Mode damping of layered ^{3}He-^{4}He films over a solid substrate

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The dispersion relation for coupled oscillations of liquid interfaces in the layered ^{3}He-^{4}He films on solid substrate is calculated by taking into account viscous dissipation and effects of the van der Waals interaction between the liquid phases and the solid substrate and among themselves. We provide a detailed analysis of the layer thickness dependence of the mode spectrum within the hydrodynamical approach by solving the linearized Navier-Stokes equation. For actual helium viscosities, we find that a mode with weak dissipation, corresponding to flexural displacements of the upper phase, does exist at large wave numbers. In the approximation of ideal fluids, we reproduce previous results obtained from different methods. We find a wide interval of wave numbers for weakly damped modes when the layer thickness of a given phase is significantly smaller than the other one. We interpret our results for weak and strong dissipation by comparing the viscous penetration depth of the modes with characteristic lengths of the system. We discuss the implications of our results for experiments to measure mode frequency and transport properties of surface electrons over the layered film.

I. INTRODUCTION

The properties of a superfluid ^{4}He surface are strongly influenced by ^{3}He atoms that accumulate near the liquid-vapor interface.[1] In particular, in past years there has been a significant interest in studying the system of ^{3}He atoms adsorbed on ^{4}He films.[2] By adding ^{3}He atoms into the liquid solution, one can go from the regime of the two-dimensional (2D) gas of ^{3}He quasiparticles (in this case the ^{3}He surface density is less than one monolayer) to the saturated regime where either the growth of a thin though *macroscopic* film of nearly pure ^{3}He near the surface of the bulk ^{3}He-^{4}He solution (microstratification) or a layering process of the mixture film on the solid substrate can occur.[3] Interesting phenomena have been investigated[4] especially for a mixture layered film on solid substrates with large van der Waals constant as, for example, graphite.[5] In particular, the solidification of a few layers of liquid helium near the surface of the substrate is found. On the other hand, for substrates with relatively low van der Waals constants, such as solid neon, the formation of solid layers is not expected and the strength of the van der Waals liquid-helium–substrate interaction is of the same order of the interphase interaction. The study of layering phenomena is, in particular, attractive in view of the possibility of a phase transition of the ^{3}He thin film into a 2D superfluid state.[6] Propagation of unusual low-damped surface modes was predicted for both the microstratified solution and the layered film when the liquids are considered ideal.[7,8] Furthermore, these modes influence significantly the properties of surface electrons localized over the helium surface. As is well known, the main scattering mechanism for $T<1$ K comes from the electron interaction with surface oscillations (ripplons) with wave number $q<2k$ where k is the electron 2D wave number.[9] For the nondegenerate electron system, typical values of k are of the order of $k_T=\sqrt{2mT}/\hbar$, with typical values $\lesssim 10^5$ cm^{-1}. As a consequence, there is a limited interval of q, which contributes to the electron-ripplon interaction, and surface mode attenuation at $q<2k_T$ may influence strongly the effectiveness of the electron scattering by these modes.

The investigation of the interfacial mode spectrum of the microstratified solution, taking into account viscosity effects, has shown that only *one* weakly damped mode can exist on the surface of the microstratified solution and that it is associated with the flexural oscillations of the ^{3}He film.[10,11] This mode has the same dispersion as ripplons in uniform liquid but with a modified surface tension coefficient $\alpha^*=\alpha_1+\alpha_{12}$, where α_1 and α_{12} are the surface tension coefficients for the free surface of the upper fluid and the stratification interface, respectively. The second branch of the spectrum that corresponds to the layer-thickness oscillations of the ^{3}He layer is overdamped for actual values of helium viscosities.

Coupled oscillations of the liquid interfaces of a layered helium film over a solid substrate have been also studied by different methods. Previous results[8,12] are almost the same when the van der Waals coupling between the substrate and liquid phases is strong. The influence of the interaction between the phases (essential for weak liquid-substrate coupling) and also capillary effects on the dispersion law of the coupled film oscillations have been studied in Ref. 8. In particular, it was shown that the velocities of two soundlike

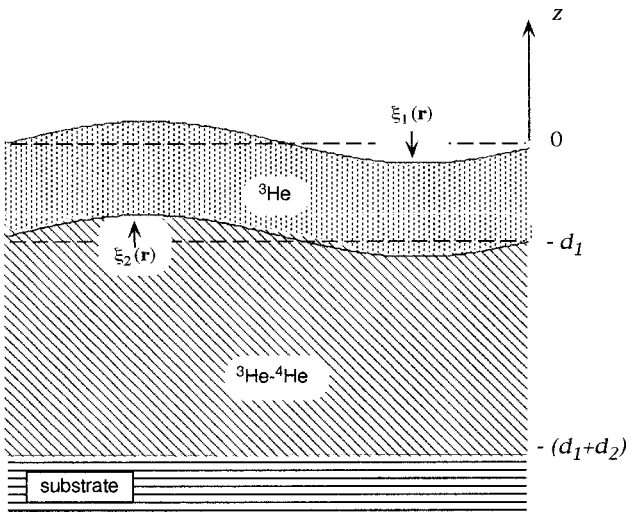

FIG. 1. Schematic view of the system.

modes are very close when the condition $d_1 \sim d_2$ is satisfied, where d_1 and d_2 are the thicknesses of the upper and lower liquid phases, respectively. However, in Refs. 8 and 12 the mode spectrum of the layered film was evaluated considering the phases as *ideal* fluids. The effect of mode damping was taken into account in Ref. 12 by setting one of the sound velocities equal to zero in the case of normal ^{3}He. The present authors showed that viscous damping effects on the spectrum of coupled surface modes depend not only on the viscosities of different phases but also on ^{3}He thickness, liquid density, and capillary forces.[10,11] One should also point out that the hydrodynamic approach becomes invalid for submonolayer films. Quite recently, it was shown that the third-sound velocity of the ^{3}He-^{4}He film is strongly dependent on the ^{3}He coverage decreasing up to 0.6 ML when the onset of the excited-state occupation occurs and the coverage dependence becomes weaker.[13]

