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Some sum rules for non-Fermi liquids: Applications taking into account the mass renormalization factor

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Restudying the non-Fermi-liquid one-particle Green functions (NFLGF) we have extended the work of Balatsky [Philos. Mag. Lett. **68**, 251 (1993)] and Yin and Chakravarty [Int. J. Mod. Phys. B **10**, 805 (1996)], among others. We use the moment approach of Nolting [Z. Phys. **255**, 25 (1972)] to compute the unknown parameters of the NFLGF's in the framework of the Hubbard model. The zeroth-order moment requires that our one-particle Green functions describe fermionic degrees of freedom. In order to satisfy the first-order sum rule, a renormalization, $\gamma \neq 1$, of the free-electron mass is called for. The second-order sum rule or moment imposes a relation between the *non-Fermi-liquid* parameter, α, the Coulomb interaction, U, and the frequency cutoff, ω_c. We have calculated the effect of the mass renormalization factor, γ, on some physical quantities, such as (i) the correlated momentum distribution function, $n_c(\vec{k})$, close to the effective chemical potential, at $T=0$; (ii) the superconducting critical temperature, T_c; and (iii) the superconducting critical interaction, $\lambda_{\rm cr}$, and compared them with analytical results found in the literature. Also, we have calculated the isotope effect, α', for non-Fermi-liquid systems, which reduces to $\alpha'=1/2$ (the BCS result) when $\alpha \rightarrow 0$. As a case of non-Fermi-liquid systems, in the Appendix, we have studied two inequivalent coupled Hubbard layers for which we calculate the one-particle spectral functions on the layers and perpendicular to them. We discuss the features which appear due to the shift in the two effective chemical potentials and propose some experiments to detect the features found from our expressions.

I. INTRODUCTION

The unusual normal state properties in high-temperature superconductors (HTSC) (Ref. 1) have led to the idea that the usual Fermi liquid theory is no longer valid. As a consequence of this, several phenomenological models[2,3] have emerged with the purpose of explaining the strange metallic behavior of the normal state phase and the behavior of T_c vs doping in the HTSC. We would like to point out that the origins of a non-Fermi-liquid ground state for a strongly correlated material ($U \geqslant 2D$, where U is the local repulsive Coulomb interaction and $2D$ is the width of the free band) in dimensions higher than 1, namely, $d>1$, are an issue that has not been solved analytically up to the present moment. The main arguments which have been cited as responsible for the failure of Landau theory are as follows:

(i) The high critical superconducting temperature is attributed to the CuO_2 planes, but it is well known that in one- or two-dimensional systems superconductivity (or any spontaneous symmetry breaking) is suppressed due to fluctuation effects for $T \neq 0$.

(ii) The superconducting phase is very close to the long-range magnetically ordered phase, and the exchange interaction J in this strange metal can be too strong.

(iii) The concentration of holes x, or the carrier number, is too low.

(iv) The Coulomb interaction may be too strong so the adiabatic assumption in Landau's theory may not apply.

(v) For optimally doped materials, the electrical resistivity on the plane $\rho_{ab} \propto T$ down to T_c. For a Fermi liquid, $\rho_{ab} \propto T^2$. There is not a consensus regarding the origin of the anomalous in-plane transport. Two broad classes of theories attribute the anomalous behavior either to singular forward or large-momentum scattering.[4] Furthermore, the Boltzmann transport equation gives an expression for the magnitude of the resistivity in terms of band parameters and a mean free path between quasiparticle collisions. At low temperatures, this expression suggests a mean free path which is much larger than the lattice constant, as in conventional metals. However, at higher temperatures the resistivity smoothly increases to large values, suggesting a mean free path which is smaller than the lattice constant, implying the breakdown of a quasiparticle picture.[5]

(vi) The overdoped materials exhibit a ratio of resistivities, $\rho_c/\rho_{ab} \neq f(T)$, as in standard anisotropic materials. On the other hand, underdoped materials exhibit a divergent out-of-plane ρ_c as T decreases, even if ρ_{ab} is metallic. The c-axis data are not as universal between different cuprate families.

(vii) The Hall coefficient, $R_H(T)$, in HTSC in the normal state shows the striking NFL behavior.[6] $R_H(T)$ follows a Curie-Weiss-type T dependence and $|R_H(T)| \gg 1/|n\,e|$ for $T \rightarrow 0$ in the underdoped compounds, where n is the carrier number. Moreover, $R_H(T)>0$ for hole-doped compounds and $R_H(T)<0$ for electron-doped ones, although each of them has a similar holelike Fermi surface.

(viii) The NMR relaxation rate $1/T_1 \propto T^0$ for these materials. Remember that for a Fermi liquid, $1/T_1 \propto T$.

(ix) In conventional metals, one observes a Drude peak in

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the optical conductivity at $\omega=0$, which broadens but persists for high temperatures. In contrast, in strongly correlated metals most of the spectral weight is in broad features at high frequencies. Furthermore, the Drude peak only exists at low temperatures.

(x) In conventional metals, the thermopower is linear in temperature, has values less than $k_B/e \approx 87 \mu V/K$, and has the same sign as the charge carriers. In strongly correlated metals, it can have a nonmonotonic temperature dependence, change sign, and have values of the order of k_B/e.

(xi) The isotope exponent, α', in HTSC is unconventional in different respects. Optimally doped samples show a very small α' of the order of 0.05 or even smaller, in contrast to the BCS value of $\frac{1}{2}$. This unusually small value in connection with the high value of T_c led to early suggestions that the pairing interaction in the HTSC cuprates might be predominantly electronic in origin with a possible small phononic contribution. However, to complicate things a little bit more, the isotope exponent factor, α', also shows an unusually strong doping dependence, reaching values of $\frac{1}{2}$, in some cases higher, in the underdoped, T_c reduced, compounds.

Among one of the models to explain the *anomalous* properties of the HTSC, we mention the one of Varma *et al.*,[2] which tries to fit the linear resistivity data of the HTSC by means of the quasiparticle lifetime of the form $\tau \approx 1/[T(\varepsilon_k - \mu)]$, where T is the absolute temperature and μ is the chemical potential. This model is strictly phenomenological. There is a second model introduced by Anderson,[3] whose starting hypothesis is the occurrence in two-dimensional $(2-d)$ systems of a state similar to the one from the one-dimensional $(1-d)$ Luttinger liquids.[7] The point of view adopted by us is that the HTSC are materials which can be treated by the Luttinger liquid model, especially in the underdoped regime. Being so, we explore some consequences of this assumption.

In such a non-Fermi system, the one-particle spectral function $A(\varepsilon_{\vec{k}},\omega) = -1/\pi \operatorname{Im}[G(\varepsilon_{\vec{k}},\omega+i0^+)]$, where $G(\vec{k},\omega)$ is the one-particle Green function, satisfies the scaling property

$$A(\Lambda \varepsilon_{\vec{k}},\Lambda \omega) = \Lambda^{\alpha-1} \times A(\varepsilon_{\vec{k}},\omega), \tag{1}$$

where α, the non-Fermi-liquid parameter, is given by the condition $0<\alpha<\frac{1}{2}$. In the case of a Fermi liquid system $\alpha=0$. We mention that Wen,[8] as we do in this paper [Eq. (15)], has shown that the exponent α is not universal, since it depends on the coupling constant between the electrons. In the case of the Hubbard model, α depends on the local Coulomb interaction. The model that we are going to study by means of the scaling relation [Eq. (1)] has poles and branch cuts. This scaling behavior is a generalization to dimensions $d>1$ of the fermionic propagator from one-dimensional systems $(d=1)$. It has been shown by different authors[9–12] that the one-particle Green function (1PGF) can be expressed as

$$G(\varepsilon_{\vec{k}},\omega) = \frac{g(\alpha) e^{i\phi} \omega_c^{-\alpha}}{(\omega-\varepsilon_{\sigma,\vec{k}})^{1/2} (\omega-\varepsilon_{\rho,\vec{k}})^{1/2-\alpha}}, \quad -\omega_c<\omega<\omega_c, \tag{2}$$

where $\varepsilon_{\sigma(\rho),\vec{k}}$ represents the spin (charge) energy spectrum of the carriers, ω_c is frequency cutoff introduced to maintain the dimension of the 1PGF correct, and $g(\alpha), e^{i\phi}$ are renormalization factors introduced in order to recover the usual properties of the Green's functions. Outside the interval $[-\omega_c;\omega_c]$ the Green function will have the usual $1/\omega$ behavior as $\omega \to \infty$.

