# Current effect on vortex-antivortex depairing in type-II superconductors

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An iterated mean-field approach is here considered to describe the contribution of the applied current to the generation of thermally excited vortices. The Lorentz term is added to the vortex-antivortex pairing potential and the obtained interaction, following the Kosterlitz-Thouless scheme, is screened by the dielectric constant. The recoursion equations are solved and an expression of the resistivity vs temperature, in the limit of low applied currents, is obtained above a current dependent critical temperature, whose expression is in good agreement with the experimental results.

## I. INTRODUCTION

The dissipation in type-II superconductors is usually attributed to the viscous motion of flux lines under the influence of the Lorentz force. In most experimental conditions the fluxons are generated by an external magnetic field and in these cases the vortices, with an hexagonal configuration (Abrikosov lattice), are mainly oriented with the applied field, producing well-known dissipative processes.[1] Without the magnetic field the resistivity is still attributed to the motion of flux lines, but their origin and dynamics are still an open question. Widely accepted proposals consider flux generation as due to the thermal fluctuations of the order parameter phase $\theta$.[2] According to the Ginsburg-Landau theory, the supercurrent $j$ is related to $\theta$ and to the potential vector $\mathbf{A}(\mathbf{r})$ as $\mathbf{j}(\mathbf{r})=(\hbar\pi n_s/2m)[\nabla\theta(\mathbf{r})-(eh/c)\mathbf{A}(\mathbf{r})]$. The previous equation with $\int\nabla\theta(\mathbf{r})\cdot\mathbf{dr}=\pm 2\pi$ defines the current vortices, whose dimensions are of the order of the penetration depth $\lambda$. Each vortex encloses a normal region core with a quantum magnetic flux $\phi_0$ and a radius equal to the coherence length $\xi$.

These excitations are more probable in planar systems, where the reduced dimensionality favors thermal fluctuation.[1] This happens in films when $d_0<\xi$ ($d_0$ being the thickness), or in layered compounds, like cuprates, if the distance $z$ between the noninteracting planes is larger than $\xi_c$ ($\xi_c$ being the $c$-axis coherence length); in these cases the core radius is $\xi_{ab}$.

Because of the total zero magnetic moment, the formation of vortices ($v$) and antivortices ($a$) is energetically favored in two-dimensional (2D) systems. At low temperatures these vortices are coupled by a logarithmic field:[3,4]

$$U_0(r)=q_0^2\ln\left(\frac{r}{\xi}\right), \tag{1}$$

where $r$ is the $v$-$a$ pair size ($r>\xi$). This interaction provides an analogy with the 2D Coulomb gas, where a temperature-dependent effective charge is associated to each vortex: $q_0(T)=\pm\sqrt{\pi\hbar n_s(T)/2m}$ ($q_0>0$ for vortices; $q_0<0$ for antivortices),[5] with $n_s(T)=n_s(0)(1-T/T_{co})$ the areal supercarrier density ($n_s(T)=n_s^{3d}(T)d$), $T_{co}$ the Ginzburg-Landau critical temperature. Then each pair is thought of as an electrical dipole and, at large density, the screening of smaller pairs, polarized in the field of the larger ones, must be taken into account. Kosterlitz and Thouless[6] (KT) propose to consider, for a pair with size $r$, a dielectric constant $\varepsilon(r)$ which reduces the interaction (1), giving rise to an effective charge: $q^2(T)=q_0^2(T)/\varepsilon(r)$. By increasing the temperature the $v$-$a$ density becames larger and consequently also the screening effect. At a temperature $T_k^0$ (KT critical temperature) the interaction is completely canceled (full renormalization) and the system exhibits a phase transition between a paired configuration and unpaired one.

Different electrical properties are expected in the two phases, corresponding to the mechanisms invoked to produce moving vortices with a resistivity proportional to the free vortex density $n_f$:[7]

$$\rho=2\pi\xi^2\rho_n n_f, \tag{2}$$

where $\rho_n$ is the normal-state resistivity.