The aim of the present work is to describe theoretically the spectrum of the ^{3}He-^{4}He layered film. Both real and imaginary parts of the dispersion law for the oscillations of the layered macroscopic film of ^{3}He-^{4}He mixture are calculated by taking into account the effects of viscous damping. We consider temperatures above 0.1 K where the upper liquid phase is the almost pure normal (viscous) ^{3}He, and Fermi liquid effects are negligible. In particular, we disregard the possibility that pairs of ^{3}He atoms should dimerize on the surface of ^{4}He.[14] Under the above assumptions, the lower phase (the ^{3}He-^{4}He mixture) can also be described, on the contrary of pure superfluid ^{4}He, by classical hydrodynamics at $T \lesssim 1$ K.[15] The solid substrate is located at $z < -(d_1+d_2)$, as shown in Fig. 1. The blanket of normal liquid over the superfluid film should strongly suppress the third-sound[16] damping due to the liquid evaporation and condensation caused by relative oscillations of normal and superfluid densities of the film and thermal conductivity in the gas phase of ^{4}He over the superfluid film.[17] In this situation, mode attenuation should be only due to viscosity. The hydrodynamical approach for describing viscous phenomena is valid if the perturbation wavelength λ is significantly larger than the mean free path of thermal excitations. One can guess this condition is satisfied in the ^{3}He-^{4}He solution where the mean free path of phonons is limited by quasiparticles of ^{3}He at $T<1$ K and the contribution of rotons is negligible at these temperatures. Hence, the viscosity of the solution of helium isotopes does not alter the hydrodynamical regime even at low enough T in contrast with pure superfluid ^{4}He, whose hydrodynamical description is not valid at $T<1$ K. The viscosity of the ^{3}He upper phase does not also change the hydrodynamical regime at T down to 0.1 K and Fermi-liquid effects become relevant only at lower T.[18]

The organization of the paper is as follows. In Sec. II, we point out the main relations and derive the dispersion law. In Sec. III, we analyze the numerical solution of the dispersion equation in a wide range of wave numbers and layer thicknesses of the liquid phases, and limiting cases are recovered. In Sec. IV, we summarize our results.

II. DISPERSION EQUATION

To find out the dispersion law for the coupled oscillations of the free liquid surface at $z=0$ and the stratification interface at $z=-d_1$, we solve the linearized Navier-Stokes equations for each phase with appropriate boundary conditions for the liquid interfaces and for the surface of the solid substrate at $z=-(d_1+d_2)$. We also have ignored in a straightforward way the superfluid velocity $\vec{v}_{2s}=\vec{\nabla}\Phi_2$ (where Φ_2 is the hydrodynamical potential) in the linearized Navier-Stokes equation for the lower superfluid phase.[19] The liquid phases are described by the equations

$$\rho_1 \frac{\partial \mathbf{v}_1}{\partial t} + \nabla P_1 - \rho_1 \mathbf{g} = \eta_1 \nabla^2 \mathbf{v}_1 , \tag{1a}$$

$$\rho_{2n} \frac{\partial \mathbf{v}_{2n}}{\partial t} + \nabla P_{2n} - \rho_2 \mathbf{g} = \eta_2 \nabla^2 \mathbf{v}_{2n} , \tag{1b}$$

where numeric labels correspond to the quantities in their respective phases. Here $\mathbf{g}$ is gravity acceleration, ρ_i and η_i $(i=1,2)$ are the densities and viscosities of both liquids, and ρ_{2n} is the density of the normal component in phase 2, $\rho_{2s}=\rho_2-\rho_{2n}$ is the density of the superfluid component in the lower phase, and $P_{2n}=P_2+\rho_{2s}\partial\Phi_2/\partial t$ where P_1 and P_2 are the pressures in the liquids.

The solutions of Eqs. (1) can be obtained straightforwardly following the same steps of Ref. 11. The only difference is the new boundary condition for velocities: $v_{2nx}=v_{2nz}=0$ at $z=-(d_1+d_2)$. The contribution of the van der Waals interaction of the liquid phases with the solid substrate and among themselves has been also taken into account by modifying the equilibrium conditions at $z=-d_1$ and $z=-(d_1+d_2)$. As shown in Ref. 7, the van der Waals interaction leads to extra coupling of the displacements of the liquid interfaces. This contribution comes from the additional potential energy due to the interaction between the stratified liquid phases that exists for finite layer thicknesses and gives no contribution to the surface-tension coefficients of both liquid interfaces.

The dispersion relation (DR) $\omega(q)=\omega_q - i\tau_q^{-1}$ is given by the roots of the dispersion equation, which is a consequence of the compatibility condition of the equations of motion describing the displacements of the liquid interfaces from the equilibrium positions. The dispersion equation can be written as

$$F_{11}(q)F_{22}(q)-\kappa^2F_{12}(q)F_{21}(q)=0, \tag{2}$$

where $\kappa=1/\cosh(qd_1)$ and

$$F_{11}=\omega_{01}^2-\left(\frac{\eta_1}{\rho_1}\right)^2\kappa^2\frac{[q^2+\lambda_1^2-2\Omega_1^2]^2}{\Delta(q)}+\left(\frac{\eta_1}{\rho_1}\right)^2[(q^2+\lambda_1^2)^2-4q^2\Lambda_1^2],$$

$$F_{12}=\omega_{12}^2+\left(\frac{\eta_1}{\rho_1}\right)^2[(q^2+\lambda_1^2)^2-4q^2\Omega_1^2]-\left(\frac{\eta_1}{\rho_1}\right)^2(q^2+\lambda_1^2-2\Omega_1^2)\Delta^{-1}(q)\times\left\{q^2+\lambda_1^2-2\Lambda_1^2+\left[\frac{\eta_2\rho_1}{\eta_1\rho_{2n}}\beta_1\beta_2(q-\lambda_2)^2+(\beta_1\beta_2-\beta_3)\frac{\lambda_2}{q}(q^2-\lambda_1^2)\right]\tanh(qd_1)\right\},$$

