

ARTICLES

Theory of spin Coulomb drag in spin-polarized transport

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(Received 20 March 2000)

We introduce a distinctive feature of spin-polarized transport, the spin Coulomb drag: there is an *intrinsic* source of friction for spin currents due to the Coulomb interaction between spin “up” and spin “down” electrons. We calculate the associated “spin *trans*-resistivity” in a generalized random-phase approximation and show that, to the leading order in the interactions, it has no contribution from correlated impurity scattering. We show that, in an appropriate range of parameters, such resistivity is measurable, and we propose an experiment to measure it.

Interest in spin-polarized transport has been growing dramatically in the past few years, spurred by the hope of realizing practical spin-electronic devices in a not too distant future.¹ In particular, it has been shown that spin coherence can be maintained over large distances $\delta_s \gtrsim 100 \mu\text{m}$ and for long times $T_2 \sim 10^{-9} - 10^{-8}$ s both in metals² and in semiconductors.³

In this paper we introduce a distinctive feature of spin-polarized transport: in a conductor, due to the Coulomb interaction, there is an *intrinsic* mechanism for friction between electrons of different spin, the “*spin Coulomb drag*” (SCD). For simplicity, we shall restrict our discussion to the case in which the spin state of each electron can be classified as “up” or “down” relative to the z axis. In the absence of impurities, the total momentum $\mathbf{P} = \sum_i \mathbf{p}_i$, where \mathbf{p}_i is the momentum of the i th electron, is a conserved quantity. On the contrary, the “up” and the “down” components of the total momentum, $\mathbf{P}_\uparrow = \sum_i \mathbf{p}_{i\uparrow} (1 + \hat{\sigma}_{zi})/2$ and $\mathbf{P}_\downarrow = \sum_i \mathbf{p}_{i\downarrow} (1 - \hat{\sigma}_{zi})/2$, where $\hat{\sigma}_{zi}$ is the Pauli matrix for the z component of the i th electron’s spin, are *not separately conserved* even in the absence of impurities: Coulomb scattering can transfer momentum between spin-up and spin-down electrons thereby effectively introducing a “friction” for relative motion of the two spin components. If, for example, one of the two spin components is set into motion relative to the other, it will tend to drag the latter in the same direction. Or, if a finite spin current is set up through the application of an external field, then the Coulomb interaction will tend to equalize the net momenta of the two spin components, causing the difference $\langle P_\uparrow \rangle - \langle P_\downarrow \rangle$ to decay to zero when the external field is turned off.

The most dramatic manifestation of the SCD is the appearance of a finite *trans-resistivity* defined as the ratio of the gradient of the spin-down electrochemical potential to the spin-up current density when the spin-down current is zero. This is completely analogous to the *trans-resistivity* measured in Coulomb drag experiments with electrons in two separate layers,⁴⁻⁶ but in this case what makes the two electron populations distinguishable is not a physical separation but the different spin. In SCD, the nonconservation of the

spin, caused mainly by the spin-orbit interaction, represents a “leakage” mechanism analogous to the interlayer tunneling in the usual Coulomb drag.

First of all, let us describe the SCD from a phenomenological point of view. Let $\mathbf{E}_\uparrow(t)$ and $\mathbf{E}_\downarrow(t)$ be uniform effective electric fields that couple to spin \uparrow and spin \downarrow electrons, respectively. These fields are sums of the ordinary electrostatic field plus the gradient of the local chemical potential, which can be spin-dependent. We restrict ourselves to the linear-response regime, assume weak electron-electron and electron-impurity scattering, and ignore spin-flipping processes altogether. If \mathbf{v}_σ is the velocity of the center of mass of electrons of spin σ , and N_σ is the number of such electrons, the phenomenological equation of motion has the form

$$mN_\sigma \dot{\mathbf{v}}_\sigma = -eN_\sigma \mathbf{E}_\sigma + \mathbf{F}_{\sigma\bar{\sigma}} - \frac{m}{\tau_D} N_\sigma \mathbf{v}_\sigma, \quad (1)$$

where τ_D is the Drude scattering time and $\mathbf{F}_{\sigma\bar{\sigma}}$ is the Coulomb force exerted by spin $\bar{\sigma}$ ($= -\sigma$) electrons on spin σ electrons. By Newton’s third law $\mathbf{F}_{\sigma\bar{\sigma}} = -\mathbf{F}_{\bar{\sigma}\sigma}$ and by Galilean invariance this force can only depend on the relative velocity of the two components. Hence, for weak Coulomb coupling we write

