Orbital magnetic susceptibility of a quasi-one-dimensional electron gas system in tilted magnetic fields

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We study the magnetic susceptibility of a quasi-one-dimensional electronic system in tilted magnetic fields. Physical features of the susceptibility at finite temperature, according to the relative strength of the electrostatic potential parameters, are investigated for a specific strength of applied magnetic field. Our results show that the magnetic susceptibility of quantum wire for the specific strength of applied magnetic field is changing from paramagnetic to diamagnetic, depending on the relative strength of the confining potential parameters. In addition, the dependence of magnetic susceptibility on the tilt angle and the strength of applied magnetic field, the temperature, the electron concentration, and the anisotropy of the effective mass of electrons is explicitly shown.

I. INTRODUCTION

Since Landau indicated¹ that the orbital motion of degenerate free electrons in the magnetic field gives rise to diamagnetic susceptibility, many investigations have been made in the corrections to the Landau susceptibility due to finitesize effect.^{2,3} All attempts reached the similar conclusion that the boundaries, regardless of their actual shapes, have small effects in finite systems with a large number of electrons. On the other hand, with recent advances in nanostructure semiconductor technology, interest has grown in the orbital magnetism of low-dimensional electron systems whose characteristic dimensions are comparable to or less than the cyclotron radius, especially for the case where a magnetic field is applied to the quasi-two-dimensional (Q2D) electronic plane at an arbitrary angle. As is well known, in this case, combined effects of electric and magnetic confinements can not be separated in general and lead to the hybridmagnetoelectric quantization of electron energies.^{4,5}

Theoretical work related to this geometry was first done by Marx and Kümmel.⁶ They calculated the magnetization of a rectangular well and found as a function of chemical potential that the in-plane magnetization oscillates with sharp jumps. This finding opened up the possibility as a dissipation-free switching device. Similar results were reported by Lee *et al.*⁷ for a triangular well of Si-MOSFET (metaloxide semiconductor field-effect transistor). Ihm *et al.*⁸ extended these studies to the parabolic quantum well and wire and found through an analytic expression that the magnetization exhibits the superposition of smooth, large-period oscillations and abrupt, small-period oscillations characterized by two hybrid eigenfunctions. Maksym and Chakraborty⁹ investigated the effect of electron-electron interactions on the magnetization of quantum dots and suggested that the study of magnetization provides a sensitive probe of interaction effects. These results are valuable since, in many cases, the far-infrared spectroscopy can only probe the center of mass motion of all electrons but is inadequate for finding any effects due to the electron-electron interaction.^{10,11}

Almost no experimental studies on this subject exist. The most up-to-date magnetization measurements comparable to our subject, to the best of our knowledge, were done by Eisenstein *et al.*¹² in 1985. They observed the oscillatory magnetization of two dimensional electron systems in high-mobility GaAs/AlGaAs single-layer and multilayer hetero-structures. However, the geometry used in their experiments, i.e., with the magnetic field perpendicular to the interface, did not allow observation of the hybrid quantization effects. The lack of experimental data on the magnetization reflects difficulties in the measurements, which are associated with the requirements of high-quality samples, sufficient sensitivity, and sufficient signal.

Our motivation is that the detailed study of the magnetic susceptibility of quasi-one-dimensional (Q1D) electronic gas system in tilted magnetic fields has not been made, as functions of various parameters characterizing the system, such as the tilt angle and the strength of applied magnetic field, electrostatic confining parameters, the temperature, and the electron concentration at finite temperature, within the framework of the canonical ensemble statistics. For this pur-

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pose, we consider a simple model of the quasi-one-dimensional electron gas systems, where the confinements induced by parabolic potentials will be adopted for simplicity. The advantage of such kind of treatment is that the eigenvalue problem can be solved exactly in the presence of a tilted magnetic field. In addition, we consider an independent electron model and the case where the boundary roughness and residual disorder of the system are neglected. Within the simplified model, we will obtain the magnetic susceptibility based on the canonical ensemble statistics and investigate the physical properties of orbital magnetism for various ratios of longitudinal mass to transverse mass, as a function of the tilt angle and the strength of applied magnetic field, electrostatic confining parameters, the temperature, and the electron concentration at finite temperature.

