Collective modes and sound propagation in a *p*-wave superconductor: Sr_2RuO_4

Hae-Young Kee,[1] Yong Baek Kim,[2] and Kazumi Maki[3]
[1]*Department of Physics, University of California, Los Angeles, California 90095*
[2]*Department of Physics, The Ohio State University, Columbus, Ohio 43210*
[3]*Department of Physics, University of Southern California, Los Angeles, California 90089*
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There are five distinct collective modes in the recently discovered *p*-wave superconductor Sr_2RuO_4; phase and amplitude modes of the order parameter, clapping mode (real and imaginary), and spin wave. The first two modes also exist in the ordinary *s*-wave superconductors, while the clapping mode with the energy $\sqrt{2}\Delta(T)$ is unique to Sr_2RuO_4 and couples to the sound wave. Here we report a theoretical study of the sound propagation in a two-dimensional *p*-wave superconductor. We identified the clapping mode and study its effects on the longitudinal and transverse sound velocities in the superconducting state. In contrast to the case of ^{3}He, there is no resonance absorption associated with the collective mode, since in metals $\omega/(v_F|\mathbf{q}|)\ll 1$, where v_F is the Fermi velocity, $\mathbf{q}$ is the wave vector, and ω is the frequency of the sound wave. However, the velocity change in the collisionless limit gets modified by the contribution from the coupling to the clapping mode. We compute this contribution and comment on the visibility of the effect. In the diffusive limit, the contribution from the collective mode turns out to be negligible. The behaviors of the sound velocity change and the attenuation coefficient near T_c in the diffusive limit are calculated and compared with the existing experimental data wherever it is possible. We also present the results for the attenuation coefficients in both of the collisionless and diffusive limits at finite temperatures.

I. INTRODUCTION

Shortly after the discovery of superconductivity in Sr_2RuO_4, the possibility of spin triplet pairing was discussed.[1] Possible pairing symmetries were also classified based on the crystal symmetry.[2] On the experimental front, there have been attempts to single out the right pairing symmetry among these possibilities. Recent measurement of ^{17}O-Knight shift in NMR for the magnetic field parallel to the *a*-*b* plane showed no change across T_c, which can be taken as the evidence of the spin triplet pairing with the $\hat{d}$ vector parallel to the *c* axis.[3] Here $\hat{d}$ is called the spin vector which is perpendicular to the direction of the spin associated with the condensed pair.[4] μSR experiment found spontaneous magnetic field in the superconducting Sr_2RuO_4, which seems to indicate broken time reversal symmetry in the superconducting state.[5] These experiment may be compatible with the following order parameter:[2]

$$\hat{\Delta}(\mathbf{k})=\Delta\hat{d}(k_1\pm ik_2), \tag{1}$$

where Δ is the magnitude of the superconducting order parameter. Notice that this state is analogous to the *A* phase of ^{3}He and there is a full gap on the Fermi surface.

On the other hand, there also exist experiments that cannot be explained by a naive application of the order parameter given by Eq. (1). Earlier specific heat measurement found residual density of states at low temperatures below T_c,[6] which provokes the ideas of orbital dependent superconductivity[7] and even a nonunitary superconducting state.[8] However, more recent specific heat experiment on a cleaner sample reports no residual density of states and it was found that the specific heat behaves as T^2 at low temperatures.[9] This result stimulated a speculation about different order parameters with line node.[10] However, since there are three bands labeled by α, β, and γ which cross the Fermi surface, it is not yet clear whether the order parameter given by Eq. (1) is compatible with more recent specific heat data or not. For example, it is possible that the pairing symmetry associated with the γ band is still given by Eq. (1) while the order parameter symmetry associated with α and β bands can be quite different. In this case, the low-temperature specific heat will be dominated by the excitations from α and β bands. In order to resolve the issue, it is important to examine other predictions of the given order parameter and compare the results with future experiments.

One way of identifying the correct order parameter among possible candidates is to investigate the unique collective modes supported by the ground state with a given pairing symmetry. The observation of the effects of these collective modes would provide convincing evidence for a particular order parameter symmetry. If we assume that the order parameter of Eq. (1) is realized in Sr_2RuO_4, the superconducting state would support unique collective modes, the so-called clapping mode and spin waves as well as the phase and amplitude modes of the order parameter which exist also in *s*-wave superconductors. Previously we studied the dynamics of spin waves.[11,12] A possible way to distinguish the order parameter of the γ band from those of the α (or β) band was also proposed in the context of spin wave dynamics.[13]

In this paper, we study the dynamics of the sound wave and its coupling to the clapping modes assuming that the order parameter is given by Eq. (1). As in ^{3}He, only the clapping mode can couple to the sound wave and affects its dynamics. Here we study the sound velocities and attenuation coefficients of the longitudinal and transverse sound waves. In particular, we identify the clapping mode with the

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frequency $\omega=\sqrt{2}\Delta(T)$, and examine the effects of this mode and disorder on the sound wave propagation.

In a recent paper, Higashitani and Nagai[14] obtained the clapping mode with the frequency $\omega=\sqrt{2}\Delta$, and discussed the possible coupling to the sound wave independent of us. However, as we shall see the coupling to the sound wave is extremely small, because $C/v_F \ll 1$ in metals. Here C is the sound velocity. Indeed the recent measurement of the sound velocity in the normal and superconducting states of Sr_2RuO_4 reported in Ref. 15 shows that $C/v_F \ll 1$. They measured the sound velocities of the longitudinal modes C_{11} ($\mathbf{q},\mathbf{u}\|[100]$) and C_{33} ($\mathbf{q},\mathbf{u}\|[001]$) and the transverse modes C_{44} ($\mathbf{q}\|[100]$, $\mathbf{u}\|[001]$) and C_{66} ($\mathbf{q}\|[100]$, $\mathbf{u}\|[010]$), where $\mathbf{q}$ and $\mathbf{u}$ are the directions of propagation and the polarization of ultrasound, respectively. They found that the longitudinal sound velocities, C_{11} and C_{33}, decrease with a kink at $T=T_c$, while the transverse sound velocities do not exhibit any effect of the onset of superconductivity. We estimate from their experimental data that $C_l/v_F \sim 10^{-2}$, where C_l is the longitudinal sound velocity. It can be also seen that the transverse sound velocity, C_t, is much smaller than the longitudinal one.[15]

Incorporating the correct limit $C/v_F \ll 1$, we obtained the sound velocities and attenuation coefficients for both collisionless and diffusive limits. In the diffusive limit, the quasiparticle scattering due to impurities should be properly taken into account. One can show that, in a metal such as Sr_2RuO_4, the collisionless limit is rather difficult to reach because it can be realized only for $\omega \sim O(1)$ GHz. For more practical range of frequencies, kHz−MHz, the diffusive limit may be easier to achieve. On the other hand, we found that it is much easier to see the effects of the coupling between the sound waves and the clapping mode in the collisionless limit. Therefore it is worthwhile to study both regimes. Here we summarize our main results.