$$\mathbf{F}_{\sigma\bar{\sigma}} = -\gamma m N_\sigma \frac{n_{\bar{\sigma}}}{n} (\mathbf{v}_\sigma - \mathbf{v}_{\bar{\sigma}}), \quad (2)$$

where n_σ is the number density of electrons of spin σ and $n = n_\uparrow + n_\downarrow$ is the total density. Equation (2) defines the *spin drag coefficient* γ . Fourier transforming Eq. (1) with respect to time, and making use of the relationship $\mathbf{j}_\sigma(\omega) = -en_\sigma \mathbf{v}_\sigma(\omega)$ between current density and velocity, we obtain

$$i\omega \mathbf{j}_\sigma(\omega) = -\frac{n_\sigma e^2}{m} \mathbf{E}_\sigma(\omega) + \left(\frac{n_{\bar{\sigma}}}{n} \gamma + \frac{1}{\tau_D} \right) \mathbf{j}_\sigma(\omega) - \frac{n_\sigma}{n} \gamma \mathbf{j}_{\bar{\sigma}}(\omega). \quad (3)$$

The resistivity matrix $\rho_{\sigma\sigma'}$ is defined as the coefficient of proportionality between the electric field and the current:

$\mathbf{E}_\sigma = \sum_{\sigma'} \rho_{\sigma\sigma'} \mathbf{j}_{\sigma'}$. A quick comparison between this definition and Eq. (3) shows that γ is directly proportional to the spin *trans*-resistivity,

$$\gamma = -\frac{ne^2}{m} \rho_{\uparrow\downarrow}. \quad (4)$$

If we want to calculate microscopically the spin *trans*-resistivity, special attention must be paid to the contributions due to the electron-impurity scattering. In the theory of the ordinary Coulomb drag¹⁰ such contributions are zero on the average because the electrons in the two layers interact with two *different* sets of impurities, which are uncorrelated to each other. In the present case, however, electrons of opposite spin interact with the *same* set of impurities, so that electron-impurity terms do not vanish upon disorder averaging. Fortunately, it will turn out that these terms cancel out exactly at low frequency ($\omega \ll E_F$) and to leading order in the electron-electron and electron-impurity interactions.

To see how this happens, let us now proceed to the microscopic calculation of the spin *trans*-resistivity. We start from the Kubo formula⁷ for the uniform conductivity matrix

$$\sigma_{\sigma,\sigma'}(\omega) = -\frac{1}{i\omega} \frac{e^2}{m} \left(n_\sigma \delta_{\sigma,\sigma'} + \frac{\langle\langle \mathbf{P}_\sigma; \mathbf{P}_{\sigma'} \rangle\rangle_\omega}{m} \right), \quad (5)$$

where $\langle\langle A; B \rangle\rangle_\omega$ represents, as usual, the retarded response function for the expectation value of A under the action of a field that couples linearly to B . The resistivity matrix is the inverse of the conductivity matrix. In the spirit of the Drude approximation, we assume that the resistivity is essentially independent of frequency for frequencies much smaller than the Fermi energy. It is therefore permissible to take the limit of weak electron-impurity and electron-electron scattering *before* taking the limit of $\omega \rightarrow 0$.⁸ When the limits are carried out in this order, the P_σ 's are almost constants of the motion and therefore the second term in the large parentheses of Eq. (5) is a small correction to the first. Inverting Eq. (5) to first order in $\langle\langle \mathbf{P}_\sigma; \mathbf{P}_{\sigma'} \rangle\rangle_\omega$ and selecting the $\uparrow\downarrow$ matrix element, we obtain

$$\rho_{\uparrow\downarrow}(\omega) = \frac{i\omega}{e^2} \frac{\langle\langle \mathbf{P}_\uparrow; \mathbf{P}_\downarrow \rangle\rangle_\omega}{n_\uparrow n_\downarrow}. \quad (6)$$