The rest of the paper is organized as follows: In Sec. II, we present a simple parabolic model of quasi-one-dimensional electron gas systems in tilted magnetic fields and obtain the magnetic susceptibilities for such a system at finite temperature. In Sec. III, we present the numerical results of magnetic susceptibilities for various ratios of longitudinal mass to transverse mass, as a function of the tilt angle and the strength of applied magnetic field, electrostatic confining parameters, the temperature, and the electron concentration at finite temperature. Conclusions will be given in the last section.

II. MODEL OF THE SYSTEM

We consider a quantum wire, such as AlGaAs/GaAs heterostructures with a split gate, in which the quasi-twodimensional (Q2D) electron gas in the heterointerface is assumed to be confined in the z direction parallel to the principal axis of an ellipsoidal energy surface by an ideal parabolic potential $\frac{1}{2}m_1\omega_z^2 z^2$, whereas the quasi-one-dimensional electron gas is assumed to be further confined in the x direction by an additional parabolic potential $\frac{1}{2}m_t\omega_x^2 x^2$ in terms of the split gate. In the presence of a magnetic field, one-particle Hamiltonian (h_e) for such Q1D electrons is expressed in a unified manner by

$$h_{e} = \frac{1}{2} [\mathbf{p} + e\mathbf{A}] \begin{pmatrix} 1/m_{t} & 0 & 0\\ 0 & 1/m_{t} & 0\\ 0 & 0 & 1/m_{l} \end{pmatrix} [\mathbf{p} + e\mathbf{A}] + \frac{1}{2} m_{t} \omega_{x}^{2} x^{2} + \frac{1}{2} m_{l} \omega_{z}^{2} z^{2}, \qquad (1)$$

where **A** is the vector potential accounting for a constant magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$, **p** is the momentum operator, and m_t and m_l represent the transverse and longitudinal mass components of the ellipsoidal energy surface of the conduction band, respectively. We shall consider the case where the magnetic field **B** is applied in the transverse tilt direction to the wire of the system: $\mathbf{B} = B(\sin \theta, 0, \cos \theta) = (B_x, 0, B_z)$, with the Landau gauge $A = (0, xB_z - zB_x, 0)$. Here, the angle θ is measured from the *z* axis in the x - z plane. Then, the oneparticle Hamiltonian (1) for those confined (Q1D) electrons subject to the transverse tilted magnetic field can be written in the new Cartesian coordinates (x', y', z') as

$$h_{e} = \frac{P_{x'}^{2}}{2m_{t}} + \frac{P_{z'}^{2}}{2m_{l}} + \frac{1}{2}m_{t}\Omega_{1}^{2}x'^{2} + \frac{1}{2}m_{l}\Omega_{2}^{2}z'^{2} - m_{t}\omega_{c}^{2}\sin\theta\cos\theta x'z' + \frac{P_{y'}^{2}}{2\tilde{m}^{*}},$$
(2)

which represents two coupled harmonic oscillators, where $\omega_c = eB/m_t$ is the cyclotron frequency, $\alpha = \omega_x/\omega_z$ is a measure of the anisotropy of the parabolic potentials, $\Omega_1^2 = \omega_c^2(\cos^2\theta + \alpha^2\gamma^2)$, $\Omega_2^2 = \omega_c^2(\sin^2\theta/M_{lt} + \gamma^2)$, and $\tilde{m}^* = m_t(1 + \sin^2\theta/(M_{lt}\gamma^2) + \cos^2\theta/\alpha^2\gamma^2)$. Here $\gamma = \omega_z/\omega_c$ and $M_{lt} = m_l/m_t$ is the anisotropic factor of the effective mass of electrons. The corresponding eigenfunction of Eq. (2) has the form of $\exp[ik_yy']\varphi(x-x_0,z-z_0)$ with $x_0 = -\hbar\omega_c\cos\theta k_y/(\tilde{m}^*\alpha\omega_1\omega_2)$ and $z_0 = \hbar\omega_c\sin\theta k_y/(\tilde{m}^*M_{lt}\omega_2^2)$. The Hamiltonian of Eq. (2) can be diagonalized by a proper rotation of coordinate with respect to the y'-axis and the resulting eigenenergy spectrum is given by