A model based on a Green function such as the one in Eq. (2) is motivated by the Luttinger-liquid model. Such a model was studied in some detail by Yin and Chakravarty,[13] following a strategy similar to the one used by Galitskii and Migdal,[14] who constructed a model for a Fermi liquid and examined its self-consistency. As a result, Yin and Chakravarty[13] obtained a non-Fermi-liquid state which is gapless and responds like a metal, but without quasiparticles. In our paper, we intended to check self-consistently the idea that the model can be a solution for the Hubbard Hamiltonian and by applying the sum rules related to such a Hamiltonian to evaluate the constants of the model [see Eq. (2)]. Let us mention that Eq. (1) was used by Chakravarty and Anderson[15] to obtain an interlayer tunneling Hamiltonian (ILT). Just recently, Chakravarty, Kee, and Abrahams[16] have used the interlayer tunneling Hamiltonian to explain the experiments of Basov *et al.*[17] on the c-axis optical sum rule in some HTSC. The structure of the paper is as follows.

In Sec. II we show that a band renormalization parameter γ is necessary to satisfy the first-order sum rule of Nolting.[18] The presence of γ seems to us very natural since when correlations are present, the free electronic band has to be renormalized, too. In the same section, we find that α, ω_c, and U are closely related. The parameter γ is going to play an important role in the following development. In Sec. III we have recalculated the following quantities: (i) the momentum distribution function $n_c(\varepsilon_{\vec{k}})$ close to the effective chemical potential, namely, $\varepsilon_{\vec{k}} \approx \mu_{\text{eff}}$; (ii) the superconducting critical temperature, T_c; and (iii) the superconducting critical interaction strength, λ_{cr}. In Sec. IV we present the calculation of the isotope exponent or coefficient, α', using the non-Fermi-liquid one-particle Green function (NFLGF). In Sec. V we present our conclusions and the outlook of our line of work.

While the semiphenomenological theory of Anderson[3] suggests anomalous exponents, i.e., $\alpha \neq 0$, a *satisfactory* derivation of them and other details await further theoretical development.[19] In the Appendix we have found an application of our theoretical treatment: two inequivalent coupled Hubbard layers (ICHL). Among the features found, we mention the appearance of an energy gap in the off-diagonal one-particle Green function. We suggest that angle-resolved photoemission spectroscopy (ARPES) experiments should be set up to measure this gap foreseen for this type of materials $[\mathrm{Y_2Ba_4Cu_7O_{15}}(247) \equiv \mathrm{YBa_2Cu_3O_7}(123) + \mathrm{YBa_2Cu_4O_8}(124)]$. The results presented in this paper, especially the ones of Sec. II, represent a first step along these lines, i.e., the use of the sum rules of Nolting[18] to determine the physical parameters of the theory.

II. CALCULATION OF THE PARAMETERS VIA SUM RULES

The model we study is the Hubbard Hamiltonian[20]

$$H = t_{i,j} c_{i\sigma}^{\dagger} c_{j\sigma} + \frac{U}{2} n_{i\sigma} n_{i\bar{\sigma}} - \mu c_{i\sigma}^{\dagger} c_{i\sigma}, \tag{3}$$

where $c^{\dagger}_{i\sigma}(c_{i\sigma})$ are creation (annihilation) operators for particles with spin σ. $n_{i\sigma}\equiv c^{\dagger}_{i\sigma}c_{i\sigma}$, U is the local interaction, and μ the chemical potential (we work in the grand-canonical ensemble). We have adopted the Einstein convention for repeated indices, i.e., for the N_s sites (labeled by i), the z nearest-neighbor (n.n.) sites j and for spin up and down $(\sigma=-\overline{\sigma}=\pm 1)$. $t_{i,j}=-t$ for n.n. and zero otherwise.

In this section we will use the first sum rules or moments of Nolting[18] applied to the Hubbard model of Eq. (3) to find some conditions on the parameters of the theory. Before applying the sum rules, let us present the spectral function, $A(\vec{k},\omega)$. It is given by

$$A(k,\omega)=-\frac{g(\alpha)}{\pi\omega_c^{\alpha}}\left[\frac{\sin[\phi-\pi(1-\alpha)]}{(\xi_{\vec{k}}-\omega)^{1/2-\alpha}(\eta\xi_{\vec{k}}-\omega)^{1/2}}\Theta(\eta\xi_{\vec{k}}-\omega)\right.$$
$$+\frac{\sin[\phi-\pi(1/2-\alpha)]}{(\xi_{\vec{k}}-\omega)^{1/2-\alpha}(\omega-\eta\xi_{\vec{k}})^{1/2}}\Theta(\omega-\eta\xi_{\vec{k}})\Theta$$
$$\left.\times(\xi_{\vec{k}}-\omega)+\frac{\sin\phi}{(\omega-\xi_{\vec{k}})^{1/2-\alpha}(\omega-\eta\xi_{\vec{k}})^{1/2}}\Theta(\omega-\xi_{\vec{k}})\right] \tag{4}$$

if $0<\omega<\omega_c$. In this equation, $\xi_{\vec{k}}\equiv\varepsilon_{\vec{k}}-\mu_{\mathrm{eff}}$, $\eta=u_{\rho}/u_{\sigma}$ represents the ratio of the charge and spin velocities, and $\Theta(x)$ is the usual theta function. The effective chemical potential is defined in Eq. (8). A similar equation can be obtained also for the case $-\omega_c<\omega<0$. The spectral function [Eq. (4)] has to satisfy the time-reversal symmetry, a condition which leads to $\phi=-\pi\alpha/2$, a result already obtained by Yin and Chakravarty.[13] Equation (4) is assumed to be the many-body solution to the Hubbard Hamiltonian [Eq. (3)].

