Low-temperature magnetic penetration depth in *d*-wave superconductors: Zero-energy bound state and impurity effects

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We report a theoretical study on the deviations of the Meissner penetration depth $\lambda(T)$ from its London value in *d*-wave superconductors at low temperatures. The difference arises from low-energy surface Andreev bound states. The temperature dependent penetration depth is shown to go through a minimum at the temperature $T_{m0} \sim \sqrt{\xi_0/\lambda_0} T_c$ if the broadening of the bound states is small. The minimum will straighten out when the broadening reaches T_{m0}. The impurity scattering sets up the low-temperature anomalies of the penetration depth and destroys them when the mean free path is not sufficiently large. A phase transition to a state with a spontaneous surface supercurrent is investigated and its critical temperature determined in the absence of a subdominant channel activated at low temperatures near the surface. Nonlinear corrections from Andreev low-energy bound states to the penetration length are obtained and shown, on account of their broadening, to be small in the Meissner state of strong type-II superconductors.

I. INTRODUCTION

The low-temperature behavior of the magnetic penetration length in *d*-wave superconductors is in general a great deal more complicated than that of their isotropic *s*-wave cousins. The changing sign of the order parameter, according to where one looks on the Fermi surface, entails coherent zero-energy or low-energy bound states in *d*-wave superconductors localized at smooth or almost smooth surfaces or interfaces.[1–6] These bound states feature peculiar low-temperature contributions to the magnetic penetration length[7] (see also Ref. 8) and the zero-bias conductance peak[2,3,5] (see also Refs. 8–18).

A minimum in the penetration depth of $YBa_2Cu_3O_{7-\delta}$ films[7] and grain boundary junctions[8] was thus interpreted as evidence for low-energy Andreev bound states. A conventional shielding-current contribution to the Meissner effect would obviously just monotonically reduce the penetration depth when the temperature goes down. On the other hand, a paramagnetic contribution from low-energy bound states increases the penetration depth. The interplay of these two effects amounts to a minimum in the penetration depth as a function of the temperature. The characteristic temperature T_{m0} of this anomaly is shown to be the order $\sqrt{(\xi_0/\lambda_0)} T_c$ $\ll T_c$ if the broadening γ of the bound states is sufficiently small. At this temperature region the bound state contribution to the penetration depth competes with the low-temperature correction from shielding supercurrents to its zero-temperature value.

An alternative explanation of an upturn in the penetration depth is possible in compounds whose bulk paramagnetic properties grow when the temperature goes down, as in the electron-doped cuprate superconductor $Nd_{1.85}Ce_{0.15}CuO_{4-y}$. There the paramagnetism arises from Nd^{3+} ions.[19–22] We will not discuss these compounds below.

There is yet another important temperature associated with the magnetic penetration depth $T_s \sim (\xi_0/\lambda_0) T_c$. If a given crystal orientation does not carry quasiparticle Andreev bound states, a nonlocal effect can take over as a correction to the zero temperature penetration depth in the clean limit.[23] Then in other orientations which do admit Andreev states, the bound-state contribution and the spontaneous surface supercurrent in particular, can in turn overwhelm the nonlocal effect. At $T \sim T_s$ the bound state paramagnetic contribution to λ in the clean limit[24] is the order of the total London penetration depth λ_0 from the screening currents. In the absence of subdominant pairing channels, a spontaneous surface supercurrent brought about by the bound states may arise below the temperature T_s (Refs. 25,26) (see also Ref. 27 on a similar effect of spontaneous magnetization brought about by low-energy interface bound states). Having in mind high-temperature superconducting compounds, we will discuss strong type-II superconductors. Then (ξ_0/λ_0) is easily the order 0.01, and the low-temperature range splits up into at least three areas staked out by 0, T_s, and T_{m0} [$T_s \sim (\xi_0/\lambda_0) T_c \ll T_{m0} \sim \sqrt{(\xi_0/\lambda_0)} T_c \ll T_c$]. Quasiparticle scattering off impurities or surface roughness and inelastic processes may also play an important role if they bring about a broadening γ of the bound states the order or greater than the characteristic temperature T_{m0} (T_s).

We assume below that nonmagnetic impurities dominate the scattering and the broadening. Nonmagnetic impurities in superconductors with an anisotropic order parameter are known to be pair breaking. They suppress T_c analogously to what happens to isotropic superconductors with magnetic impurities. Assuming superconductors always clean within the conventional definition $\xi \ll l$, we disregard this kind of effects throughout the article. Even then impurity broadening of Andreev bound states in anisotropically paired superconductors can be significant. Since the broadening removes singularities in the density of states (for instance, δ peaks from quasiparticle bound states) as well as in other related physical quantities, superconductors can be sensitive to extremely small concentrations of impurities.[24] This is analogous to the

role of pair breaking and small anisotropy of the gap in the Riedel anomaly in isotropic *s*-wave superconductors. The Riedel anomaly is associated with the BCS singularity in the density of states. Pair breaking and small anisotropy of the gap are known to wipe out the BCS singularity in the density of states averaged over the Fermi surface, and control the height of the Riedel peak.[28]

The emphasis of the present work is on the various effects of broadening on the low-temperature anomalies of the Meissner effect. The zero-energy polelike term of what is known as the quasiclassical Green's function was exploited in the investigation. Broadening is introduced into the polelike term simply sliding the pole along the imaginary energy axis. With small broadening, relatively simple expressions are found for the penetration length in the two lowest-temperature regions defined above. If $T_{s(m0)} \lesssim \gamma \ll T_c$, the growing γ can wipe out the low-temperature anomalies. Beginning with the critical broadening $\gamma_{s(m)}$, anomalies at $T_{s(m)}$ are fully destroyed. It turns out that unitary scatterers need to come with significantly larger scattering rates $\Gamma_{s(m)}$ than Born impurities in order to achieve the critical broadening $\gamma_{s(m)}$. This effect is peculiar of the impact of impurities on the Andreev bound states as seen in the local density of states and Josephson critical currents.[29] For this reason, the requirements the mean free path must meet for the low-temperature anomalies to show up are sensitive to the strength of the impurity potential and very different in the unitary and the Born limits.

For Born scatterers, the shortest normal-state impurity mean free path l which preserves the low-temperature upturn at $T \sim T_m$ is shown to be $\lambda_0 \lesssim l$. This looks quite restrictive although conceivably compatible with the strikingly large low temperature mean free paths in some high-T_c compounds.[30–35] For the spontaneous surface supercurrent in the absence of a subdominant component at the surface, we find the threshold $\lambda_0^2/\xi_0 \lesssim l$. This demands extraordinary clean samples not available for the time being. On the other hand, the requirements set by unitary scatterers are much weaker and probably can be met. In this case surface roughness is likely to control the broadening and the experimental observability of the effects.