A. Collisionless limit

In the absence of the coupling to the clapping mode, the longitudinal sound velocity decreases in the superconducting state because the effect of the screening of the Coulomb potential increases, which happens in the s-wave superconductors as well. However, one of the important features of the p-wave order parameter in consideration is that the sound wave can now couple to the clapping mode. This effect is absent in s-wave superconductors. One can show that, among longitudinal waves, C_{11} mode can couple to the clapping mode, but C_{33} mode cannot. We found that the longitudinal sound velocity C_{11} decreases as

$$\frac{\delta C_l^{11}}{C_l^{11}} = -\lambda_l^{11}\left[\frac{1}{2}-2\left(\frac{C_l^{11}}{v_F}\right)^2 \times\left(1-f-\frac{f}{4\{1+(2\Delta(T)/v_F q)^2\}}\right)\right], \tag{2}$$

where λ_l is the couping constant and f is the superfluid density. $\delta C_l/C_l$ is the relative shift in the sound velocity. We estimated the frequency regime where one can observe the effect of the clapping mode and found that the effect is visible if $v_F|\mathbf{q}|\sim 2-3\Delta(0)$. This implies that $\omega \sim O(1)$ GHz. Since C_{33} does not couple to the clapping mode, the velocity change is simply given by

$$\frac{\delta C_l^{33}}{C_l^{33}} = -\lambda_l^{33}\left[\frac{1}{2}-2\left(\frac{C_l^{33}}{v_F}\right)^2(1-f)\right]. \tag{3}$$

Since the velocity of the transverse wave is much smaller than that of the longitudinal one and the coupling to the electron system is weaker than the longitudinal case as well, we expect that the change of the transverse sound velocity is hard to observe. In order to complete the discussion, we also present the results for these small changes in transverse velocities. Here only C_{66} mode couples to the clapping mode, and C_{44} mode does not. We found

$$\frac{\delta C_t^{66}}{C_t^{66}} = -\lambda_t^{66}\left[\frac{1}{2}+2\left(\frac{C_t^{66}}{v_F}\right)^2\left(1-f+\frac{f}{4\{1+(2\Delta/v_F q)^2\}}\right)\right], \tag{4}$$

$$\frac{\delta C_t^{44}}{C_t^{44}} = -\lambda_t^{44}\left[\frac{1}{2}+2\left(\frac{C_t^{44}}{v_F}\right)^2(1-f)\right], \tag{5}$$

where the λ_t is the transverse coupling constant. We also found that the leading contribution to the attenuation coefficient is the same as that of s-wave superconductors in the collisionless limit.

B. Diffusive limit

This case corresponds to $\omega, v_F|\mathbf{q}| \ll \Gamma$, where Γ is the scattering rate due to impurities. As in the case of the collisionless limit, in principle C_{11} and C_{66} modes couple to the clapping mode, but it turns out that the effect is almost impossible to detect. Neglecting the coupling to the clapping mode and working in the limit $4\pi T_c \gg 2\Gamma \gg v_F|\mathbf{q}|,\omega$, we obtain the following results near T_c:

$$\frac{\delta C_l}{C_l} = -\lambda_l \frac{1}{2}\left\{1-\left(\frac{\omega}{2\Gamma}\right)^2\left[1-\frac{4\pi^3}{7\zeta(3)}\frac{T}{\Gamma}\left(1-\frac{T}{T_c}\right)\right]\right\},$$

$$\frac{\alpha_l}{\alpha_l^n} = 1-\frac{2\pi^3}{7\zeta(3)}\frac{T}{\Gamma}\left(1-\frac{T}{T_c}\right), \tag{6}$$

where α and α_n are the attenuation coefficients in the superconducting and the normal states, respectively. The shift of the sound velocity and attenuation coefficient decrease linearly in $(1-T/T_c)$ as $T\to T_c$. This result for the longitudinal sound wave is consistent with the experimental observation reported in Ref. 15.

In the case of the transverse sound waves, the leading behaviors of the sound velocity and the attenuation coefficient can be obtained simply by replacing C_l and λ_l by C_t and λ_t in the diffusive limit. However, the absolute value of the transverse sound velocity is much smaller than the longitudinal one and the coupling to the electron system is also much weaker than the case of the longitudinal sound waves. Thus, it would be hard to observe any change at $T=T_c$ for the transverse wave. This may explain the experimental finding that the transverse velocity does not show any change across T_c. We also obtained the attenuation coefficient for all temperatures below T_c. It is given by Eq. (36) and Fig. 2 shows its behavior.

The rest of the paper is organized as follows. In Sec. II, the clapping mode is briefly discussed. In Sec. III, we make a short summary of the result in Ref. 16 to explain how the sound velocity and the attenuation coefficient are related to autocorrelation function of stress tensor, which help for readers to understand the physical quantities that we present in this paper. Then we show the results of the study on the sound propagation in the collisionless and diffusive limits in Secs. IV and V, respectively. We conclude in Sec. VI. Further details which are not presented in the main text are relegated to Appendices A and B.

II. COLLECTIVE MODES IN Sr_2RuO_4

As in the *s*-wave superconductors, the phase and amplitude modes of the order parameter also exist in the *p*-wave superconductors. On the other hand, due to the internal structure of the Cooper pair in the *p*-wave superconductor, there exist other types of collective mode associated with the order parameter. The nature of these modes is determined by the structure of the order parameter.

There are collective modes associated with the oscillation of the spin vector $\hat{d}$, which we have already discussed in Refs. 11 and 12. There exists another collective mode associated with the orbital part. Using the notation $e^{\pm i\phi}=(k_1 \pm ik_2)/|\mathbf{k}|$, the oscillation of the orbital part $e^{\pm i\phi}\to e^{\mp i\phi}$ gives rise to the clapping mode with $\omega=\sqrt{2}\Delta(T)$. This mode couples to the sound waves as we will see in the next section. Therefore, the detection of the clapping mode will provide a unique evidence for the *p*-wave superconducting order parameter. The derivation of the clapping mode and the coupling to the sound wave is discussed in Appendix A.