For $T>T_k^0$ unpaired vortices are thermally produced and one has an Ohmic response:[8]

$$\rho=A\rho_n\exp(-2\sqrt{b/|\tau^0|}), \tag{3}$$

where $b$ is a sample-dependent parameter and $\tau^0=(T_k^0-T)/(T_{co}-T_k^0)$ is an appropriate reduced temperature.

For $T<T_k^0$ the Lorentz force $F_L=j\phi_0/c$ dissociates loose pairs with an efficiency increasing with the applied current density: non-Ohmic behavior is then expected. In this case the total potential (1) is modified by a term due to the current contribution:

$$U_j(r)=U_o(r)-F_l\cdot r=q^2\left(\ln\frac{r}{\xi}-\frac{2mj}{\hbar n_s e}r\right). \tag{4}$$

The energy has a maximum at $r_m=(\hbar n_s e)/(2mj)$ which is overcome by a classical jump with the depairing rate $\Gamma\propto\exp[-U_j(r)/kT]=(j/j^*)^{q^2/2kT}$, where $j^*=(\hbar n_s e)/(2m\xi)$ is the Ginzburg-Landau critical current. If $\gamma$ is the recombination probability at the equilibrium: $\Gamma-\gamma n_f^2=0$ and $n_f\sim\Gamma^{1/2}$. Then the non-Ohmic contribution is described by a power law: $\rho\sim j^{a(T)}$, where $a(T)=q^2/2kT$ decreases with $T$ and it is expected to jump from 2 to 0 (universal jump condition) at the critical temperature $T_k^0$.

 

The linear dissipation in Eq. (3) and the power dependence of the $I$-$V$ characteristics are considered as the signature for the applicability of the KT theory to 2D superconductors.[3,4]

An alternative point of view considers Eq. (4) as starting point, where the pair energy is reduced by the current; the couple generation and the screening effect are consequently favored. Now the complete depairing occurs at temperature $T_k(j)$, which decreases with the current. This picture is considered by Pierson[9–11] who uses a rigorous real-space renormalization-group theory to study the critical behavior of vortices in a layered system in the presence of current. In that work a linear dependence of the critical temperature is proposed. The agreement with the experimental data covers a limited region of the $j$-$T$ plane, while a deviation when $j$ approaches to zero, remains unexplained.

Here the KT transition in 2D superconductors in the presence of applied current is studied by means of an ''iterated mean-field approximation'' approach, following the procedure used by Fischer[12] to study the interplanar coupling effect. The obtained scale recoursion equations are solved and a dependence of the critical current vs temperature in good agreement with the experiments is found.

## II. THE MODEL

In the presence of current,[4] the density of the pairs with separation $r$ is

$$n_P(r)=\left(\frac{N_0}{\xi^2}\right)^2\exp\left(-\frac{2W+U_j(r)}{kT}\right), \tag{5}$$

where $W$ is the formation energy for single vortices and $N_0$ is the probability that a vortex core is situated in a cell of area $\xi^2$.

The interaction potential for a pair of size $r$ is screened by the smaller couples and, by introducing the dielectric constant $\varepsilon(r)$, one has

$$U_j(r)=\int_\xi^r\left(\frac{q_0^2}{\varepsilon(r)}\right)\left(\frac{1}{r'}-\frac{2mj}{\hbar n_s e}\right)dr'. \tag{6}$$

As usual the dielectric constant is $\varepsilon(r)=1+4\pi\chi(r)$ and in terms of the pair polarizability $\alpha=q_0^2r^2/4kT$, one has $\varepsilon(r)=1+4\pi\int_\xi^r dr'\int_0^{2\pi}d\theta r' n_P(r',\theta)\alpha(r')$.

To know the pairing status in the function on $r$, one has to derive the coupled Eqs. (5) and (6) and it is convenient to introduce the length scale $l=\ln(r/\xi)$ and the three variables: $x(l)=[2/\pi K(l)]-1$: where $\pi K(l)=q_0^2/2\varepsilon(l)kT$ (reduced stiffness constant) is the $v$-$a$ coupling strength in the screening condition; $y(l)=\exp\{2l-[2W+U_j(l)]/2kT\}$: the pair excitation probability; $J(l)=(j/j^*)e^l$: the adimensional current contribution.

