Calculation of spatially resolved energy dissipation in the critical-state model: Supercooling and metastable states

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Metastable states of vortex matter can be converted to stable phases through the fluctuation energy dissipated during isothermal field variations. The critical state model provides a calculation of the total energy dissipated in the sample during a field cycle through the area enclosed within the *M-H* loop. We present a formalism for calculating the spatial distribution of the energy dissipation, and show that it rises quadratically near the surface.

Supercooled or metastable phases of vortex matter have been receiving attention[1–8] and it has been reported that supercooled vortex phases persist further under field cooling (FC) than under isothermal field variation.[6,7] We have proposed that this is because energy dissipation during isothermal field variations provides a fluctuation energy that causes the metastable supercooled phase to cross the free-energy barrier and transform to the stable equilibrium phase.[9] (We shall refer to this process as a ''metastable to stable transformation'' for brevity.) The total fluctuation energy produced in the sample under a cyclic field variation is given by the area within an *M-H* loop and can be calculated within the critical state model (CSM).[10–12]

Recent experiments have reported that different (metastable and equilibrium) phases can exist simultaneously in different regions of the sample.[1–3,13] Zeldov *et al.*[14] had also shown that vortex-lattice melting occurs when the local magnetic induction reaches a critical value, and different regions of the sample exhibit vortex-lattice melting at different values of applied fields. In the light of such measurements with local probes it appears that quasiequilibrium is established locally, and not globally, on experimental time scales. It is thus important to calculate the spatially resolved energy dissipation, and the fluctuation energy created locally, when a hard superconductor is subjected to an isothermal variation of magnetic field. We shall consider here the case of an infinite slab in a parallel field as this geometry has the simplest algebra among the zero-demagnetization-factor cases of infinite cylinders in a parallel field. We shall also use Bean's simplifying assumption of a field-independent J_C,[10] and shall continue with the assumption $H_{C1}=0$ followed in most papers on the CSM.[12,15]

We consider the case of a virgin zero-field-cooled (ZFC) slab with surfaces at $x=\pm R$, with an applied field along the z axis that is raised isothermally from 0 to B_m. After establishing our formalism, we shall consider the case where the external field is cycled from $-B_m$ to B_m and back to $-B_m$. As is known, field profiles (and energy dissipation) in second and subsequent cycles do not depend on the sample thermomagnetic history (e.g., whether FC or ZFC). We shall show explicitly that the value of the total energy dissipated in the sample, as obtained from our formalism, agrees with that obtained from the area within the *M-H* loop.

Bean had recognized that the energy dissipation due to bulk pinning also corresponds to electric fields, generated by the time-varying magnetic fields, being parallel to the local shielding current density $\vec{J}_S$ (of magnitude $\pm J_C$).[16] This interpretation was used by him to attack the case of rotating magnetic fields applied parallel to the surface of an infinite slab.[16] If both $\vec{J}_S(x)$ and $\vec{E}(x)$ can be obtained, this provides a scheme for calculating the spatially resolved energy dissipation through $\vec{J}\cdot\vec{E}$.

We first consider that the external field along **z** is raised from 0 to B_m and assume that this is done at a constant rate b, in time T. (We shall see later that our results for energy dissipation are independent of b or T, so that this is not a restrictive assumption.) In the region of the sample where flux is penetrating, we have $\partial\vec{B}(x)/\partial t=b\mathbf{z}$ so that curl $\vec{E}$ is nonzero and we get $(dE_y/dx)=-b$. The shielding currents are set up to the point $x_0(t)=R-(bt/J_C)$, and the region interior of $x_0(t)$ has seen no flux and provides the physical boundary condition that $\vec{E}$ must vanish in the interior of $x_0(t)$. For time $t<T=B_m/b$, we thus have, for $|x|>x_0(t)=R-(bt/J_C)$,

$$\vec{B}(x,t)=[bt-J_c(R-x)]\mathbf{z},$$

$$\vec{J}_s(x,t)=-J_C\mathbf{y},$$

$$\frac{\partial\vec{E}(x,t)}{\partial x}=-b\mathbf{y},$$

$$E_y(x,t)=-\int_{x_0(t)}^{x} b\,dx. \tag{1}$$

The right-hand side of all the expressions vanishes for $|x|<x_0(t)$. For simplicity we shall only present expressions for positive x in the rest of the paper. The local rate of energy dissipation per unit volume is given by $\vec{J}\cdot\vec{E}$ and we get the total energy $P_d(x)$ dissipated at x, as the external field is raised from 0 to $B_m=bT$, as

$$P_d(x)=\int_0^T \vec{J}(x)\cdot\vec{E}(x)\,dt=\int_{t_1}^T \vec{J}(x)\cdot\vec{E}(x)\,dt, \tag{2}$$

