Analytic model for the development of bamboo microstructures in thin film strips undergoing normal grain growth

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The kinetics of the transformation of polygranular thin film strips to bamboo structures through two-dimensional (2D) normal grain growth are studied. A differential model for the evolution of the polygranular cluster length distribution is developed. It is observed, and demonstrated using a dimensional analysis, that the rate of bamboo-segment nucleation per unit time and unit of untransformed length is proportional to μ/w^3, and is negligible in the growth-dominated steady state. It is also demonstrated that the cluster shrinkage velocity reaches a constant steady-state value proportional to μ/w (assuming constant and uniform μ). This is shown to lead to a time-invariant, steady-state exponential cluster length distribution with an average cluster length proportional to the strip width, and a cluster length fraction decaying exponentially with $\tau=\mu t/w^2$. The analytic model is validated through comparison with data generated using a 2D computer simulation of grain growth. The distribution of grain lengths in the resulting final bamboo grain structure is well fit by a log-normal distribution, with a median grain length scaling with the linewidth, and a linewidth-independent normalized deviation in the grain length.

I. INTRODUCTION

It has been well established in experiments that the geometry and microstructure of on-chip integrated-circuit Al-based interconnects have a direct impact on their electromigration-limited reliability.[1–4] Interconnects can have polygranular structures for which there are continuous grain boundary paths along the length of the interconnect (see the first figure in Fig. 1). This is likely when the median in-plane grain size of the film from which an interconnect is patterned, D_{50}, is smaller than the linewidth, w. Post-patterning annealing can lead to grain growth that results in bamboo structures for which all grain boundary planes are normal to the direction of the interconnect length (see the last figure in Fig. 1). At intermediate stages, interconnects can have near-bamboo structures for which polygranular clusters with grain boundaries along the interconnect length are separated by one or more grains which span the width of the line (see Fig. 1). Lengths of line in which one or more neighboring grains span the linewidth constitute bamboo segments.

In interconnects with near-bamboo structures, electromigration-induced diffusion occurs much faster along grain boundaries in the polygranular segments than in the bamboo segments, where diffusion occurs primarily along the Al-oxide or Al-intermetallic interfaces.[5,6] Because grain boundary diffusivities are orders of magnitude higher than diffusivities in bamboo segments, the magnitudes and statistics of electromigration-induced failure times and statistics depend strongly on the distributions of the lengths of polygranular clusters and bamboo segments.[7,8] As the ratio of the linewidth to the median grain size of the initial metal film decreases, the polygranular clusters in as-patterned films become shorter and an increase in the median time to electromigration-induced failure is observed, along with an increase in the deviation in the time to failure.[4,7,8] However, if such lines are annealed, their reliability can be improved by orders of magnitude due to post-patterning grain growth.[9,10] This improvement is related to the change in microstructure outlined above, during which polygranular clusters shrink through formation of new spanning grains in the cluster interiors, and through motion of the boundaries at the edge of the clusters. In recent work,[7,8] Knowlton *et al.* used a 2D computer simulation of grain growth,[11,12] in conjunction with an electromigration computer simulation,[13] to build a quantitative analysis of the impact of cluster length and cluster spacing, and the associated statistics, on electromigration failure time statistics. More specifically, they were able to reproduce the experimentally observed dependence of failure time statistics on the linewidth and on the post-patterning annealing history. Post-patterning annealing-induced grain structure evolution has also been observed in experiments on scanned laser annealing of Al interconnects, and was successfully predicted using the grain growth simulation.[14]

Although reliable in its grain structure prediction capability, the grain growth simulation used is, as simulations in general are, limited in speed, especially for the generation of large populations, necessary to obtain reliable statistics. The goal of this paper is to develop an analytic model for the evolution from polygranular to bamboo structures in strips. Such compact analytic models are needed for efficient generation of large populations of realistic process and

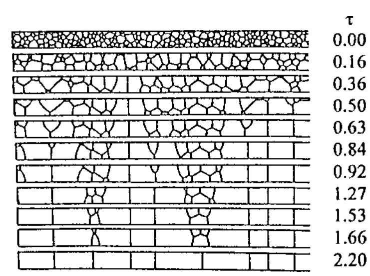

FIG. 1. Evolution of the grain structure within a strip with $w/D_{50}=3.0$. w is the linewidth and D_{50} is the median grain size in the film from which the line is etched. $\tau=\mu t/w^2$ is the normalized dimensionless time.

geometry-sensitive interconnect structures, in order to predict the effects of geometry and processing on the rates of electromigration-induced damage and on interconnect reliability.