We also examine what the Andreev bound states do to the nonlinear Meissner effect. At low temperatures $T \ll T_c$, the field $\tilde{H}_0$ at which the nonlinear response of the bound states saturates in the clean limit[24] is much weaker than the one from the screening current. Ignoring the broadening, $\tilde{H}_0$ is a linear function of the temperature. With $T \lesssim T_s$, nonlinear effects from the bound states become important already in the Meissner state. Close to the transition to the state with a spontaneous surface supercurrent, a nonlinear term entering into the Landau mean-field free energy is important also in a weak external field. The broadening γ introduces another field, $\tilde{H}_\gamma$ characterizing the nonlinear consequences of the bound states at $\pi T_s \lesssim \gamma$. For sufficient broadening $\pi T_s \ll \gamma$, we get $H_{c1} \ll \tilde{H}_\gamma$ and the nonlinear terms are shown always to be small in the Meissner state.

II. THE UPTURN IN THE LOW-TEMPERATURE DEPENDENCE OF THE PENETRATION DEPTH

Our considerations are based on the quasiclassical matrix Green function which describes quasiparticle excitations in

thermal equilibrium. The quasiclassical propagator $\hat{g}(p_f, x, \varepsilon_n)$ satisfies Eilenberger's equations, which have a 2×2 particle-hole matrix form

$$\left[\left(i\varepsilon_n + \frac{e}{c} v_f \cdot \mathbf{A}(\mathbf{R}) \right) \hat{\tau}_3 - \hat{\Delta}(p_f, \mathbf{R}) \right.$$

$$\left. - \hat{\sigma}(p_f, \mathbf{R}; \varepsilon_n), \hat{g}(p_f, \mathbf{R}; \varepsilon_n) \right] + i v_f \cdot \nabla_\mathbf{R} \hat{g}(p_f, \mathbf{R}; \varepsilon_n) = 0,$$

$$(1)$$

$$\hat{g}^2(p_f, \mathbf{R}; \varepsilon_n) = -\pi^2 \hat{1}, \qquad (2)$$

where $\varepsilon_n = (2n+1)\pi T$ are the Matsubara energies, p_f the momentum on the Fermi surface, v_f the Fermi velocity, $\mathbf{A}$ the vector potential, $\hat{\Delta}$ the order parameter matrix, and $\hat{\sigma}$ the impurity self-energy. A symbol with a hat denotes a matrix in the Nambu space.

The propagator $\hat{g}$ and the order parameter matrix $\hat{\Delta}$ parametrize as

$$\hat{g} = \begin{pmatrix} g & f \\ f^+ & -g \end{pmatrix} \quad \text{and} \quad \hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta* & 0 \end{pmatrix}. \qquad (3)$$

The gap function $\Delta(p_f, \mathbf{R})$ is related to the anomalous Green function f and must be determined self-consistently. The diagonal part $g(p_f, \mathbf{R}, \varepsilon_n)$ of the full matrix propagator $\hat{g}$ carries information on the electrical current density

$$j(\mathbf{R}) = 2eTN_f \sum_{\varepsilon_n} \langle v_f g(p_f, \mathbf{R}, \varepsilon_n) \rangle_{S_f}. \qquad (4)$$

Here N_f is the normal state density of states per spin direction and $\langle \cdots \rangle_{S_f}$ means averaging over quasiparticle states at the Fermi surface.

Let an anisotropic singlet strong type-II superconductor occupy the right half-space $x > 0$. A magnetic field is applied along the z axis. The induced supercurrent and the vector potential [in the gauge div $\mathbf{A}(\mathbf{R}) = 0$ and vanishing in the bulk] have only y components. The linear current-field relation in general has a nonlocal form, i.e., $j(x) = -\int_0^{+\infty} Q(x, x', T) A(x') dx'$.

For strongly type-II superconductors with nodes in the order parameter, a nonlocal current-field relation can be of importance only at very low temperatures $T \lesssim T_s$.[23] Hence, a study of the penetration depth at low temperatures $T_s \ll T \sim T_{m0} \ll T_c$ may be carried out disregarding nonlocal effects. Then a magnetic field enters into Eq. (4) only together with the Matsubara frequencies $\varepsilon_n - i(e/c)v_{f,y}A(x)$ in the argument of the Green's function. The kernel $Q(x, T)$ can then be written

$$Q(x, T) = \frac{2ie^2 TN_f}{c} \sum_{n=-\infty}^{+\infty} \left\langle v_{f,y}^2(p_f) \frac{\partial g(p_f, x, \varepsilon_n)}{\partial \varepsilon_n} \right\rangle_{S_f}. \qquad (5)$$

In the presence of zero-energy surface bound states, the polelike term in the propagator becomes dominating at temperatures $T \ll T_c$. Surface bound states as well as their paramagnetic response are localized on the scale of the coherence length at the surface, however, while the conventional

screening current has an avenue of the huge thickness of the penetration depth. That is why the zero-energy bound-state contribution to the penetration depth remains a small low-temperature correction to $\lambda_0 \equiv \lambda_0(T=0)$ at all temperatures $T \gg T_s$ (in particular, at $T \sim T_{m0}$). The contribution from surface bound states must be viewed together with a low temperature correction from the screening current as small low-temperature imports to the zero-temperature London penetration depth λ_0. Then the total kernel of the form $Q(x,T) = c/4\pi\lambda_0^2 + \delta Q(x,T)$ includes only the lowest order corrections in $\delta Q(x,T)$.

Solving the Maxwell equation

$$A''(x) - \frac{1}{\lambda_0^2}A(x) - \frac{4\pi}{c}\delta Q(x,T)A(x) = 0 \qquad (6)$$

perturbatively with respect to the last term delivers a first order approximation to the vector potential

$$A(x) = A^{(0)}(0)\left[\exp\left(-\frac{x}{\lambda_0}\right) - \frac{2\pi\lambda_0}{c}\int_0^{+\infty}dx'\right.$$
$$\left.\times \exp\left(-\frac{|x-x'|}{\lambda_0}\right)\delta Q(x',T)\exp\left(-\frac{x'}{\lambda_0}\right)\right]. \qquad (7)$$

The kernel $\delta Q(x,T)$ incorporates only a contribution from the bound states and a low-temperature correction from the screening current.