$$F_{21}=\omega_{12}^2+\left(\frac{\eta_1}{\rho_1}\right)^2[(q^2+\lambda_1^2)^2-4q^2\Omega_1^2]-\left(\frac{\eta_1}{\rho_1}\right)^2(q^2+\lambda_1^2-2\Omega_1^2)\Delta^{-1}(q)\left[q^2+\lambda_1^2-2\Lambda_1^2+\frac{\eta_2\rho_1}{\eta_1\rho_{2n}}\beta_4(q-\lambda_2)^2\tanh(qd_1)\right],$$

$$F_{22}=\omega_{02}^2-\frac{\rho_{2s}}{\rho_1}\omega^2\frac{\tanh(qd_1)}{\tanh(qd_2)}+\left(\frac{\eta_1}{\rho_1}\right)^2[(q^2+\lambda_1^2)^2-4q^2\Lambda_1^2]+\left(\frac{\eta_2^2}{\rho_1\rho_{2n}}\right)[\beta_2\beta_4(q^2+\lambda_2^2)(q-\lambda_2)-2\beta_5q\lambda_2(q+\lambda_2)]\times(q-\lambda_2)\tanh(qd_1)-\left(\frac{\eta_1\eta_2}{\rho_1\rho_{2n}}\right)(q-\lambda_2)^2\left[\beta_1\beta_2+\beta_4+(\beta_1\beta_2-\beta_3)\frac{\lambda_2}{q}\left(\frac{q+\lambda_2}{q-\lambda_2}\right)\right][q^2+\lambda_1^2-2\Lambda_1^2]\frac{\tanh(qd_1)}{\Delta(q)}-\left(\frac{\eta_2}{\rho_{2n}}\right)^2\beta_4(q-\lambda_2)^3[\beta_1\beta_2(q^2+\lambda_2^2)-\beta_3\lambda_2(q+\lambda_2)]\frac{\tanh^2(qd_1)}{q\Delta(q)}-\left(\frac{\eta_1}{\rho_1}\right)^2[q^2+\lambda_1^2-2\Lambda_1^2]^2\frac{1}{\Delta(q)},$$

with

$$\Delta(q)=1-\frac{\Lambda_1^2}{q^2}+\frac{\rho_1}{\rho_{2n}}\beta_1\left(1-\frac{\lambda_2}{q}\right)\tanh(qd_1).$$

In the above expressions, $\Omega_1^2=q\lambda_1\sinh(qd_1)/\sinh(\lambda_1d_1)$ and $\Lambda_1^2=q\lambda_1\tanh(qd_1)/\tanh(\lambda_1d_1)$ with $\lambda_1^2=q^2-i\omega\rho_1/\eta_1$ and $\lambda_2^2=q^2-i\omega\rho_{2n}/\eta_2$ and

$$\omega_{01}^2=\left(\frac{\alpha_1}{\rho_1}q^3+\sigma_1q\right)\tanh(qd_1),$$

$$\omega_{02}^2=\left(\frac{\alpha_{12}}{\rho_1}q^3+\sigma_2q\right)\tanh(qd_1),$$

$$\omega_{12}^2=\frac{\sigma q}{2}(qd_1)^2K_2(qd_1)\sinh(qd_1),$$

where $K_2(x)$ is the modified Bessel function. The accelerations σ_1 and σ_2 are given by

$$\sigma_1=\sigma\left[1-\left(\frac{d_1}{d_1+d_2}\right)^4\right]+\frac{3n_1a_1}{\rho_1(d_1+d_2)^4}+g,$$

$$\sigma_2=\sigma\left[1-\left(\frac{d_1}{d_2}\right)^4\right]+\frac{3(n_2a_2-n_1a_1)}{\rho_1d_2{}^4}+\left(\frac{\rho_2}{\rho_1}-1\right)g,$$

where $\sigma=\pi n_1(n_2b_{12}-n_1b_{11})/2\rho_1d_1^4$. In the above equations, a_1 and a_2 are the van der Waals coupling constants describing the interaction of helium atoms with the solid substrate; n_1 and n_2 are volume concentrations of atoms in liquid phases. The coefficients b_{ij} are related to the strength of interatomic interaction, $u_{ij}\simeq-b_{ij}/[|\vec{r}_i-\vec{r}_j|^2+(z_i-z_j)^2]^3$.[20] Note that the van der Waals constants for the solid substrate of inert elements[21] are approximately described by $a_i=(\pi/6)(n_sb_{is}-n_ib_{ii})$, where n_s and b_{is} are the atomic concentration of the substrate and b_{is} gives the interaction between the atom of substrate and helium atoms in phases 1 and 2. The cumbersome functions $\beta_i(q)$, $i=1,\ldots,5$ are depicted in the Appendix for the sake of completeness. Equation (2) contains all information about both real and imaginary parts of the dispersion law for surface oscillations of the layered film of ^{3}He-^{4}He liquid solution.

III. RESULTS AND DISCUSSION

In the limit of large qd_1 and qd_2 (small wavelengths), Eq. (2) splits into two independent equations, $F_{11}(q)=0$ and $F_{22}(q)=0$, corresponding to the dispersion laws for the oscillations of the free surface of ^{3}He and the stratified interface of two infinite liquids, respectively. The functions $F_{12}(q)$ and $F_{21}(q)$, which describe the coupling between the oscillations of two liquid interfaces, can be discarded in this limit but are essential in the opposite case $qd_1, qd_2\ll1$. For $d_2\to\infty$, we reproduce the results of Refs. 10 and 11 with $F_{12}=F_{21}$.

Analytical expressions for the spectrum of surface oscillations of uniform liquid with density ρ and viscosity η can be obtained either in the limit of weak dissipation $q^2\ll\omega\rho/\eta$ or in the opposite limit when nonoscillatory damping takes place.[19] In the two-phase system discussed here, there are two parameters $q^2\eta_1/\rho_1\omega$ and $q^2\eta_2/\rho_{2n}\omega$ that can be of the order of one at large enough q. Furthermore, two small parameters qd_1 and $qd_2\ll1$ can appear in Eq. (2) in the long-wavelength limit. Hence, to find out the mode spectrum for a wide range of q, we must solve Eq. (2) numerically.