The first moment (or zeroth-order sum rule) $M_o(\vec{k})$ is given by

$$M_o(\vec{k})=\int_{-\infty}^{+\infty}A(k,\omega)d\omega=1. \tag{5}$$

Let us say that the condition given by Eq. (5) represents the equal-time anticommutation relation of fermions as pointed out by Yin and Chakravarty.[13] Equation (5) is valid for any fermionic theory, independent of the model used. Furthermore, the area of the distribution is 1, i.e., it is normalized. Doing the integration of $A(k,\omega)$ and using the expression given in Eq. (4), we find

$$\frac{g(\alpha)}{\pi}\sin\frac{\pi\alpha}{2}\left\{\frac{1}{\alpha}\left[\left(1+\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha}+\left(1-\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha}\right]+(1-\eta)\right.$$
$$\left.\times\frac{\xi_{\vec{k}}}{\omega_c}\,\frac{\alpha-1/2}{\alpha-1}\left[\left(1+\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha-1}-\left(1-\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha-1}\right]\right\}$$
$$+\frac{g(\alpha)}{\pi}(1-\eta)^{\alpha}\frac{\xi_{\vec{k}}^{\alpha}}{\omega_c^{\alpha}}\left[\cos\frac{\pi\alpha}{2}B(1/2,\alpha+1/2)\right.$$
$$\left.+\sin\frac{\pi\alpha}{2}\,\frac{\alpha^2-3/2\alpha+1}{\alpha(\alpha-1)}\right]=1, \tag{6}$$

where $B(x,y)$ is the usual beta Euler function. If we restrict ourselves to regions close to the effective chemical potential, i.e., $\xi_{\vec{k}}\approx 0$, then the normalization factor, $g(\alpha)$, is independent of $\vec{k}$ and is given by

$$g(\alpha)\approx\frac{\pi\alpha}{2\sin\left(\frac{\pi\alpha}{2}\right)}, \tag{7}$$

which reduces to 1 when $\alpha\to 0$.

The first-order sum rule is given by

$$M_1(\vec{k})=\int_{-\infty}^{+\infty}\omega A(k,\omega)d\omega=\varepsilon_{\vec{k}}-\mu+\rho U\equiv\xi_{\vec{k}};$$
$$\mu_{\mathrm{eff}}\equiv\mu-\rho U. \tag{8}$$

The integral equal to $\varepsilon_{\vec{k}}-\mu+\rho U$ is an exact result within the working scheme of the Hubbard model (model-dependent[8]). In Eq. (8), ρ is the carrier number per lattice site and per spin. We work in the paramagnetic phase, namely, $\rho_{\uparrow}=\rho_{\downarrow}=\rho$. Combining Eqs. (4) and (8), i.e., doing the integral of $\omega A(k,\omega)$ to find the center of the distribution, we get

$$\frac{g(\alpha)}{\pi\omega_c^{\alpha}}\sin\frac{\pi\alpha}{2}\times\left\{-\frac{\omega_c^{\alpha+1}}{\alpha+1}\left[\left(1+\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha+1}-\left(1-\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha+1}\right]-\xi_{\vec{k}}[(1-\eta)(\alpha-1/2)-\eta]\frac{\omega_c^{\alpha}}{\alpha}\left[\left(1+\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha}+\left(1-\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha}\right]\right.$$
$$\left.+\xi_{\vec{k}}^{2}\eta(1-\eta)(\alpha-1/2)\frac{\omega_c^{\alpha-1}}{\alpha-1}\left[\left(1+\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha-1}-\left(1-\frac{\eta\xi_{\vec{k}}}{\omega_c}\right)^{\alpha-1}\right]\right\}+\frac{g(\alpha)}{\pi\omega_c}\xi_{\vec{k}}^{\alpha+1}(1-\eta)^{\alpha}$$
$$\times\left\{\cos\frac{\pi\alpha}{2}\eta B\left(\alpha+\frac{1}{2},\frac{1}{2}\right)F\left(-1,\frac{1}{2};\alpha+1;-\frac{1-\eta}{\eta}\right)+\sin\frac{\pi\alpha}{2}\left[\frac{\eta(\alpha-1/2)}{\alpha-1}-\frac{1-\eta}{\alpha+1}-\frac{\eta-(\alpha-1/2)(1-\eta)}{\alpha}\right]\right\}=\xi_{\vec{k}}, \tag{9}$$

where $F(\alpha,\beta;\gamma;z)$ is the hypergeometric function. Using an expansion around $\xi_{\vec{k}}\approx 0$ as previously, we find that

$$\frac{2g(\alpha)\sin[\pi\alpha/2]}{\pi\alpha}\left[\eta-(1-\eta)\left(\alpha-\frac{1}{2}\right)-\eta\alpha\right]\xi_{\vec{k}}\approx\xi_{\vec{k}}, \tag{10}$$

which leads to the following relation between the anomalous coefficient α and the spin-charge characteristic ratio η:

$$\alpha=\frac{\eta-1}{2}. \tag{11}$$

If we analyze Eq. (11), we can see that as long as $0<\alpha<1/2$ and $0<\eta<1$, the only possibility is that $\alpha=0$ and $\eta=1$, which is actually characteristic for the usual noninteracting Fermi liquid. As a conclusion, in order to satisfy the first-order sum rule, we have to introduce a new coefficient γ related to the band renormalization factor. With this coefficient, the energy $\xi_{\vec{k}}$ will be renormalized as $\gamma\xi_{\vec{k}}$. By applying again the first-order sum rule, we get

$$\gamma=\frac{2}{\eta+1-2\alpha} \tag{12}$$

so the band renormalization factor will be a function of the two previous parameters α and η. We have to mention that the introduction of the band renormalization factor does not affect the time-reversal symmetry and the zeroth-order sum rule. We remark that this relation had not been previously calculated in the literature because only the first sum rule [Eq. (5)] had been used. Now, we are going further and we will apply the second-order sum rule or the width of the distribution function, $A(\vec{k},\omega)$, to calculate α. Indeed, what we will find is that this sum rule imposes a condition on the three remaining parameters of the theory, namely, ω_c, α, and U. The interaction U is coming from the Hubbard model. As the parameter α is not really independent of the Coulomb interaction, our result evidently departs from the approach adopted by Ţifrea[21] in his Ph.D. thesis and related works[22,23] [see the discussion after Eq. (15)].

The second-order moment, $M_2(\vec{k})$, is given as

$$M_2(\vec{k})=\int_{-\infty}^{+\infty}\omega^2 A(k,\omega)\,d\omega=(\varepsilon_{\vec{k}}-\mu)^2+2U(\varepsilon_{\vec{k}}-\mu)+\rho U^2$$
$$\equiv\xi_{\vec{k}}^2+\rho(1-\rho)U^2. \tag{13}$$

This is an exact relation and the model dependence is visible through the local Coulomb interaction. Performing the integral of $\omega^2A(k,\omega)$ to find the width of the distribution and using the definition of the one-particle spectral function given in Eq. (4), we find that in the limit $\xi_{\vec{k}}\approx 0$,

$$\frac{2g(\alpha)\sin[\pi\alpha/2]}{\pi\alpha}\,\frac{\alpha}{\alpha+2}\,\omega_c^2\approx\rho(1-\rho)U^2, \tag{14}$$

which gives

$$\frac{\alpha}{\alpha+2}\approx\rho(1-\rho)\left(\frac{U}{\omega_c}\right)^2. \tag{15}$$

Thus, as we already pointed out, Eq. (15) puts a strong condition on the remaining physical variables of the theory. We immediately see that $\alpha=0$ for $U\equiv 0$, as it should be. Also, another parameter entering in the constraint is the electron number/spin. As $0<\alpha<\frac{1}{2}$, we see that $U_{\max}\approx\omega_c$, for $\rho\neq 0;1$. Here we appreciate the difference from the work of Ţifrea[21] in his Ph.D. thesis, in which he calculates the superconducting critical temperature, T_c, in the presence of repulsive local Coulomb interaction. The non-Fermi-liquid parameter, α, continues to be a free parameter in Ţifrea's approach. Before we leave this section, we say that our energy scales are ordered in the following way: $T_c<\Delta(0)<\omega_D\ll\omega_c\ll D$, where $\Delta(0)$ is the superconducting gap at zero temperature and ω_D is the Debye frequency giving origin to the superconducting critical temperature, T_c.