The penetration depth is defined as $\lambda = \int_0^{+\infty}H(x)dx/H(0) = -A(0)/A'(0)$. Expanding this to first order in δQ and extracting the low-temperature correction from the screening current for the case of a superconductor with a line of nodes

$$\lambda(T) = \lambda_0 + a\lambda_0\frac{T}{T_c} - \frac{4\pi\lambda_0^2}{c}\int_0^\infty Q^{\text{bound}}(x,T)dx. \qquad (8)$$

Here a is a coefficient of the order of unity which depends on the shape of the Fermi surface and on an angular slope of the order parameter near the nodes. For instance, for a quasi-two-dimensional $d_{x^2-y^2}$ tetragonal superconductor with a cylindrical Fermi surface (with a principal axis z) and order parameter $\Delta(\phi) = \Delta_0\cos(2\phi - 2\alpha)$, one gets $a \approx 0.32$.

Kernel $Q^{\text{bound}}(x,T)$ takes negative values. It is a paramagnetic contribution from zero-energy bound states to Eq. (5). One obtains $Q^{\text{bound}}(x,T)$ from Eq. (5) substituting instead of the full expression for $g(\mathbf{p}_f,x,\varepsilon_n)$ only its singular part (polelike term) $g_s(\mathbf{p}_f,x,\varepsilon_n)$. Associated with zero energy surface bound states, this term vanishes in the bulk on the scale of the coherence length ξ_0. It has longer tails only towards the nodes. Node contributions do not dominate, however, in the following expressions. The presence of zero-energy surface bound states is crucial in the reasoning. All sectors of the Fermi surface associated with a sign change of the order parameter in a quasiparticle reflection from the surface, contribute significantly to the results. This allows us to neglect, to a good accuracy, the factor $\exp(-2x/\lambda_0)$ under the integral sign in Eq. (8).

The analytic expression for the polelike term has been found in the clean limit and for a smooth surface in Ref. 36:

$$g_s(\mathbf{p}_f,x,\varepsilon_n) = \frac{-2\pi i}{\varepsilon_n}\frac{|\tilde{\Delta}(\mathbf{p}_f,0)||\tilde{\Delta}(\underline{\mathbf{p}}_f,0)|}{|\tilde{\Delta}(\mathbf{p}_f,0)| + |\tilde{\Delta}(\underline{\mathbf{p}}_f,0)|}\Theta(\mathbf{p}_f)$$
$$\times \exp\left(-\frac{2}{|\mathrm{v}_{f,x}(\mathbf{p}_f)|}\int_0^x|\Delta(\mathbf{p}_f,x')|dx'\right). \qquad (9)$$

The effective surface order parameter $|\tilde{\Delta}(\mathbf{p}_f,0)|$ introduced in Eq. (9), is defined

$$\frac{1}{|\tilde{\Delta}(\mathbf{p}_f,0)|} = \frac{2}{|\mathrm{v}_{f,x}(\mathbf{p}_f)|}\int_0^\infty \exp\left(-\frac{2}{|\mathrm{v}_{f,x}(\mathbf{p}_f)|}\right.$$
$$\left.\times \int_0^x|\Delta(\mathbf{p}_f,x')|dx'\right)dx. \qquad (10)$$

Here we distinguish between incoming $\mathbf{p}_f$ and outgoing $\underline{\mathbf{p}}_f$ quasiparticle momenta in a reflection event. For specular reflection, the momentum parallel to the interface is conserved. Function $\Theta(\mathbf{p}_f)$ is equal to unity where zero energy bound states occur on the Fermi surface (i.e., where the order parameter in the bulk taken for incoming $\mathbf{p}_f$ and outgoing $\underline{\mathbf{p}}_f$ momentum directions have opposite signs), and vanishes elsewhere.

Substituting Eq. (9) in Eq. (5), one can easily sum over the Matsubara frequencies. Integration over the space coordinate x in Eq. (8) then yields the penetration depth

$$\lambda(T) = \lambda_0 + a\frac{T}{T_c}\lambda_0 + \frac{\pi^2 e^2 N_f\lambda_0^2}{c^2 T}\langle \mathrm{v}_{f,y}^2(\mathbf{p}_f)|\mathrm{v}_{f,x}(\mathbf{p}_f)|\Theta(\mathbf{p}_f)\rangle_{S_f},$$
$$(11)$$
$$T_c\frac{\xi_0}{\lambda_0} \ll T \ll T_c.$$

For a three-dimensional superconductor with a spherical Fermi surface one has the relation $\lambda_0^2 = 3c^2/(8\pi e^2\mathrm{v}_f^2 N_f)$. Then the coefficient in front of the third term in Eq. (11) is $3\pi/8T\mathrm{v}_f^2$. Analogously, for a simple model of a quasi-two-dimensional superconductor with a cylindrical Fermi surface, $\lambda_0^2 = c^2/(4\pi e^2\mathrm{v}_f^2 N_f)$ and the coefficient $\pi/4T\mathrm{v}_f^2$.

In particular, for a $d_{x^2-y^2}$-wave superconductor with a cylindrical Fermi surface, we get from Eq. (11)

$$\lambda(T) = \lambda_0 + a\frac{T}{T_c}\lambda_0 + \frac{\mathrm{v}_f}{6T}||\sin^3\beta| - |\cos^3\beta||,$$
$$(12)$$
$$T_c\frac{\xi_0}{\lambda_0} \ll T \ll T_c,$$

where $\beta = \alpha + (\pi/4)$ is the angle between the surface normal and the direction to a node of the order parameter, while α is the angle between the surface normal and the crystalline a axis along its positive lobe.

We note that the correction from zero energy bound states to the penetration depth [the third term in Eq. (11)] has a quite universal form. It is independent both of the spatial profile of the order parameter near a surface and its particular anisotropic structure (basis functions). Therefore, this correction depends only on the type of pairing, which determines

regions on the Fermi surface with opposite signs of the order parameter. For example, expression (12) is valid irrespective of a particular form of a momentum direction dependence of the basis function for a d-wave order parameter of given symmetry.

The ratio of a supercurrent density at the surface $j_s^{\text{bound}}(x=0,T)$ to the one $j_s(x_{\text{scr}},T)$ at a characteristic distance x_{scr} ($\xi_0 \ll x_{\text{scr}} \ll \lambda_0$) from the surface can be estimated for a clean superconductor[24] at $T \ll T_c$ and a smooth surface as $\quad |j_s^{\text{bound}}(x=0,T)/j_s(x_{\text{scr}},T)| \sim 4\pi\lambda_0^2 |Q^{\text{bound}}(x=0,T)|/c$ $\sim T_c/T$. This verifies that at low temperatures $T \ll T_c$ the paramagnetic current $j_s^{\text{bound}}(x,T)$ dominates over the shielding current near the surface within a relatively small characteristic scale ξ_0.