For ideal fluids ($\eta_1=0$ and $\eta_2=0$), one has $\tau_q^{-1}=0$. In this situation, the DR of the coupled surface oscillations con-

sists of two branches, which were obtained previously.[8] We consider $d_1, d_2 \lesssim 10^{-5}$ cm, where the gravity effect on the mode spectrum is negligible even though it was taken into account in the numerical calculations. As it was shown in Ref. 8, the dispersion laws depend strongly on the relation between d_1 and d_2. In the limit $d_2 = \infty$, the results of numerical calculations coincide with the mode spectrum of the microstratified ^{3}He-^{4}He solution (see Fig. 2 of Ref. 10). By decreasing d_2, but keeping $d_1/d_2 \ll 1$, the anticrossing point q_c of the modes, which appears in the long wavelength limit $q \ll d_1^{-1}, d_2^{-1}$, becomes larger than for the spectrum of the microstratified mixture and one obtains for $q \ll q_c$

$$\omega_{1q}^2 \simeq \sigma d_1 q^2, \tag{3a}$$

$$\omega_{2q}^2 \simeq \left[\frac{\alpha^*}{\rho_2} q^2 + \frac{3 n_2 a_2}{\rho_2 d_2^4}\right] d_2 q^2, \tag{3b}$$

and for $q \gg q_c$

$$\omega_{1q}^2 \simeq \left[\frac{\alpha^*}{\rho_2} q^2 + \frac{3 n_2 a_2}{\rho_2 d_2^4} + \frac{\rho_2}{\rho_1}\left(\frac{d_1}{d_2}\right)^2 \sigma\right] d_2 q^2, \tag{4a}$$

$$\omega_{2q}^2 \simeq \left[\frac{\alpha_1 \alpha_{12}}{\rho_1(\alpha_1 + \alpha_{12})} q^2 + \sigma\right] d_1 q^3. \tag{4b}$$

Far away from the anticrossing point, the spectrum behavior is quite different. The mode described by ω_{1q} is soundlike and corresponds to layer-thickness oscillations of the upper phase at $q \ll q_c$ and to flexural oscillations of upper phase in the opposite limit. On the contrary, the mode with ω_{2q}, which is a flexural mode at small q, transforms into the mode of layer-thickness oscillations at $q \gg q_c$, as one can see in Fig. 2(a). Note that the sound velocity $s = (3 n_2 a_2/\rho_2 d_2^3)^{1/2}$ for layer-thickness oscillations coincides with third-sound velocity in uniform film of superfluid ^{4}He over a solid substrate at $T < 1$ K, where the entropic contribution in sound velocity is negligible and the superfluid density practically coincides with the total liquid density.[16] By further decreasing d_2, one observes in Fig. 2(b) that q_c moves to higher q, the soundlike branch extends over a larger interval of q, and $|\omega_{1q} - \omega_{2q}|$ decreases at large q. The minimum $|\omega_{1q} - \omega_{2q}|$ is observed for $d_1 \approx d_2$, as shown in Fig. 2(d). The displacements of the liquid interfaces in these modes are of the same order and can be treated as layer-thickness oscillations, even though the velocities of the interface motion have the same direction in the mode ω_{1q} and opposite direction in the mode ω_{2q}. The DR's for these modes can be written as $\omega_{1(2)q} \simeq c_{1(2)} q$ with sound velocities given by[8]

$$c_{1,2}^2 = \frac{d_1}{2D}(\sigma_1(1+D) + \sigma_2 - 2\sigma \pm \{[\sigma_1(1+D) - \sigma_2]^2 + 4(\sigma_1 - \sigma)[\sigma_2 - \sigma(1+D)]\}^{1/2}), \tag{5}$$

where $D = \rho_2 d_1/\rho_1 d_2$. For a substrate with large van der Waals constant a_2 (for example, graphite), one can disregard the contribution σ of the interaction between helium phases in accelerations σ_1 and σ_2. In this approximation, the sound velocities, given by Eq. (5), are the same as those obtained in Ref. 12. However, for solid neon, used in the present work, the contribution from σ must be taken into account.