In the following sections we will first present a Johnson-Mehl-Avrami analysis in which the formation of a linewidth-spanning grain is treated as a nucleation event and the shrinkage of polygranular segments corresponds to growth of bamboo segments. After discussing the limitations of this approach, we will develop a differential model which allows line-geometry-sensitive prediction of the evolution of the polygranular-cluster-length distribution during normal grain growth. Finally, we will use data generated with a 2D grain-growth simulation to validate our analytic models.

II. NUCLEATION AND GROWTH JOHNSON-MEHL-AVRAMI ANALYSIS

Figure 1 depicts the transformation in which we are interested. As grain growth proceeds, a few grains grow to span the width of the line, forming bamboo grains and corresponding to ''nucleation'' of bamboo segments. The boundaries shared with grains in polycrystalline segments continue to move and the bamboo grains continue to grow unless two growing bamboo grains meet. From this perspective, the kinetics of the bamboo segment formation can be treated using a Johnson-Mehl-Avrami (JMA) analysis of one-dimensional (1D) nucleation and growth.[15] Taking L to be the total line length, L_c the total length of polygranular segments in the line, and L_b the total length of bamboo segments ($L_b = L - L_c$) with an initial value L_{b0}, we can apply the JMA analysis to the portion of the line, of length $L - L_{b0}$, initially available for nucleation and growth. We define the extended bamboo length L_b^e as the bamboo length expected to be created when the effects of impingement of growing bamboo segments are ignored. If $n(t)$ is the rate of bamboo nucleation per unit time and untransformed length L_c, so that $nL_c dt$ new nuclei form between times t and $t+dt$, $v(t)$ is the velocity at which bamboo segments grow, and t_0 is the time at which the transformation starts, then, following the JMA analysis,

$$L_b^e = \int_{t_0}^{t} (L - L_{b0}) l_\tau(t) n(\tau) d\tau, \quad \text{with} \quad l_\tau(t) = \int_{\tau}^{t} \nu(\tau') d\tau'. \tag{1}$$

$l_\tau(t)$ is the length at time t of a bamboo segment nucleated at time τ. The actual bamboo length dL_b created in an interval dt is only a fraction $(L - L_b)/(L - L_{b0})$ of the extended one:

$$dL_b = \frac{L - L_b}{L - L_{b0}} dL_b^e, \tag{2}$$

which after integration gives

$$1 - \underline{L}_b = (1 - \underline{L}_{b0}) \exp\left(- \frac{L_b^e}{L - L_{b0}} \right), \tag{3}$$

where $\underline{L}_b = L_b / L$ and $\underline{L}_{b0} = L_{b0}/L$ are the bamboo length fractions, initially and at time t. When the initial condition is characterized by a value of $\underline{L}_{b0}$ different from zero, and an initial number of bamboo segments (or, equivalently, number of polygranular clusters), N_0 (not equal to zero), the nucleation rate $n(t)$ is given by

$$n(t) = \frac{N_0}{L - L_{b0}} \delta(t - t_0) + a(t), \tag{4}$$

where $\delta(t)$ is the zero-centered Dirac distribution and $a(t)[L - L_b(t)]dt$ is the number of bamboo nucleation events occurring between $t > t_0$ and $t + dt$. The first term in the expression of $n(t)$ accounts for the growth of the bamboo sections present initially. In the special case that the nucleation rate ''a'' as well as the growth rate ''v'' are constant, Eqs. (1), (3), and (4) lead to

$$1 - \underline{L}_b = (1 - \underline{L}_{b0}) \exp\left(- \frac{N_0}{L - L_{b0}} v(t - t_0) - \tfrac{1}{2} v a (t - t_0)^2 \right). \tag{5}$$

It is also possible to evaluate the evolution of the number of bamboo grains N_b (N_b is larger, and generally strictly larger, than the number of bamboo segments and the number of polygranular clusters, N). Using a similar analysis, we obtain

$$N_b = N_{b0} + \int_{t_0}^{t} (L - L_b) a(\tau) d\tau, \tag{6}$$

where N_{b0} is the initial number of bamboo grains. The JMA analysis allows characterization of the evolution of the total length fraction of bamboo or cluster regions, but does not allow characterization of the evolution of the statistical characteristics of individual cluster lengths, which is essential for the prediction of the impact of grain structure on interconnect failure-time statistics. In the following section, we will present a geometry-sensitive model that can accomplish this task.