The temperature dependent terms in Eq. (11) behave in very different fashions from each other. They come from the conventional shielding currents and from the zero-energy bound states. Growing with decreasing temperature, the diamagnetic screening currents monotonically reduce the penetration depth. On the other hand, Andreev surface-bound states respond paramagnetically and increase the penetration depth when the temperature goes down. Disregarding the broadening effects, Eq. (11) delivers the following estimate for the field of the low-temperature minimum of the penetration depth:

$$T_{m0} = \zeta \sqrt{\frac{\xi_0}{\lambda_0}} T_c, \qquad (13)$$

where ζ is of the order of unity for crystalline orientations with sufficient amount of momentum directions admitting zero-energy bound states. Otherwise ζ is a small quantity. For a d-wave superconductor $\zeta \propto \||\sin^3\beta| - |\cos^3\beta|\|^{1/2}$ and vanishes for $\beta = 45°$ (i.e., for $\alpha = 0$), when there are no zero energy bound states.

Broadening of the bound states can substantially modify the conditions for the presence of a minimum in the low-temperature dependence of the penetration depth. For a small broadening $\gamma(p_f) \ll T_c$ we simply replace the factor $1/\varepsilon_n$ in Eq. (9) for the polelike term with $1/[\varepsilon_n + \gamma(p_f)\text{sgn}(\varepsilon_n)]$. Taking into account the broadening Eq. (11) is generalized to the following form:

$$\lambda(T) = \lambda_0 + a\frac{T}{T_c}\lambda_0 + \frac{2e^2 N_f \lambda_0^2}{c^2 T}$$

$$\times \left\langle v_{f,y}^2(p_f)|v_{f,x}(p_f)|\Theta(p_f)\psi'\left(\frac{1}{2} + \frac{\gamma(p_f)}{2\pi T}\right)\right\rangle_{s_f}.$$

$$(14)$$

Here and below $\psi(x)$ is the digamma function and $\psi'(x)$ is its derivative.

Equation (14) is a reasonable representation of the role of a broadening in the low-temperature anomaly of the penetration depth. The minimum lies at $T_{m0} \approx 1.8\sqrt{\xi_0/\lambda_0}T_c$ for momentum independent broadening in a $d_{x^2-y^2}$ superconductor in the clean limit $\gamma \ll \pi T$ with the orientation $\alpha = 45°$. With increasing broadening it drifts to lower temperatures (becoming less pronounced at the same time) until $T_{m\gamma} \approx 0.4\sqrt{\xi_0/\lambda_0}T_c$ at $\gamma \approx 0.96 T_{m0}$, where it evaporates. As an
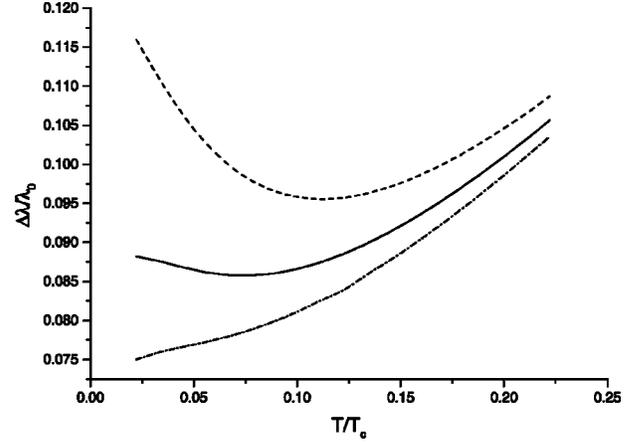

FIG. 1. Low-temperature correction to the penetration depth (in units of λ_0) in a $d_{x^2-y^2}$ superconductor with a cylindrical Fermi surface and the orientation $\alpha = 45°$. The temperature is measured in units of T_c. The parameter ξ_0/λ_0 is chosen to be ≈ 0.01, where $\xi_0 = (v_f/2\pi T_c)$. The curves are given for three values of the broadening: $\gamma = 0.10 T_c$ (dashed line), $\gamma = 0.15 T_c$ (solid line), and $\gamma = 0.19 T_c$ (dashed-dotted line).

example, the low-temperature correction to the penetration depth is shown in Fig. 1 in the vicinity of $T_{m\gamma}$ for various values of the momentum independent broadening.

There are various contributions to the broadening of the bound states associated, in particular, with surface roughness, nonmagnetic and magnetic impurities, and inelastic scattering. We now pin-point the origin of the broadening, assuming that nonmagnetic impurities dominate the scene.

With Born scatterers $\gamma_b \approx \sqrt{T_c/\tau}$ (see Ref. 29) and the coefficient of the order of unity can be estimated within the simple model of spatially constant order parameter. Then we easily get the shortest normal-state impurity mean free path l which admits a low-temperature upturn $\lambda_0 \lesssim l$. In high-temperature superconductors one should distinguish between l and the actual mean free path in the normal state at T_c incorporating significant contributions from inelastic processes. Impurity scattering dominates there at low temperatures already in the superconducting state where the collapse of inelastic scattering takes place. For instance, below 20 K in $YBa_2Cu_3O_{7-\delta}$ there is a regime of extremely long and weakly temperature dependent quasiparticle scattering times[30-35] usually interpreted as due to feeble impurity scattering in high-purity samples.

For scatterers with sufficient strength of impurity potential there are practically no restrictions on the impurity scattering rate in contrast to what was found above for Born impurities. For unitary scatterers with scattering rates $\Gamma_u \ll T_c$ the broadening of the zero-energy bound states is exponentially small:[29] $\gamma_u = B\sqrt{\Delta_0 \Gamma_u}\exp(-b\Delta_0/\Gamma_u)$. A scattering rate Γ_u which leads to a given broadening γ_u is almost independent of a constant coefficient B in the pre-exponential factor, while it is sensitive to the model dependent parameter b in the argument of the exponential function. Within the simple model considered in Ref. 29, one gets $b \sim 1$.

For temperatures $T \lesssim \sqrt{\Gamma_u \Delta_0}$, the share of the penetration depth from the shielding currents must be modified for unitary scatterers. This leads instead of the linear term in Eqs.

(8), (11), (12), (14) to a quadratic low-temperature correction of the form $\lambda_0 T^2/(\Gamma^{1/2}\Delta_0^{3/2})$ to within a factor the order of unity.[37,31] Correspondingly, T_{m0} given in Eq. (13) is valid for unitary scatterers only if $T_{m0} > \sqrt{\Gamma_u\Delta_0}$, which sets an upper limit on the scattering rate $\Gamma_u < (\xi_0/\lambda_0)\Delta_0$.

