Random field and random anisotropy effects in defect-free three-dimensional *XY* models

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Monte Carlo simulations have been used to study a vortex-free *XY* ferromagnet with a random field or a random anisotropy on simple cubic lattices. In the random field case, which can be related to a charge-density wave pinned by random point defects, it is found that long-range order is destroyed even for weak randomness. In the random anisotropy case, which can be related to a randomly pinned spin-density wave, the long-range order is not destroyed and the correlation length is finite. In both cases there are many local minima of the free energy separated by high entropy barriers. Our results for the random field case are consistent with the existence of a Bragg glass phase of the type discussed by Emig, Bogner, and Nattermann.

I. INTRODUCTION

The effects of random pinning on systems of charge-density waves (CDW) or spin-density waves (SDW) and related problems, like the pinning of the Abrikosov vortex lattice, have been studied for a long time.[1] In real laboratory samples, there are always defects which create such pinning forces. Nevertheless, fundamental issues remain controversial. In 1970, Larkin[2] presented an argument which shows that, if the unpinned system is translation-invariant (which means that we are ignoring any effects of a periodic crystal-lattice potential), then weak random pinning forces will destroy the long-range order (LRO) of a CDW in four or fewer spatial dimensions. A simpler domain-wall energy argument was later presented by Imry and Ma,[3] and under some conditions it can be made mathematically rigorous.[4] One unresolved issue is whether these arguments can be extended to the SDW case, where there are experiments which indicate the stability of LRO in the presence of pinning.[5] Another controversy involves the existence of the proposed ''Bragg glass'' phase,[6] which has quasi-long-range order (QLRO).

If we ignore amplitude fluctuations,[7,8] we can transform the pinned CDW problem into an *XY* model in a random field, whose Hamiltonian is usually taken to have the form

$$H_{RFXY} = -J\sum_{\langle ij\rangle} \cos(\theta_i - \theta_j) - G\sum_i \cos(\theta_i - \phi_i). \quad (1)$$

Site i is at position $\mathbf{r}_i$, the sites form a lattice, and $\langle ij\rangle$ indicates a sum over nearest neighbors. Each θ_i is a dynamical variable representing the phase of the CDW at site i, and can take on values in the interval $[-\pi,\pi)$. Each ϕ_i is a representation of the random pinning energy arising from lattice defects. Since the defect sites are assumed to be immobile, the ϕ_i do not change with time. We also assume that the ϕ_i on different sites are uncorrelated, and that the probability distribution for each ϕ_i is uniform on $[-\pi,\pi)$.

We can generalize Eq. (1) to study *XY* models in random p-fold fields, where p is any positive integer:

$$H_{rp} = -J\sum_{\langle ij\rangle} \cos(\theta_i - \theta_j) - G\sum_i \cos[p(\theta_i - \phi_i)]. \quad (2)$$

For Eq. (2), each ϕ_i can be chosen to be in the interval $[-\pi/p,\pi/p)$, but θ_i still takes on values in $[-\pi,\pi)$. The $p=2$ case, which is often simply referred to as the *XY* model with random anisotropy, is related[9] to a linearly polarized SDW pinned by local moment impurities in the same way that the $p=1$ case is related to a pinned CDW. Note, in particular, that for $p=2$ the Hamiltonian preserves a twofold inversion symmetry, in contrast to the $p=1$ case. One consequence of this is that for $p=2$ (or more), unlike $p=1$, the time average of the local magnetization, $\langle \mathbf{M}(\mathbf{r}_i,t)\rangle_t = \langle\{\cos[\theta_i(t)],\sin[\theta_i(t)]\}\rangle_t$, must be zero for every i in the paramagnetic phase.