III. DYNAMICS OF SOUND WAVE VIA STRESS TENSOR

In ordinary liquids, the sound wave is a density wave. In superconductors, the density is not only coupled to the longitudinal component of the normal velocity, but also to the superfluid velocity and to temperature. The role of these couplings and their consequences in the dynamics of sound wave can be studied by looking at the autocorrelation function $\langle[\tau_{ij},\tau_{ij}]\rangle$ of stress tensor τ_{ij}:

$$\langle[\tau_{ij},\tau_{ij}]\rangle(\mathbf{r}-\mathbf{r}',t-t')\equiv -i\theta(t-t')\langle[\tau_{ij}(\mathbf{r},t),\tau_{ij}(\mathbf{r}',t')]\rangle, \tag{7}$$

where

$$\tau_{ij}(\mathbf{r},t)=\sum_{\sigma}\left[\frac{(\nabla-\nabla')_i}{2i}\frac{(\nabla-\nabla')_j}{2im}\psi^{\dagger}_{\sigma}(\mathbf{r},t)\psi_{\sigma}(\mathbf{r}',t)\right]_{\mathbf{r}'=\mathbf{r}}. \tag{8}$$

Here $\psi^{\dagger}_{\sigma}$ is the electron creation operator with spin σ.

The other operators whose correlation functions are needed for the ultrasonic attenuation and the sound velocity change are the density operator

$$n(\mathbf{r},t)=\sum_{\sigma}\psi^{\dagger}_{\sigma}(\mathbf{r},t)\psi_{\sigma}(\mathbf{r},t) \tag{9}$$

and the current operator

$$\mathbf{j}(\mathbf{r},t)=\sum_{\sigma}\left[\frac{(\nabla-\nabla')_j}{2im}\psi^{\dagger}_{\sigma}(\mathbf{r},t)\psi_{\sigma}(\mathbf{r}',t)\right]_{\mathbf{r}'=\mathbf{r}}. \tag{10}$$

Assuming that the wave vector of the sound wave is in the $\hat{\mathbf{x}}$ direction, $\mathbf{q}=q\hat{x}$, the sound velocity shift, δC, at low frequencies can be computed from

$$\frac{\delta C_l}{C_l}=\frac{C_l(\omega)-C_l}{C_l}\bigg|_{\omega=C_l|\mathbf{q}|}=-\frac{\omega}{m_{\mathrm{ion}}C_l|\mathbf{q}|}\mathrm{Re}\langle[h_l,h_l]\rangle(\mathbf{q},\omega)\bigg|_{\omega=C_l|\mathbf{q}|},$$

$$\frac{\delta C_t}{C_t}=\frac{C_t(\omega)-C_t}{C_t}\bigg|_{\omega=C_t|\mathbf{q}|}=-\frac{\omega}{m_{\mathrm{ion}}C_t|\mathbf{q}|}\mathrm{Re}\langle[h_t,h_t]\rangle(\mathbf{q},\omega)\bigg|_{\omega=C_t|\mathbf{q}|}, \tag{11}$$

where

$$h_l(\mathbf{r},t)=\frac{q}{\omega}\tau_{xx}(\mathbf{q},t)-\frac{\omega m}{q}n(\mathbf{r},t),$$

$$h_t(\mathbf{r},t)=\frac{q}{\omega}\tau_{xy}(\mathbf{q},t)-mj_y(\mathbf{r},t). \tag{12}$$

Here C_l and C_t represent the longitudinal and transverse sound velocities in the normal state, respectively. m_{ion} and m are the mass of ions and the mass of electron, respectively. On the other hand, the attenuation coefficient α at low frequencies is obtained from

$$\alpha_l=\frac{\omega}{m_{\mathrm{ion}}C_l}\mathrm{Im}\langle[h_l,h_l]\rangle(\mathbf{q},\omega)\bigg|_{\omega=C_l|\mathbf{q}|},$$

$$\alpha_t=\frac{\omega}{m_{\mathrm{ion}}C_t}\mathrm{Im}\langle[h_t,h_t]\rangle(\mathbf{q},\omega)\bigg|_{\omega=C_t|\mathbf{q}|}. \tag{13}$$

These relations are extensively discussed in the work of Kadanoff and Falko.[16]

IV. SOUND PROPAGATION IN THE COLLISIONLESS LIMIT

As discussed in the Introduction, in a metal such as Sr_2RuO_4 the collisionless limit is somewhat difficult to reach because we need the sound wave with the frequency $\omega \sim O(1)$ GHz. However, we will also see that this is the regime where one has the best chance to observe the effect of the collective mode.

In superconductors the density wave is not only coupled to the longitudinal component of the normal velocity, but also to the superfluid velocity and to temperature. The role of these couplings and their consequences in the dynamics of sound wave can be studied by looking at the autocorrelation function $\langle[\tau_{ij},\tau_{ij}]\rangle$ of stress tensor τ_{ij} presented in the previous section.

We will use the finite temperature Green's function technique[17] to compute these correlation functions. The single particle Green's function $G(i\omega_n,\mathbf{k})$ in the Nambu space is given by

$$G^{-1}(i\omega_n,\mathbf{k})=i\omega_n-\xi_{\mathbf{k}}\rho_3-\Delta(\hat{k}\cdot\hat{\rho})\sigma_1, \tag{14}$$

where ρ_i and σ_i are Pauli matrices acting on the particle-hole and spin space, respectively, $\omega_n=(2n+1)\pi T$ is the fermionic Matsubara frequency, and $\xi_{\mathbf{k}}=\mathbf{k}^2/2m-\mu$. Then, for example, the irreducible correlation function can be computed from

$$\begin{aligned}\langle[\tau_{ij},\tau_{ij}]\rangle_{00}(i\omega_\nu,\mathbf{q})&=T\sum_n\sum_{\mathbf{p}}\mathrm{Tr}\Bigg[\left(\frac{p_ip_j}{m}\right)^2\rho_3G(\mathbf{p},\omega_n)\\&\quad\times\rho_3G(\mathbf{p}-\mathbf{q},i\omega_n-i\omega_\nu)\Bigg],\end{aligned} \tag{15}$$

where $\omega_\nu=2\nu\pi T$ is the bosonic Matsubara frequency.