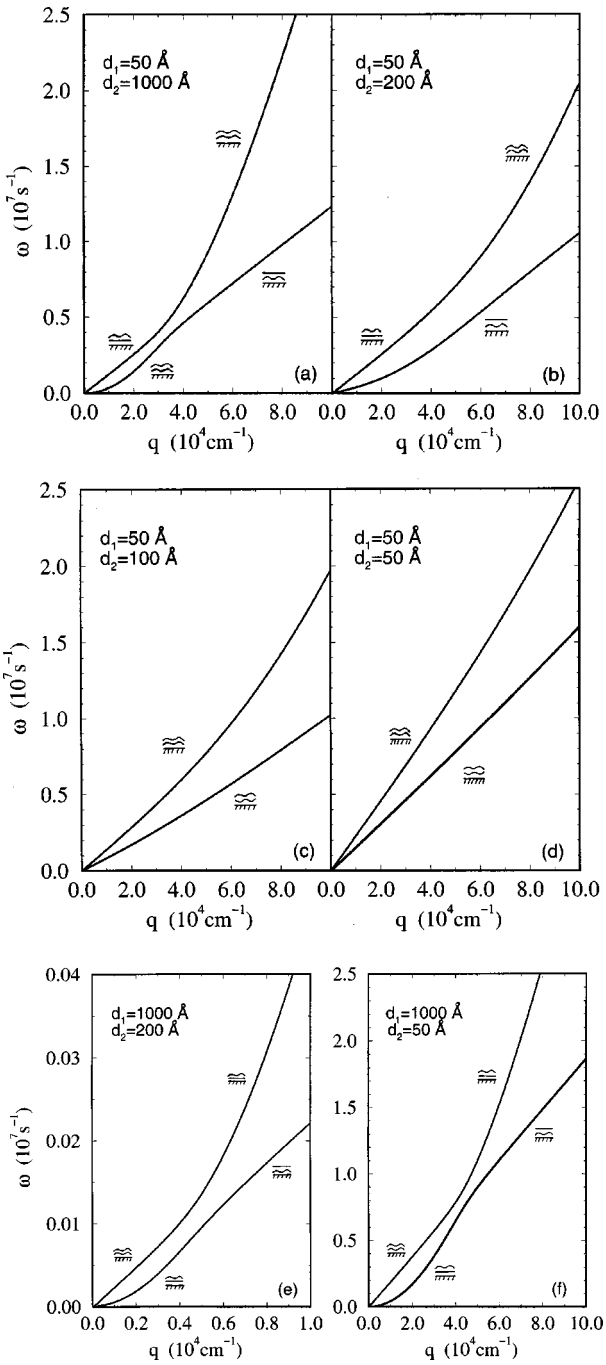

FIG. 2. Mode spectrum of the coupled oscillations of the layered ^{3}He-^{4}He film in the *ideal* case for different values of layer thicknesses. The substrate is solid neon. The displacements of the liquid interfaces at $z = 0$ and $z = -d_1$ are sketched.

The spectrum of collective modes becomes rather interesting for $d_1 \gg d_2$ as shown in Figs. 2(e) and 2(f). In this limit, the dispersion curves become similar to those obtained in the opposite limit $d_1 \ll d_2$ [Figs. 2(a) and 2(b)]. However, the physical description of the spectrum changes drastically. For $q \ll q_c$, the DR's are

$$\omega_{1q}^2 \simeq \left[\frac{3(n_2 a_2 - n_1 a_1)}{\rho_2 d_2^4} + \frac{\rho_1}{\rho_2}\left(\frac{d_1}{d_2}\right)^4 \sigma\right] d_2 q^2, \tag{6a}$$

$$\omega_{2q}^2 \simeq \left(\frac{\alpha_1}{\rho_1}q^2+\sigma_1\right)d_1q^2, \tag{6b}$$

and for $q \gg q_c$

$$\omega_{1q}^2 \simeq \left[\left(\frac{\alpha_1}{\rho_1}+\frac{\sigma d_1^2}{2D}\right)q^2+\sigma_1+\frac{\sigma_2}{D^2}\right]d_1q^2, \tag{7a}$$

$$\omega_{2q}^2 \simeq \frac{\rho_1}{\rho_2}\left[\left(\frac{\alpha_{12}}{\rho_1}+\sigma d_1^2\right)q^2+\frac{\alpha_{12}}{\alpha_1}\sigma_1+\sigma_2\right]d_2q^2. \tag{7b}$$

Now the mode ω_{1q} has a flexural character for $q \ll q_c$, and the mode ω_{2q} corresponds to the layer-thickness oscillation in this limit. At $q \gg q_c$, both modes are related to layer-thickness oscillations of the upper phase.

The unusual DR's of collective modes of the layered films in the ideal case make especially interesting the study of mode dissipation at real helium viscosities. To obtain the DR $\omega(q)=\omega_q - i\tau_q^{-1}$ containing both real and imaginary parts, one has to consider the numerical solutions of Eq. (2) for a given temperature. Hereafter, we restrict our analysis for $T=0.4$ K and $\eta_1=5\times10^{-5}$ Ps and $\eta_2=3\times10^{-5}$ Ps, with densities and surface tensions taken from Ref. 15. We have also evaluated the DR's for $T=0.1$ K, and the results are qualitatively the same.

We start the discussion with the case $d_1/d_2 \ll 1$. Numerical results give strong evidence that only one of the film modes is weakly damped. The spectrum of this mode is presented in Fig. 3 for two values of d_1 and different values of d_2. As it is clearly seen, the mode has weak dissipation at large enough q and is overdamped (ω_q is of the same order or can be neglected in comparison with τ_q^{-1}) at q smaller than a threshold value $q_0 \gtrsim q_c$. Let us define Δq as the wave-number interval in which the mode is weakly damped. With decreasing d_2 for a given d_1, Δq is displaced to larger q and ω_q starts very abruptly from zero at $q=q_0$, and τ_q^{-1} becomes larger than ω_q for $q<q_0$ showing a kink near q_0. The dispersion law of this mode coincides with that of the flexural mode of the microstratified solution ($d_2=\infty$) for $q \gg q_0$. Hence, we conclude that the upper phase *flexural* mode exhibits small dissipation. Note, however, that we cannot use the asymptotic expression given by Eq. (4a) for the frequency ω_q for $q \gg q_0 \gtrsim q_c$. Indeed, Eq. (4a) is valid for $q \ll d_1^{-1}, d_2^{-1}$. Meantime, ω_q for this mode starts to approach its asymptotic limit for $d_2 \to \infty$ at $q \gg q_0$ where $q > d_2^{-1}$. In order to obtain an analytical estimate for the DR of the weakly damped flexural mode, we have to analyze Eq. (2) in the limit of weak dissipation ($\eta_1, \eta_2 \to 0$) when $d_1 \ll d_2$ and $d_2^{-1} \ll q \ll d_1^{-1}$. As a result, the following asymptotic of DR of the flexural mode can be written as

$$\omega_q^2 \simeq (\alpha^*/\rho_2)q^3\tanh(qd_2). \tag{8}$$

Note that for $q \gg d_2^{-1}$, the above DR is the same as that for the flexural mode of the microstratified ^{3}He-^{4}He solution, i.e., $\omega_q^2 \simeq (\alpha^*/\rho_2)q^3$.[10,11] For $q<d_2^{-1}$, Eq. (8) reproduces Eq. (4a) for $q \gg q_c$, where the contribution from the van der Waals force is small compared with the capillary one, which is proportional to α^*/ρ_2. For this reason, Eq. (8) is the DR of the flexural mode in the case of ideal fluids in a wide Δq and