The T^2 term instead of the linear one in Eq. (11) delivers an estimate for the location T_{md} of the low-temperature minimum of the penetration depth modified by unitary scatterers

$$T_{md} \approx \left(\frac{\xi_0}{\lambda_0}\right)^{1/3} (\Gamma_u\Delta_0^5)^{1/6}, \quad \Gamma_u > \frac{\xi_0}{\lambda_0}\Delta_0. \quad (15)$$

This expression replaces Eq. (13) in the case of unitary scatterers with the scattering rate $\Gamma_u > (\xi_0/\lambda_0)\Delta_0$. The minimum slowly drifts to higher temperatures with increasing Γ_u. It does not melt away at any $\Gamma_u \ll T_c$. The normal-state impurity mean free path must just be large on the scale of the coherence length.

We conclude that observation of the low-temperature upturn of the penetration depth in samples with $l < \lambda_0$ is evidence for both Andreev bound states and a sufficiently large strength of the bulk impurity potential in the superconducting compounds. For unitary impurities one needs to take into account the broadening that arises from surface roughness which then very probably controls the total broadening. The same effect with Born scatterers demands the normal-state impurity mean free path larger than the London penetration depth.

III. ZERO ENERGY BOUND STATES AND SPONTANEOUS SURFACE CURRENT

Throughout this section the broadening of the zero-energy bound states is assumed small. We look at a clean d-wave superconductor with a smooth surface. Its crystal-to-surface orientation shall admit zero-energy surface bound states and feature an upturn in the penetration depth. Below the upturn temperature T_{m0}, imagine a great deal of space for λ to grow, first as described by the perturbative result Eq. (11). Then a second order phase transition occurs at $T \sim T_s \ll T_{m0}$ into a state which carries a spontaneous surface supercurrent.[25-27] We shall find an analytic expression for the transition temperature and discuss the impact of impurities on the effect. The transition implies the absence of subdominant channels activated at low temperatures close to the surface on account of the presumably large surface pair breaking in the dominant component of the order parameter. Otherwise a spontaneous current can arise at higher temperatures.[4,38,5,26]

There is experimental evidence[9] for a phase transition on the (110) surface in $YBa_2Cu_3O_{7-\delta}$ at $T=7$ K $\gg (\xi_0/\lambda_0)T_c$. It was interpreted as associated with an activated near surface subdominant channel of the order parameter.[5] For some other crystal-to-surface orientations, however, a subdominant component can be not present near a surface.[4,38] Zero-energy bound states can still arise for a noticeable part of quasiparticle trajectories. Our theoretical study is relevant to these cases.

In order to find an equation for the transition temperature, one has to admit a paramagnetic contribution to the penetration depth at least as large as the diamagnetic one. Then a

perturbation treatment of the preceding section is not adequate. In this context we develop an approach based on the integral form of Eq. (6) and take into account only the terms in $\delta Q(x,T)$ brought about by the bound states. In other words, a contribution only from the polelike term Eq. (9) needs to be taken into account in Eqs. (5) for the kernel which enters into Eq. (6). The kernel $\delta Q(x,T)$ varies on the characteristic scale ξ_0 and is associated in the clean limit[24] with large contributions to the magnetic field at the surface at temperatures $T \lesssim (\xi_0/\lambda_0)T_c$. We therefore disregard the nonlocal temperature correction from the Meissner current to $\delta Q(x,T)$.

We transform Eq. (6) into the integral form

$$A(x) = \left[A(0) - \frac{2\pi\lambda_0}{c} \int_0^{+\infty} dx' Q^{\text{bound}}(x',T)A(x') \right.$$

$$\left. \times (e^{x'/\lambda_0} - e^{-x'/\lambda_0}) \right] e^{-x/\lambda_0}$$

$$- \frac{2\pi\lambda_0}{c} \int_x^{+\infty} dx' Q^{\text{bound}}(x',T)A(x')$$

$$\times (e^{(x-x')/\lambda_0} - e^{-(x-x')/\lambda_0}). \quad (16)$$

The two terms on the right hand side of this equation obey very different scales. The first decays exponentially in the depth on the scale λ_0 while the last term vanishes for $x \gg \xi_0$ along with the kernel $Q^{\text{bound}}(x,T)$. The kernel $Q^{\text{bound}}(x=0,T)$ can be estimated (see preceding section) for a clean superconductor and a smooth surface as $2\pi\lambda_0^2 Q^{\text{bound}}(x=0,T)/c \sim T_c/T$. Then, in accordance with Eq. (16), the approximate formula $[1 - A(x_{\text{scr}})/A(0)]$ $\sim \xi_0^2 T_c/\lambda_0^2 T$ is established for a relative deviation of the vector potential $A(x_{\text{scr}})$ taken at the distance x_{scr} ($\xi \ll x_{\text{scr}} \ll \lambda_0$) from its value $A(0)$ at the surface. The deviation reflecting the bound state contribution to the vector potential turns out to be small at all temperatures $T \gg (\xi_0^2/\lambda_0^2)T_c$, in particular, for $T \sim T_s \sim (\xi_0/\lambda_0)T_c$. Varying on the scale ξ_0, small terms in the expression for the vector potential at temperatures $T \sim T_s$ are of importance only when differentiating $A(x)$. After that they can already noticeably contribute to the expression for the magnetic field.

Indeed, a spatial differentiation of Eq. (16) leads to

$$H(x) = -\frac{1}{\lambda_0} \left[A(0) - \frac{2\pi\lambda_0}{c} \int_0^{+\infty} dx' Q^{\text{bound}}(x',T)A(x') \right.$$

$$\left. \times (e^{x'/\lambda_0} - e^{-x'/\lambda_0}) \right] e^{-x/\lambda_0}$$

$$- \frac{2\pi}{c} \int_x^{+\infty} dx' Q^{\text{bound}}(x',T)A(x')$$

$$\times (e^{(x-x')/\lambda_0} + e^{-(x-x')/\lambda_0}). \quad (17)$$

The second term in the square brackets remains negligibly small $\sim (\xi_0^2 T_c/\lambda_0^2 T)A(0) \ll A(0)$ as compared with $A(0)$ for $T \gg (\xi_0^2/\lambda_0^2)T_c$. The last term of Eq. (17) is the order of $(\xi_0 T_c/\lambda_0 T)[A(0)/\lambda_0]$. For a deviation of the magnetic field at $x=x_{\text{scr}}$ from its value at $x=0$: $[H(x_{\text{scr}})/H(0)-1]$

$\sim (\xi_0 T_c / \lambda_0 T)$. Hence, the bound state contribution to the magnetic field can be viewed as a small perturbation as compared with the shielding contribution unless $T \lesssim T_s$. Considering $(\xi_0^2/\lambda_0^2)T_c \ll T \lesssim T_s$, we can discard the second term in the square brackets but have to keep track of the last term in Eq. (17). Choosing $x=0$ in Eq. (17), the small terms x'/λ_0 in the exponential functions under the integral sign can be taken to vanish. For the same reason and within the same accuracy one can treat the vector potential under the integral sign in Eq. (17) as constant in space $A(0)$ discarding small terms in the vector potential which vary on the scale ξ_0. All this results in an explicit relation between $A(0)$ and $H(0)$ and therefore

$$\lambda = \frac{\lambda_0}{1 + \frac{4\pi\lambda_0}{c} \int_0^{+\infty} Q^{\text{bound}}(x,T)dx}. \qquad (18)$$