A. Longitudinal sound wave

Let us consider the longitudinal wave with $\mathbf{u}\|\mathbf{q}\|\mathbf{x}$, which corresponds to the C_{11} mode. Since the stress tensor couples to the density, the autocorrelation function $\langle[h_l,h_l]\rangle$ is renormalized by the Coulomb interaction. In the long wavelength limit and for $s\equiv\omega/v_F|\mathbf{q}|\ll1$, the renormalized correlation function $\langle[h_l,h_l]\rangle_0$ can be reduced to

$$\begin{aligned}\mathrm{Re}\langle[h_l,h_l]\rangle_0&\approx\left(\frac{q}{\omega}\right)^2\mathrm{Re}\frac{p_F^4}{4m^2}\langle\cos(2\phi),\cos(2\phi)\rangle\\&=\frac{p_F^4}{4m^2C_l^2}N(0)\left[\frac{1}{2}-2s^2(1-f)\right],\end{aligned} \tag{16}$$

where

$$\langle A,B\rangle\equiv T\sum_n\sum_{\mathbf{p}}\mathrm{Tr}[A\rho_3G(\mathbf{p},\omega_n)B\rho_3G(\mathbf{p}-\mathbf{q},i\omega_n-i\omega_\nu)], \tag{17}$$

with A and B being some functions of ϕ or operators. Here ϕ is the angle between $\mathbf{p}$ and $\mathbf{q}$, $N(0)=m/2\pi$ is the density of states at the Fermi level and f is the superfluid density in the static limit ($\omega\ll v_F|\mathbf{q}|$) given by

$$f=2\pi T\Delta^2\sum_{n=0}^{\infty}\frac{1}{\omega_n^2+\Delta^2}\frac{1}{\sqrt{\omega_n^2+\Delta^2+(v_Fq)^2/4}}. \tag{18}$$

The derivation of the result in Eq. (16) is given in Appendix B 1. Therefore, the sound velocity shift δC_l is given by

$$\frac{\delta C_l}{C_l}=-\lambda_l\left[\frac{1}{2}-2s^2(1-f)\right], \tag{19}$$

where $\lambda_l=p_F^4/(8\pi mm_{\mathrm{ion}}C_l^2)$ is the longitudinal coupling constant. Here we set $s=\omega/v_F|\mathbf{q}|=C_l/v_F$, where C_l is the longitudinal sound velocity. In Sr_2RuO_4, $s=10^{-2}\ll1$ which is very different from $s\gg1$ of ^{3}He.

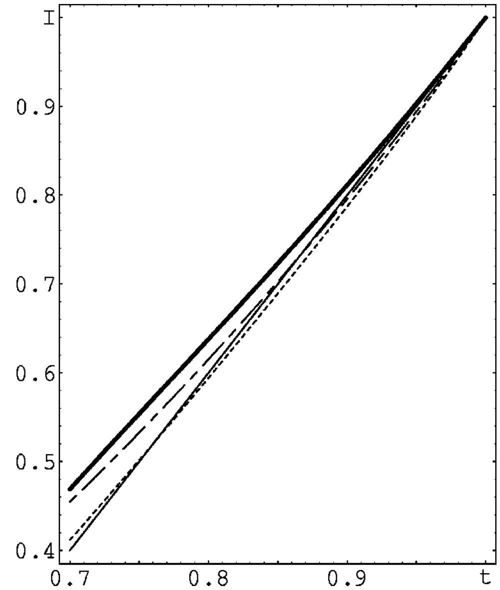

FIG. 1. The function I representing the reduction in the sound velocity as a function of the reduced temperature $t=T/T_c$ with $0.7<t<1.0$ for $v_F|\mathbf{q}|/\Delta(0)=0$ (thin solid line), 1 (dotted line), 2 (dashed line), 3 (thick solid line).

Now let us consider the correction due to the collective modes. The additional renormalization of $\langle[h_l,h_l]\rangle$ (in the C_{11} mode) due to the collective mode is computed in Appendix B 2 and the result is given by

$$\begin{aligned}\langle[h_l,h_l]\rangle&=\left(\frac{q}{\omega}\right)^2\frac{p_F^4}{4m^2}N(0)\Bigg[\frac{1}{2}-2s^2\\&\quad\times\left(1-f-\frac{f(v_F|\mathbf{q}|)^2}{4\{(v_F|\mathbf{q}|)^2+4\Delta(T)^2-2\omega^2\}}\right)\Bigg]\\&\quad+i\frac{m_{\mathrm{ion}}C_l^{11}}{\omega}\alpha_l(\omega).\end{aligned} \tag{20}$$

As one can see from the above equation, there is no resonance because $\omega\ll v_F|\mathbf{q}|$. However, we will be able to see a shadow of the collective mode in the sound velocity change, which we discuss in the following.

In the limit $s\ll1$ and setting $s=C_l^{11}/v_F$, the above equation leads to the sound velocity shift given by

$$\frac{\delta C_l^{11}}{C_l^{11}}=-\lambda_l^{11}\left[\frac{1}{2}-2\left(\frac{C_l^{11}}{v_F}\right)^2\left(1-f-\frac{f}{4\{1+(2\Delta/v_Fq)^2\}}\right)\right]. \tag{21}$$

Note that the sound wave gets soften more by the collective mode. In Fig. 1, we show

$$I=1-f-\frac{f}{4[1+(2\Delta(T)/v_F|\mathbf{q}|)^2]}$$

for $v_F|\mathbf{q}|/\Delta(0)=0,1,2,3$ for $0.7<t<1.0$ where $t=T/T_c$. Note that the coupling to the collective mode can be observed for $v_F|\mathbf{q}|\sim2-3\Delta(0)$ which corresponds to $\omega\sim O(1)$ GHz.

The attenuation coefficient α_l is given by

$$\frac{\alpha_l(\omega)}{\alpha_l^n(\omega)}=\frac{1}{\omega}\int_\Delta^\infty d\omega'\frac{\omega'(\omega'+\omega)-\Delta^2}{\sqrt{\omega'^2-\Delta^2}\sqrt{(\omega'+\omega)^2-\Delta^2}}\times\left(\tanh\frac{\omega+\omega'}{2T}-\tanh\frac{\omega'}{2T}\right)-\theta(\omega-2\Delta)\frac{1}{\omega}\times\int_\Delta^{\omega-\Delta}d\omega'\frac{\omega'(\omega'-\omega)-\Delta^2}{\sqrt{\omega'^2-\Delta^2}\sqrt{(\omega'-\omega)^2-\Delta^2}}\times\left(\tanh\frac{\omega'}{2T}\right), \tag{22}$$

where α_l^n is the attenuation coefficient in the normal state. This form is the same as the one in the *s*-wave superconductors.

We can carry out a parallel analysis for the longitudinal wave with $\mathbf{u}\|\mathbf{q}\|\mathbf{z}$ which corresponds to the C_{33} mode. Unfortunately, this sound wave does not couple to the clapping mode. Therefore, the velocity shift of this sound wave is simply given by

$$\frac{\delta C_l^{33}}{C_l^{33}}=-\lambda_l^{33}\left[\frac{1}{2}-2\left(\frac{C_l^{33}}{v_F}\right)^2(1-f)\right]. \tag{23}$$

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B. Transverse sound wave

Here we consider first the C_{66} mode that has $\mathbf{u}\|\mathbf{y}$ and $\mathbf{q}\|\mathbf{x}$. In this case, the sound velocity change can be obtained from the evaluation of $\langle[h_t,h_t]\rangle$. Assuming that the current contribution is negligible at low frequencies and following the same procedure used in the case of the longitudinal sound wave, we obtain

$$\frac{\delta C_t^{66}}{C_t^{66}}=-\lambda_t^{66}\left[\frac{1}{2}+2\left(\frac{C_t^{66}}{v_F}\right)^2\left(1-f+\frac{f}{4\{1+(2\Delta/v_Fq)^2\}}\right)\right], \tag{24}$$

where the $\lambda_t=p_F^4/(8\pi m m_{\rm ion}C_t^2)$ is the transverse coupling constant. Note that the transverse sound velocity increases upon entering the superconducting state. However, due to the fact that the transverse velocity is rather small and the coupling to the electron system is also weak compared to the longitudinal case, it will be hard to observe the change of the transverse sound velocity at $T=T_c$.