It is convenient to recast this equation in a form that emphasizes the importance of the nonconservation of P_\uparrow and P_\downarrow . To this end we make use twice of the general equation of motion

$$\langle\langle A; B \rangle\rangle_\omega = \frac{1}{\omega} (\langle\langle [A, B] \rangle\rangle + i \langle\langle \dot{A}; B \rangle\rangle_\omega), \quad (7)$$

where $\dot{A} \equiv -i[A, H]$ is the time derivative of the operator A and $\langle\langle \rangle\rangle$ denotes the thermal average. Thus, Eq. (6) can be rewritten as

$$\rho_{\uparrow\downarrow}(\omega) = \frac{i}{e^2 n_\uparrow n_\downarrow} \frac{\langle\langle \dot{\mathbf{P}}_\uparrow; \dot{\mathbf{P}}_\downarrow \rangle\rangle_\omega + i \langle\langle [\dot{\mathbf{P}}_\uparrow, \mathbf{P}_\downarrow] \rangle\rangle_\omega}{\omega}. \quad (8)$$

The commutator term controls the high-frequency behavior of $\rho_{\uparrow\downarrow}(\omega)$ and can be expressed in terms of ground-state properties.⁹ This term, however, gives a purely imaginary contribution to the *trans*-resistivity. Our present interest is in the real part of the *trans*-resistivity, which is controlled by the imaginary part of the force-force response function.

The force operator is given by

$$\dot{\mathbf{P}}_\sigma = -\frac{i}{V} \sum_{\mathbf{q}} \mathbf{q} v_{\mathbf{q}} \rho_{\mathbf{q}\bar{\sigma}} \rho_{-\mathbf{q}\sigma} - \frac{i}{V} \sum_{\mathbf{q}} \mathbf{q} v_{\mathbf{q}}^{e-i} \rho_{\mathbf{q}}^i \rho_{-\mathbf{q}\sigma}, \quad (9)$$

where $v_{\mathbf{q}} = 4\pi e^2/q^2$ is the Fourier transform of the Coulomb interaction, $v_{\mathbf{q}}^{e-i}$ is the Fourier transform of the electron-impurity interaction, $\rho_{\mathbf{q}\sigma}$ is the electronic spin density fluctuation operator, $\rho_{\mathbf{q}}^i$ is the Fourier transform of the impurity density (a number), and V is the volume of the system.

We are now ready to evaluate the contribution of correlated impurity scattering to the *trans*-resistivity. Let us insert the expression (9) for $\dot{\mathbf{P}}_\sigma$ into Eq. (8). The impurity-impurity contribution takes the form

$$\text{Re } \rho_{\uparrow\downarrow}^{I-I}(\omega) = -\frac{N^i}{n_\uparrow n_\downarrow e^2 \omega V} \sum_{\mathbf{q}} \frac{\mathbf{q}^2}{3} |v_{\mathbf{q}}^{e-i}|^2 \text{Im} \langle\langle \rho_{-\mathbf{q}\uparrow}; \rho_{\mathbf{q}\downarrow} \rangle\rangle_\omega \quad (10)$$

$$\approx -\frac{N^i}{n_\uparrow n_\downarrow e^2 \omega V} \sum_{\mathbf{q}} \frac{\mathbf{q}^2}{3} |v_{\mathbf{q}}^{e-i}|^2 v_{\mathbf{q}} [\chi'_{0\uparrow}(\mathbf{q}, \omega) \times \chi''_{0\downarrow}(\mathbf{q}, \omega) + \chi''_{0\uparrow}(\mathbf{q}, \omega) \chi'_{0\downarrow}(\mathbf{q}, \omega)], \quad (11)$$

where the last equality is valid to leading order in the Coulomb and electron-impurity interactions, $\chi'_{0\sigma}$ is the real (imaginary) part of the noninteracting density-density response function, and N^i is the number of impurities. In deriving this equation we have made use of the fact that, to the first order in the Coulomb interaction and zero order in the electron-impurity interaction, one has

$$\chi_{\uparrow\downarrow}(\mathbf{q}, \omega) \equiv \langle\langle \rho_{-\mathbf{q}\uparrow}; \rho_{\mathbf{q}\downarrow} \rangle\rangle_\omega \approx v_{\mathbf{q}} \chi_{0\uparrow}(\mathbf{q}, \omega) \chi_{0\downarrow}(\mathbf{q}, \omega) \quad (12)$$