$$E_{n,l}(k_y) = (n+1/2)\hbar\omega_+ + (l+1/2)\hbar\omega_- + \hbar^2 k_y^2/(2\tilde{m}^*)$$
$$\equiv E_{n,l} + \hbar^2 k_y^2/(2\tilde{m}^*), \qquad (3)$$

where the frequencies ω_{\pm} indicate the hybrid effect between magnetic and electric confinements, i.e., the hybridmagnetoelectric effect, which are respectively given by $\omega_{\pm}^{2} = [\Omega_{1}^{2} + M_{lt}\Omega_{2}^{2} \pm \sqrt{(\Omega_{1}^{2} - M_{lt}\Omega_{2}^{2})^{2} + \omega_{c}^{4}\sin^{2}2\theta}]/2.$ The quantum numbers *n* and *l* denote Landau level indices and k_y is the quasicontinuous wave vector in the y direction, where the maximum value of k_{y} is determined by the requirement that the center of the cyclotron orbit $(x_0 \text{ and } z_0)$ in Eq. (2) falls within the specimen having a rectangular parallelepiped of sides L_x , L_y , and L_z , i.e., $-\sqrt{L_x^2 + L_z^2}/2 < \sqrt{x_0^2 + z_0^2}$ $<\sqrt{L_x^2+L_z^2}/2$. Note that we can see the dimensional crossover between Q2D and Q1D systems if ω_x or ω_z in Eq. (3) is taken as 0 and Eq. (3) is reduced to Ihm *et al.*'s result⁸ if $M_{lt} = 1$, where they considered the isotropic effective mass of electrons. In addition, if a triangular well in the z direction is taken in Eq. (1), instead of the ideal parabolic well, the model of the system is more realistic. However, we believe that the parabolic well enables us to understand various interesting physical properties of the system, such as magnetization or magnetic susceptibility, because the electrostatic confining potential parameter ω_{τ} are closely related to the bias field given in the triangular well.¹³

For the case where a magnetic field is applied to the quasi-two-dimensional (Q2D) electronic plane at an arbitrary angle, most theoretical works^{6–9} proposed so far deal with the grand canonical ensemble statistics having the chemical potential fixed to explain the thermodynamic properties, such as magnetic susceptibility, in low-dimensional electronic systems, because the chemical potential is adjusted to have an average number of electrons equal to *N*. In typical experiments, however, thermodynamic properties should correspond to the canonical ensemble statistics since the number of electrons in each specimen is fixed due to the charge neutrality. To compare with actual experiment, therefore, it is desirable to describe the thermodynamic properties in the canonical ensemble statistics. For this purpose, the number

of electrons of the system is calculated introducing the Fermi-Dirac distribution function f(E), to give

$$N = \int_{-\infty}^{\infty} f(E)g(E)dE = \zeta \sum_{n,l} \frac{1}{1 + \exp[\beta(E_{n,l} - \mu)]}, \quad (4)$$

where $\beta = 1/k_B T$ with k_B being the Boltzmann constant and *T* temperature, μ is the chemical potential, and g(E) is the density of state given by⁶

$$g(E) = 2\sum_{n,l,k_y} \delta(E - E_{n,l}) = \zeta \sum_{n,l} \delta(E - E_{n,l})$$
(5)

with the degeneracy factor

$$\zeta = \tilde{m}^* \omega_2 L_y \sqrt{L_x^2 + L_z^2} / (\pi \hbar \omega_c \sqrt{\cos^2 \theta / \omega_1^2 \alpha^2 + \sin^2 \theta / \omega_2^2 M_{lt}^2}).$$

Here, the twofold spin degeneracy of the electrons has been included explicitly and the fact that $E_{n,l}$ is degenerate in k_y has been taken into account. The condition of constant electron concentration per length, $(n_e = N/L_y)$, of Eq. (4) leads to a basic equation for the chemical potential. The chemical potential is generally determined by Eqs. (3) and (4) and it depends on the temperature, the electron concentration, the strength of the confining potential parameters ω_x and ω_z in the *x* and *z* directions, the strength and/or the tilt angle of the applied magnetic field, and the anisotropic factor of the effective mass of electrons.

