguish between charge and spin response couplings due to their different coherence factors. Therefore, in the superconducting state, the propagator splits into two contributions given by²¹

$$\begin{aligned} t_c(\mathbf{q}, \Omega) &= \frac{1}{2} \frac{U^2\chi_c(\mathbf{q}, \Omega)}{1 + U\chi_c(\mathbf{q}, \Omega)}, \\ t_s(\mathbf{q}, \Omega) &= \frac{3}{2} \frac{U^2\chi_s(\mathbf{q}, \Omega)}{1 - U\chi_s(\mathbf{q}, \Omega)} - U^2\chi_s(\mathbf{q}, \Omega), \end{aligned} \quad (3)$$

with $\chi_{c,s}$ is the charge and spin susceptibilities, respectively. In the following we perform calculations in the continuum limit and neglect lattice effects. This is certainly important in order to capture strong scattering via qp exchange of antiferromagnetic reciprocal lattice vector momenta \mathbf{Q} . However, the incipient magnetic instability via paramagnon exchange nevertheless is reflected via the Stoner criterion. Albeit a naive approach to the borocarbides or other materials with strong antiferromagnetic fluctuations, the results allow us to qualitatively estimate the effects of disorder on qp inelastic scattering from paramagnons.

The gauge-invariant charge polarization χ_c has been calculated in disordered superconductors in Ref. 22. It has the structure $\chi(\mathbf{q}, \omega) = B(\mathbf{q}, \omega) + B_c(\mathbf{q}, \omega)$. Here B is the density response function in the pair approximation,²³ while B_c contains the collective excitation (the Anderson-Bogoliubov mode) that restores gauge invariance. It was shown that for

$k_F\xi \gg 1$ collective effects can be ignored and that the ‘‘pair approximation’’ for the polarization is adequate, where $\xi = \sqrt{1/m\pi\Delta\hat{\rho}^{-1}}$ is the dirty-limit coherence length, for $T = 0$ the polarization can be written as

$$\begin{aligned} \chi_c''(\mathbf{q}, \omega) &= \phi''(\mathbf{q}, \sqrt{\omega(\omega - 2\Delta)})\Theta(\omega - 2\Delta) \\ &\times \left[(\omega + 2\Delta)E(\alpha) - \frac{4\Delta\omega}{\omega + 2\Delta}K(\alpha) \right], \end{aligned} \quad (4)$$

while the spin susceptibility is given by the Mattis-Bardeen result²⁴

$$\begin{aligned} \chi_s''(\mathbf{q}, \omega) &= \phi''(\mathbf{q}, \sqrt{\omega(\omega - 2\Delta)})\Theta(\omega - 2\Delta) \\ &\times [(\omega + 2\Delta)E(\alpha) - 4\Delta K(\alpha)]. \end{aligned} \quad (5)$$

Here, $\alpha = (\omega - 2\Delta)/(\omega + 2\Delta)$, and E and K are complete elliptical integrals of the first and second kinds, respectively. ϕ'' is the spectrum of the density Kubo function for noninteracting electrons. It can be calculated by a variety of techniques for various limits of disorder. For clean metals, the spectrum is white,

$$\phi''(\mathbf{q}, \epsilon) = \frac{m^2}{4\pi q}, \quad \text{clean}, \quad (6)$$

while for diffusive qp dynamics, ϕ'' is given by a diffusion pole

$$\phi''(\mathbf{q}, \epsilon) = N_F \frac{Dq^2}{(Dq^2)^2 + \epsilon^2}, \quad \text{diffusive}, \quad (7)$$

with D the diffusion constant. Here we have neglected Cooper propagator renormalization, which can be shown to give a smaller contribution to the scattering rate than diffusion propagator renormalization by a factor of $1/k_F\xi$.