III. DYNAMICAL AND GLOBAL QUANTITIES

In this section we will calculate some *dynamical* properties of the theory, namely, the momentum distribution function, $n_c(\vec{k})$, in the normal phase at $T=0$. Also, we will recalculate the superconducting critical temperature, T_c, and the superconducting critical interaction strength, λ_{cr}, paying due attention to the presence of the new parameter of the theory, $\gamma=2/(\eta+1-2\alpha)$ [Eq. (12)].

A. Calculation of $n_c(\varepsilon_{\vec{k}})$ at $T=0$

The correlated momentum distribution function is given by the following expression:

$$n_c(\varepsilon_{\vec{k}})\equiv\int_{-\infty}^{+\infty}d\omega\,\frac{A(\varepsilon_{\vec{k}},\omega)}{\exp(\omega/T)+1}. \tag{16}$$

At $T=0$ we have to look carefully to this integral because as long as $\omega>0$, the exponential function is infinity, which implies a zero contribution from the integral. We still have to integrate over the region $\omega<0$ where the exponential function is zero.

Performing the integrals,[24,25] we end up with

$$\lim_{|\xi_{\vec{k}}|\to 0} n_c(\varepsilon_{\vec{k}})=\frac{1}{2}\Bigg\{1-\mathrm{sgn}(\xi_{\vec{k}})\Bigg[\left(\frac{2\eta}{\eta+1-2\alpha}\,\frac{|\xi_{\vec{k}}|}{\omega_c}\right)^{\alpha}$$
$$-2^{1-\alpha}\alpha(\alpha-1/2)B\left(\frac{1}{2},1-\alpha\right)$$
$$\times\left(\frac{2(1-\eta)}{\eta+1-2\alpha}\,\frac{|\xi_{\vec{k}}|}{\omega_c}\right)^{\alpha}+\alpha(\alpha-1/2)$$
$$\times\frac{1-\eta}{\eta}\left(\frac{2\eta}{\eta+1-2\alpha}\,\frac{|\xi_{\vec{k}}|}{\omega_c}\right)f(\alpha,\eta)\Bigg]\Bigg\}, \tag{17}$$

where $f(\alpha,\eta)=\int_0^1 dz\,z^{-1/2}[z+(1-\eta)/\eta]^{\alpha-3/2}$.

As we see from Eq. (17), the correlated momentum distribution function has been calculated close to the effective chemical potential. Also, we observe that for $\alpha=0$ and $\eta=1$, we recover the jump at the chemical potential, as it is the case for a Fermi liquid. For $\alpha\neq 0$ and $\eta=1$, this jump has gone away, but the derivative at the effective chemical potential is discontinuous. A calculation of the renormalization factor Z, defined as $Z=n_c(\xi_{\vec{k}}^{+})-n_c(\xi_{\vec{k}}^{-})$, gives $Z=0$, a result which implies that our theory is a non-Fermi-liquid one. In the other case, $\alpha=0$ and $\eta\neq 1$, we obtain the case of a Fermi liquid with spin-charge separation $(Z=1)$. Another conclusion that we reach by looking at Eq. (17) is that the parameter $\gamma=2/(\eta+1-2\alpha)$ modifies the results of Yin and Chakravarty[13] in the following way: in order to study non-Fermi-liquid systems, we have to consider that the frequency cutoff is effectively smaller, i.e., $\omega_c\rightarrow(1-\alpha)\omega_c$.

B. Calculation of T_c and λ_c in an s-wave superconductor

In the following, we will restrict ourselves to the study of the non-Fermi-liquid system $(\alpha\neq 0,\eta=1)$, where the Green's function according to our previous calculations is given by

$$G_0(k,\omega)=\frac{g(\alpha)e^{-i\pi\alpha/2}}{\omega_c^{\alpha}(\omega-\gamma\xi_{\vec{k}})^{1-\alpha}} \qquad (18)$$

with $\gamma=1/(1-\alpha)$. The order-parameter equation in the framework of the Gorkov equations is given by

$$1=V\sum_{\vec{k}}\frac{1}{\beta}\sum_{\omega_n}\frac{1}{G_0^{-1}(k,i\omega_n)G_0^{-1}(-k,-i\omega_n)-|\Delta_{\vec{k}}|^2}, \qquad (19)$$

where $\beta=1/T$. The critical temperature will be obtained from Eq. (19) with the condition $\Delta_{\vec{k}}\rightarrow 0$, and the difficult problem of evaluating the sum over the Matsubara frequency will be solved by using a contour integral similar to the one used in Refs. 21 and 22. In the limit $\beta\omega_D\ll 1$, the critical temperature can be obtained exactly as

$$T_c^{2\alpha}=\frac{1}{C(\alpha)}\left[D(\alpha)(\gamma\omega_D)^{2\alpha}-\frac{\gamma}{g^2(\alpha)}\frac{\omega_c^{2\alpha}}{\lambda A(\alpha)}\right], \qquad (20)$$

where $\lambda=1/N(0)V$ and $A(\alpha)$, $C(\alpha)$, and $D(\alpha)$ have the same meaning as in the paper of Muthukumar *et al.*[22] We have to mention that our critical temperature is different from the one obtained in Ref. 22, and include the renormalization factor $g(\alpha)$ and the band renormalization factor. As a result, we can see a decrease of the critical temperature due to the effective Debye frequency $\omega_D^{\mathrm{eff}}=\gamma\omega_D$. We also obtained a modified critical coupling constant λ_{cr},

$$\lambda_{\mathrm{cr}}=\frac{\gamma}{g^2(\alpha)}\left(\frac{\omega_c}{\gamma\omega_D}\right)^{2\alpha}\frac{1}{A(\alpha)D(\alpha)}, \qquad (21)$$

which for the same reason as an enhanced effective Debye frequency seems to be enhanced. In Fig. 1 we present a plot of the critical coupling constant versus the non-Fermi parameter α. As we can see in the limit $\alpha\rightarrow 0$, we recover the usual BCS result[21] $(\lambda_{\mathrm{cr}}=0)$.

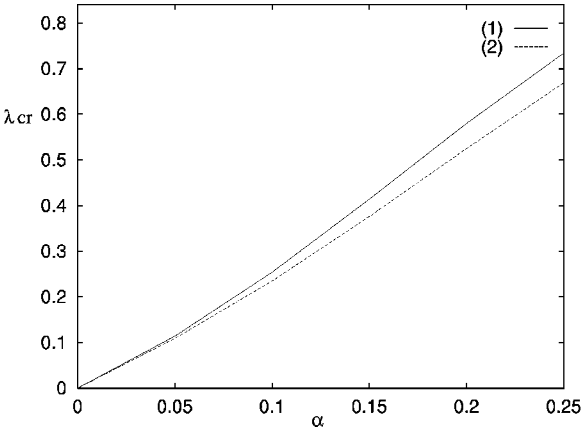

FIG. 1. The critical coupling constant versus the non-Fermi parameter α. The two lines correspond to the renormalized (1) $\gamma=1/(1-\alpha)$ and nonrenormalized (2) $\gamma=1$ case.

IV. THE ISOTOPE EFFECT FOR NON-FERMI LIQUIDS

The isotope effect exponent is an important physical parameter since, in the low-temperature superconductors (LTSC), it was used to determine the origin of the pairing mechanism. In the HTSC, the isotope effect has been widely discussed, in particular to detect the influence of the phonon degrees of freedom on T_c. As Kishore[26] points out, the isotope effect in the cuprates is sensible to various factors such as (i) the form of the free density of states, (ii) the Coulomb interaction, (iii) the carrier concentration, (iv) presence of impurities, (v) anharmonicity, and (vi) the symmetry of the order parameter, among others. From our Eq. (15) we can account for the first three dependences. Here we will calculate T_c and the isotope exponent, α', for an *s*-wave superconductor. Thus, our expression for α' could be approximately applicable to the cuprates to fit the data. We believe that the numerical values of global quantities such as α' do not depend too much on the symmetry of the order parameter. It appears that $La_{2-x}SrCuO_4$ presents an *s*-wave symmetry order parameter. So, it is the natural candidate to apply the ideas worked out here. On the other side, $Bi_2Sr_2CaCu_2O_8$ (Ref. 27) holds a *d*-wave symmetry order parameter.