III. DIRECT DIFFERENTIAL MODEL FOR CLUSTER STATISTICS EVOLUTION

The guide to the development of our analytic model is a front-tracking 2D simulation of boundary-curvature-driven grain growth.[16–18] The velocity of points on grain boundaries is taken to be proportional to the local curvature κ, such that $v = \mu\kappa$, where the mobility μ is the temperature-dependent product of the grain boundary mobility and the grain boundary energy. At grain boundary triple junctions, a local force balance is enforced so that grain boundaries meet at 120°. Normal 2D grain growth in thin film or cellular structures has been investigated elsewhere,[16–19] and Ref. 19 demonstrates the detailed quantitative agreement between quasi-ideal systems such as froths and our simulation. It has been shown that 2D normal grain growth leads to a uniquely defined grain structure, evolving in a geometrically and statistically self-similar fashion, with an average grain area increasing linearly with both time t and mobility μ (assuming that μ is uniform and constant). The transition to this steady-state regime occurs soon after growth has been initiated, typically before the grains have undergone a 20% increase in area.

2D normal grain growth in strips is simulated by requiring the contact angles of grain boundaries with strip edges to be 90°.[11] This leads to the evolution of a bamboo structure as illustrated in Fig. 1. Two strips of different width etched from a film with a pre-etching grain size smaller than the linewidths will undergo similar grain structure evolution (their initial structure is similar to a 2D cellular structure). Differences in grain structure evolution in strips of different widths can be accounted for through kinetic and geometric scaling. As suggested by Fig. 1, the evolution towards bamboo is governed by three phases. During a first *incubation* phase, grains in the initially polygranular structure grow to a size comparable to the strip width. As the evolution proceeds, some grains grow large enough to span the width of the strip, creating sections of the strip with a bamboo structure. This is the *nucleation* period, during which bamboo sections continue to form, increasing the number of polygranular clusters, until these polygranular clusters separating the bamboo segments achieve geometrically stable configurations with grains having four sides within the strip interior and one side defined by the strip edge. The structure then undergoes a *growth-dominated* evolution during which almost no nucleation is observed, with the evolution occurring exclusively at the boundaries between grain clusters and bamboo segments, with the bamboo segments growing to consume the polycrystalline clusters. Given the growth conditions discussed previously and the fact that in polygranular lines the initial structures have an average area that is negligible compared to the square of the width w, the duration of these phases will be proportional to w^2/μ. We can therefore define a dimensionless time $\tau=\mu t/w^2$ that unifies the kinetic analysis for strips with different widths. The grain structures of two lines with different widths will be statistically identical at a given τ, after accounting for geometric magnification. It is therefore also possible to define reduced dimensionless variables to account for geometric scaling in the evolution of the number of bamboo grains in a line N_b ($\underline{N}_b=N_b w/L$), the total cluster length in a line L_c ($\underline{L}_c=L_c/L$), the total number of clusters in a line N ($\underline{N}=Nw/L$), the average cluster length $l_{\rm av}=L_c/N$ ($\underline{l}_{\rm av}=l_{\rm av}/w=\underline{L}_c/\underline{N}$), and the average bamboo length l_b ($\underline{l}_b=l_b/w$). All of these geometric parameters scale as a function of the reduced time $\tau=\mu t/w^2$. The evolution of these dimensionless variables is expected to be independent of line geometry, provided the initial width is larger than the pre-etching grain size.