The limiting behavior for finite temperatures with $T \ll \Delta$ is given as

$$\begin{aligned} \chi_{c,s}''(\mathbf{q}, \Omega \ll \Delta) &\approx \Omega \phi''(\mathbf{q}, \sqrt{2\Delta\Omega}) \\ &\times e^{-\Delta/T} \begin{cases} 1, & \text{charge} \\ (\Delta/T)\ln(4T/\Omega), & \text{spin}, \end{cases} \end{aligned} \quad (8)$$

$$\begin{aligned} \chi_{c,s}''(\mathbf{q}, \Omega \geq 2\Delta) &\approx \Delta \phi''(\mathbf{q}, 2\sqrt{2\Delta}) \\ &\times \sqrt{\pi T/\Delta} e^{-\Delta/T} \begin{cases} 1/2, & \text{charge} \\ 1, & \text{spin}. \end{cases} \end{aligned} \quad (9)$$

Thus the behavior of the spin and charge susceptibilities yields different contributions to the paramagnon scattering in the charge and spin channel.

The paramagnon contribution to the self-energy can be split in the usual way into an anomalous and even and odd normal pieces. It has been shown for the case of long-range Coulomb interactions that the even part of the normal self-energy contribution can be ignored, and can be shown for the spin-fluctuation case as well.¹⁵ Expanding near the qp pole in the BCS Green's function,²⁵ we obtain the expression for the on-shell inelastic scattering rate due to paramagnons exchange in the charge channel Γ_c and spin channel Γ_s ,

$$\begin{aligned}
\Gamma_{c,s}(\omega) = & -\frac{1}{Z'} \sum_{\mathbf{q}} \int \frac{d\epsilon}{\pi N_F} \phi''(\mathbf{q}, \epsilon - \omega) \\
& \times \int_0^\infty \frac{dx}{\pi} [f(x + \omega) + n(x)] t_c''(\mathbf{q}, x) \\
& \times [G''(\epsilon, \omega + x) \pm \Delta / \omega F''(\epsilon, \omega + x)] + (\omega \rightarrow -\omega),
\end{aligned} \tag{10}$$

where n and f are Bose and Fermi distributions, respectively, G'' and F'' are the imaginary parts of the bare normal and anomalous BCS Green's functions, respectively, Z' is the real part of the qp renormalization, and $(\omega \rightarrow -\omega)$ denotes the addition of terms that differ from the ones written only by the sign of ω .

Substituting Eqs. (3)–(9) into Eq. (10), we obtain the inelastic scattering rate $\tau_s^{-1} = 2\Gamma_s$. It can be shown that the contribution to the scattering rate from the charge channel yields a subdominant contribution for all values of disorder and interaction $I < 1$ compared to the long-range Coulomb contribution calculated in Ref. 15. Therefore, for the remainder of the paper we neglect Γ_c and focus on Γ_s . The scattering rate is dominated by qp population at the gap edge. For $T=0$, an injected qp must have enough energy to give up to break a Cooper pair (3Δ) and for $(\Omega - 3\Delta)/(\Omega + 3\Delta) \ll 1$ we obtain

$$\begin{aligned}
\Gamma_s^{T=0}(\Omega \geq 3\Delta) = & \frac{3I^2 \pi^2}{16(1-I)^{3/2}} \frac{\Delta}{Z'} \frac{\Delta}{\Omega} \frac{\Delta}{E_F} F\left(\frac{\Omega}{\Delta}\right) \\
& \times \begin{cases} \frac{1}{\sqrt{1-I}}, & \text{clean} \\ \left(\frac{3\hat{\rho}}{\pi}\right)^{3/2} \sqrt{\frac{E_F}{\Delta}}, & \text{dirty,} \end{cases}
\end{aligned} \tag{11}$$

with $F(x) = x(x^2/2 - x - 1)\sqrt{(x-2)^2 - 1} + (x/2 - 2)\ln[x - 2 + \sqrt{(x-2)^2 - 1}]$. At finite temperatures, the Cooper pair recombination rate is dominated by the kinematic factor $\Gamma^R \propto e^{-2\Delta/T}$ and qp scattering rate $\Gamma^S \propto e^{-\Delta/T}$. For a qp at the gap edge, the dominant contribution to the recombination rate is given by

$$\begin{aligned}
\Gamma_s^R(\Delta \geq T) = & \frac{3\pi^2 I^2}{8\sqrt{2}(1-I)^{3/2}} \frac{T^2}{Z' E_F} \\
& \times e^{-2\Delta/T} \begin{cases} \frac{1}{\sqrt{1-I}}, & \text{clean} \\ \left(\frac{3\hat{\rho}}{\pi}\right)^{3/2} \sqrt{\frac{E_F}{\Delta}}, & \text{dirty,} \end{cases}
\end{aligned} \tag{12}$$