The isotope effect exponent is defined as $T_c\propto M^{-\alpha'}$. Then, it is given by

$$\alpha'=-\frac{\partial\ln T_c}{\partial\ln M}, \qquad (22)$$

where M is the isotope mass. Another relation that we will use is the relation between the Debye frequency and the isotope mass, namely, $\omega_D\propto M^{-1/2}$. Now, Eq. (20) can be rewritten as

$$\frac{1}{\lambda}=g^2(\alpha)A(\alpha)F(\omega_D,\alpha,T_c),$$

$$F(\omega_D,\alpha,T_c)=\left[\gamma^{2\alpha-1}D(\alpha)\left(\frac{\omega_D}{\omega_c}\right)^{2\alpha}-\frac{C(\alpha)}{\gamma}\left(\frac{T_c}{\omega_c}\right)^{2\alpha}\right]. \qquad (23)$$

Deriving Eq. (23) with respect to M, we get that

$$\alpha'=-\frac{\omega_D\left(\frac{\partial F}{\partial\omega_D}\right)}{2T_c\left(\frac{\partial F}{\partial T_c}\right)}. \qquad (24)$$

Let us mention that Eq. (24) is valid for the case that $\alpha\neq f(\omega_D)$. The case $\alpha=f(\omega_D)$ will be discussed later on [Eq. (26)]. Performing the partial derivatives, we come down to the following expression:

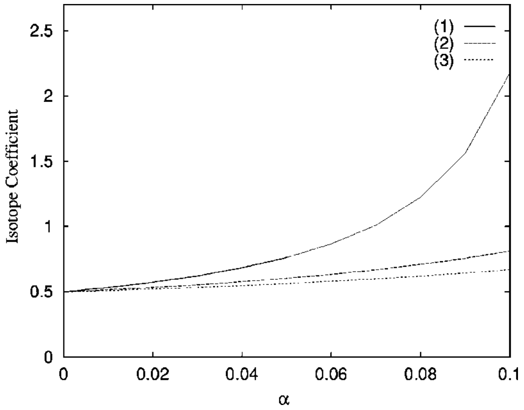

FIG. 2. The isotope coefficient versus the non-Fermi parameter α for different values of the coupling constant [(1) $\lambda = 0.33$, (2) $\lambda = 0.66$, (3) $\lambda = 1$].

$$\alpha' = \frac{1}{2}\left[1 - \frac{\gamma}{g^2(\alpha)}\left(\frac{\omega_c}{\gamma\omega_D}\right)^{2\alpha}\frac{1}{\lambda A(\alpha)D(\alpha)}\right]^{-1}. \tag{25}$$

From Eq. (25) we gain the BCS case, i.e., $\alpha' = 1/2$ when $\alpha = 0$. In Fig. 2 we plot the isotope coefficient versus the non-Fermi parameter α for different values of the coupling constant. As we expected, there is a deviation for the usual BCS result as α increases. Also, we mention that the isotope coefficient depends on the coupling constant, a result similar with the one obtained for the $2\Delta(0)/T_c$ ratio.[21]

While leaving this section, let us mention that if α would depend on ω_D, then the isotope effect exponent should be obtained from the following expression:

$$\alpha' = -\frac{\omega_D\left[\dfrac{\partial F}{\partial \omega_D} + \dfrac{\partial F}{\partial \alpha}\,\dfrac{\partial \alpha}{\partial \omega_D}\right]}{2T_c\left(\dfrac{\partial F}{\partial T_c}\right)}. \tag{26}$$

We could make $\alpha = f(\omega_D)$ by choosing $\omega_c \propto \omega_D$, as Ţifrea has done in his Ph.D. thesis.[21] This contribution is important to get full agreement with the experimental data, namely, that $\alpha' \approx 0$ at optimal doping. We argue that the lowering of α' from the BCS result should come from the second contribution in the numerator in Eq. (26). We leave this task for the future. However, the result found in the present section should be applicable in the underdoped regime of the HTSC's.

V. CONCLUSIONS AND OUTLOOK

We have applied the first three sum rules of Nolting[18] for the NFLGF of Eq. (4), which is assumed to be a solution of the Hubbard model [Eq. (3)] for frequencies $|\omega| \leqslant \omega_c$. This NFLGF is anomalous due to the presence of the non-Fermi-liquid exponent, α. Due to the requirement that the first-order sum rule of the spectral function be satisfied, a new parameter has to be called for. γ, the so-called mass renormalization factor, plays an important role in the theory. Due to its presence, we have recalculated some results found in the literature and pointed out the role of γ. Also, an additional quantity, the isotope effect exponent or α', has been calculated for non-Fermi-liquid systems. α' reduces to the BCS case when the Coulomb interaction is zero, namely, for $\alpha = 0$. The fact that $\alpha \propto U^2$ comes out from the application of the second-order sum rule to the spectral function. Due to Eq. (15), α is a model-dependent parameter, a result found previously in the literature by Wen.[8] According to our view, T_c decreases with α (Ref. 21) and $\alpha \propto U^2$; then we can conclude that the local Coulomb repulsive interaction is detrimental to superconductivity. This result has been found in other approaches.[29,30]

We would like to point out that our spectral function [Eq. (4)] does not reduce to the well known Dirac δ function when $\alpha = 0$, $\eta = 1$. In order to reobtain the usual δ form of the spectral function, we have to set $\alpha = 0$ and $\eta = 1$ in our starting Green function. Even without doing that for $\alpha = 0$ and $\eta = 1$, the spectral function [Eq. (4)] will satisfy the usual scaling relation for a Fermi-liquid system $A(\Lambda[k - k_F], \Lambda\omega) = \Lambda^{-1}A([k - k_F], \omega)$. We used this property in some parts of the paper and we have recovered known features, such as (i) the jump at μ of the momentum distribution function, $n_c(\vec{k})$, which is a Fermi-liquid behavior; (ii) the superconducting critical interaction is zero, namely, $\lambda_{\mathrm{cr}} = 0$; and (iii) the BCS isotope exponent, $\alpha' = \frac{1}{2}$.