Another model which is often considered is the elastic glass,[10,6]

$$H_{eg} = -J\sum_{\langle ij\rangle} (\theta_i - \theta_j)^2 - G\sum_i \cos[p(\theta_i - \phi_i)]. \quad (3)$$

For the elastic glass, the ϕ_i again have values in the interval $[-\pi/p,\pi/p)$, but the θ_i are now defined on $(-\infty,\infty)$. The dependence of Eq. (3) on p is trivial, since it can be removed by a rescaling of the variables.[10] Therefore, after making the appropriate scaling, the behavior of the elastic glass must be the same for all p. Giamarchi and Le Doussal[6] have performed an analytical calculation which shows that in three dimensions at zero temperature this model has a structure factor,[11]

$$S_\theta(\mathbf{k}) = \frac{1}{L^3}\left|\sum_j^{L^3} \theta_j \exp(i\mathbf{k}\cdot\mathbf{r}_j)\right|^2, \quad (4)$$

which diverges like $1/|\mathbf{k}|^3$ at small $|\mathbf{k}|$. This result has recently been confirmed by a numerical simulation.[12]

It is argued[10,13] that in the absence of topological defects (i.e., vortex lines), Eq. (1) should have the same continuum limit as Eq. (3), and that this should be true for Eq. (2) as well. The implication is that the behavior of Eq. (2) should be essentially the same for all p, just as is the case for Eq. (3). However, the phase space for Eq. (3) is simply connected, while that of Eq. (2) is not, even in the absence of defects. Since it is known that the numerical simulation results for the $p=3$ case[14] of Eq. (2) do not show the same behavior as the numerical simulation[12] of Eq. (3), it appears

that this difference in the topology of the phase space for the two models invalidates the mapping. One can map the energies from Eq. (2) into Eq. (3), but the entropies are different, even in the absence of vortex lines.

The difficulty is most transparent in the large G/J limit considered by Fisher.[13] In this limit, for the elastic glass, Eq. (3), each θ_i can still assume a countably infinite number of values, for any p. However, for the random-field model, Eq. (2), each θ_i has only p distinct allowed values in this limit. The $1/|\mathbf{k}|^3$ divergence of $S_\theta(\mathbf{k})$ for the elastic glass arises from the unbounded variations of the θ_i. This cannot occur in the random-field model, where θ_i is defined on a compact manifold.

II. RANDOM *P*-FOLD FIELDS

In the remainder of this work we will consider a version of the model of Eq. (2), in the $p=1$ and $p=2$ cases. In a ferromagnetic phase, where time-reversal symmetry is spontaneously broken, the configuration averaged value of $\mathbf{M}(\mathbf{r},t)$ in a single sample, $\langle \mathbf{M}(\mathbf{r},t)\rangle_{\mathbf{r}}$, is not zero. Thus, naively,[15,16] one would expect that if LRO is destroyed by a weak random $p=1$ field, then it would also be destroyed by a weak random p-fold field for any p. This argument can be made explicit within a perturbation theory[15] for small G/J which is exact to leading order in $1/N$, where N is the number of spin components.

The cause of the Larkin-Imry-Ma instability in the random-field model may be seen by studying the magnetic structure factor, whose form in three dimensions is

$$S(\mathbf{k},t)=\frac{1}{L^3}\left|\sum_j^{L^3}\mathbf{M}(\mathbf{r}_j,t)\exp(i\mathbf{k}\cdot\mathbf{r}_j)\right|^2. \tag{5}$$

In equilibrium, $S(\mathbf{k},t)$ becomes independent of the time t when L becomes infinite. If the system is not ergodic, however, there may be multiple equilibrium states, each with a different $S(\mathbf{k})$. In a ferromagnetic phase $S(\mathbf{k})$ shows a δ-function peak at $\mathbf{k}=\mathbf{0}$. For the random $p=1$ field one shows[3] that the presence of this δ function induces a $1/|\mathbf{k}|^4$ peak in $S(\mathbf{k})$ at small $|\mathbf{k}|$. Such a peak is impossible in four spatial dimensions or less. Due to the norm-preserving property of the Fourier transform,

$$\sum_{\mathbf{k}} S(\mathbf{k})=L^3, \tag{6}$$

where the sum over $\mathbf{k}$ runs over the Brillouin zone. Since the square of the length of each spin is one in this model, Eq. (6) merely states that the total cross section in a scattering experiment is equal to the number of spins in the scattering volume times the cross section of one spin. There is no corresponding sum rule for $S_\theta(\mathbf{k})$.