The magnetic susceptibility at finite temperature T can be calculated from the Helmholtz free energy,

$$F = N\mu - k_B T \sum_{n,l,k_y} \ln\{1 + \exp[(\mu - E_{n,l})/k_B T]\}, \quad (6)$$

where the chemical potential for a quantum wire in tilted magnetic fields can be generally given by Eq. (4). Differentiating the free energy twice with respect to the magnetic field, we can obtain the susceptibility at finite temperature T given by

$$\chi = -\left(\frac{\partial^2 F}{\partial B^2}\right)_{T.N}.$$
(7)

It is noted that the susceptibility depends on the temperature, the electron concentration, the strength of the confining potential parameters ω_x and ω_z in the *x* and *z* directions, the strength and/or the tilt angle of the applied magnetic field, and the anisotropic factor of the effective mass of electrons.

III. NUMERICAL RESULTS

In this section we present the numerical results of the magnetic susceptibility χ depending on the tilt angle and the strength of the applied magnetic field, the values of α and γ , the temperature, and the electron concentration, using Eq. (7), where we consider the anisotropic factors of the effective mass of electrons to be $M_{lt}=0.8$, 1.0, and 1.2, respectively, in order to see their characteristics depending on the relative effective mass. For numerical calculation, some parameters have been taken as follows: $L_x = L_z = 100$ A, $L_y = 5,000$ A.

The angular dependence of the magnetic susceptibility for



FIG. 1. The angular dependence of the negative magnetic susceptibility for various values of $\alpha = \omega_x/\omega_z$ and several anisotropic factors of the effective mass of electrons at $\gamma = \omega_z/\omega_c = 1$, T = 100 K, B = 10 T, and $n_e = 10^8$ /m.

a specific strength of applied magnetic field, B = 10 T, is illustrated by Fig. 1, where we plot the susceptibility in Eq. (7) as a function of the tilt angle of applied magnetic field at $\alpha = \omega_x / \omega_z = 0.4, 1, 1.6, \ \gamma = \omega_z / \omega_c = 1, \text{ and } T = 100 \text{ K.}$ Here the electron concentration was taken as $n_e = 1 \times 10^8$ /m. It is clearly seen from this figure that the system has two different magnetic susceptibilities for a specific strength of applied magnetic field and a specific relative strength of the confining potential parameters, ω_x and ω_z , according to the tilt angle of applied magnetic field. For $\alpha \ge 1$, i.e., $\omega_r \ge \omega_z$, the system becomes paramagnetic, independent of the tilt angle, and the paramagnetism decreases slightly with increasing the tilt angle, whereas for $\alpha < 1$, i.e., $\omega_x < \omega_z$, the magnetic susceptibility of the system is changing from diamagnetic to paramagnetic, depending on the tilt angle. Moreover, there is a tendency to larger paramagnetism for larger α . It is understood that the occurrence of paramagnetism arises from incomplete circular motion due to the changes of the value of α and the tilt angle of applied magnetic field. This appearance of paramagnetism is also observed^{14,15} in small metal particles whose linear dimension is comparable to or less than the cyclotron radius. For given α , the critical angles whose magnetism disappears during transition from diamagnetism to paramagnetism are slightly influenced by the anisotropic factor of the effective mass of electrons. Thus, the susceptibilities are very sensitive to the confining potential parameters ω_x and ω_z , the tilt angle of applied magnetic field, and the anisotropic factor of the effective mass of electrons. The interesting thing is that by changing the tilt angle for a specific relative strength of the confining potential parameters ω_x and ω_z , and a specific strength of applied magnetic field, the system can be paramagnetic or diamagnetic.