while for the scattering rate we obtain to leading order

$$\begin{aligned}
\Gamma_s^S(\Delta \geq T) = & \frac{3\pi I^2 \ln(2)}{8(1-I)^{3/2}} \frac{\Delta}{Z'} \sqrt{\frac{\pi T}{2\Delta}} \\
& \times e^{-\Delta/T} \begin{cases} \frac{1}{\sqrt{1-I}}, & \text{clean} \\ \left(\frac{3\hat{\rho}}{\pi}\right)^{3/2} \sqrt{\frac{E_F}{\Delta}}, & \text{dirty.} \end{cases}
\end{aligned} \tag{13}$$

We see a similar behavior between the paramagnon and long-range Coulomb contributions to the inelastic scattering rate.¹⁵ Γ_s possesses the same temperature dependence as the Coulomb contribution, with the exponential temperature dependence reflecting the necessity of two qp's per scattering event. Further, we see the same disorder enhancement [$\hat{\rho}^{3/2}(E_F/\Delta)^{1/2}$] relative to the clean case as in the long-range Coulomb case. Lastly, we note that the energy gap Δ acts as a cutoff for the divergence of the rate that occurs in the 2- d dirty-normal calculation,¹⁷ just as in the long-range Coulomb case.¹⁵

On top of the disorder enhancement, there is the Stoner enhancement relative to the Coulomb contributions due to the nearness of a magnetic instability. In materials close to the instability, this contribution will be dominant over the Coulomb and phonon terms except for very low temperatures, where the power-law temperature dependence of the phonon contribution takes over.^{15,25} We note that our expression is valid for $\Delta/E_F \ll 1 - I \ll 1$, i.e., provided that one is not too close to the Stoner criterion for magnetism, $I=1$. At the instability, the rate saturates as it does in the case of a normal metal near the metal-insulator transition²⁶. However, in order to accurately describe the dynamics at the magnetic transition, one needs to use a more sophisticated spin-fluctuation propagator than the one derived here from random-phase approximation diagrams only, which tend to overestimate paramagnon effects.²⁷

Finally, we can compare the results to the values of the scattering rates inferred from STM data on clean and thin films of LuNi₂B₂C. To our knowledge, a temperature dependence of the scattering rate has not yet been published, nor has a reliable estimate of the scattering rate been made from optical (Raman or infrared) or Hall probes as has been done in the high- T_c cuprates. Moreover, no systematic study of the effects of impurities and doping has been made concerning the scattering rate. Nevertheless we can estimate if inelastic scattering from paramagnons in an s -wave superconductor is sufficient to explain the broadening observed in STM measurements.⁴ As a rough estimate for $1/\tau_s$, we take $I \sim 2/3$, $Z' = 1/2$, Fermi velocity $v_F \sim 3.5 \times 10^7$ cm/s, Fermi energy $E_F \sim 0.3$ eV given from local-density approximation estimates for LuNi₂B₂C from Ref. 28. STM data taken at low temperatures in Ref. 4 give $\Delta = 18$ cm⁻¹, which is consistent with Raman measurements.⁸ This yields a scattering rate for clean systems at $T=0.5T_c$ from Eq. (13) of $1/\tau_s = 1.3 \times 10^{-3}$ meV, or $1/\tau_s \Delta = 6 \times 10^{-4}$, which is clearly too small to match experiments. Either the scattering is most likely due to electron-phonon collisions²⁹ or perhaps due to large gap anisotropy.