At low depairing conditions ($y\ll 1$) the scaling equations are

$$dx/dl=8\pi^2y^2, \tag{7a}$$

$$dy/dl=2y(x+J)/(x+1), \tag{7b}$$

$$dJ/dl=J. \tag{7c}$$

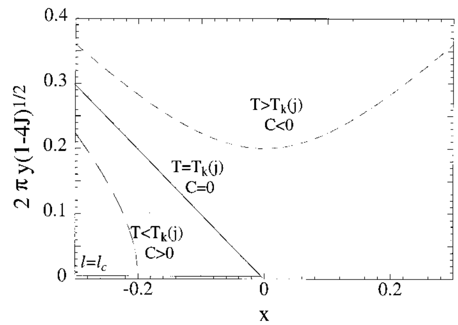


FIG. 1. Hyperbolic trends of $2\pi y(1-4J)^{1/2}$ vs $x$, as indicated in the text, for three values of temperature. That corresponding to $C=0$ ($T=T_k$) separates the two regions: that of the coupled $v$-$a$ pairs ($C>0$, $T<T_k$) and that of free vortices ($C<0$, $T>T_k$).

Equations (7) are different from the more rigorous Eqs. (4) and (5) of Ref. 9, but in the limit of low current and near the critical point the conclusions are similar.

Near the critical condition ($T\sim T_k, |x|\ll 1$), the solution of Eqs. (7) is

$$x^2-4\pi^2y^2=C(T,j)-I_1. \tag{8}$$

The quantity $I_1=2\int J dx=16\pi^2\int y^2(l')J(l')dl'$ of Eq. (8) should be evaluated by an iterative method, however, since $J(l)$ grows exponentially with $l$, in fair approximation, for $J(l)\ll 1$ one has $I_1=16\pi^2y^2J$.

In Eq. (8), $C=x_0^2-4\pi^2y_0^2+16\pi^2y_0^2J_0$ ($x_0$ and $y_0$ being the initial values of $x$ and $y$ for $l=0$ and $J_0=j/j^*$) and in the low current conditions ($j\ll j^*$), as usual in the experiments, is the same integration constant of the KT theory.[6] At temperatures near $T_k(j), C$ is expanded in power of the temperature: $C=B\tau(\tau\ll 1)$ with $C(0)=0$ and $B=C'(0)=-2x_0x_0'+8\pi^2y_0y_0'$ and $\tau=[T_k(j)-T]/[T_{co}-T_k(j)]$. Then $C=0$ for $T=T_k(j)$ and it changes its sign from $T<T_k(j)$ ($C>0$) to $T>T_k(j)$ ($C<0$).

By substituting $I_1$ in Eq. (8) one has

$$4\pi^2y^2=\frac{x^2-C}{1-4J}. \tag{9}$$

Since $x^2-C$ is always positive,[3,4,6] $y$ real values are assured if $J\leqslant 1/4$ and for $J=1/4$ the maximum length value $l_c=\ln(j^*/4j)$ may be considered as a cutoff in the integral (6). Equation (9) represents a set of hyperboles (Fig. 1) on the plane $(x,2\pi y\sqrt{(1-4J)})$, one for each temperature, along which $l$ varies. The behavior of the system is indicated by the limit of $x$ and $y$ when $l\rightarrow l_c$; in this way one takes into consideration all the pairs.

For $J=0$ Eqs. (7), (8), and (9) are the same as the KT theory and $l_c\rightarrow\infty$, being the integration upper limit of the KT recoursion equations.

Then $T_k(j)=T_k^0$ and when $C=0$ ($T=T_k^0$) Eq. (9) represents a straight line which, for $l\rightarrow\infty$, gives $x=0$ ($\pi K=2$) (universal jump condition). This critical behavior delimits two regimes:

(a) when $C>0$ ($T<T_k^0$), $x$ approaches to a finite value and $y\rightarrow 0$, meaning a nonzero coupling strength and all vortices paired.

(b) when $C<0$ ($T>T_k^0$), $x\to\infty$ and $y\to\infty$, indicating a zero coupling strength and a complete dissociation.