Figure 2 shows the dependence of the magnetic susceptibility on the value of α for several tilt angles and a specific strength of applied magnetic field at $\gamma = 1$, $n_e = 1 \times 10^8$ /m, and T = 100 K. From the figure, one can find the difference between the magnetic susceptibility of the quasi-onedimensional electron gas system and that of the quasi-twodimensional electron gas system for a specific strength of applied magnetic field. For $\alpha = 0$ corresponding to the quantum-well case with no split gate¹⁶ in the *x* direction, the system becomes diamagnetic, except for $\theta = 87^{\circ}$. This means that the magnetic susceptibility of the two-dimensional elec-



FIG. 2. The dependence of the negative susceptibility on the value of $\alpha = \omega_x/\omega_z$ for various tilt angles and several anisotropic factors of the effective mass of electrons at $\gamma = \omega_z/\omega_c = 1$, T = 100 K, B = 10 T, and $n_e = 10^8$ /m.

tron gas system for a specific strength of applied magnetic field can change from diamagnetic to paramagnetic, depending on the tilt angle of applied magnetic field. Same phenomena take place in the quasi-one-dimensional electron gas system, as can be seen from Fig. 1 and for small α in Fig. 2. However, when the value of α for a specific angle and a specific strength of applied magnetic field increases, i.e., the electrostatic confinement in the x direction becomes stronger, the quantum wires are changing from diamagnetic to paramagnetic, which is unlike to the quantum-well case for α =0. The critical value of α in which the transition of magnetic susceptibility takes place is closely related to the tilt angle. It is shown in the figure that the critical value of α has maximum value at zero angle. In addition, as the value of α increases, i.e., the electrostatic confinement in the x direction becomes strong, we can see that the magnetic susceptibility approaches zero, except for $\theta = 87^{\circ}$. This is valid for $\gamma = 1$.

The γ dependence of the magnetic susceptibility for a specific strength of applied magnetic field at $n_e = 1 \times 10^8$ /m, and T = 100 K is shown in Fig. 3, where we consider the $\omega_x = \omega_c$ case for various tilt angles. From this figure, one can see the difference between the susceptibility of the quasi-two-dimensional electron gas system and that of the quasi-one-dimensional electron gas system. For $\gamma = 0$



FIG. 3. The dependence of the negative susceptibility on the value of $\gamma = \omega_z / \omega_c$ for various tilt angles and several anisotropic factors of the effective mass of electrons at $\omega_x = \omega_c$, T = 100 K, B = 10 T, and $n_e = 10^8$ /m.



FIG. 4. The magnetic-field dependence of the negative susceptibility for various tilt angles, various values of $\alpha = \omega_x/\omega_z$, and several anisotropic factors of the effective mass of electrons at $\gamma = \omega_z/\omega_c = 1$, T = 100 K, and $n_e = 10^8$ /m.

corresponding to the quantum-well case with a split gate³ in the x direction, the system shows diamagnetic for a specific strength of applied magnetic field and a specific tilt angle, except for $\theta = 0^{\circ}$. However, the quantum wires for a specific strength of applied magnetic field and a specific tilt angle are changing from diamagnetic to paramagnetic as the value of γ increases, i.e., the electrostatic confinement in the z direction becomes strong. The critical value of γ in which the transition of magnetic susceptibility takes place is closely related to the tilt angle. It is seen from the figure that the critical value increases with increasing the tilt angle and it is slightly influenced by the anisotropic factor of the effective mass of electrons. Moreover, when the value of γ is large, i.e., the electrostatic confinement in the z direction becomes strong, the magnetic susceptibility for $\theta = 87^{\circ}$ approaches zero. However, it seems that the reason why all magnetic susceptibilities in Figs. 2 and 3 do not approach zero for strong confinements is due to the fact that our numerical results of magnetic susceptibility are restricted to the case where the confining potential parameters are comparable to the magnetic confinements is small or large. In the truly quasi-onedimensional limit, the orbital effects of the magnetic field should vanish. The validity of our model for the quasi-onedimensional electron gas system can be checked from Eqs. (3)–(7). If the confining potential parameters, ω_x and ω_z , in Eq. (3) are very larger than the cyclotron frequency, the magnetic-field dependent term can be negligible and hence the eigenvalues in Eq. (3) and the Helmholtz free energy in Eq. (6) are independent of magnetic field. As a result, the magnetic susceptibility (or the magnetization given in the first derivative of the Helmholtz free energy) becomes zero. The remarkable thing here is that the quantum wires are changing from diamagnetic to paramagnetic for a specific strength of applied magnetic field, according to the changes of the electrostatic potential parameters. This is unlike to the quantum-well case for $\alpha = 0$ or $\gamma = 0$.