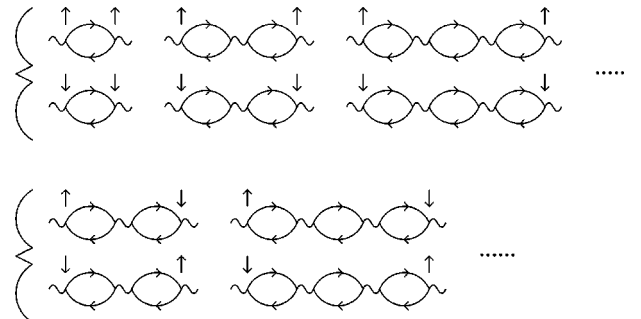


FIG. 1. The two series of “bubble” diagrams for the four-point response function $\chi_{4\rho}$ in the RPA. The vertices represent spin-density fluctuations $\rho_{q\sigma}$ as labeled.

and that for a random distribution of impurities

$$\langle \rho_{\mathbf{q}}^i \rho_{-\mathbf{q}'}^i \rangle = N^i \delta_{\mathbf{q}, \mathbf{q}'}. \quad (13)$$

The Coulomb-impurity contributions to Eq. (8) can also be calculated easily. We note that in the limit of weak Coulomb scattering the spin-up and spin-down components become decoupled so that

$$\langle \langle \rho_{-\mathbf{q}\uparrow} \rho_{\mathbf{q}\downarrow}; \rho_{-\mathbf{q}'\downarrow} \rangle \rangle_{\omega} \sim \delta_{\mathbf{q}, \mathbf{q}'} \langle \rho_{-\mathbf{q}\uparrow} \rangle \langle \langle \rho_{\mathbf{q}\downarrow}; \rho_{-\mathbf{q}'\downarrow} \rangle \rangle_{\omega}. \quad (14)$$

The equilibrium value of the spin-up electron density fluctuation is given, to first order in the electron-impurity interaction and zeroth order in the Coulomb interaction, by

$$\langle \rho_{\mathbf{q}\uparrow} \rangle = \chi_{0\uparrow}(\mathbf{q}, 0) v_q^{e-i} \rho_{\mathbf{q}}^i, \quad (15)$$

where $\chi_{0\sigma}(\mathbf{q}, 0)$ is the *static* noninteracting density-density response function for spin σ .

Making use of these results in Eq. (8), we obtain

$$\begin{aligned} \text{Re } \rho_{\uparrow\downarrow}^{C-I}(\omega) &= \frac{1}{n_{\uparrow} n_{\downarrow} e^2 \omega V} \sum_{\mathbf{q}} \frac{q^2}{3} v_q v_q^{e-i} \\ &\times \langle \rho_{-\mathbf{q}\uparrow} \rangle \text{Im} \langle \langle \rho_{\mathbf{q}\downarrow}; \rho_{-\mathbf{q}\downarrow} \rangle \rangle_{\omega} \rho_{\mathbf{q}}^i \\ &+ (\text{spin}\uparrow \rightarrow \text{spin}\downarrow) \\ &\approx \frac{N^i}{n_{\uparrow} n_{\downarrow} e^2 \omega V} \sum_{\mathbf{q}} \frac{q^2}{3} |v_q^{e-i}|^2 v_q \\ &\times [\chi_{0\uparrow}'(\mathbf{q}, 0) \chi_{0\downarrow}''(\mathbf{q}, \omega) \\ &+ \chi_{0\downarrow}'(\mathbf{q}, 0) \chi_{0\uparrow}''(\mathbf{q}, \omega)]. \end{aligned} \quad (16)$$

$$\begin{aligned} \text{Re } \rho_{\uparrow\downarrow}(\omega, T) &= \frac{1}{n_{\uparrow} n_{\downarrow} V e^2} \sum_{\mathbf{q}} \frac{q^2}{3} v_q^2 \frac{(e^{-\beta\omega} - 1)}{\omega} \\ &\times \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{[\chi_{\uparrow\uparrow}''(q, \omega') \chi_{\downarrow\downarrow}''(q, \omega - \omega') - \chi_{\uparrow\downarrow}''(q, \omega') \chi_{\downarrow\uparrow}''(q, \omega - \omega')]}{(e^{-\beta\omega'} - 1)(e^{-\beta(\omega - \omega')} - 1)}. \end{aligned} \quad (19)$$

Here $\beta = 1/k_B T$, with k_B the Boltzmann constant, and $\chi_{\sigma\sigma'}''(q, \omega)$ is the imaginary part of the RPA spin-resolved density-density response function, which is related to the noninteracting response function $\chi_{0\sigma}(q, \omega)$ as follows:

$$[\chi^{-1}(q, \omega)]_{\sigma\sigma'} = [\chi_{0\sigma}]^{-1}(q, \omega) \delta_{\sigma\sigma'} - v_q. \quad (20)$$