Since $\rho(T=0)$ increases quickly as Co is doped in,⁹ rising by over an order of magnitude for 15% Co doping,⁴ it

may be feasible that the disorder enhancement for $1/\tau_s$ in an s -wave scenario could lead to spectral weight at low frequencies observed via magnetic-field anisotropy⁷ or Raman⁸ measurements. An estimate for the Mott number is difficult since the parameters v_F , k_F and the other parameters entering in Eqs. (12) and (13) are presumably disorder dependent, and it is not clear where the metal-insulator transition occurs for this compound. A conservative estimate from the Ioffe-Regel criterion in Ref. 4 gives $\rho_M \sim 400 \mu\Omega \text{ cm}$. Therefore, taking $\rho(T=0) \sim 100 \mu\Omega \text{ cm}$ as in Ref. 4 into Eq. (13) only gives $1/\tau_s \Delta \sim 10^{-3}$, which is clearly too small to account for the large broadening observed via STM, even in relatively clean films, nor is it sufficient to account for the substantial spectral weight observed at low frequencies via Raman scattering. It is tempting to therefore conclude that the large broadening comes either from nodal qp's in conventional (extended s -) or unconventional (d -) pair states.

However, there are problems in each scenario. A small amount of Co doping (on the few percent level) quickly

pushes these materials into the dirty limit ($\xi/l \ll 1$).⁹ If the gap possessed extended s -wave symmetry, the disorder would be sufficient to wash out any remaining anisotropy and necessarily lead to sharp threshold behavior, which is not observed. On the other hand, the disorder would also lead to a sharp drop in T_c if the gap possessed d -wave symmetry and unconventional superconductivity would be expected to be completely suppressed³⁰ for 15% Co doping, which again is not observed. Therefore, it is unclear from current data whether superconductivity is conventional or not, and perhaps the situation is clouded by the presence of additional nonsuperconducting bands, which would also yield a nonvanishing zero bias conductance and low-frequency spectral weight. It would thus be extremely useful to study impurity and cation dopings further to determine if the enhanced scattering rates are responsible for the behavior indicative of unconventional pairing as the disorder is increased. Raman scattering measurements would be very useful in this regard and remain a topic for further investigation.