In the Appendix we have applied the formalism developed in this paper to two inequivalent coupled Hubbard layers (ICHL) extending the calculation of Yin and Chakravarty[13] for equivalent planes. Hildebrand *et al.*[28] have applied the FLEX formalism to ICHL's. In particular, our expressions for $G_{11}(\vec{k}, i\omega_n)$, $G_{22}(\vec{k}, i\omega_n)$, and $G_{12}(\vec{k}, i\omega_n) = G_{21}(\vec{k}, i\omega_n)$ are different from the ones of Ref. 13 due to the presence of a shift in the effective chemical potential, i.e., $\mu_{1,\mathrm{eff}} = \mu_{2,\mathrm{eff}} + \delta$. Experiments should be designed to detect the results found in our work (see the Appendix). In particular, the presence of the theoretical gap in the off-diagonal one-particle spectral function, $A_{12}(\vec{k}, \omega)$, calls for ARPES experiments to be performed in these materials.[31]

We have assumed that the non-Fermi-liquid parameter, α, is independent of $\vec{k}$. This seems not to be the case as Meden[32] has pointed out. He concludes that the asymptotic behavior of the one-particle Green functions of Luttinger liquids at large space-time distances is not *universal*. Namely, along certain directions the exponent of the asymptotic power law is not given by the Luttinger-liquid parameters. Due to this consideration, α could depend on $\vec{k}$. This possibility is outside the scope of the present paper. Among one of the possibilities we would like to explore is the calculation of pressure effects[33] in some HTSC materials, when correlation is important, i.e., $\alpha \neq 0$. Another possibility worth considering, for the case of inequivalent coupled Hubbard layers, is $\alpha_1 \neq \alpha_2$, as the two non-Fermi-liquid parameters on the two planes. This approach is likely more demanding than the one followed in the paper. We leave for the future theself-consistent calculation of ρ_1 and ρ_2 on the two layers [see Eq. (A5)]. On the other hand, the presence of the mass renormalization parameter, γ, will modify the zero temperature order parameter, $\Delta(0)$. However, its calculation has been left out of the present work. Of course, with $\Delta(0)$, we could calculate the ratio $2\Delta(0)/T_c = f(U) \neq 3.5$, seeing its dependence on U (or α). Another aspect which should be addressed in the future is the superconducting properties of

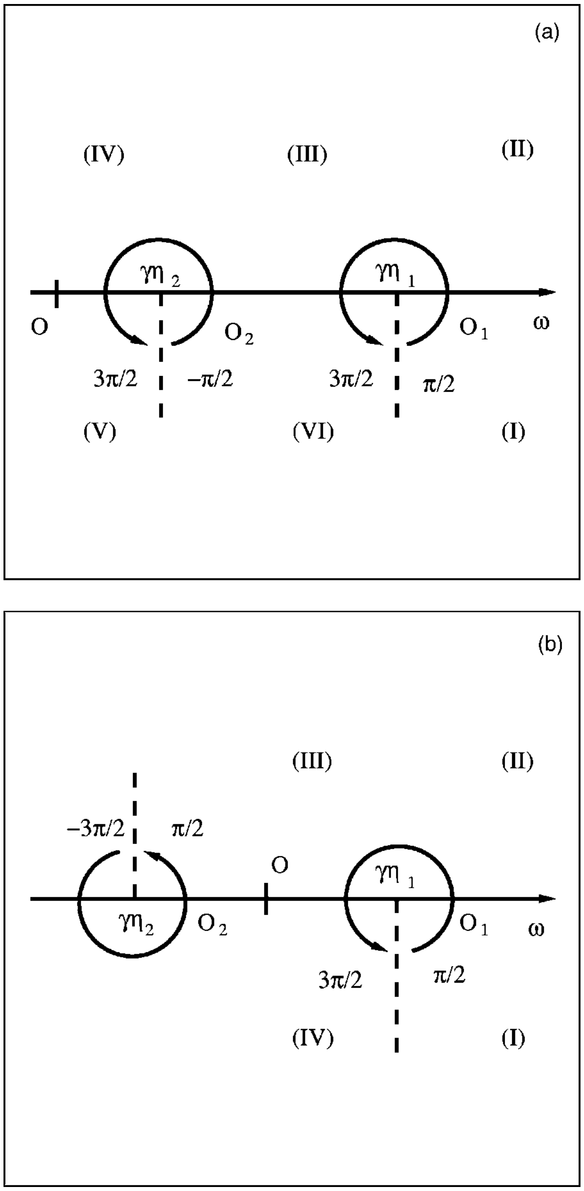

FIG. 3. (a) For $\omega>0$, $\gamma\eta_{1,\vec{k}}>\gamma\eta_{2,\vec{k}}$, and $\eta_{(1,2,)\vec{k}}>0$, we have divided the complex plane in six regions, showing the branch points ($\gamma\eta_{2,\vec{k}}$ and $\gamma\eta_{2,\vec{k}}$) and branch cuts. The branch cuts are given by the dashed lines. The solutions for $\omega>0$ are in regions (1) *I*, namely, $n=0$; and (2) *V*. (b) For $\omega>0$, $\gamma\eta_{1,\vec{k}}>\gamma\eta_{2,\vec{k}}$, and $\eta_{1,\vec{k}}>0$, $\eta_{2,\vec{k}}<0$, we have divided the complex plane in four regions, showing the branch points and branch cuts. For $\omega>0$, the solutions are in region *I*.

two inequivalent coupled Hubbard layers. In this case, our mean-field Hamiltonian becomes a 4×4 matrix in the Nambu formalism, giving rise to pairing on the planes and perpendicular to them. We would like to end by saying that the nature of the superconducting transition is strongly related to how anomalous (non-Fermi-liquid-like) the normal state spectral function is, and as such, is dependent upon the doping level.[34] The anomalous properties of the normal state spectral function are visible in the underdoped regime of the HTSC.

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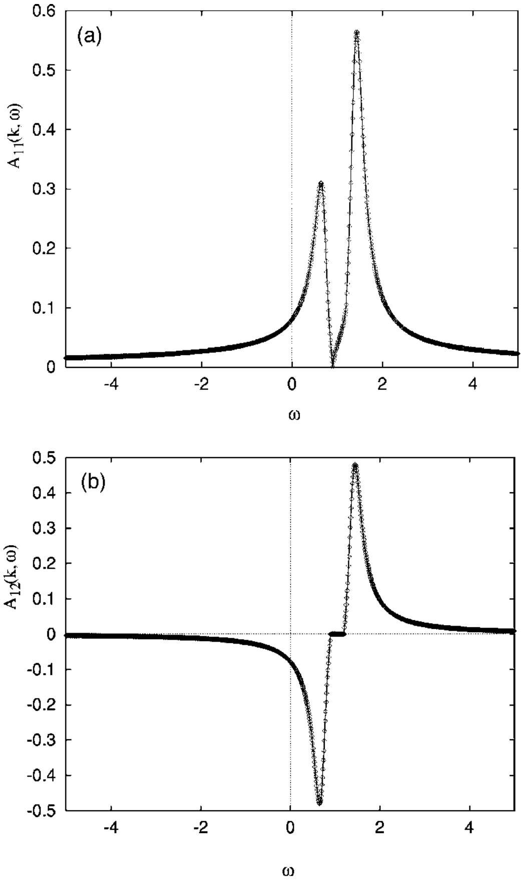

FIG. 4. (a) The diagonal one-particle spectral function, $A_{11}(\vec{k},\omega)$, vs ω, for some values of $\gamma\eta_{1,\vec{k}}$ and $\gamma\eta_{2,\vec{k}}$, namely, $\gamma\eta_{1,\vec{k}}=1.2$ and $\gamma\eta_{2,\vec{k}}=0.9$. Compare with $F_{11}(x)$ of Ref. 13. We notice that the symmetry around $\omega=0.9$ is lost. This is a realization of inequivalent coupled Hubbard layers. In a similar form, $A_{22}(\vec{k},\omega)$ vs ω should not be symmetric around $\omega=1.2$, in accord with the given parameters. (b) The off-diagonal one-particle spectral function, $A_{12}(\vec{k},\omega)$, vs ω. Same values as in (a). Compare with $F_{12}(x)$ of Ref. 13. Now we have a gap in the energy spectrum between $\omega=0.9$ and $\omega=1.2$. This is also a consequence of having inequivalent coupled Hubbard layers. Both spectral functions are calculated for $\alpha=0.25$.