It is possible to show by simple but tedious algebraic calculations that this expression for the spin *trans*-resistivity $\rho_{\uparrow\downarrow}(\omega, T)$ reduces, in the case of finite temperature and $\omega = 0$, to the well known result of memory function and diagrammatic theories for the Coulomb drag.^{10,13} Furthermore, for $T=0$ and $\omega \neq 0$, the RPA is equivalent to the decoupling approximation for the four-point response function used in

Comparing Eq. (11) and Eq. (17), it is clear that, in the limit of $\omega \rightarrow 0$, the Coulomb-impurity term *exactly* cancels the impurity-impurity contribution. Thus, at low frequency ($\omega \ll E_F$) and to leading order in the electron-electron and electron-impurity interactions, the contribution of the impurities disappears and the real part of the spin *trans*-resistivity takes the form

$$\begin{aligned} \text{Re } \rho_{\uparrow\downarrow}(\omega) &= \frac{1}{n_{\uparrow} n_{\downarrow} e^2 \omega V^2} \sum_{\mathbf{q}\mathbf{q}'} \frac{\mathbf{q} \cdot \mathbf{q}'}{3} v_q v_{q'} \\ &\times \text{Im} \langle \langle \rho_{-\mathbf{q}\uparrow} \rho_{\mathbf{q}\downarrow}; \rho_{\mathbf{q}'\uparrow} \rho_{-\mathbf{q}'\downarrow} \rangle \rangle_{\omega}. \end{aligned} \quad (18)$$

It must be stressed that the cancellation of correlated impurity scattering effects has been proved here only within the frame of the Drude-Boltzmann theory defined by Eq. (6), which is the result of interchanging the natural order of the $\omega \rightarrow 0$ limit and the weak scattering limit. A more sophisticated treatment of quantum effects in correlated impurity scattering¹¹ suggests that the spin drag would be *even larger* than predicted by the present theory at temperatures so low that $k_B T \ll \hbar/\tau_D$. The temperature range in which these quantum corrections are important shrinks to zero in the limit of weak impurity scattering.

We have calculated the four-point response function $\chi_{4\rho}(\mathbf{q}, \mathbf{q}', \omega) \equiv \langle \langle \rho_{-\mathbf{q}\uparrow} \rho_{\mathbf{q}\downarrow}; \rho_{\mathbf{q}'\uparrow} \rho_{-\mathbf{q}'\downarrow} \rangle \rangle_{\omega}$ at finite temperature in a generalized random-phase approximation (RPA). The selected diagrams are shown in Fig. 1. Because of its infinite range, the Coulomb interaction must be treated to infinite order, even when weak. The sum of the RPA diagrams has been evaluated by standard methods¹² with the following result:

Ref. 14 to calculate the dynamical exchange-correlation kernel. Thus our calculation demonstrates that those two approximations, quite different at first sight, are simply RPAs performed in different limits.

Let us focus on the low-temperature and low-frequency regime $k_B T \ll E_F$ and $\omega \ll E_F$, with E_F the Fermi energy. In this regime the imaginary part of the density-density response functions $\chi_{\sigma\sigma'}''(q, \omega)$ is a linear function of ω . In the limit of vanishing impurity concentration, $\chi_{0\sigma}(q, \omega)$ is simply the Lindhard function, whose imaginary part, at low frequency, is given by $\chi_{0\sigma}''(\mathbf{q}, \omega \rightarrow 0) = -(m^2/4\pi)(\omega/q)$ and whose real part can be approximated by its value at $\omega=0$. Making use of this limiting form, the calculation of $\rho_{\uparrow\downarrow}$ can be carried in an essentially analytical fashion. The result is