Another transverse sound mode, C_{44}, that has $\mathbf{u}\|\mathbf{z}$ and $\mathbf{q}\|\mathbf{x}$, does not couple to the clapping mode. Thus the sound velocity change in this case is given by

$$\frac{\delta C_t^{44}}{C_t^{44}}=-\lambda_t^{44}\left[\frac{1}{2}+2\left(\frac{C_t^{44}}{v_F}\right)^2(1-f)\right]. \tag{25}$$

V. THE DIFFUSIVE LIMIT

In the frequency range kHz–MHz, the diffusive limit is more realistic. In this limit, the incorporation of the quasiparticle damping is very important. Here we assume for simplicity that the quasiparticle scattering is due to impurities. Unlike the case of *s*-wave superconductors, we treat the impurity scattering in the unitary limit. Then the effect of the impurity is incorporated by changing ω_n to $\tilde{\omega}_n$ (renormalized Matsubara frequnecy) in Eq. (14).[18] The impurity renormalized complex frequency $\tilde{\omega}_n$ is determined from

$$\tilde{\omega}_n=\omega+\Gamma\frac{\sqrt{\tilde{\omega}_n^2+\Delta^2}}{\tilde{\omega}_n}, \tag{26}$$

where Γ is the quasiparticle scattering rate and the quasiparticle mean free path is given by $l=v_F/(2\Gamma)$. In order to compare the results in the normal state and those in the superconducting state, let us first work out the correlation functions in the normal state, where $\Delta=0$.

A. Normal state

We can use Eq. (B5) to compute $\langle[h_l,h_l]\rangle_0$. In the limit of $\omega,v_Fq\ll 2\Gamma$, we get

$$\langle\cos(2\phi),\cos(2\phi)\rangle=\left\langle\cos^2(2\phi)\left(1-\frac{\omega}{\omega+2i\Gamma-\zeta}\right)\right\rangle\approx\frac{1}{2}\left[1-\left(\frac{\omega}{2\Gamma}\right)^2+i\frac{\omega}{2\Gamma}\right]. \tag{27}$$

One can show that $\langle\cos(2\phi),1\rangle$ is of higher order in $\omega/2\Gamma$ and $v_F|\mathbf{q}|/2\Gamma$ while $\langle 1,1\rangle\approx 2\langle\cos(2\phi),\cos(2\phi)\rangle$ to the lowest order. Thus, as in the previous section, $\langle[h_l,h_l]\rangle$ is well approximated by $\langle\cos(2\phi),\cos(2\phi)\rangle$ times a multiplicative factor. This gives us

$$\frac{\delta C_l^n}{C_l}\approx-\lambda_l\frac{1}{2}\left[1-\left(\frac{\omega}{2\Gamma}\right)^2\right],\quad \alpha_l^n\approx\lambda_l|\mathbf{q}|\left(\frac{\omega}{2\Gamma}\right), \tag{28}$$

where ω is set to $C_l|\mathbf{q}|$.

It is not difficult to see that the results of the transverse sound wave is essentially the same as the longitudinal case up to the lowest order with a simple replacement of λ_l and C_l by λ_t and C_t. Therefore, in the diffusive limit, the longitudinal and transverse sound velocities have the same form with different coupling constants.

B. Superconducting state near T_c

Now we turn to the case of the superconducting state near T_c, where the correlation functions can be computed from Eq. (B6) after replacing ω_n by $\tilde{\omega}_n$. In this section, we will assume $4\pi T_c\gg 2\Gamma\gg v_F|\mathbf{q}|$ and use $\Delta/2\pi T\ll 1$ near T_c. As in the previous sections, the leading contribution in $\langle[h_l,h_l]\rangle$ can be computed from $\langle\cos(2\phi),\cos(2\phi)\rangle$.

After some algebra, we finally obtain

$$\langle\cos(2\phi),\cos(2\phi)\rangle \approx \frac{1}{2}-\frac{1}{2}\left(\frac{\omega}{2\Gamma}\right)^2\left[1-\frac{\Delta^2}{\pi\Gamma T}\left\{\psi^{(1)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)-\frac{\Gamma}{8\pi T}\psi^{(2)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right\}\right]+i\frac{1}{2}\left(\frac{\omega}{2\Gamma}\right)\left[1-\frac{\Delta^2}{2\pi\Gamma T}\times\left\{\psi^{(1)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)-\frac{\Gamma}{4\pi T}\psi^{(2)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right\}\right], \tag{29}$$

where

$$\psi^{(n)}(z)=\left(\frac{d}{dz}\right)^n\psi(z)=(-1)^{n+1}n!\sum_{k=0}^{\infty}\frac{1}{(z+k)^{n+1}}. \tag{30}$$

Here $\psi^{(n)}(z)$ is the poly-gamma function and $\psi(z)$ is the di-gamma function. This leads to

$$\frac{\delta C_{l,t}}{C_{l,t}}=-\lambda_{l,t}\frac{1}{2}\left[1-\left(\frac{\omega}{2\Gamma}\right)^2\left(1-\frac{\Delta^2}{\pi\Gamma T}\psi^{(1)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right)\right],$$

$$\frac{\alpha_{l,t}}{\alpha_{l,t}^n}=1-\frac{\Delta^2}{2\pi\Gamma T}\psi^{(1)}\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right), \tag{31}$$

where α_n is the ultrasonic attenuation coefficient in the normal state. Here we combined the subscripts l and t, because the above analysis applies to the case of the transverse sound wave as well. Only the coupling constants $\lambda_{l,t}$ are different. In particular, when $\Gamma/2\pi T\approx\Gamma/2\pi T_c\ll 1$, the above equations can be further reduced to

$$\frac{\delta C_{l,t}}{C_{l,t}}=-\lambda_{l,t}\frac{1}{2}\left\{1-\left(\frac{\omega}{2\Gamma}\right)^2\left[1-\frac{4\pi^3}{7\zeta(3)}\frac{T}{\Gamma}\left(1-\frac{T}{T_c}\right)\right]\right\},$$

$$\frac{\alpha_{l,t}}{\alpha_{l,t}^n}=1-\frac{2\pi^3}{7\zeta(3)}\frac{T}{\Gamma}\left(1-\frac{T}{T_c}\right). \tag{32}$$

Note that the sound velocity change and the attenuation coefficients decrease linearly in $(1-T/T_c)$ as $T\to T_c$. This result, Eq. (32), for the longitudinal sound wave is consistent with the experimental observation reported in Ref. 15 However, in the experiment, the transverse sound velocity does not show any change across T_c. The absolute value of the transverse sound velocity is much smaller than the longitudinal one and the coupling to the electron system is also much weaker than that of the longitudinal sound waves. Therefore, it is difficult to observe any change at $T=T_c$ for the transverse wave, which may explain the experimental results.