In previous work,[12] Walton *et al.* analyzed the kinetics of the transformation of the structure of a polygranular strip to a bamboo structure. The three phases of evolution were identified, and it was argued that bamboo ''nucleation'' occurred randomly within the polygranular regions, leading to an exponential distribution of polygranular cluster lengths. Walton *et al.* then focused on the growth-dominated phase to show that it was characterized by an exponential decay of N and L_c, as well as by a constant value for the average cluster length in the strip. This behavior is consistent with an exponential distribution for the length of clusters, and with cluster shrinkage at a constant rate v.[12] Although it is true that the growth-dominated phase is the one that governs the kinetics of the bamboo transformation through the shrinkage and elimination of polygranular clusters, knowledge of the rate of polygranular cluster shrinkage alone does not allow complete characterization of the evolution of the grain structure statistics. To accomplish this, one also needs to know the cluster statistics (number and length) at the beginning of the growth-dominated phase. This, in turn, depends on what happens during the nucleation phase.

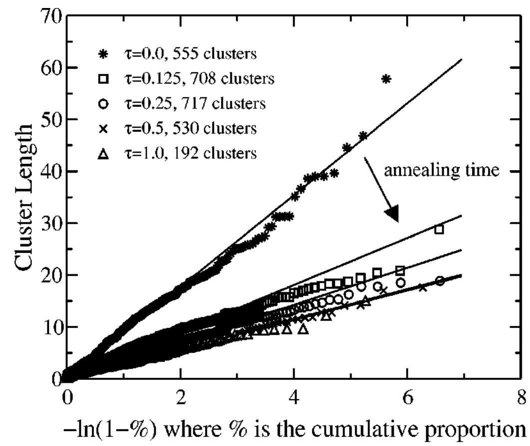

FIG. 2. Exponential plot of cluster length distributions at various times during simulated evolution of the grain structure in a thin film strip with $w/D_{50}=1.0$. Solid lines represent the best-fitting exponential distributions. The plot is consistent with the structure reaching a steady state where the average cluster length is constant.

To capture the essentials of the structural evolution during both the nucleation and growth-dominated phases, we will first show that at any time during the evolution, the grain structure statistics are well described by exponential distributions. We will then develop a simple analytic model of the cluster length and number evolution that allows the prediction of cluster and bamboo length statistics at any point in time during post-patterning annealing.

Figure 2 shows an exponential plot of the polygranular cluster length distribution for a strip with a linewidth-to-initial-median-grain size ratio $w/D_{50}=1.0$, at many different times during all phases of the evolution. When plotting the cluster length l_c as a function of $-\ln[1-F(l_c)]$, where $F(l_c)$ is the proportion of clusters shorter than l_c, data that fall on a straight line are fit by an exponential distribution function. Figure 2 shows that the polygranular cluster length distribution is well fit by an exponential distribution function at all times. The lines overlap for $\tau>0.5$, which indicates a constant average cluster length in this regime.

If we consider that during annealing-induced evolution, the polygranular cluster length distribution is, and remains, exponential, the problem of predicting the structure statistics is reduced to the determination of the evolution with time of both the average cluster length $l_{\rm av}(t)$ and the total number of clusters $N(t)$, or equivalently, one of these variables and the total cluster length $L_c(t)$. Assuming that the effect of bamboo nucleation on the total cluster length is negligible, only cluster shrinkage will account for the variations in L_c, so that

$$\frac{dL_c}{dt}=-v(t)N(t), \tag{7}$$

where $v(t)$ is the average value of the rate of polygranular cluster shrinkage at time t. The variation in the total number of clusters is caused by the increase due to bamboo nucleation inside a polygranular cluster, or, equivalently, cluster splitting events, and the decrease following cluster disappearance by shrinkage. If $a(t)$ is the rate of splitting events per unit cluster length at time t, as defined previously, then assuming an exponential distribution of cluster length $f_t(l_c)=[1/l_{av}(t)]\exp[-1_c/l_{av}(t)]$, the number of clusters that disappear by shrinkage between t and $t+dt$ can be evaluated to be $v(t)N(t)dt/l_{av}(t)$, which leads to the second evolution equation:

$$\frac{dN}{dt}=a(t)L_c(t)-v(t)\frac{N^2(t)}{L_c(t)}. \tag{8}$$

These equations can be recast in terms of the reduced dimensionless variables previously defined and two dimensionless parameters, $\underline{v}=(w/\mu)v$ and $\underline{a}=(w^3/\mu)a$. At this point, knowledge of the initial conditions $\underline{L}_c(0)$ and $\underline{N}(0)$ and the profiles of $\underline{a}(\tau)$ and $\underline{v}(\tau)$ allows solving Eqs. (7) and (8) to determine $L_c(t)$ and $N(t)$. It is important to note that, as mentioned earlier in the section, the evolution of the normalized variables in the case $w/D_{50}\gg 1$ is independent of line geometry, which implies that $\underline{a}$ and $\underline{v}$ depend on geometry only through τ. This, in turn, proves that the rate of cluster splitting "a" is proportional to μ/w^3 and the shrinkage velocity v is proportional to μ/w.