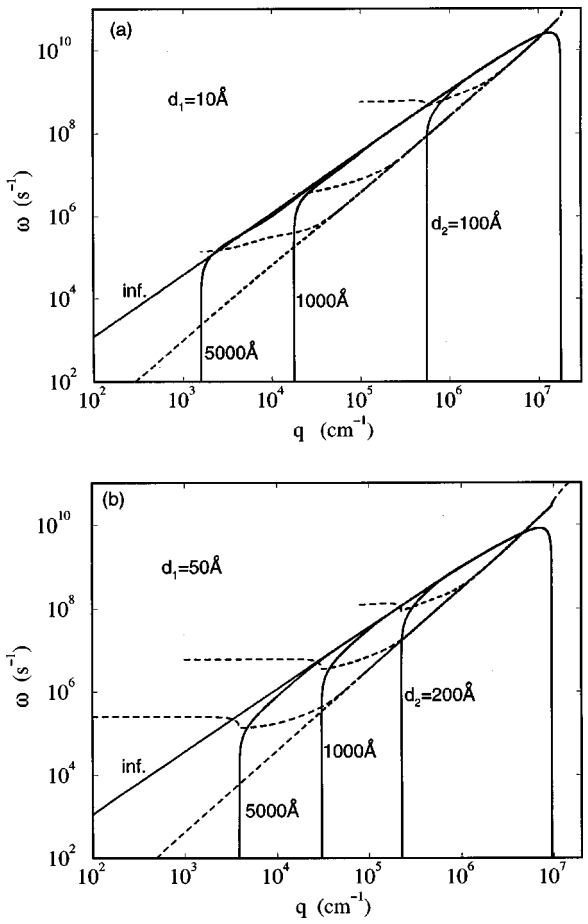

FIG. 3. Real (solid line) and imaginary (dashed line) parts of the dispersion relation corresponding to the flexural mode of the film for some lower-phase thicknesses d_2 and two values of the upper-phase thickness (a) $d_1=10$ Å and (b) $d_1=50$ Å.

for arbitrary d_2. The attenuation frequency $\tau_q^{-1} \simeq 2(\eta_2/\rho_2)q^2$, indicated by the dashed curve in Fig. 3, is typical of weak dissipation[19] and coincides with that of the microstratified solution.[11]

As it is well known,[19] strong mode dissipation should be found when the viscous penetration depth δ is larger than any characteristic length of the system. The dissipation of the flexural mode for $q \ll q_0$, where we have $qd_2<1$, can be explained by comparing δ for this mode with d_2 taken as the characteristic length. Extrapolating the DR $\omega_q^2 \simeq (\alpha^*/\rho_2)q^3\tanh(qd_2)$ for $q \lesssim q_0$, where the dissipation is strong, one can write $\delta=(\eta_2/\rho_{2n}\omega_q)^{1/2}$ for this mode. We obtain that *strong* dissipation of the flexural mode takes place for $q \ll q_0^{(1)}$ where $q_0^{(1)}=(\eta_2/\rho_{2n})^{1/2}(\rho_2/\alpha^*)^{1/4}d_2^{-5/4}$. Weak dissipation should appear in the opposite limit. Numerical values of $q_0^{(1)}$ vary from 6.4×10^3 to 8.5×10^5 cm^{-1} as d_2 changes from 5×10^{-5} to 10^{-6} cm, whereas q_0 varies from 4×10^3 to 1.2×10^5 cm for the same d_2 and $d_1=5\times10^{-7}$ cm [see Fig. 3(b)]. On the other hand, one can compare δ, for same values of d_2, calculated for the frequency $\omega_q^2 \simeq (\alpha^*/\rho_2)q^3$ of the weakly damped flexural mode for $q \gg q_0$. *Weak* dissipation can be expected when $\delta \ll d_2$, which gives $q \gg q_0^{(2)}$, where $q_0^{(2)}=(\eta_2/\rho_{2n})^{2/3}(\rho_2/\alpha^*)^{1/3}d_2^{-4/3}$. This is a quite different result from the other case. Obviously, one can expect strong

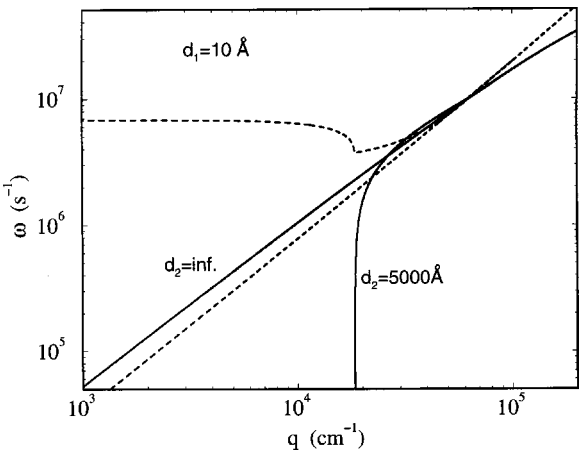

FIG. 4. The same as Fig. 3 but for the overdamped soundlike mode for $d_1 \ll d_2$. The spectrum in the case of microstratification ($d_2=\infty$) is also presented for $d_1=10$ Å.

dissipation for $q \ll q_0^{(2)}$. When d_2 decreases from 5×10^{-5} to 10^{-6} cm, $q_0^{(2)}$ changes from 4.4×10^3 to 8.0×10^5 cm^{-1}, which is closer to q_0 than $q_0^{(1)}$. These estimates agree reasonably well with our numerical results and explain the existence of weak dissipation of the modes at large q and the dependence of q_0 on d_2 shown in Fig. 3. We observe also in Fig. 3 the weak dependence of q_0 on d_1. For a given d_2, q_0 moves to smaller q as d_1 decreases. For $q \gg \omega_q \rho_{2n}/\eta_2$, the DR is independent of d_2 and exhibits a maximum and a strong decrease of ω_q in comparison with τ_q^{-1}, indicating that the mode with $\omega_q^2 \simeq (\alpha^*/\rho_2)q^3$ is overdamped.