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- ¹ *Superconductivity in Ternary Compounds*, edited by M. B. Maple and O. Fisher (Springer-Verlag, Berlin, 1982).
- ² P. C. Canfield *et al.*, Phys. Today **51** (10), 40 (1998).
- ³ C. Bernhard, J. L. Tallon, C. Niedermayer, T. Blasius, A. Golnik, E. Brücher, R. K. Kremer, D. R. Noakes, C. E. Stronach, and E. J. Ansaldo, Phys. Rev. B **59**, 14 099 (1999); A. C. McLaughlin, W. Zhou, J. P. Attfield, A. N. Fitch, and J. L. Tallon, *ibid.* **60**, 7512 (1999).
- ⁴ Y. De Wilde, M. Iavarone, U. Welp, V. Metlushko, A. E. Koshelev, I. Aranson, G. W. Crabtree, and P. C. Canfield, Phys. Rev. Lett. **78**, 4273 (1997); R. Vaglio, A. Andreone, C. Aruta, A. Cassinese, F. Fontana, G. W. Crabtree, M. Iavarone, Y. De Wilde, L. Maritato, C. Attanasio, C. Coccorese, M. Salluzzo, and M. Salvato, Phys. Rev. B **56**, 934 (1997); T. Kohara, T. Oda, K. Ueda, Y. Yamada, A. Mahajan, K. Elankumaran, Z. Hussran, L. C. Gupta, R. Nagarajan, R. Vijayaraghavan, and C. Mazumdar, Phys. Rev. B **51**, 3985 (1995).
- ⁵ F. Bommeli, L. Degiorgi, P. Wachter, B. K. Cho, P. C. Canfield, R. Chau, and M. B. Maple, Phys. Rev. Lett. **78**, 547 (1997).
- ⁶ T. Terashima, C. Haworth, H. Takeya, S. Uji, H. Aoki, and K. Kadowaki, Phys. Rev. B **56**, 5120 (1997).
- ⁷ G. Wang and K. Maki, Phys. Rev. B **58**, 6493 (1998).
- ⁸ I.-S. Yang, M. V. Klein, S. L. Cooper, P. C. Canfield, B. K. Cho, and S.-I. Lee, cond-mat/9910087 (unpublished); I.-S. Yang, M. V. Klein, T. P. Devereaux, I. R. Fisher, and P. C. Canfield, cond-mat/9912492 (unpublished); (private communication).
- ⁹ H. Schmidt and H. F. Braun, Phys. Rev. B **55**, 8497 (1997); K. O. Cheon, I. R. Fisher, V. G. Kogan, P. C. Canfield, P. Miranović, and P. L. Gammel, *ibid.* **58**, 6463 (1998).
- ¹⁰ C. C. Hoellwarth, P. Klavins, and R. N. Shelton, Phys. Rev. B **53**, 2579 (1996).
- ¹¹ R. C. Dynes, J. P. Garno, G. B. Hertel, and T. P. Orlando, Phys. Rev. Lett. **53**, 2436 (1984); A. E. White, R. C. Dynes, and J. P. Garno, Phys. Rev. B **33**, 3549 (1986); D. S. Pyun and T. R. Lemberger, *ibid.* **43**, 3732 (1991); J. M. Valles, R. C. Dynes, and J. P. Garno, Phys. Rev. Lett. **69**, 3567 (1992); R. P. Barber, L. M. Merchant, A. La Porta, and R. C. Dynes, Phys. Rev. B **49**, 3409 (1994).
- ¹² M. Fibich, Phys. Rev. Lett. **14**, 561 (1965); **14**, 621 (1965).
- ¹³ P. Allen and D. Rainer, Nature (London) **349**, 496 (1991); O. V. Dolgov, A. A. Golubov, and A. E. Koshelev, Solid State Commun. **72**, 81 (1989).
- ¹⁴ T. P. Devereaux, Z. Phys. B: Condens. Matter **90**, 65 (1993).
- ¹⁵ T. P. Devereaux and D. Belitz, J. Low Temp. Phys. **77**, 319 (1989); Phys. Rev. B **44**, 4587 (1991); **43**, 3736 (1991).
- ¹⁶ S. Wermbter and L. Tewordt, Phys. Rev. B **43**, 10 530 (1991); S. M. Quinlan, P. Hirschfeld, and D. J. Scalapino, *ibid.* **49**, 1470 (1994).
- ¹⁷ M.-C. Chang and E. Abrahams, Phys. Rev. B **32**, 1315 (1985).
- ¹⁸ A. B. Pippard, Philos. Mag. **46**, 1104 (1955); A. Schmid, Z. Phys. **259**, 421 (1973); M. Yu. Reizer and A. V. Sergeev, Zh. Éksp. Teor. Fiz. **90**, 1056 (1986) [Sov. Phys. JETP **63**, 616 (1986)].
- ¹⁹ A. Schmid, Z. Phys. **271**, 251 (1974).
- ²⁰ N. F. Berk and J. R. Schrieffer, Phys. Rev. Lett. **17**, 433 (1966).
- ²¹ C.-H. Pao and N. E. Bickers, Phys. Rev. B **44**, 10 270 (1991).
- ²² D. Belitz, S. De Souza-Machado, T. P. Devereaux, and D. W. Hoard, Phys. Rev. B **39**, 2072 (1989).
- ²³ R. E. Prange, Phys. Rev. **129**, 2495 (1963).
- ²⁴ D. C. Mattis and J. Bardeen, Phys. Rev. **111**, 412 (1958).
- ²⁵ S. B. Kaplan, C. C. Chi, D. N. Langenberg, J. J. Chang, S. Jafarey, and D. J. Scalapino, Phys. Rev. B **14**, 4854 (1976).
- ²⁶ D. Belitz and K. I. Wysokinski, Phys. Rev. B **36**, 9333 (1987).
- ²⁷ S. Nakajima, Prog. Theor. Phys. **50**, 1101 (1973).
- ²⁸ W. E. Pickett and D. J. Singh, Phys. Rev. Lett. **72**, 3702 (1994).
- ²⁹ An estimate for the electron-phonon contribution for $1/\tau(\Delta)\Delta$ from Ref. 25 using the parameters in Ref. 28 gives ~ 0.06 .
- ³⁰ See, e.g., P. Fulde, J. Keller, and G. Zwirnagl, Solid State Phys. **41**, 1 (1988); M. Sigrist and K. Ueda, Rev. Mod. Phys. **63**, 239 (1991); N. Grewe and F. Steglich, in *Handbook of the Physics and Chemistry of Rare Earths*, edited by K. A. Gschneider, Jr. and L. Eyring (North-Holland, Amsterdam, 1991), Vol. 14.