We proceed by separating the time-dependent and time-independent parts of $\mathbf{M}$. Without loss of generality, we can rewrite Eq. (5) in the form

$$S(\mathbf{k},t)=\frac{1}{L^3}\left|\sum_j^{L^3}\left[\langle\mathbf{M}(\mathbf{r}_j,t)\rangle_t+\delta\mathbf{M}(\mathbf{r}_j,t)\right]\exp(i\mathbf{k}\cdot\mathbf{r}_j)\right|^2, \tag{7}$$

where $\langle\delta\mathbf{M}(\mathbf{r}_j,t)\rangle_t=0$. Performing the Fourier transform and taking a time average yields

$$\langle S(\mathbf{k},t)\rangle_t=|\langle\mathbf{M}(\mathbf{k},t)\rangle_t|^2+\langle|\delta\mathbf{M}(\mathbf{k},t)|^2\rangle_t. \tag{8}$$

If the system is not ergodic, then we will find a different $\langle S(\mathbf{k},t)\rangle_t$ for each equilibrium state.

To evaluate Eq. (8), Imry and Ma[3] ignore the fixed-length-spin constraint, and assume that for small G/J they can use a linear response spin-wave perturbation theory. The second term of Eq. (8) is the standard contribution from dynamical fluctuations of the spins. In linear response theory it gives a contribution to $S(\mathbf{k})$ of Lorentzian form, proportional to $1/(|\mathbf{k}|^2+1/\xi^2)$, where ξ is the correlation length. When $G=0$, then ξ is infinite in the ferromagnetic phase, and within perturbation theory this remains true for small G/J.

In a ferromagnetic phase, the linear-response spin-wave theory for the first term of Eq. (8) generates a $1/|\mathbf{k}|^4$ peak whose amplitude is proportional to G^2 times the square of the order parameter, $\langle\langle|\mathbf{M}(\mathbf{r}_i,t)|^2\rangle_{\mathbf{r}}\rangle_t$. Such a peak is impossible in four dimensions or less, due to the sum rule on $S(\mathbf{k})$. If the decoupling of the different Fourier modes assumed in the spin-wave approximation were valid, this would indicate the instability of ferromagnetism in four dimensions or less in the presence of the random p-fold field. As discussed by Fisher,[17] this decoupling is adequate when the number of dimensions is large, but it breaks down in four dimensions or less.

For $p=1$ the random fields cause each $\langle\mathbf{M}(\mathbf{r}_i,t)\rangle_t$ to become nonzero even in the paramagnetic phase. Thus for $p=1$ the spin-wave theory result in the paramagnetic phase for the first term of Eq. (8) is a ''Lorentzian-squared'' peak, of the form $G^2/(|\mathbf{k}|^2+1/\xi^2)^2$. The sum rule on $S(\mathbf{k})$ then implies that ξ must be finite. This Lorentzian-squared peak also occurs for the random-field Ising model,[3,18] and is not related to the existence of massless spin waves.

There is no Lorentzian-squared peak in the paramagnetic phase for $p=2$, since in this case each $\langle\mathbf{M}(\mathbf{r}_i,t)\rangle_t=0$, so that the first term of Eq. (8) then makes no contribution to $S(\mathbf{k})$. Therefore the sum rule on $S(\mathbf{k})$ cannot prevent ξ from diverging for $p=2$, and the existence of a QLRO phase in this case was proposed in 1980 by Aharony and Pytte.[19]

The domain-wall energy scaling argument given by Imry and Ma,[3] which is nonperturbative, compares the relative strengths of the exchange energy term and the random pinning term as a function of length scale. If the effective value of the coupling G/J scales to infinity at large length scales, then we know that for $p=1$ the model cannot be ferromagnetic.