For ideal fluids, the spectrum of collective oscillations of the layered film contains also the soundlike mode of layer-thickness oscillations of the upper phase for $d_1 \ll d_2$ as given by Eqs. (3a) and (4b).[8] This is not the case for real fluids, since we did not obtain roots of Eq. (2) with $\omega_q \gg \tau_q^{-1}$ for any q except those given by Eq. (8). In Fig. 4, the dispersion law of the soundlike mode is presented for $d_1=10^{-7}$ cm and $d_2=5\times10^{-7}$ cm. For comparison, we plotted also the soundlike mode spectrum in the limit $d_2=\infty$ (microstratification).[11] As it is seen in Fig. 4, the dissipation of this mode in the layered film is stronger than in the microstratificated situation. One can conclude that the mode corresponding to layer-thickness oscillations of upper phase with the thickness d_1 is overdamped for all q investigated both in the case of ^{3}He-^{4}He solution with microstratification ($d_2 \to \infty$) and for layered film with finite values of d_2. The strong dissipation of the soundlike mode means that viscous forces acting upon the particles moving with different velocities in this mode are so essential at real helium viscosities that the oscillatory motion of the phase thickness cannot exist. On the other hand, as the velocities of two liquid interfaces are equal at $z=0$ and $z=-d_1$, a flexural mode with weak dissipation survives in the presence of liquid viscosity.

As is clearly seen in Fig. 3, Δq becomes narrower as d_2 decreases. This behavior is clearly evidenced in Fig. 5, where the mode spectrum is depicted for d_2 close to d_1. One can see that Δq becomes shorter when d_2 changes from 2×10^{-6} to 5×10^{-7} cm but also τ_q^{-1} approaches ω_q. For $d_2=d_1=5\times10^{-7}$ cm, ω_q and τ_q^{-1} are of the same order, which means that this mode is overdamped and cannot be observed.

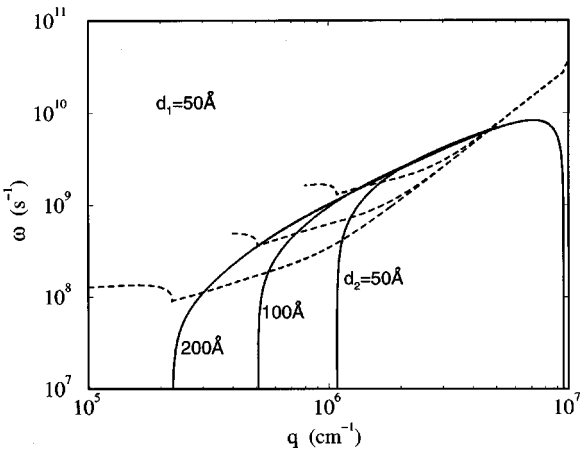

FIG. 5. The same as in the previous figures but now for the flexural mode for $d_1 \cong d_2$.

The situation is rather interesting for $d_1/d_2 \gg 1$ as presented in Fig. 6. Here Δq is larger than that in Fig. 5, $|\omega_q - \tau_q^{-1}|$ also increases, and the mode spectrum becomes formally similar to the DR in the opposite limit $d_1/d_2 \ll 1$. One of the branches starts abruptly from q_0 and is weakly damped for $q \gg q_0 \gtrsim q_c$ and $d_1^{-1} \ll q \ll d_2^{-1}$. The asymptotic solution of Eq. (2) for this branch can be written for weak dissipation as

$$\omega_q^2 \simeq (\alpha_1/\rho_1)q^3 \tanh(qd_1), \tag{9}$$

which is the well-known DR for capillary waves propagating on the surface of a ^{3}He film with thickness d_1. For $q \gg q_0$, one has $\omega_q^2 \simeq (\alpha_1/\rho_1)q^3$, which corresponds to the DR of a capillary wave on the free surface of bulk ^{3}He. Our numerical values of ω_q for this branch agree rather well with Eq. (9). Note that for $qd_1 < 1$, Eq. (9) is formally identical to Eq. (7a) for the mode ω_{1q}. For this reason, we added the term $\tanh(qd_1)$ in Eq. (9), which tends to unity for $q > q_0$. The asymptotic expression $\tau_q^{-1} \simeq 2(\eta_1/\rho_1)q^2$ corresponding to weak dissipation of capillary waves on the free surface of bulk ^{3}He well describes our results in Fig. 6 for $q > q_0$. The dependence of q_0 on d_1 can be explained along the same lines as the dependence of q_0 on d_2 for $d_1/d_2 \ll 1$. Now we have to compare δ relative to the wave propagation along the

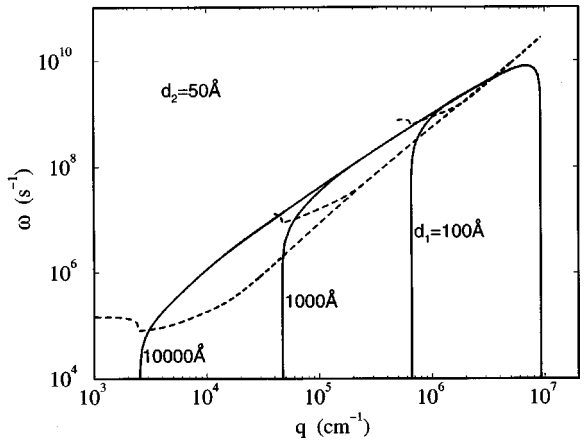

FIG. 6. The same as Fig. 5 but now for the capillarylike mode of upper-thickness oscillations for $d_2=50$ Å and different d_1.

^{3}He free surface with thickness d_1. In this case, we found $q_0^{(1)}=(\eta_1/\rho_1)^{1/2}(\rho_1/\alpha_1)^{1/4}d_1^{-5/4}$ and $q_0^{(2)}=(\eta_1/\rho_1)^{2/3}(\rho_1/\alpha_1)^{1/3}d_1^{-4/3}$ for $d_1/d_2\gg 1$, which agree quantitatively with our numerical results. Indeed, as it is seen from Fig. 6, q_0 varies from 2.6×10^3 cm^{-1} to 6.5×10^5 cm^{-1} as d_1 changes from 10^{-4} cm to 10^{-6} cm. Correspondingly, for the same d_1, one obtains $2.1\times10^3<q_0^{(1)}<6.6\times10^5$ cm^{-1} and $1.2\times10^3<q_0^{(2)}<5.7\times10^5$ cm^{-1}.