APPENDIX: NON-FERMI-LIQUID ONE-PARTICLE GREEN FUNCTIONS FOR TWO INEQUIVALENT COUPLED HUBBARD LAYERS

In Ref. 28, Hildebrand *et al.* have studied the case of two inequivalent coupled Hubbard layers for the case of $Y_2Ba_4Cu_7O_{15}(247)\equiv YBa_2Cu_3O_7(123)+YBa_2Cu_4O_8(124)$. We will restudy this system using NFLGF for each one of the layers with the purpose of finding new theoretical consequences of this assumption. For example, we are going to compare with the results of Refs. 13 and 28. The Hamiltonian of the system is given by

$$H=\begin{pmatrix} G_1^{-1}(\varepsilon_{\vec k},i\omega_n) & t_\perp \\ t_\perp & G_2^{-1}(\varepsilon_{\vec k},i\omega_n) \end{pmatrix},$$

where $t_\perp$ is the coupling matrix element between the two inequivalent Hubbard layers and $G_j(\varepsilon_{\vec k},i\omega_n)$, $j=1,2$, are the normal state non-Fermi-liquid one-particle Green functions of the layers. They are given by

$$G_j(\varepsilon_{\vec k},i\omega_n)=\frac{g(\alpha)}{\omega_c^{\alpha}e^{\pm i\pi\alpha/2}}\,\frac{1}{(i\omega_n-\gamma\eta_{j,\vec k})^{1-\alpha}}\,\Theta(\pm\omega_n), \tag{A1}$$

with $\eta_{1,\vec k}\equiv\varepsilon_{\vec k}-\mu_{\mathrm{eff}}$ and $\eta_{2,\vec k}\equiv\eta_{1,\vec k}-\delta$, i.e., we have included a shift between the two effective chemical potentials. A simple calculation allows us to calculate the diagonal and off-diagonal one-particle Green functions. They are

$$Gjj(\varepsilon k,i\omega n)$$
$$=Gj-1(\varepsilon k,i\omega n)G1-1(\varepsilon k,i\omega n)G2-1(\varepsilon k,i\omega n)-t\perp 2\,;$$

$$j=1,2;\quad \overline{1}=2;\quad \overline{2}=1, \tag{A2}$$

$$G_{12}(\varepsilon_{\vec k},i\omega_n)=\frac{t_\perp}{G_1^{-1}(\varepsilon_{\vec k},i\omega_n)G_2^{-1}(\varepsilon_{\vec k},i\omega_n)-t_\perp^2},$$

$$G_{21}(\varepsilon_{\vec k},i\omega_n)=G_{12}(\varepsilon_{\vec k},i\omega_n).$$

The excitation spectrum is determined by the roots of the denominator of Eq. (A2). There are branch points at $\omega=\gamma\eta_{j,\vec k}$. Let us assume that $\eta_{1,\vec k}>\eta_{2,\vec k}>0$. Thus, for $\omega>0$, we must divide the complex plane in six regions named *I–II–III–IV–V–VI*, as shown in Fig. 3(a). The poles are given by the solutions of

$$e^{i\pi\alpha}\omega_c^{2\alpha}g^{-2}(\alpha)(\omega-\gamma\eta_{1,\vec k}+i0^+)^{1-\alpha}(\omega-\beta\eta_{2,\vec k}+i0^+)^{1-\alpha}$$
$$=t_\perp^2e^{2in\pi};\quad n\quad \text{an integer}. \tag{A3}$$

Following the analysis performed by Yin and Chakravarty,[13] we conclude that, for $\eta_{1,\vec k}>\eta_{2,\vec k}>0$, there are solutions only in the regions denoted by *I* and *V*. In the case that $0<\eta_{1,\vec k}>\eta_{2,\vec k}<0$, still for $\omega>0$, we divide the complex plane in four regions *I–II–III–IV* as shown in Fig. 3(b). Similarly to the previous analysis, we conclude that there is a solution in region *I* only. The analysis is similar for the case $\eta_{1,\vec k}<\eta_{2,\vec k}<0$.

The poles are localized at

$$\omega_{1,2}=\frac{\gamma(\eta_{1,\vec k}+\eta_{2,\vec k})\pm[\gamma^2(\eta_{1,\vec k}-\eta_{2,\vec k})^2+4t_{\perp,\mathrm{eff}}^2e^{\pm[i\pi\alpha/(1-\alpha)]}]^{1/2}}{2};\quad t_{\perp,\mathrm{eff}}\equiv g(\alpha)t_\perp\left(\frac{g(\alpha)t_\perp}{\omega_c}\right)^{\alpha/(1-\alpha)}. \tag{A4}$$

From Eq. (A4) we recover the case of equivalent coupled planes of Yin and Chakravarty[13] by making $\eta_{1,\vec k}=\eta_{2,\vec k}$. Now, we are in a position of calculating the one-particle spectral functions, namely, $A_{i,j}(\vec k,\omega)$, with $i,j=1,2$. The results are (with $\gamma\eta_{1,\vec k}>\gamma\eta_{2,\vec k}$)

$$A_{11}(\vec k,\omega)=\frac{\sin(\pi\alpha/2)X_2^{1-\alpha}}{\pi t_\perp}\begin{cases}\dfrac{1+(X_2X_1)^{1-\alpha}}{1+(X_2X_1)^{2(1-\alpha)}-2\cos(\pi\alpha)(X_2X_1)^{1-\alpha}} & \text{if }\ \omega<\gamma\eta_{2,\vec k}<\gamma\eta_{1,\vec k}\quad\text{or}\quad \gamma\eta_{2,\vec k}<\gamma\eta_{1,\vec k}<\omega\\[2ex] \dfrac{1}{1+(X_2X_1)^{1-\alpha}}\quad \text{if }\ \gamma\eta_{2,\vec k}<\omega<\gamma\eta_{1,\vec k}, \end{cases}$$

where $X_j\equiv|\omega-\gamma\eta_{j,\vec k}|/|t_{\perp,\mathrm{eff}}|$,

$$A_{22}(\vec k,\omega)=\frac{\sin(\pi\alpha/2)X_1^{1-\alpha}}{\pi t_\perp}\begin{cases}\dfrac{1+(X_2X_1)^{1-\alpha}}{1+(X_2X_1)^{2(1-\alpha)}-2\cos(\pi\alpha)(X_2X_1)^{1-\alpha}} & \text{if }\ \omega<\gamma\eta_{2,\vec k}<\gamma\eta_{1,\vec k}\quad\text{or}\quad \gamma\eta_{2,\vec k}<\gamma\eta_{1,\vec k}<\omega\\[2ex] \dfrac{1}{1+(X_2X_1)^{1-\alpha}}\quad \text{if }\ \gamma\eta_{2,\vec k}<\omega<\gamma\eta_{1,\vec k}, \end{cases}$$

$$A_{12}(\vec k,\omega)=A_{21}(\vec k,\omega)=\frac{\sin(\pi\alpha)(X_1X_2)^{1-\alpha}}{\pi t_\perp}\begin{cases}\dfrac{1}{1+(X_2X_1)^{2(1-\alpha)}-2\cos(\pi\alpha)(X_2X_1)^{1-\alpha}} & \text{if }\ \omega>\gamma\eta_1\\[2ex] \dfrac{-1}{1+(X_2X_1)^{2(1-\alpha)}-2\cos(\pi\alpha)(X_2X_1)^{1-\alpha}} & \text{if }\ \omega<\gamma\eta_2\\[2ex] 0\quad \text{if }\ \gamma\eta_1<\omega<\gamma\eta_2. \end{cases}$$