The rate of cluster shrinkage has been investigated previously.[12] In the growth-dominated regime, most polygranular clusters are bound by pairs of four- and five-sided grains (counting the strip edge as a side) with a series of five-sided edge grains between them. The Mullins–von Neumann law can be used to show that the rate of cluster shrinkage is constant and proportional to $1/w$:

$$v=2\frac{1}{w}\frac{dA_4}{dt}=\frac{2\pi}{3}\frac{\mu}{w}, \tag{9}$$

where A_4 is the area of a four-sided edge grain and where it should be noted that when using the Mullins–von Neumann analysis for the rate of shrinkage of individual grains in strips, the strip edge counts as two sides.[20] The time evolution of the normalized velocity as defined in Eq. (7) is depicted, for several linewidths, in Fig. 3. The plot confirms that the average shrinkage rate is constant in the steady-state regime, and the fact that all curves overlap confirms that this rate is proportional to μ/w. The average value of the shrinkage rate in the steady state exceeds the predictions of Eq. (9) by about 20%, a difference that is accounted for by the high velocities associated with bamboo nucleation at the edge of a cluster. At the early, nucleation-dominated stage, the values of v are variable and higher, which is expected since, following Eq. (7), it is v and not a that accommodates the topology-driven cluster length variations due to random nucleation events.

An analytic assessment of the rate of cluster splitting per unit time and unit length, $a(t)$, is more complex. This rate is expected to depend on the average grain size at a given time, as well as on the deviation in the grain size, since it is the number of grains that are bigger than a certain width-related threshold that affects the number of nucleation events. However, as can be seen in Fig. 4, and will be discussed in the next section, simulations indicate that significant cluster splitting through nucleation occurs only during the *nucleation* period, and is essentially absent during the steady-state growth regime. This behavior is expected since during the steady-state phase, clusters have stable geometries which are immune to an internal bamboo nucleation. Therefore, a relatively good approximation would be to take $a(t)$ to be constant during the nucleation period in the time interval $[t_0,t_1]$, and zero at other times. This is confirmed by the data in Fig. 4, for the evolution of the simulated normalized bamboo nucleation rate $(1/\underline{L}_c)d\underline{N}_b/d\tau$, an overestimate of the normalized cluster splitting rate $\underline{a}=(w^3/\mu)a$, as a function of $\tau=\mu t/w^2$. The plots coincide within statistical variations for initially polygranular lines $(w/D_{50}>2)$, confirming that the nucleation rate is proportional to μ/w^3.

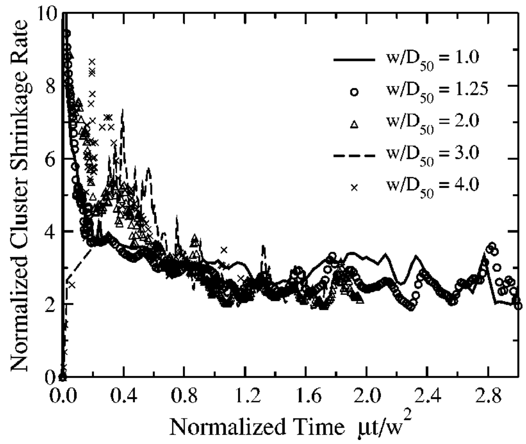

FIG. 3. Evolution of the normalized cluster shrinkage rate $\underline{v}=-(1/\underline{N})d\underline{L}_c/d\tau$ with $\tau=\mu t/w^2$. The plots coincide for initially polygranular lines $(w/D_{50}>2)$, confirming that the shrinkage rate is proportional to μ/w.