Here we neglect the coupling to the collective mode. Indeed, even in the diffusive limit, C_{11} and C_{66} modes do couple to the collective mode. However, our investigation showed that the coupling to the collective mode in these cases is almost impossible to detect although we do not present the details of the analysis here.

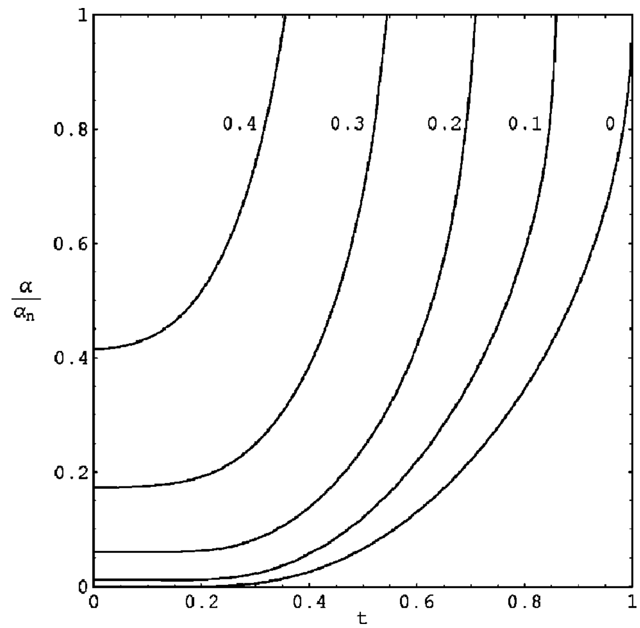

FIG. 2. The normalized attenuation coefficient as a function of the reduced temperature $t=T/T_c$ for $\Gamma/\Gamma_c=0,0.1,0.2,0.3,0.4$.

C. Ultrasonic attenuation for all temperature regimes

The general expression of the sound attenuation coefficient for $T<T_c$ can be obtained by following the procedure of Kadanoff and Falko and Tsuneto.[16] To obtain the ultrasonic attenuation coefficient, α_t, we compute the imaginary part of the correlation function $\langle[\tau_{xy},\tau_{xy}]\rangle$. We finally arrive at

$$\mathrm{Im}\langle[\tau_{xy},\tau_{xy}]\rangle=\frac{p_F^4}{m^2}N(0)\omega\int_{-\infty}^{\infty}d\omega\left(-\frac{\partial n_F}{\partial\omega}\right)\times\frac{g(\tilde{\omega})}{\mathrm{Im}\sqrt{\tilde{\omega}^2-\Delta^2}}\frac{1+y^2/2-\sqrt{1+y^2}}{y^4}, \tag{33}$$

where $y=v_Fq/2\mathrm{Im}\sqrt{\tilde{\omega}^2-\Delta^2}$ and $n_F(\omega)=1/(e^{\omega/T}+1)$ is the Fermi distribution function. The coherence factor $g(\tilde{\omega})$ is given by

$$g(\tilde{\omega})=\frac{1}{2}\left(1+\frac{|\tilde{x}|^2-1}{|\tilde{x}^{\,2}-1|}\right), \tag{34}$$

where $\tilde{x}=\tilde{\omega}/\Delta$ is determined from

$$\tilde{x}=\frac{\omega}{\Delta}+i\frac{\Gamma}{\Delta}\frac{\sqrt{\tilde{x}^{\,2}-1}}{\tilde{x}}. \tag{35}$$

Similar analysis can be also done for α_l.

In the limit of $|\mathbf{q}|l\ll 1$, the above result leads to the following ratio between the attenuation coefficients in the superconducting state $\alpha_{l,t}$ and the normal state $\alpha_{l,t}^n$.

$$\frac{\alpha_{l,t}}{\alpha_{l,t}^n}=\frac{\Gamma}{2\Delta}\int_0^{\infty}\frac{d\omega}{T}\,\mathrm{sech}^2\left(\frac{\omega}{2T}\right)\frac{g(\tilde{\omega})}{\mathrm{Im}\sqrt{\tilde{x}^2-1}}. \tag{36}$$

Notice that the Eq. (36) applies for both of the transverse and longitudinal sound waves. This result is evaluated numerically and shown in Fig. 2 for several Γ/Γ_c where $\Gamma_c = \Delta(0)/2$ is the critical scattering rate which drives T_c to zero.

VI. CONCLUSION

We have identified a unique collective mode called the clapping mode in a p-wave superconductor with the order parameter given by Eq. (1). This collective mode couples to the sound wave and affects its dynamics.

The effect of the clapping mode on the sound waves was calculated in the collisionless limit. However, unlike the case of ^{3}He, the detection of the collective mode appears to be rather difficult. One needs, at least, the high frequency experiment with $\omega \sim O(1)$ GHz.

In the diffusive limit, we worked out the sound velocity change near $T=T_c$ and found that it decreases linearly in $1-T/T_c$ which is consistent with the experiment reported by Matsui *et al.*[15] We also obtained the ultrasonic attenuation coefficient for the whole temperature range, which can be tested experimentally. On the other hand, the coupling of the collective mode is almost invisible in the diffusive limit.

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APPENDIX A: CLAPPING MODE AND ITS COUPLING TO THE STRESS TENSOR.