Proceeding as in the derivation of Eq. (11) above, we find that the paramagnetic (negative) sign of Q^{bound} leads to a divergence of λ at the temperature

$$T_s = \frac{\pi^2 e^2 N_f \lambda_0}{c^2} \langle \text{v}_{f,y}^2(\boldsymbol{p}_f)|\text{v}_{f,x}(\boldsymbol{p}_f)|\Theta(\boldsymbol{p}_f)\rangle_{S_f}. \qquad (19)$$

For the model d-wave superconductor with a cylindrical Fermi surface one gets from Eq. (19)

$$T_s = \frac{\pi\xi_0}{3\lambda_0} ||\sin^3\beta| - |\cos^3\beta||T_c, \qquad (20)$$

where $\xi_0 = \text{v}_f / 2\pi T_c$.

The divergence of λ implies the existence of a nontrivial solution to Eq. (16) in a vanishing external magnetic field. Indeed, if we let $H(0)=0$, $A(0)\neq 0$, then Eq. (17) transforms, with the same approximation as above, into the relation $1 = -(4\pi\lambda_0/c)\int_0^{+\infty} Q^{\text{bound}}(x',T)dx'$, which defines the transition temperature T_s into a state with a spontaneous surface supercurrent.

The nontrivial solution at T_s is a result of interplay between the paramagnetic supercurrent which originates in the zero energy bound states localized within ξ_0 on the one hand and the diamagnetic supercurrent distributed over the region $x \sim \lambda_0$ on the other. The latter compensates for the magnetic field from the bound states at the surface in order to satisfy the boundary conditions in the absence of an external magnetic field. Then $\int_0^{+\infty} j(x)dx=0$ always applies being a consequence of the full screening of the spontaneous surface magnetic field in the bulk of a superconductor. Under this condition the Bloch theorem, in general, admits spontaneous surface currents.[39] The magnetic part of the superconducting free energy $(1/8\pi)\int_0^{+\infty}[A'^2(x)+(4\pi/c)Q(x,T)A^2(x)]dx$ vanishes at $T=T_s$ and becomes negative below T_s on account of negative sign of the paramagnetic kernel Q^{bound} (Gibbs and Helmholtz free energies coincide in zero external magnetic field). The result is an energetically favorable state with a spontaneous surface supercurrent below T_s.

The broadening γ of the bound states modifies Eq. (19):

$$T_s = \frac{2e^2 N_f \lambda_0}{c^2} \left\langle \text{v}_{f,y}^2(\boldsymbol{p}_f)|\text{v}_{f,x}(\boldsymbol{p}_f)|\Theta(\boldsymbol{p}_f)\psi'\left(\frac{1}{2} + \frac{\gamma(\boldsymbol{p}_f)}{2\pi T_s}\right)\right\rangle_{S_f}. \qquad (21)$$

The broadening prevents the appearance of a spontaneous surface current unless $\gamma \lesssim (\xi_0/\lambda_0)T_c$. This is a very strong restriction. If Born impurities control the broadening, they admit spontaneous surface supercurrent only with extremely large values of the mean free path $\lambda_0^2/\xi_0 \sim 100\lambda_0 \lesssim l$, unrealistic for high-temperature superconductors. Unitary scatterers impose a much weaker restriction $\Gamma_u \lesssim 2b\Delta_0 / \ln[\lambda_0^2/\xi_0^2 \ln(\lambda_0/\xi_0)] \sim 0.1T_c$. Then, however, surface roughness probably dominates the broadening and can destroy the state with a spontaneous surface supercurrent.

IV. NONLINEAR MEISSNER EFFECT FROM LOW ENERGY BOUND STATES

It is important in the derivation of Eq. (18) that the kernel Q^{bound} varies much faster in space than the screening currents. Then contributions of the paramagnetic current carried by surface Andreev-bound states at temperatures $(\xi_0/\lambda_0)^2 T_c \ll T$, can result in significant spatial variations of the magnetic field near the surface while in the weakly spatially dependent vector potential. This leads to Eq. (18) on the basis of the local current-field relation.

It is straightforward to show within the same framework that a nonlinear penetration depth $\lambda_{nl}(T,H)$ incorporating contributions both from screening currents and from zero-energy bound states is described as

$$\lambda_{nl}(T,H) = \frac{\lambda_{nl}^{\text{scr}}(T,H_{\text{scr}})}{1 + \frac{4\pi\lambda_{nl}^{\text{scr}}(T,H_{\text{scr}})}{c} \int_0^{+\infty} Q_{nl}^{\text{bound}}(x,T)dx}, \qquad (22)$$

where $\lambda_{nl}^{\text{scr}}(T,H_{\text{scr}})$ is a contribution from screening supercurrents to $\lambda_{nl}(T,H)$, taken at an effective value of the field

$$H_{\text{scr}} = H(0) - (4\pi/c)\int_0^{+\infty} H(x)dx \int_0^{+\infty} Q_{nl}^{\text{bound}}(x',T)dx'.$$

Here $H(0)$ is the external magnetic field. The second term describes the field of the zero-energy bound states inside the superconductor at distances $x \approx x_{\text{scr}}$ ($\xi_0 \ll x_{\text{scr}} \ll \lambda_0$), as can be seen in Eq. (17). A paramagnetic response of zero-energy bound states ($Q_{nl}^{\text{bound}} < 0$) increases the field to be screened by diamagnetic supercurrents $[H_{\text{scr}} \equiv H(x_{\text{scr}}) > H(0)]$. This leads, in general, to more pronounced nonlinear terms in $\lambda_{nl}^{\text{scr}}(T,H_{\text{scr}})$ as compared to disregarding the contribution from zero-energy bound states. In the case of spontaneous surface supercurrent H_{scr} differs from zero even in the absence of an external field. We assume the condition $H_{\text{scr}} < H_{c1}$ for the Meissner state to be stable in the magnetic field on account of a paramagnetic influence of the bound states.

Small nonlocal low-temperature corrections to the penetration depth from screening currents can be taken into account in Eq. (22) as perturbations to $\lambda_{nl}^{\text{scr}}(T,H)$. For a non-

local current-field relation a penetration depth $\lambda_{nl}(T,H)$ is actually a functional of the spatial profile of the magnetic field.