Using, as a simplification, a cluster-splitting rate $a(t)$ such as the one defined above, and a constant shrinkage velocity, the coupled equations (7) and (8) can be solved analytically to obtain the following:
for $t_0<t<t_1$,

$$L_c=L_{c0}\exp\left(-\frac{1}{l_{av0}}v(t-t_0)-\tfrac{1}{2}va(t-t_0)^2\right) \tag{10}$$

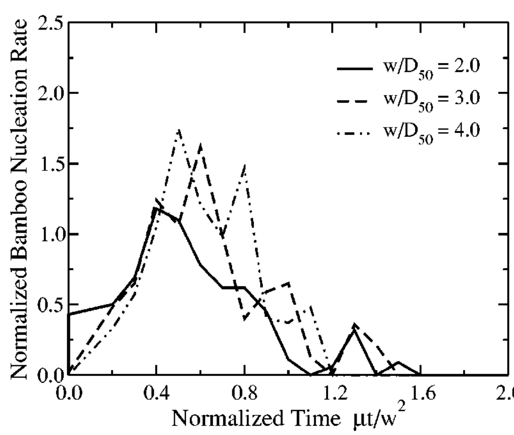

FIG. 4. Evolution of the normalized bamboo nucleation rate $(1/\underline{L}_c)d\underline{N}_b/d\tau$ with $\tau=\mu t/w^2$. These plots have been obtained after averaging highly variable instantaneous rates over longer times. The plots coincide within statistical variations for initially polygranular lines $(w/D_{50}>2)$ showing that bamboo nucleation is only significant during the nucleation phase and that the nucleation rate is proportional to μ/w^3.

TABLE I. The parameters for simulations of grain structure evolution in lines with different widths w that minimize the error $e=\langle \log^2(\underline{L}_c/\underline{L}_{c\_\mathrm{fit}})\rangle+\langle \log^2(\underline{l}_{\mathrm{av}}\underline{l}_{\mathrm{av\_fit}})\rangle+\langle \log^2(\underline{N}/\underline{N}_{\mathrm{fit}})\rangle$.

w/D_{50}	$\tau_0=\mu t_0/w^2$	$\tau_1=\mu t_1/w^2$	$\underline{v}=vw/\mu$	$\underline{a}=aw^3/\mu$.	Error
7.05	0.16	0.75	3.96	0.74	0.026
5.81	0.16	0.75	3.84	0.66	0.020
5.0	0.18	0.75	3.88	0.64	0.022
4.0	0.15	0.75	3.50	0.80	0.013
3.0	0.12	0.75	3.76	0.84	0.019
2.0	0.0	0.75	3.38	0.60	0.018
1.25	0.0	0.45	3.34	0.78	0.024
1.0	0.0	0.40	3.84	0.70	0.035
0.75	0.0	0.35	4.44	0.52	0.055
0.5	0.0	0.35	4.92	0.28	0.096

and

$$l_{\mathrm{av}}(t)=\left(\frac{1}{l_{\mathrm{av0}}}+a(t-t_0)\right)^{-1}, \tag{11}$$

and for $t>t_1$,

$$L_c=L_c(t_1)\exp\left(-\frac{1}{l_{\mathrm{av1}}}v(t-t_1)\right) \tag{12}$$

and

$$l_{\mathrm{av}}(t)=l_{\mathrm{av}}(t_1). \tag{13}$$

That the average cluster length reaches a constant value ($\underline{l}_{\mathrm{av}}$ reaching a constant value means also that l_{av} reaches a constant value proportional to w) is in agreement with Walton's results.[11] We also note, as a final remark before discussing simulation results, that Eq. (5) obtained through the JMA analysis and Eq. (10) above are identical, as would be expected for nucleation at a constant rate and growth at a constant velocity.

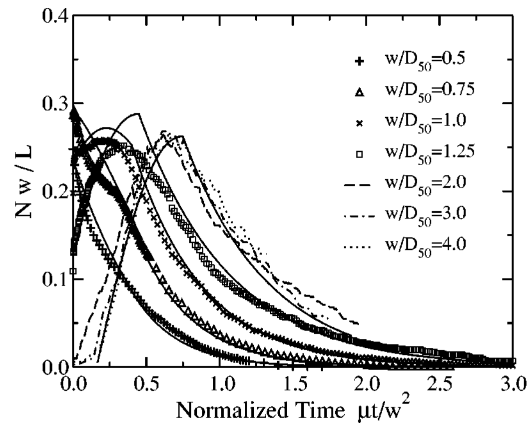

FIG. 6. Simulated evolution of the normalized number of clusters $\underline{N}=Nw/L$ with $\tau=\mu t/w^2$. The plots overlap for initially polygranular lines. Solid lines represent the predictions of the analytic model.