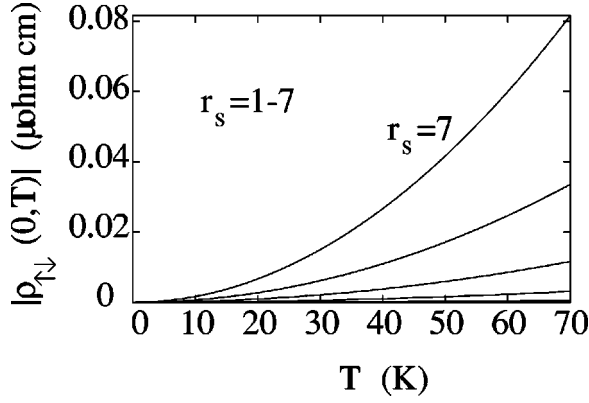


FIG. 2. Temperature and density dependence of $|\rho_{\uparrow\downarrow}(0, T)|$ in a paramagnetic metal. The top line corresponds to $r_s=7$. The electron-gas parameter is decremented by 1 starting from the top.

$$\text{Re } \rho_{\uparrow\downarrow}(\omega, T) = -\frac{\hbar a}{e^2} \frac{4\pi^2(k_B T)^2 + \hbar^2 \omega^2}{6(\text{Ry})^2} \times \frac{1}{24\pi^3 \bar{n}_\downarrow \bar{n}_\uparrow} \int_0^{2k_F a d \bar{q}} \frac{1}{\bar{q}^2 |\epsilon(\bar{q}/a, 0)|^2}, \quad (21)$$

where $a \equiv \hbar^2/m_e^2$ is the effective Bohr radius, $\text{Ry} = e^2/2a$ is the effective Rydberg, where $k_F \equiv \min(k_{F\uparrow}, k_{F\downarrow})$, with $k_{F\sigma}$ being the δ spin population Fermi wave vector, $\bar{q} \equiv qa$, $\bar{n}_\sigma \equiv n_\sigma a^3$, and $\epsilon(q, \omega) = 1 - v_q \chi_{0\uparrow}(q, \omega) - v_q \chi_{0\downarrow}(q, \omega)$ is the RPA dielectric function. Equation (21) shows that, in the absence of impurities, $\rho_{\uparrow\downarrow}(\omega, T)$ is proportional to ω^2 for $k_B T \ll \omega$ and to T^2 for $\omega \ll k_B T$.

Modifications in the form of $\chi_{0\sigma}(q, \omega)$ due to the presence of impurities can be taken into account through Mermin's approximation scheme.¹⁵ These modifications amount to replacing ω/qv_F by ω/Dq^2 ($D = v_F^2 \tau/3$) being the diffusion constant) for $\omega < 1/\tau$ and $q < 1/v_F \tau$, where v_F is the Fermi velocity and τ is the electron-impurity mean scattering time. The ω and T dependences of Eq. (21) are not affected.

Writing explicitly in Eq. (21) the dependence over $r_{s\sigma}$ [where $r_{s\sigma} = (4\pi n_\sigma a^3/3)^{-1/3}$ is the usual electron gas parameter] one can also see that $\rho_{\uparrow\downarrow}(\omega, T) \sim r_{s\uparrow}^3 r_{s\downarrow}^3$, so that $|\rho_{\uparrow\downarrow}|$ will strongly increase with decreasing electron density.¹⁶ In Fig. 2 we plot $|\rho_{\uparrow\downarrow}(\omega=0, T)|$ as a function of the temperature, for $n_\uparrow = n_\downarrow$ and in the density range $1 < r_s < 7$. The figure shows that, for metallic densities correspondent to $r_s \geq 5$ and temperatures of the order of 40–60 K (at which, for example, experiments on spin relaxation time using spin-polarized currents have been performed²), the spin *trans*-resistivity is appreciable [$|\rho_{\uparrow\downarrow}(\omega=0, T)| \geq 0.01 \mu\Omega \text{ cm}$].

Using this result, we can check *a posteriori* the consistency of neglecting spin-flip processes. We estimate the spin-diffusion time by δ_s/v_F and the Coulomb scattering time γ^{-1} from Eq. (4). For $r_s = 5$ we obtain $\gamma^{-1} \approx 10^{-13} \text{ s}$ and $\delta_s/v_F \approx 10^{-10} \text{ s}$: γ^{-1} is indeed several orders of magnitude smaller than the spin-diffusion time, so our approximation is fully justified.