The fluctuation of the order parameter corresponding to the clapping mode can be written as $\delta\Delta\rho_3 \sim e^{\pm 2i\phi}\sigma_1\rho_3$. The relevant correlation functions for the couplings are

$$\begin{aligned}\langle \delta\Delta, \cos(2\phi)\rangle(i\omega_\nu,\mathbf{q}) &\equiv T\sum_n\sum_{\mathbf{p}} \mathrm{Tr}[\delta\Delta\rho_3 G(\mathbf{p},\omega_n) \\ &\quad\times \cos(2\phi)\rho_3 G(\mathbf{p}-\mathbf{q}, i\omega_n - i\omega_\nu)],\\ \langle \delta\Delta,\delta\Delta\rangle(i\omega_\nu,\mathbf{q}) &\equiv T\sum_n\sum_{\mathbf{p}} \mathrm{Tr}[\delta\Delta\rho_3 G(\mathbf{p},\omega_n)\\ &\quad\times \delta\Delta\rho_3 G(\mathbf{p}-\mathbf{q}, i\omega_n - i\omega_\nu)].\end{aligned} \tag{A1}$$

After summing over $\mathbf{p}$, we get

$$\begin{aligned}\langle \delta\Delta,\cos(2\phi)\rangle &= \Bigg\langle \pi T N(0)\sum_n \left(\frac{-i\omega_\nu\Delta}{2\sqrt{\omega_n^2+\Delta^2}\sqrt{\omega_{n+\nu}^2+\Delta^2}}\right)\\ &\quad\times\frac{\sqrt{\omega_n^2+\Delta^2}+\sqrt{\omega_{n+\nu}^2+\Delta^2}}{(\sqrt{\omega_n^2+\Delta^2}+\sqrt{\omega_{n+\nu}^2+\Delta^2})^2+\zeta^2}\Bigg\rangle,\\ \langle \delta\Delta,\delta\Delta\rangle &= \Bigg\langle \pi T N(0)\sum_n\left(1+\frac{\omega_n\omega_{n+\nu}}{\sqrt{\omega_n^2+\Delta^2}\sqrt{\omega_{n+\nu}^2+\Delta^2}}\right)\\ &\quad\times\frac{\sqrt{\omega_n^2+\Delta^2}+\sqrt{\omega_{n+\nu}^2+\Delta^2}}{(\sqrt{\omega_n^2+\Delta^2}+\sqrt{\omega_{n+\nu}^2+\Delta^2})^2+\zeta^2}\Bigg\rangle,\end{aligned} \tag{A2}$$

where $\langle \cdots \rangle$ on the right hand side of the equations represents the angle average. Now summing over ω_n and analytic continuation $i\omega_\nu \rightarrow \omega + i\delta$ lead to

$$\begin{aligned}\langle \delta\Delta,\cos(2\phi)\rangle &= N(0)\left\langle \frac{\omega}{4\Delta}F\right\rangle,\\ \langle \delta\Delta,\delta\Delta\rangle &= g^{-1} - N(0)\left\langle \frac{\zeta^2+2\Delta^2-\omega^2}{4\Delta^2}F\right\rangle,\end{aligned} \tag{A3}$$

where F is given by

$$F(\omega,\zeta) = 4\Delta^2(\zeta^2-\omega^2)\int_\Delta^\infty dE\,\frac{\tanh(E/2T)}{\sqrt{E^2-\Delta^2}}\,\frac{(\zeta^2-\omega^2)^2-4E^2(\omega^2+\zeta^2)+4\zeta^2\Delta^2}{[(\zeta^2-\omega^2)^2+4E^2(\omega^2-\zeta^2)+4\zeta^2\Delta^2]^2-16\omega^2E^2(\zeta^2-\omega^2)^2}. \tag{A4}$$

In the limit of $\omega \ll v_F|\mathbf{q}|$, the contribution (in $\langle [h_l,h_l]\rangle$; see Appendix B 2) due to the coupling with the clapping mode becomes

$$\begin{aligned}\frac{\langle \delta\Delta,\cos(2\phi)\rangle^2}{g^{-1}-\langle \delta\Delta,\delta\Delta\rangle} &= N(0)\,\frac{\omega^2\langle F\rangle^2}{4\langle(\zeta^2+2\Delta^2-\omega^2)F\rangle}\\ &\approx N(0)\frac{s^2 f}{2[1+(2\Delta/v_F q)^2-2s^2]},\end{aligned} \tag{A5}$$

where $s=\omega/v_F|q|$ and $f=\lim_{q\to 0}\lim_{\omega\to 0}\langle F\rangle$ is the superfluid density and given by Eq. (18). We can see that the frequency of the clapping mode is given by $\sqrt{2}\Delta$ from $\langle \delta\Delta,\delta\Delta\rangle$.

APPENDIX B:

1. Longitudinal sound wave in the collisionless limit

The longitudinal sound velocity shift is given by the real part of $\langle [h_l,h_l]\rangle$. The irreducible correlation function for the stress tensor can be obtained from

$$\langle[\tau_{xx},\tau_{xx}]\rangle_{00}=T\sum_{n}\sum_{\mathbf{p}}\mathrm{Tr}\bigg[\frac{p_F^4}{m^2}(\cos\phi)^4\rho_3 G(\mathbf{p},\omega_n)\times\rho_3 G(\mathbf{p}-\mathbf{q},i\omega_n-i\omega_\nu)\bigg],\tag{B1}$$

where ϕ is the angle between $\mathbf{p}$ and $\mathbf{q}$. Since the stress tensor couples to the density, the correlation function is renormalized as

$$\langle[h_l,h_l]\rangle_0=\langle[h_l,h_l]\rangle_{00}+\frac{V(\mathbf{q})\langle[h_l,n]\rangle\langle[n,h_l]\rangle}{1-V(\mathbf{q})\langle[n,n]\rangle},\tag{B2}$$

where $V(\mathbf{q})=2\pi e^2/|\mathbf{q}|$ is the Coulomb interation. This equation can be simplified in the long wave length limit ($|\mathbf{q}|\to 0$) as

$$\langle[h_l,h_l]\rangle_0\approx\left(\frac{q}{\omega}\right)^2\left[\langle[\tau_{xx},\tau_{xx}]\rangle_{00}-\frac{\langle[\tau_{xx},n]\rangle\langle[n,\tau_{xx}]\rangle}{\langle[n,n]\rangle}\right].\tag{B3}$$

It is useful to define the following quantity for notational convenience:

$$\langle A,B\rangle=T\sum_{n}\sum_{\mathbf{p}}\mathrm{Tr}[A\rho_3 G(\mathbf{p},\omega_n)B\rho_3 G(\mathbf{p}-\mathbf{q},i\omega_n-i\omega_\nu)],\tag{B4}$$

where A and B can be some functions of ϕ or operators. Using this notation, Eq. (B3) can be rewritten as

$$\begin{aligned}\langle[h_l,h_l]\rangle_0&\approx\left(\frac{q}{\omega}\right)^2\frac{p_F^4}{m^2}\left[\langle\cos^2\phi,\cos^2\phi\rangle-\frac{\langle\cos^2\phi,1\rangle\langle 1,\cos^2\phi\rangle}{\langle 1,1\rangle}\right]\\&=\left(\frac{q}{\omega}\right)^2\frac{p_F^4}{4m^2}\left[\langle\cos(2\phi),\cos(2\phi)\rangle-\frac{\langle\cos(2\phi),1\rangle\langle 1,\cos(2\phi)\rangle}{\langle 1,1\rangle}\right].\end{aligned}\tag{B5}$$