since $E(x,t)$ vanishes at x for $t<t_1(x)=(R-x)J_c/b$. Thus using Eq. (1) we get $P_d(x)=J_C b\int_{t_1(x)}^{T}dt\int_{x_0(t)}^{x}ds$. When B_m is smaller than the field for full penetration, $B^*=J_C R$, we get

$$P_d(x)=(J_C b/2)[T-t_1(x)][x-x_0(T)] \tag{3}$$

and $P_d(x)$ vanishes for $x<x_0(T)=R-(B_m/J_C)$. Substituting for T and $t_1(x)$, we get for $B_m<B^*$

$$\begin{aligned}P_d(x)&=(J_C/2)[B_m-(R-x)J_C][x-R+(B_m/J_C)]\\&=(1/2)B(x)^2\end{aligned} \tag{4}$$

and we note that $B(x)$ is also the total change in B at x as the external field varies from 0 to B_m. For $B_m>B^*$, we similarly obtain

$$P_d(x)=(1/2)J_C^2x^2+J_Cx(B_m-B^*) \tag{5}$$

and $P_d(x)$ is finite for all x. The total change in B at x is now from zero to $B(x)=J_Cx+(B_m-B^*)$, and $P_d(x)$ is no longer simply related to $B(x)$. We note from Eqs. (4) and (5) that $P_d(x)$ is independent of the rate of ramping the external field from 0 to B_m. The ramping could have also taken place in smaller steps of unequal rate with no change in the total energy dissipated at x.

We now consider the case of a sample with applied field $B_{ext}=-B_m$ which has been prepared by reducing field isothermally from above B^*. The field is now raised to B_m and lowered back to $-B_m$. Following the method outlined above, we get the spatial distribution of the energy dissipated during this complete cycle as

$$P_d(x)=2J_C^2(x-x_0)^2=2[\delta B(x)]^2 \quad \text{for} \quad B_m\leqslant B^*, \tag{6}$$

where $\delta B(x)$ is the amplitude of the excursion in $B(x)$. Again $P_d(x)$ vanishes for $x<[R-(B_m/J_C)]$. And, for $B_m\geqslant B^*$, we get

$$P_d(x)=2J_C^2x^2+4J_Cx[B_m-B^*] \tag{7}$$

and again $P_d(x)$ is finite for all x. As a cross-check, we calculate the total energy dissipated over the entire samples as $Q=(1/R)\int_0^R P_d(x)dx$, and get

$$\begin{aligned}Q&=(2/3)B_m^3/B^* \quad \text{for} \quad B_m\leqslant B^*\\&=2B^*B_m-(4/3)B^{*2} \quad \text{for} \quad B_m\geqslant B^*.\end{aligned} \tag{8}$$

The results in Eq. (8) agree with standard results[11] of the CSM obtained from the area within the *M-H* loop. This completes a necessary cross-check on our main results in Eqs. (4)–(7).

We have assumed here that J_C is independent of B. The above expressions for $P_d(x)$ thus remain unchanged if the applied field is cycled around a bias field B_0, i.e., between B_0-B_m and B_0+B_m. Our formalism goes through when $J_C(B)$ is not a constant. $P_d(x)$ would then depend on B_0, but the leading x dependence would be still given by Eqs. (6) and (7).

We now discuss the transformation from a metastable supercooled state, to the stable equilibrium state, under isothermal field variation. As discussed in Ref. 9, this field variation produces a fluctuation energy e_f and the transformation will occur when $e_f+k_BT\approx f_B(T)$, where f_B is the free-energy barrier surrounding the supercooled state. As mentioned in the introduction, quasiequilibrium in vortex matter appears to be established only locally on experimental time scales. The metastable to stable transformation would thus occur in the neighborhood of x when

$$P_d(x)+k_BT\approx f_B(T) \tag{9}$$

and the x dependence of $P_d(x)$ is similar for unidirectional [Eqs. (4) and (5)] and cyclic [Eqs. (6) and (7)] variations of the applied field. For small variations, $P_d(x)$ rises quadratically close to the surface, and this can explain why metastable to stable transformations are triggered progressively from the surface.[3] Further, under cyclic fields with $B_m \ll B^*$, the metastable phase will continue to exist near the center of the sample even under repeated cycling because $P_d(x)$ vanishes for $x<[R-(B_m/J_C)]$.

To conclude, we have presented a calculation of the spatially resolved energy dissipation when a hard superconductor is subjected to an isothermal field variation. This is an experimentally relevant quantity because measurements with local probes have shown that vortex matter attains quasiequilibrium locally, and not globally over the sample. Our results show that this local energy dissipation rises quadratically with x as x rises towards the sample surface, implying that metastable to stable transformations would, under ac fields, be triggered from the surface.

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