In the remaining part of this paper, we describe an experiment aimed at detecting the effect of the spin Coulomb drag

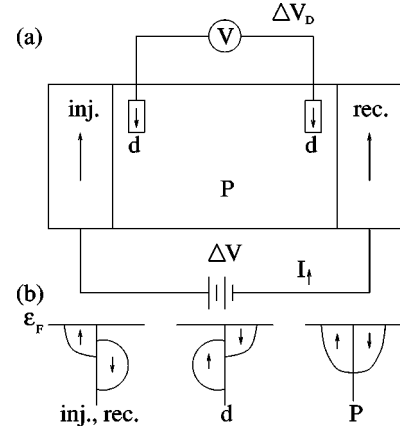


FIG. 3. (a) Experimental setup to detect the SCD effect: the voltage ΔV is applied between two parallel half-metallic ferromagnets [injector (inj.) and receiver (rec.)] that sandwich a paramagnet (P). The voltage ΔV_D is detected using two ferromagnetic electrodes (d) similar to the injector and the receiver, but polarized in the *opposite* direction. (b) Schematic band structure of injector, receiver, d , and P .

and measuring the spin *trans*-resistivity. The setup is shown in Fig. 3: a paramagnetic metal film of thickness L is sandwiched between two ferromagnets polarized in the same direction. A battery is connected to the ferromagnets inducing a *spin-polarized current*² from the first ferromagnet (“injector”) through the paramagnet and toward the second ferromagnet (“receiver”). The injector and receiver are chosen to be half-metals, i.e., they have only electron states of spin \uparrow at the Fermi level (see Fig. 3). It follows that the injected current \mathbf{j}_\uparrow is carried *only* by spin \uparrow electrons. If we choose $L \ll \delta_s$, where δ_s is the spin relaxation length, we can safely neglect spin-flipping processes and the polarized current entering the paramagnet will not relax before reaching the receiver. Spin relaxation lengths are relatively large in some materials ($\delta_s \approx 100 \mu\text{m}$ in Al),² so the condition $L \ll \delta_s$ is not particularly restrictive. Due to the SCD, the injected \mathbf{j}_\uparrow will drag spin \downarrow electrons toward the junction with the receiver. But, since there is no conduction band available in the receiver for spin \downarrow electrons, the circuit will behave as an *open circuit* for spin \downarrow electrons, i.e., $\mathbf{j}_\downarrow = \mathbf{0}$. The vanishing of \mathbf{j}_\downarrow is an indication that the Coulomb drag force is exactly balanced by the gradient of the electrochemical potential for spin down,

$$-e\mathbf{E}_\downarrow + m\gamma \frac{\mathbf{j}_\uparrow}{n_\uparrow} = 0, \quad (22)$$

where $\mathbf{E}_\downarrow = \nabla \mu_\downarrow/e + \mathbf{E}$ is the sum of the electrostatic field \mathbf{E} and the gradient of the chemical potential μ_\downarrow . What Eq. (22) tells us is that due to the SCD there will be a measurable electrochemical potential difference $eE_\downarrow l = em\gamma j_\uparrow l/n_\uparrow$ for spin \downarrow electrons between two points within the metal separated by a distance l along the direction of the current.

To measure this potential difference, a second circuit including a voltmeter of very large resistance is connected to the regions of the paramagnet close to the junctions (see Fig. 3). Our purpose is to measure E_\downarrow , so this second circuit must be driven by the spin \downarrow electrochemical potential only. In

order to accomplish this, we propose to use as contacts two half-metallic ferromagnetic electrodes (“detectors”), similar to the injector and the receiver, but polarized in the *opposite* direction. In this way, for the same reasons explained before, the detection circuit will be “open” as far as spin \uparrow electrons are concerned, and the current flowing in the voltmeter will be exclusively driven by the electrochemical potential difference of spin \downarrow electrons. The spin *trans*-resistivity will then be given by $\rho_{\uparrow\downarrow} = (\Delta V_D / I_{\uparrow})(A/l)$, where ΔV_D is the voltage measured by the meter, A is the cross section of the paramagnetic metal, l is the distance between the detectors, and I_{\uparrow} the current flowing between injector and receiver. As shown by our calculations, we expect a resistivity of the

order of $10^{-2} \mu\Omega \text{ cm}$ that is proportional to T^2 for $k_B T \gg \omega$.

In summary, we have pointed out a novel effect in spin-polarized transport, the spin Coulomb drag, and we have proposed an experiment to observe it. We hope that this paper will stimulate experimental work aimed at the detection of this effect.

This research was supported by NSF Grant No. DMR-9706788. We thank Shufeng Zhang for very valuable discussions.

Note added in proof: After submitting the paper, we became aware that some features of SCD had been discussed in Ref. 17 and Ref. 18.

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