Nonlinear corrections from the shielding supercurrent to the Meissner effect can be given in terms of the dimensionless ratio $\rho=(H/H_0)$, where H_0 is usually the order of the thermodynamic critical field $\sim\Phi_0/(\lambda_0\xi_0)$. Hence, they are always small in strong type-II superconductors in the Meissner state. In isotropic s-wave superconductors, the first nonlinear correction to the penetration depth $\propto\rho^2$. In superconductors with nodes in the order parameter (for instance, d-wave) a term linear in ρ can arise for particular crystal orientations at low temperatures.[40,41] The linear term, however, is quite sensitive to nonlocal effects[42] and the impurity influence, in particular, at sufficiently large strength of impurity potentials.[41,43]

A nonlinearity in the magnetic response of low energy Andreev surface bound states has, in general, a very different field scale $\tilde{H}_0$. In a clean limit $\tilde{H}_0(T)=(\Phi_0 T/\lambda v_f)$, where λ is determined by Eq. (22). At low temperatures $T\ll T_c$ one always gets $\tilde{H}_0(T)\ll H_0$. For instance, $\tilde{H}_0(T_{m0})\sim\sqrt{(\xi_0/\lambda_0)}H_0\sim0.1H_0$, $\tilde{H}_0(T_s)\sim(\lambda_0/\lambda)H_{c1}\lesssim H_{c1}\sim0.01H_0$. Moreover, at sufficiently low temperatures $T\ll T_s$, i.e., well below the transition to the state with a spontaneous surface supercurrent, a paramagnetic response from the bound states may become seriously nonlinear already in the Meissner state $\tilde{H}_0(T)\sim H<H_{c1}$.[25,27] We will show, however, that the broadening of the bound states introduces a new field scale $\tilde{H}_\gamma=(\gamma\lambda_0/T_c\lambda)H_0$ coming into play at $\pi T\lesssim\gamma$. For $\gamma\gg(\xi_0/\lambda_0)T_c\sim0.01T_c$ nonlinear corrections from Andreev low-energy bound states to the penetration length turn out always to be small in the Meissner state, even at $T=0$.

As a polelike term Eq. (9) decays exponentially on the scale $\sim\xi_0$ for almost all momentum directions admitting bound states, we consider a local nonlinear current-field relation

$$j(x)=2eTN_f\sum_{\varepsilon_n}\left\langle v_f g_s\left(p_f,x,\varepsilon_n-i\frac{e}{c}v_{f,y}A(x)\right)\right\rangle_{S_f}$$

$$(23)$$

for the current via the bound states. One can also set the vector potential $A(x)$ in the kernel equal to $A(x=0)$. Then we easily generalize the reasoning in the derivation of the third term in Eq. (14). Substituting into Eq. (23) the expression Eq. (9) for the polelike term with the pole shifted in accordance with the broadening, we find

$$\int_0^\infty Q_{nl}^{\text{bound}}(x,T)dx$$

$$=\frac{ieN_f}{A(0)}\left\langle v_{f,y}(p_f)|v_{f,x}(p_f)|\Theta(p_f)\right.$$

$$\times\left.\psi\left(\frac{1}{2}+\frac{\gamma(p_f)+i(e/c)v_{f,y}(p_f)A(0)}{2\pi T}\right)\right\rangle_{S_f}.$$

$$(24)$$

In Eq. (24) the Fermi surface is assumed symmetric in reflections across the xz plane. Then averages over the Fermi surface of odd powers of $v_{f,y}$ vanish, no matter whether they are multiplied by $|v_{f,x}(p_f)|\Theta(p_f)$ or not. This applies, in particular, to a d-wave superconductor with a cylindrical Fermi surface whose principal axis z is parallel to the boundary for arbitrary orientations of the two other crystal axes x_0, y_0.

If $(e/c)v_f A(0)\ll\max(2\pi T,\gamma)$, one can expand the ψ function in Eq. (24) in powers of the small parameter $\min[H(0)/\tilde{H}_0(T),H(0)/\tilde{H}_\gamma]$. Considering nonlinear corrections to the penetration depth from screening currents $\Delta\lambda_{nl}^{\text{scr}}$ and bound states $\Delta\lambda_{nl}^b$ to be small, one can represent them in the first approximation as additive contributions to the total penetration depth $\lambda_{nl}(T,H)\approx\lambda(T)+\Delta\lambda_{nl}^{\text{scr}}+\Delta\lambda_{nl}^b$. The nonlinear correction from screening currents takes the form

$$\Delta\lambda_{nl}^{\text{scr}}=\frac{\lambda^2(T)}{\lambda_0^2(T)}\left[\lambda_{nl}^{\text{scr}}\left(T,H(0)\frac{\lambda(T)}{\lambda_0(T)}\right)-\lambda_0(T)\right].$$

Quantities $\lambda(T)$ and $\lambda_0(T)$ being the zero-field values of $\lambda_{nl}(T,H)$, $\lambda_{nl}^{\text{scr}}(T,H)$, respectively, satisfy Eq. (18). Bound states renormalize nonlinear response from screening currents already in this approximation. Thus, the explicit analysis of $\Delta\lambda_{nl}^{\text{scr}}$ can be done combining the results of the preceding section and Refs. 41–43. Apart from too close to the transition temperature T_s, the nonlinear correction to the penetration depth from the bound states is

$$\Delta\lambda_{nl}^b\approx-\frac{e^4\lambda^4(T)N_f H^2(0)}{12\pi^2 c^4 T^3}\left\langle v_{f,y}^4(p_f)|v_{f,x}(p_f)|\Theta(p_f)\right.$$

$$\times\left.\psi^{(3)}\left(\frac{1}{2}+\frac{\gamma(p_f)}{2\pi T}\right)\right\rangle_{S_f}.$$

$$(25)$$

In the limit $(e/c)v_f A(0)\gg2\pi T$ when the argument of the ψ function in Eq. (24) is large, we obtain

$$\int_0^\infty Q_{nl}^{\text{bound}}(x,T)dx=-\frac{eN_f}{A(0)}\left\langle v_{f,y}(p_f)|v_{f,x}(p_f)|\right.$$

$$\times\left.\Theta(p_f)\arctan\left(\frac{ev_{f,y}(p_f)A(0)}{c\gamma(p_f)}\right)\right\rangle_{S_f}.$$

$$(26)$$

Then the broadening rather than the temperature fixes the bound state contribution to the penetration depth.