Then, each correlation function can be computed from

$$\begin{aligned}\langle 1,1\rangle(i\omega_\nu,\mathbf{q})&=T\sum_{n}\sum_{\mathbf{p}}\mathrm{Tr}[\rho_3 G(\mathbf{p},\omega_n)\times\rho_3 G(\mathbf{p}-\mathbf{q},i\omega_n-i\omega_\nu)],\\ \langle 1,\cos(2\phi)\rangle(i\omega_\nu,\mathbf{q})&=T\sum_{n}\sum_{\mathbf{p}}\mathrm{Tr}[\cos(2\phi)\rho_3 G(\mathbf{p},\omega_n)\times\rho_3 G(\mathbf{p}-\mathbf{q},i\omega_n-i\omega_\nu)],\\ \langle\cos(2\phi),\cos(2\phi)\rangle(i\omega_\nu,\mathbf{q})&=T\sum_{n}\sum_{\mathbf{p}}\mathrm{Tr}[\cos^2(2\phi)\rho_3\sigma_1 G(\mathbf{p},\omega_n)\times\rho_3 G(\mathbf{p}-\mathbf{q},i\omega_n-i\omega_\nu)].\end{aligned}\tag{B6}$$

Summing over $\mathbf{p}$ leads to

$$\langle 1,1\rangle=\left\langle\pi TN(0)\sum_n\left(1-\frac{\omega_n\omega_{n+\nu}+\Delta^2}{\sqrt{\omega_n^2+\Delta^2}\sqrt{\omega_{n+\nu}^2+\Delta^2}}\right)\times\frac{\sqrt{\omega_n^2+\Delta^2}+\sqrt{\omega_{n+\nu}^2+\Delta^2}}{(\sqrt{\omega_n^2+\Delta^2}+\sqrt{\omega_{n+\nu}^2+\Delta^2})^2+\zeta^2}\right\rangle,\tag{B7}$$

where $\zeta=\mathbf{v}_F\cdot\mathbf{q}$ and $N(0)=m/2\pi$ is the two-dimensional density of states. Similar equations can obtained for $\langle\cos(2\phi),1\rangle$ and $\langle\cos(2\phi),\cos(2\phi)\rangle$ with additional angle factors $\cos(2\phi)$ and $\cos^2(2\phi)$, respectively. After summing over ω_n and analytic continuation $i\omega_\nu\to\omega+i\delta$, we get the following results in the limit of $\omega\ll v_F|\mathbf{q}|$.

$$\langle 1,1\rangle=N(0)\left\langle\frac{\zeta^2-\omega^2 f}{\zeta^2-(\omega+i\delta)^2}\right\rangle\approx N(0)\left[1-i\frac{s(1-f)}{\sqrt{1-s^2}}\right],$$

$$\langle\cos(2\phi),1\rangle=N(0)\left\langle\cos(2\phi)\frac{\zeta^2-\omega^2 f}{\zeta^2-(\omega+i\delta)^2}\right\rangle\approx N(0)\left[2s^2(1-f)\left(1+i\frac{1-2s^2}{2s\sqrt{1-s^2}}\right)\right],$$

$$\langle\cos(2\phi),\cos(2\phi)\rangle=N(0)\left\langle\cos^2(2\phi)\frac{\zeta^2-\omega^2 f}{\zeta^2-(\omega+i\delta)^2}\right\rangle\approx N(0)\left[\frac{1}{2}-2s^2(1-f)-i\frac{(1-f)s}{2\sqrt{1-s^2}}\right].\tag{B8}$$

We find that the second term in the last line of Eq. (B5) is of higher order in $\omega/v_F|\mathbf{q}|(\equiv s)$ so that we can ignore it. In Sr_2RuO_4 or metals, $s\ll 1$. Thus the effect of the coupling to the density is merely to change the vertex associated with τ_{xx} from $(p_F^2/m)\cos^2\phi$ to $(p_F^2/2m)\cos(2\phi)$ as far as the lowest order contribution is concerned. Evaluation of $\langle\cos(2\phi),\cos(2\phi)\rangle$ leads to

$$\begin{aligned}\mathrm{Re}\langle[h_l,h_l]\rangle_0&\approx\left(\frac{q}{\omega}\right)^2\mathrm{Re}\frac{p_F^4}{4m^2}\langle\cos(2\phi),\cos(2\phi)\rangle\\&=\left(\frac{q}{\omega}\right)^2\frac{p_F^4}{4m^2}N(0)\left[\frac{1}{2}-2s^2(1-f)\right],\end{aligned}\tag{B9}$$

where f is the superfluid density.

2. Contribution coming from the coupling to the clapping mode

The correction due to the collective mode leads to the renormalized correlation function as follows:

$$\langle [h_l , h_l] \rangle = \langle [h_l , h_l] \rangle_0 + \frac{\langle [h_l , \delta\Delta\rho_3] \rangle \langle [\delta\Delta\rho_3 , h_l] \rangle}{g^{-1} - \langle [\delta\Delta\rho_3 , \delta\Delta\rho_3] \rangle}, \tag{B10}$$

where g is the coupling constant between the stress tensor and the collective mode. $\delta\Delta\rho_3$ represents the fluctuation associated with the clapping mode. Using the fact that $\langle 1, e^{2i\phi} \rangle = 0$ and $\delta\Delta \sim e^{2i\phi}\sigma_1$, the above equation can be further reduced to

$$\begin{aligned}\langle [h_l , h_l] \rangle &= \left(\frac{q}{\omega}\right)^2 \frac{p_F^4}{4m^2} \left[\langle \cos(2\phi), \cos(2\phi) \rangle + \frac{\langle \cos(2\phi), \delta\Delta \rangle \langle \delta\Delta, \cos(2\phi) \rangle}{g^{-1} - \langle \delta\Delta, \delta\Delta \rangle} \right] \\ &= \left(\frac{q}{\omega}\right)^2 \frac{p_F^4}{4m^2} N(0) \left[\frac{1}{2} - 2 \left(\frac{C_l}{v_F}\right)^2 \times \left(1 - f - \frac{f (v_F |\mathbf{q}|)^2}{4\{(v_F|\mathbf{q}|)^2 + 4\Delta(T)^2 - 2\omega^2\}} \right) \right] \\ &\quad + i \frac{m_{\text{ion}} C_l}{\omega} \alpha_l(\omega),\end{aligned} \tag{B11}$$

where f is the superfluid density and given by Eq. (18).

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