One can conclude that the capillary mode with the same DR as in the free surface of bulk ^{3}He has weak dissipation at $q>q_0$ for $d_1\gg d_2$. However, as it is seen in Fig. 6, strong dissipation takes place at $q\gg q_0$ when $\lambda\ll\delta$. According to Eqs. (6a)–(7b), the second branch of the DR corresponds to flexural displacements of the upper phase at $q<q_c$ and capillary waves on the interface between a thick upper and a thin lower phase ($d_2\ll d_1$) for $q>q_c$. We did not find weakly damped solutions of Eq. (2) in this case, and hence we conclude that the second branch of the spectrum is overdamped for all q investigated.

IV. CONCLUSIONS

In the present work, we have studied the mode damping in the layered ^{3}He-^{4}He film over a solid substrate. We have calculated the dispersion law of the coupled oscillations of two liquid interfaces taking into account viscous dissipation. The calculations are carried out for layer thicknesses $\lesssim10^{-5}$ cm where the van der Waals interaction between the helium phases and with solid substrate plays a crucial role in the mode spectrum. We have shown that flexural oscillations of the upper phase for $d_1\ll d_2$ and the capillarylike oscillations of thick film of upper liquid phase in the opposite limit ($d_1\gg d_2$) do exist in the mode spectrum at large $q>q_0$. The critical q_0 depends strongly on the layer thicknesses of the phases. No weakly damped modes were found when the layer thicknesses are approximately equal. There are two ''bottlenecks'' of layer thicknesses for weak dissipation where the layer thickness of one of the phases is significantly larger than the thickness of the other phase. By fixing the layer thickness of one phase and changing continuously the other, one should reach experimentally the situation $d_1\simeq d_2$, where the modes are overdamped and cannot propagate for all q. The strong attenuation of the modes at small q can be explained by assuming that the penetration depth related to the modes exceeds the thickness of broader phase. The possibility of strong dissipation for small q may be a serious obstacle for experimental observation of the mode when the upper phase is normal ^{3}He. This makes preferable the experimental study of these modes at very low T where transition to the superfluid state of ^{3}He can happen. In such a condition, one can expect weakly damped oscillations of the stratified film propagating through superfluid components with DR given in Refs. 12 and 8 and shown in Fig. 2.

Note also that mode damping may affect strongly the properties of surface electrons over the layered film. For $q<2k_T\sim10^5$ cm^{-1}, the electron-ripplon interaction should be weakened, especially at $d_1\ll d_2$ when the weakly damped mode can be observed only at $q\gtrsim2k_T$. In this case, the main electron-scattering mechanism should be residual helium atoms, which is negligible for $T<1$ K in the case of bulk ^{4}He. For $d_1\gg d_2$, where the DR is given by Eq. (9), the electron-ripplon scattering is the same as that over bulk ^{3}He. On the other hand, mobility measurements over bulk ^{3}He have evidenced that ripplons on the free ^{3}He surface can unexpectedly contribute to the electron scattering at T around 0.1 K despite the strong dissipation of capillary waves in ^{3}He for values of q, which contributes to the electron-ripplon interaction.[22] We hope this paper will shed some light on the study of transport phenomena of surface electrons over the stratified film of helium isotopes at low temperatures.

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APPENDIX

For the sake of completeness, we list here the functions β_i presented in the expressions for F_{ij} of Eq. (2), which defines the dispersion equation of the coupled oscillations of the layered film over a solid substrate. The functions are as follows:

$$\beta_1=\frac{(1+\lambda_2^2/q^2)\sinh(\lambda_2 d_2)\sinh(qd_2)-2(\lambda_2/q)[\cosh(\lambda_2 d_2)\cosh(qd_2)-1]}{(1-\lambda_2/q)[\sinh(\lambda_2 d_2)\cosh(qd_2)-(\lambda_2/q)\cosh(\lambda_2 d_2)\sinh(qd_2)]},$$

$$\beta_2=\frac{(1+\lambda_2^2/q^2)[\sinh(\lambda_2 d_2)-(\lambda_2/q)\cosh(\lambda_2 d_2)]+2(\lambda_2/q)\exp(-qd_2)}{(1+\lambda_2^2/q^2)[\sinh(\lambda_2 d_2)-(\lambda_2/q)\cosh(\lambda_2 d_2)+(\lambda_2/q)\exp(-qd_2)]},$$

$$\beta_3=\frac{\cosh(\lambda_2 d_2)-(\lambda_2/q)\sinh(\lambda_2 d_2)-\exp(-qd_2)}{\sinh(\lambda_2 d_2)-(\lambda_2/q)[\cosh(\lambda_2 d_2)-\exp(-qd_2)]},$$

$$\beta_4=\frac{(1+3\lambda_2^2/q^2)\sinh(\lambda_2 d_2)\sinh(qd_2)-(\lambda_2/q)(3+\lambda_2^2/q^2)[\cosh(\lambda_2 d_2)\cosh(qd_2)-1]}{(1-\lambda_2/q)^2[\sinh(\lambda_2 d_2)\cosh(qd_2)-(\lambda_2/q)\cosh(\lambda_2 d_2)\sinh(qd_2)]},$$

$$\beta_5=\frac{\cosh(\lambda_2 d_2)-(\lambda_2/q)\sinh(\lambda_2 d_2)-(1/2)(1+\lambda_2^2/q^2)\exp(-qd_2)}{\sinh(\lambda_2 d_2)-(\lambda_2/q)[\cosh(\lambda_2 d_2)-\exp(-qd_2)]}.$$

As $d_2\to\infty$, all β_i approach 1. In the limit of $\eta_2\to 0$, $\beta_4\simeq 1/\tanh(qd_2)$, and $\beta_i\simeq 1$ for $i\neq 4$.

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