In Fig. 4 we show the behavior of the diagonal and off-diagonal one-particle spectral functions. From Fig. 4(a) we observe that the symmetry of the diagonal spectral function, $A_{11}(\vec{k},\omega)$, is lost around the frequency $\omega=\gamma\eta_{2,\vec{k}}=0.9$. To realize this feature, we refer the reader to the respective figure $[F_{11}(x)]$ in Ref. 13. Also, the symmetry of $F_{22}(x)$ in Ref. 13 is lost around $\omega=\gamma\eta_{1,\vec{k}}=1.2$. In consequence, ARPES experiments shedding light on layer 1 or layer 2 should detect these fine details. Namely, we will have two symmetry breakings, $1\rightarrow 1$ and $2\rightarrow 2$. Now, from Fig. 4(b) we see that there is an interval of energy where $A_{12}(\vec{k},\omega)=0$. This feature was not obtained for the off-diagonal one-particle spectral function for equivalent planes [see $F_{12}(x)$ in Ref. 13]. Another aspect which we would like to point out is the fact that in the one-particle spectral functions there is not a unique variable to describe the *data*. We have two relevant energy scales in the problem, for each value of $\vec{k}$. For the type of materials we are studying, it seems natural to find, on theoretical grounds, a gap in the energy spectrum. ARPES experiments should be designed to detect the gap found from our expressions.[31] We mention that the gap found in $A_{12}(\vec{k},\omega)$ depends on the relative carrier number in the two inequivalent coupled Hubbard layers. Therefore, the presence of a shift between the two effective chemical potentials produces new theoretical results for the spectral densities. In particular, non-Fermi-liquid quasiparticles acquire a more complex structure [Eq. (A4)] than the case of equivalent planes.

In order to calculate the carrier number per site per spin per plane, at $T=0$, we have to perform the following integrals:

$$\rho_{1(2)}=\frac{1}{4}\int_{-D}^{+D}d\varepsilon\int_{-\omega_c}^{0}N(\varepsilon)A_{11(22)}(\varepsilon,\omega)d\omega, \qquad (A5)$$

which have to be computed numerically. Work is in progress to solve Eq. (A5) self-consistently. $N(\varepsilon)$ is the uncorrelated density of states. To conclude this appendix, we say that our results generalize the case of Yin and Chakravarty.[13] With respect to the results of Ref. 28, they have not discussed the two one-particle spectral functions, $A_{11}(\vec{k},\omega)$ and $A_{12}(\vec{k},\omega)$, around the effective chemical potential. Most likely the results of Ref. 28 are valid for large values of frequencies.

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[1] G. Bednorz and K.A. Muller, Z. Phys. **64**, 189 (1986).

[2] C.M. Varma, P.B. Littlewood, S. Schmitt, E. Abrahams, and A.E. Ruckestein, Phys. Rev. Lett. **63**, 1996 (1989).

[3] P.W. Anderson, Science **256**, 1526 (1992); *The Theory of Superconductivity in High-T_c Cuprates* (Princeton University Press, Princeton, 1997).

[4] R. Hlubina, Phys. Rev. B **60**, 3068 (1999).

[5] J. Merino and R.H. McKenzie, Phys. Rev. B **61**, 7996 (2000).

[6] H. Kontani, K. Kanki, and K. Ueda, Phys. Rev. B **59**, 14 723 (1999).

[7] J.D. Mahan, *Many-Particle Physics*, 2nd ed. (Plenum, New York, 1990), Chap. 9.

[8] X.G. Wen, Phys. Rev. B **42**, 6623 (1990).

[9] V. Meden and K. Schonhammer, Phys. Rev. B **46**, 15 753 (1992).

[10] J. Voit, Phys. Rev. B **47**, 6740 (1993).

[11] M. Fabrizio and A. Parola, Phys. Rev. Lett. **70**, 226 (1993).

[12] A.V. Balatsky, Philos. Mag. Lett. **68**, 251 (1993).

[13] L. Yin and S. Chakravarty, Int. J. Mod. Phys. B **10**, 805 (1996).

[14] V. Galitskii and A. Migdal, Zh. Éksp. Teor. Fiz. **34**, 139 (1958) [Sov. Phys. JETP **7**, 96 (1958)].

[15] S. Chakravarty and P.W. Anderson, Phys. Rev. Lett. **72**, 3859 (1994).

[16] S. Chakravarty, H.Y. Kee, and E. Abrahams, Phys. Rev. Lett. **82**, 2366 (1999).

[17] D. Basov *et al.*, Science **283**, 49 (1999).

[18] W. Nolting, Z. Phys. **255**, 25 (1972); *Grundkurs: Theoretische Physik. 7 Viel-Teilchen-Theorie* (Verlag Zimmermann-Neufang, Ulmen, 1992); H. Herrmann and W. Nolting, J. Magn. Magn. Mater. **170**, 253 (1997); M. Pottoff, T. Herrmann, and W. Nolting, Eur. Phys. J. B **4**, 485 (1998).

[19] G. Baskaran, cond-mat/9906183 (unpublished).

[20] J. Hubbard, Proc. R. Soc. London, Ser. A **276**, 238 (1963); **281**, 401 (1964).

[21] I. Ţifrea, Ph.D. thesis, University of Cluj, 1998.

[22] M. Crişan, C.P. Moca, and I. Ţifrea, cond-mat/9809152 (unpublished); Phys. Rev. B **59**, 14 680 (1999); I. Ţifrea, I. Grosu, and M. Crişan, Physica B **259-261**, 464 (1999); V.N. Muthukumar, D. Sa, and M. Sardar, Phys. Rev. B **52**, 9647 (1995); I. Grosu, I. Ţifrea, and M. Crişan, J. Supercond. **11**, 339 (1998).

[23] A. Sudbo, Phys. Rev. Lett. **74**, 2575 (1995).

[24] I.S. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. (Academic Press, San Diego, 1994), p. 401.

[25] N.N. Lebedev, *Special Functions and their Applications* (Dover, New York, 1972).

[26] R. Kishore, in *Studies of High Temperature Superconductors*, edited by A.V. Narlikar (Nova Science Publ., Commack, NY, 1999), Vol. 29, pp. 23–58.

[27] H.M. Fretwell *et al.*, Phys. Rev. Lett. **64**, 2308 (1990); H. Ding *et al.*, *ibid.* **76**, 1533 (1996); J. Mesot *et al.*, cond-mat/9910430 (unpublished); M.R. Norman *et al.*, Nature (London) **392**, 157 (1998); M.R. Norman, M. Randeria, H. Ding, and J.C. Campuzano, Phys. Rev. B **57**, R11 093 (1998); M.R. Norman *et al.*, *ibid.* **60**, 7585 (1999); M.R. Norman, M. Randeria, H. Ding, and J.C. Campuzano, *ibid.* **59**, 11 191 (1999).

[28] G. Hildebrand, E. Arrigoni, J. Schmalian, and W. Hanke, cond-mat/9807132 (unpublished).

[29] T. Domański and K.I. Wysokiński, Phys. Rev. B **59**, 173 (1999).

[30] A.A. Schmidt and J.J. Rodríguez-Núñez, cond-mat/9907448 (unpublished); J.J. Rodríguez-Núñez and A.A. Schmidt (unpublished).

[31] Z.X. Shen *et al.*, Phys. Rev. Lett. **70**, 1553 (1993); Science **267**, 343 (1993); Phys. Rev. Lett. **76**, 4841 (1996).

[32] V. Meden, Phys. Rev. B **60**, 4571 (1999).

[33] E.V.L. de Mello and C. Acha, Phys. Rev. B **56**, 466 (1997); Physica C **282-287**, 1819 (1997).

[34] M.R. Norman, M. Randeria, B. Jankó, and J.C. Campuzano, Phys, Rev. B **61**, 14 742 (2000).