An analogous argument does not suffice for $p>1$, however, because even for strong random anisotropy the mean-field theory[20] has a ferromagnetic phase. The domain-wall energy argument does not account for the exact p-fold symmetry of the Hamiltonian which exists for $p>1$. For $p>1$ one cannot show that the random term uniquely determines the large-scale structure of the low-energy states. Thus the rigorous proof which works for $p=1$ cannot be applied for larger p.[21]

Because the spin-wave argument assumes replica symmetry,[3,15] its lack of rigor has long been recognized. More recently, Mézard and Young[22] have shown explicitly

that when one calculates beyond the leading order in $1/N$ for $p=1$, the replica symmetry is broken in the ferromagnetic phase, and that, therefore, the randomness should cause ξ to be finite. Since the randomness destroys translation invariance, it is not surprising that it should also cause the long wavelength spin waves to become massive. Presumably this replica-symmetry breaking will also occur for $p>1$.

Within spin-wave perturbation theory,[15,17] the effect of a random anisotropy on the ferromagnetic phase appears to be the same as the effect of a random field. It seems reasonable that one should be able to study the properties of a single minimum of the free energy, and that, at least for small G/J, the behavior of the system in this local minimum should not depend on p. Because the replica symmetry is broken[22] for finite N, however, we know that this can fail.

There are a number of results which support the existence of LRO for XY models (i.e., $N=2$) with random anisotropy in four dimensions or less. The first are the experiments[5] on SDW alloys which appear to have LRO. The second is the high-temperature susceptibility series for the random anisotropy XY model,[23] which gives no indication of an instability of ferromagnetism in four dimensions. The third is the computer simulations[14] for the $p=3$ case in three dimensions, which show that the $p=3$ random anisotropy does not destroy the transition to ferromagnetism, but the transverse correlation length in the ferromagnetic phase becomes finite.

In this work we present the results of a computer simulation study of a toy model which we believe preserves the essential features of Eq. (2). For this model we find that in three dimensions a $p=1$ random field perturbs the structure factor of finite lattices in a manner consistent with the destruction of ferromagnetism for any strength of the random field, as predicted by the domain-wall energy scaling argument. In the corresponding $p=2$ case, however, we find no evidence for the destruction of ferromagnetism. Instead, we find that for this model the $p=2$ random anisotropy causes ξ to become finite without destroying the LRO.

III. TOY MODEL FOR RANDOM FIELDS

Large-scale computer simulations of the random-field[24,25] and random anisotropy[9] XY models have been performed in the last few years. While the results of these simulations are quite instructive, it has been difficult to study the behavior at weak randomness and low temperature. This is due to the limited size of the lattices which can be studied, and the difficulty of making transitions over energy barriers. In order to improve the effectiveness of the simulations, one may either try to develop new techniques for studying Eq. (2), or else one may try to find a modification of the Hamiltonian which preserves the essential features, but is easier to study. In this work we adopt the second approach. We will describe and study a model in which there are no energy barriers.

It was shown by Kohring, Shrock, and Wills[26] that if one adds a large vortex fugacity term to the XY model on a simple cubic lattice, then the model retains a ferromagnetic equilibrium state even in the absence of any explicit exchange energy. It was later shown[27] that this ''vortex-free'' XY model, in which all allowed spin configurations have the same energy, behaves in most respects like a normal XY model at some finite temperature within the ferromagnetic phase. Here, we will study the effects of adding random fields and random anisotropies to the vortex-free model. In order to retain the property that all allowed states of the model have the same energy, we replace the random term of Eq. (2) by constraints on each θ_i.