Then $T_k$ divides the totally paired configuration from the unpaired one (KT transition).[6]

For $J\neq 0$ the solutions of Eqs. (7) are

(a) for $C=0$ $[T=T_k(j)]$:

$$x(l)=\frac{x_0}{1-(2l+I_2)x_0}, \tag{10a}$$

$$2\pi\sqrt{1-4J}y(l)=x(l). \tag{10b}$$

On the $[x,y(1-4J)^{1/2}]$ plane, one has a straight line reaching the $x$ axis ($l\to lc$) at $x=0$ ($\pi K=2$; universal jump condition).

(b) for $C>0$ $[T<T_k(j)]$:

$$x(l)=-\sqrt{C}\coth(D), \tag{11a}$$

$$2\pi\sqrt{1-4J}y(l)=\sqrt{C}\operatorname{csc}h(D), \tag{11b}$$

where $D=2\sqrt{C}(l+I_2)+\coth^{-1}(x_0/\sqrt{C})$. On the $[x,y(1-4J)^{1/2}]$ plane, the curves have downward concavity. For $l\to lc$ one has $x\to-\sqrt{C}$ and $y\to 0$. The coupling strength is nonzero and all vortices are paired.

(c) for $C<0$ $[T>T_k(j)]$:

$$x(l)=\sqrt{|C|}\tan(D'), \tag{12a}$$

$$2\pi\sqrt{1-4J}y(l)=\sqrt{|C|}\sec(D'), \tag{12b}$$

where $D'=2\sqrt{|C|}(l+I_2)+\tan^{-1}(x_0/\sqrt{|C|})$. On the $[x,y(1-4J)^{1/2}]$ plane, the curves have upward concavity. For $l\to lc$ one has $x\to\infty$ and $y\to\infty$. The coupling strength is zero ($\pi K=0$) and all vortices are dissociated.

Here $I_2=\int_0^l I_1/(x^2-C)dl'=2\ln[(j^*-4j)/(j^*-4je^l)]$. For $j=0$ this term disappears and the solutions (10)–(12) are the same as the KT theory.

As in the KT case, $T_k(j)$ separates the paired and the free configuration but, in this case, it is a decreasing function in the $T$-$j$ plane, with two dissipative regions: for $T<T_k(j)$ the paired vortices do not contribute to the dissipation, while for $T>T_k(j)$ free vortices generates a resistivity following Eq. (2).

For $T>T_k(j)$, one may define a characteristic length $\xi_+$ $[l_+=\ln(\xi_+/\xi)]$, corresponding to the scale at which the vortices begin to unbound; then the free vortex density is $n_F=1/(2\pi\xi_+^2)$ and from Eq. (2) the resistivity is

$$\rho=\rho_0\exp(-2l_+). \tag{13}$$

The dependence of $l_+$ on the temperature is indirectly obtained by considering that the generated vortices at this scale are all free, that is $y_0(T)=y(l_+,T)$. Then

$$\sqrt{\frac{1-4J_0}{1-4J}}\sec\left[2\sqrt{|C|}(l_++I_2)+\tan^{-1}\left(\frac{x_0}{\sqrt{|C|}}\right)\right]-\sec\left[\tan^{-1}\left(\frac{x_0}{\sqrt{|C|}}\right)\right]=0. \tag{14}$$

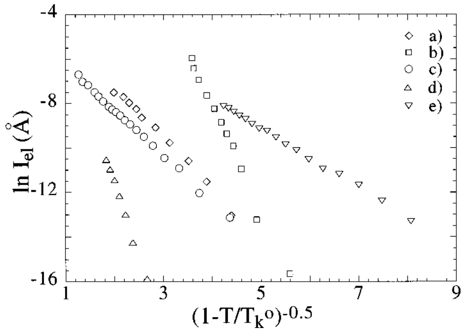


FIG. 2. Plots of the natural logarithmic of $I_{el}$ vs $(1-T/T_k^0)^{-0.5}$ for the planar samples reported in Table I. The currents $I_{el}$ correspond to the voltage $V_{el}$ indicated in Table I. The data of the sample (b) are shifted by two units in order to avoid superposition. The obtained linear trends indicate the good approximation of Eq. (16).