Figure 4 shows the dependence of the magnetic susceptibility on the strength of applied magnetic field for various tilt angles and several relative strengths of the confining potential parameters at $\gamma = 1$, $n_e = 1 \times 10^8$ /m, and T = 100 K. The dependence of Figs. 1, 2, and 3 on the strength of applied magnetic field can be understood from Fig. 4. When the applied magnetic field is increased, paramagnetism or dia-



FIG. 5. The dependence of the negative susceptibility on the temperature for various tilt angles, various values of $\alpha = \omega_x/\omega_z$, and several anisotropic factors of the effective mass of electrons at $\gamma = \omega_z/\omega_c = 1$, B = 10 T, and $n_e = 10^8$ /m.

magnetism shown in Fig.2 decreases, in the case of two different tilt angles $\theta = 0^{\circ}$ and 45° . For $\theta = 90^{\circ}$, the system shows diamagnetic with the increase of the magnetic field. The temperature dependence of the magnetic susceptibility for various tilt angles and several relative strengths of the confining potential parameters at $\gamma = 1$, $n_e = 1 \times 10^8$ /m, and B = 10 T is presented in Fig. 5. From this figure, we can understand the temperature dependence of Figs. 1, 2, and 3. The increase of the temperature leads to the increase of paramagnetism or diamagnetism shown in Fig. 2 for a specific strength of applied magnetic field. Figure 6 shows the dependence of the magnetic susceptibility on the electron concentration for various tilt angles and several relative strengths of the confining potential parameters at $\gamma = 1$, $n_e = 1 \times 10^8$ /m, and B = 10 T. We can see from the figure that paramagnetism or diamagnetism shown in Fig. 2 increases with the increase of the electron concentration.

IV. CONCLUSIONS

So far, we have investigated the magnetic susceptibility of quasi-one-dimensional electronic system whose confining potential energies in two directions are comparable to $\hbar \omega_c$ in strong tilted magnetic fields, as a function of the tilt angle and the strength of the applied magnetic field, the temperature, the confining potential parameters ω_x and ω_z , and the electron concentration for various anisotropic factors of the effective mass of electrons. Our results show that the magnetic susceptibility of the quantum wire for a specific strength of applied magnetic field is quite different from that of the quantum well. As shown in Figs. 2 and 3, the quantum well for a specific strength of applied magnetic field shows diamagnetic or paramagnetic, depending on the tilt angle, whereas the quantum wire is changing from diamagnetic to paramagnetic for given tilt angles, depending on the relative strength of the confining potential parameters, ω_x and ω_z . Their critical values in which the transition from diamagne-



FIG. 6. The dependence of the negative susceptibility on the electron concentration for various tilt angles, various values of $\alpha = \omega_x / \omega_z$, and several anisotropic factors of the effective mass of electrons at $\gamma = \omega_z / \omega_c = 1$ and B = 10 T.

tism to paramagnetism takes place are closely related to the tilt angle and the anisotropic factor of the effective mass of electrons. It is interesting to note that the parabolic well modeled in this study can be replaced by the triangular well¹⁶ in an actual system such as AlGaAs/GaAs heterostructures with a split gate.¹⁷ In that case, the confining potential parameter ω_z corresponds to the bias field which is controlled by the gate voltage.¹³ This means that the quantum wire for a specific strength of applied magnetic field can be diamagnetic or paramagnetic, by changing the gate voltage in real system. From a technological point of view, the present system can be a candidate for control devices using the magnetic susceptibility as a function of the relative strength of confining potential parameters or the gate voltage, in addition to the application of the known dissipation freeswitching device⁶ utilizing the sharp jumps of the magnetization as a function of the chemical potential in tilted fields. In addition to the confining potential parameters discussed above, the factors such as the temperature, the tilt angle and the strength of applied magnetic field, the anisotropic factor of the effective mass of electrons, and the electron concentration also play an important role in the susceptibility.

Throughout this work, the single-particle picture has been used. Therefore, it is worth pointing out that if one wishes to compare quantitatively theoretical results with experimental measurements, further theoretical calculations including disorder and electron-electron interactions are needed. This is the subject of ongoing research, which will be presented in a separate paper later.

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