IV. SIMULATION RESULTS AND DISCUSSION

We have used the grain growth simulation discussed in Ref. 11, 12, and 20 to generate grain structures in strips with different widths, and compared the evolution of the polygranular cluster statistics observed using the simulation with predictions made using the analytic model derived above. Table I shows values for various error-minimizing parameters obtained using the simulation.

The simulation shows that during 2D normal grain growth in strips, the incubation period t_0 is approximately $0.15w^2/\mu$ for wide, initially polygranular lines ($w/D_{50}\geqslant 3.0$). The incubation time decreases rapidly with strip width, and is already zero for $w/D_{50}\leqslant 2.0$. The nucleation period lasts until $t_1\approx 0.75w^2/\mu$ for all polygranular lines. The fact that $\underline{v}$ and $\underline{a}$ have constant values ($\underline{v}\approx 3.8\pm 0.2$ and $\underline{a}\approx 0.7\pm 0.1$) for all polygranular lines ($w/D_{50}\geqslant 3.0$), regardless of the linewidth in the growth-dominated regime, validates the assumptions of the analytic model. The value of $\underline{v}$ is consistent with the Mullins–von Neumann analysis described above after accounting for the high velocities associated with bamboo nucleation events. Figures 5, 6, and 7 show the evolution of

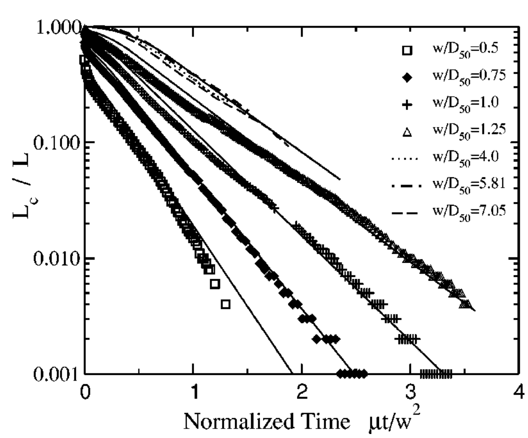

FIG. 5. Evolution of the normalized total cluster length $\underline{L}_c=L_c/L$ with $\tau=\mu t/w^2$. The plots coincide for initially polygranular lines ($w/D_{50}>2$). Solid lines represent the evolution predicted using the analytic model.

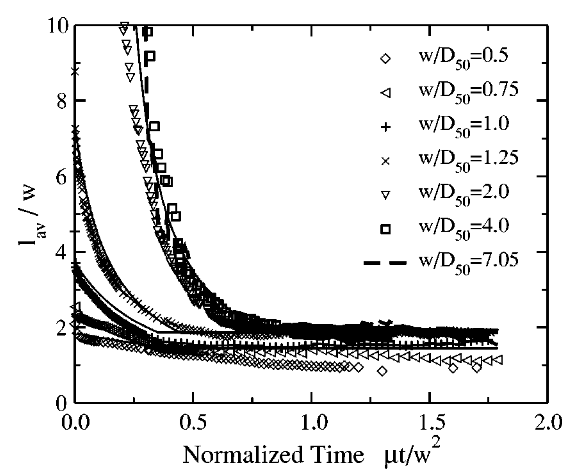

FIG. 7. Simulated evolution of the normalized average cluster length $\underline{l}_{\mathrm{av}}=l_{\mathrm{av}}/w$ with $\tau=\mu t/w^2$. Solid lines represent the predictions of the analytic model.