As shown above, there is no state with a spontaneous surface current with $\gamma\gg T_s$. Then $\lambda\sim\lambda_0$ and $ev_f A(0)/c\gamma\ll H(0)/H_{c1}$. Since $H(0)<H_{c1}$ in the Meissner state we estimate $ev_f A(0)/c\gamma\ll1$ and obtain in this limit from Eq. (26)

$$\Delta\lambda_\gamma^b=\frac{4\pi e^2 N_f\lambda_0^2}{c^2\gamma}\langle v_{f,y}^2(p_f)|v_{f,x}(p_f)|\Theta(p_f)\rangle_{S_f}$$

$$-\frac{4\pi e^4\lambda_0^4 N_f H^2(0)}{3c^4\gamma^3}\langle v_{f,y}^4(p_f)|v_{f,x}(p_f)|\Theta(p_f)\rangle_{S_f}.$$

$$(27)$$

For a given $\lambda_{nl}^{\rm scr}(T,H)$, Eq. (22) in general should be solved with respect to $\lambda_{nl}(T,H)$ in accordance with Eq. (24), since $A(0)=-\lambda_{nl}(T,H)H(0)$. This is particularly important close to the transition temperature T_s, where the Landau theory of second order phase transitions is applicable. Then first nonlinear term turns out to be the order of the zero-field paramagnetic contribution in the denominator in Eq. (22). Ignoring a weak field dependence of $\lambda_{nl}^{\rm scr}(T,H)$ stipulated by screening currents, we obtain from Eqs. (22), (24) the following equation for $\lambda_{nl}(T,H)$:

$$\left(\frac{T}{T_s}-1\right)\kappa\lambda_{nl}(T,H)+\eta H^2\lambda_{nl}^3(T,H)=\lambda_{nl}^{\rm scr}(T_s), \quad (28)$$

where $H=H(0)$,

$$\eta=\frac{e^4\lambda_{nl}^{\rm scr}(T_s)N_f}{12\pi^2c^4T_s^3}$$

$$\times\left\langle v_{f,y}^4(\boldsymbol{p}_f)|v_{f,x}(\boldsymbol{p}_f)|\Theta(\boldsymbol{p}_f)\right.$$

$$\times\psi^{(3)}\left.\left(\frac{1}{2}+\frac{\gamma(\boldsymbol{p}_f)}{2\pi T_s}\right)\right\rangle_{S_f},$$

$$\kappa=1-\left.\frac{d\tilde{T}_s(T)}{dT}\right|_{T=T_s}.$$

T_s is described by Eq. (21) and $\tilde{T}_s(T)$ is the result of the substitution $T_s\rightarrow T$ on the right hand side of Eq. (21). The broadening is assumed to be sufficiently small for admitting the phase transition.

The role of the order parameter in the phase transition can be played by a surface magnetization

$$m_S=\int_0^{+\infty}[M(x)-M_\infty]dx=(1/c)\int_0^{+\infty}dxxj_s(x)$$

$$=(1/4\pi)\int_0^{+\infty}H(x)dx=(1/4\pi)\lambda_{nl}(T,H)H(0),$$

which, for simplicity, we choose constant in space along a smooth surface. The magnetization $\mathbf{M}$ enters by the conventional definition $\mathbf{j}=c\,{\rm curl}\,\mathbf{M}$, and $M_\infty=-H(0)/4\pi$. Then the Landau free energy per unit surface $\mathcal{F}_S$ which leads to the same equation for m_S as implied in Eq. (28) has the form

$$\mathcal{F}_S=\tilde{\alpha}\left(\frac{T}{T_s}-1\right)m_S^2+\tilde{\beta}m_S^4-m_SH, \quad (29)$$

where $\tilde{\alpha}=2\pi\kappa/\lambda_{nl}^{\rm scr}(T_s)$, $\tilde{\beta}=16\pi^3\eta/\lambda_{nl}^{\rm scr}(T_s)$, H is the external field. As for a conventional order parameter in a strong field near T_s one gets $m_S(T_s,H)\propto H^{1/3}$, which entails $\lambda_{nl}(T_s,H)\propto H^{-2/3}$.

Finally, in the limit of very small broadening $\gamma\ll[H(0)/H_0]T_c$, Eqs. (22) and (26) give

$$\Delta\lambda_H^b=\lambda_{nl}(T,H)-\lambda_{nl}^{\rm scr}(T,H)$$

$$=\frac{4\pi eN_f\lambda_0}{c|H(0)|}\langle|v_{f,y}(\boldsymbol{p}_f)v_{f,x}(\boldsymbol{p}_f)|\Theta(\boldsymbol{p}_f)\rangle_{S_f} \quad (30)$$

at temperatures $(\xi_0^2/\lambda_0^2)T_c\ll T\ll[H(0)/H_0]T_c$. Since $\Delta\lambda_H^b$ is at least the order of λ_0, we put here $\lambda_{nl}^{\rm scr}(T,H)\approx\lambda_0$ disregarding small nonlinear corrections from the screening currents in the Meissner state. The approximate inverse proportionality of the penetration length the magnetic field implies the presence of a spontaneous surface magnetization weakly dependent on H.

V. CONCLUSION

We have examined the paramagnetic contribution from surface zero-energy Andreev bound states to the low-temperature penetration length of d-wave superconductors in the Meissner state. The paramagnetic current is localized within several coherence lengths near the surface and grows larger in the clean limit when the temperature goes down. A broadening of the bound states chokes their contribution and determines their actual role in shaping the penetration length. We found that the upturn in the low-temperature penetration depth lies at $T_{m0}\sim\sqrt{\xi_0/\lambda_0}T_c$ in the clean limit where the paramagnetic contribution from the bound states can be handled with perturbation theory same as small low-temperature corrections to the penetration depth from the screening current. The minimum broadening capable of straightening out the upturn is $\gamma\approx T_{mo}$.

Furthermore, we examined the penetration depth when the bound states must be kept track of beyond perturbation theory. A divergence of $\lambda(T)$ was found at the phase transition to a state with spontaneous surface supercurrent. This transition occurs only with smallish broadening, $\gamma<(\xi_0/\lambda_0)T_c$. In the clean limit[24] and at low temperatures, there is a nonlinear regime of the paramagnetic current already in magnetic fields substantially weaker than the fields for the nonlinear effects to show up in response of shielding supercurrents. The broadening of the bound states modifies and weakens the nonlinear response.

Specifying an origin of the broadening as associated with nonmagnetic impurity scattering, we obtained restrictions on the mean free path admitting the low-temperature anomalies. The conditions turn out to be sensitive to the strength of the impurity potential and very different in the unitary and in the Born limits. The Born impurities are shown to easily prevent the anomalies of the penetration depth taking place at least well below T_{m0}. By contrast, unitary scatterers with sufficiently small normal-state scattering rate $\Gamma_u\ll T_c$ admit the transition to a state with spontaneous surface supercurrent at $T_s\sim(\xi_0/\lambda_0)T_c$. In the latter case, however, surface roughness very probably dominates the broadening and controls the bound state contribution to the low-temperature penetration length.

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