A numerical solution is necessary to obtain the $l_+$ values from the previous equation; on the other hand, at low density current, when $J\ll 1$ and near $T_k(x_0\ll 1)$, an approximate analytical solution is given $l_+=2\pi/\sqrt{|C|}=\pi/2\sqrt{\tau/2B}$. From Eq. (13) the resistivity is

$$\rho=A\rho_n\exp(-2\sqrt{b/|\tau|}). \tag{15}$$

This result is qualitatively the same as the case without applied current with $\tau^0=\tau$ and $b=8\pi^2/B$, but in this case, $T_k(j)$ is current dependent and the resistivity is non-Ohmic. This result is in agreement with the experimental data in several planar samples.[13]

For $T<T_k$, a characteristic length $\xi_-$ which represents the mean distance between the vortices of a pair, is usually defined.[3,4] It corresponds to a length scale $l_-\cong\frac{1}{2}\pi\sqrt{b'[1/(1-T/T_k^0)]}$, where $b'$ is a sample-dependent parameter. If $j=0$ the couple is broken when $l_-\to\infty$, that is at $T=T_k^0$, but for $j>0$ the depairing takes place for $l=l_c$ and $T_k(j)$ is obtained when $l_-(T)=l_c$. By reversing $T_k(j)$, on the $j$-$T$ plane one has

$$j_k(T)=\frac{j^*(T)}{4}\exp\left(-\frac{\sqrt{b'}}{2\pi}\sqrt{\frac{1}{1-T/T_k^0}}\right), \tag{16}$$

where $j^*(T)=j^*(0)(1-T/Tco)^{3/2}$,[2] which may be considered as a constant if $T\ll Tco$.

The current $j_k(T)$ plays a role of a temperature-dependent critical current at which the dissipative process takes place. Figure 2 shows the logarithmic plot of $I_{el}$ vs $\sqrt{1}/(1-T/T_k^0)$ for five planar samples listed in Table I, which includes one superlattice (sample b),[14] two thin films [samples (a) and (c)],[14,15] and two Josephson arrays [samples (d) and (e)].[16,17] $I_{el}$ is the current at the lowest experimental voltage $V_{el}$ indicated in Table I as obtained from the $I$-$V$ characteristics. $I_{el}$ is assumed to be a good approximation of the critical current. As is usually done, this can be considered a good approximation of the critical current. The obtained straight line confirms the validity of Eq. (16) in a large range of temperature.

TABLE I. List of the examined samples and their significant parameters, whose meaning is explained in the text.

| Sample | Composition | Ref. | Thickness (nm) | $V_{el}(V)$ | $T_k^0$ (K) | $I^*(0)(A)$ | $\frac{\sqrt{b'}}{2\pi}$ |
|---|---|---|---|---|---|---|---|
| *a* | Hg-Xe | 4 | 10 | $1.2\times10^{-8}$ | 3.5 | 0.47 | 2.55 |
| *b* | YBCO/$PrBa_2Cu_3O_{7-x}$ | 14 | 2.4/10 | $1.0\times10^{-7}$ | 4.97 | 21.3 | 4.88 |
| *c* | YBCO monolayer | 15 | 1.2 | $1.0\times10^{-7}$ | 26.38 | 0.06 | 2.05 |
| *d* | Nb-Au-Nb junction array | 16 | | $5.6\times10^{-10}$ | 8.26 | 20.41 | 6.58 |
| *e* | YBCO Ag junction array | 17 | | $1.0\times10^{-8}$ | 79.44 | 0.33 | 1.33 |

## III. CONCLUSIONS

The current contribution to the thermal generation of vortex-antivortex couples is here considered, as due to the pair potential reduction. The increased pair density gives rise to a larger screening effect, which produces a phase transition with a current dependent critical temperature, separating a nondissipative regime from a dissipative one.

Following the KT renormalization scheme, one obtains, in the limit of low currents, a resistivity vs temperature, similar to that for $j=0$, where $T_k$ is substituted by $T_k(j)$. At high current a numerical solution of Eq. (14) is necessary.

A dependence of critical current on temperature is obtained in good agreement with the experimental results in several planar superconductors.

## ACKNOWLEDGMENT

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