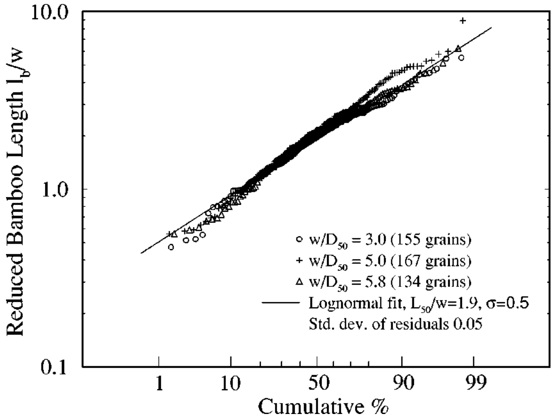

FIG. 8. Log-normal plot of normalized bamboo grain length l_b/w distributions in the bamboo structures, resulting from simulated prolonged annealing of polygranular strips with different widths. The overlapping curves for different values of w/D_{50} show that the distribution of the values of l_b/w is linewidth-independent. The solid line represents the best-fitting log-normal distribution.

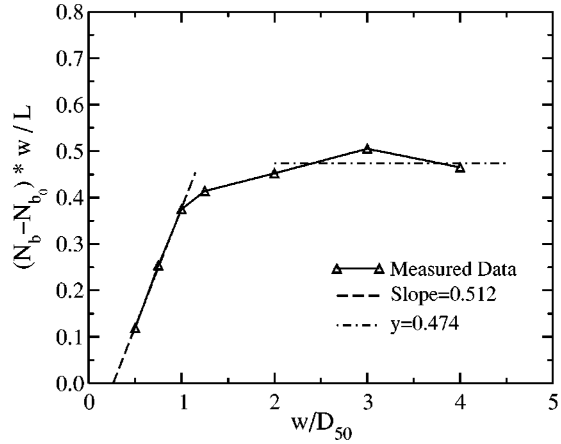

FIG. 9. Normalized total number of bamboo nucleation events as a function of w/D_{50}.

$\underline{L}$, $\underline{N}$, and $\underline{l}_{av}$ for both simulation data and model predictions in lines with different widths. We note the exponential decay of $\underline{L}$ and $\underline{N}$ during the steady-state phase ($\tau > \tau_1$), while $\underline{l}_{av}$ reaches a constant value of about 2 (corresponding to an average cluster length of twice the linewidth). The coincidence of the curves for the wide lines ($w/D_{50} \geqslant 3.0$) is a validation of the geometric scaling analysis presented earlier. The time to reach 90% bamboo structure (in terms of length fraction) for wide lines is about $1.85w^2/\mu$, and the final bamboo structure has grains with an average length of about 2.1 times the linewidth. Figure 8 shows that the distribution of the final bamboo grain lengths normalized by the linewidth is width-independent for initially polygranular lines, as expected with geometric scaling, and is well fit by a log-normal distribution. Figure 9 shows the normalized total number of bamboo nucleation events $(N_b - N_{b0})w/L$ as a function of w/D_{50}, demonstrating that the total number of bamboo nucleation events is proportional to L/D_{50} for initially near-bamboo lines ($w/D_{50} < 2.0$) and L/w for initially polygranular lines ($w/D_{50} > 2.0$), the latter result being consistent with the geometric scaling discussed earlier.

Because cellular evolution in froths and grain growth are both driven by capillary forces, studies of froth evolution have been extensively compared to simulations of 2D grain growth.[19,21,22] We have recently completed experimental studies of froth evolution in rectangular prisms, in which detailed agreement is demonstrated with both simulation results and the analytic model developed above, when 2D evolution occurs.[23]

V. SUMMARY

We have developed a geometry-sensitive analytic model for the evolution of the grain structures of initially polygranular thin film strips to bamboo grain structures. The model is in agreement with the partial results predicted using a modified Johnson-Mehl-Avrami analysis of the transformation. The model predicts the line-geometry dependence of the two transformation parameters, the bamboo nucleation rate and the polygranular cluster shrinkage velocity. The model also allows prediction of the time and geometry dependence of the polygranular and bamboo segment length distributions. For initially near-bamboo lines ($w/D_{50} < 2.0$), the model's predictions are still in agreement with the simulation, but the geometry dependence of the nucleation rate and the cluster shrinkage velocity is expectedly not the same as for initially polygranular lines. Evolution from the near-bamboo regime can be analyzed using parameters derived from simulations.[24] With these compact analytic models it is possible to generate appropriately varying grain structures for simulations of interconnect reliability, eliminating the need for time-consuming multiple simulations of grain structure evolution.

ACKNOWLEDGMENTS

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