To obtain a random-field-type model, for each i we choose a random arc of the circle of some fixed size, and declare that θ_i cannot take on values within that arc. The fraction of the circle which is removed at each site is then a parameter which measures the strength of the random field. In order to maintain the vortex-free constraint everywhere, it is sufficient that the fraction removed, R_1, be less than 1/2. To see this, note that any state in which all spins have values on the same half of the circle, so that there is some axis for which the projection of all spins is in the same direction, is vortex-free. We will refer to such a state as a ''semicircle state.''

For a random-anisotropy-type model we perform the same procedure, except that we symmetrically remove two arcs from opposite sides of the circle at each site. In this case it is possible to satisfy the vortex-free constraint even if at each site we only allow two points. For the random anisotropy model it is clear that the allowed states come in pairs, so that time-reversal symmetry is not explicitly broken by the Hamiltonian.

We expect the qualitative behavior of the constraint-type random fields and random anisotropies to be the same as the corresponding $p=1$ or $p=2$ random terms of Eq. (2). It is somewhat less clear that our replacement of the exchange term in Eq. (2) by the vortex-free constraint will not make any qualitative difference. One can argue that for small G/J the low-energy states of Eq. (2) should be vortex-free, but it is difficult to prove this. In the simulation of Gingras and Huse[24] it was observed that the vortex loops disappeared rapidly as the random field was made weaker at low temperature. It was suggested by them that in the absence of vortex loops an XY model with a random field would possess a QLRO phase, in which two-point correlations have a power-law decay as a function of distance. For the vortex-free model we can test this conjecture in a straightforward manner.

IV. MONTE CARLO CALCULATION

The Monte Carlo program was a modified version of one used earlier[27] to study the vortex-free model without randomness. It approximates the circle by a 256 state discretization, and uses a simple cubic lattice with periodic boundary conditions. Two linear congruential pseudorandom number generators are used, one for assigning the random fields, and a different one for flipping the spins. The initial state of each lattice is chosen to be a semicircle state. Moves are rejected if they would violate the vortex-free constraint or the local random-field constraint.

A brief study of $L\times L\times L$ lattices as a function of size L and the strength of the randomness showed that for the $p=1$ case increasing the strength of the randomness caused a progressive decrease of the equilibrium magnetization, $\langle\langle|\mathbf{M}(L)|\rangle_{\mathbf{r}}\rangle_t$, as expected, with $\langle\langle|\mathbf{M}|\rangle_{\mathbf{r}}\rangle_t$ extrapolating to zero for large L. For the $p=2$ case, however, there was no evidence of a decrease of $\langle|\mathbf{M}|\rangle_{\mathbf{r}}$ as the randomness was

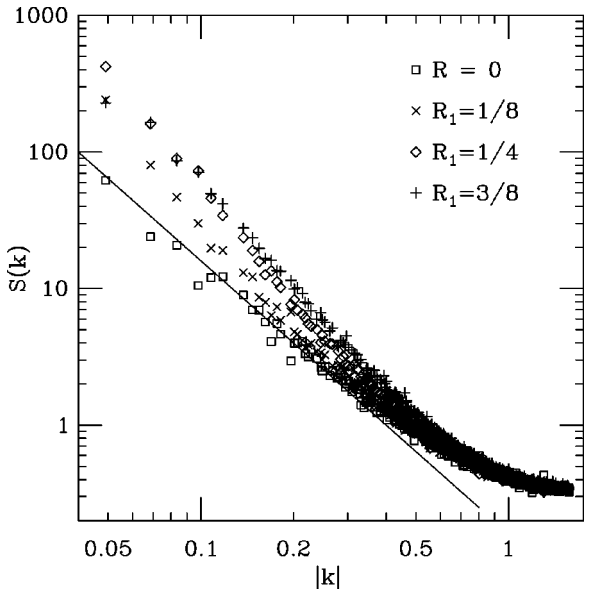

FIG. 1. Angle-averaged magnetic structure factor (per spin component) for the vortex-free XY model with random fields on $64\times64\times64$ simple cubic lattices, log-log plot. Each data set shows an average of data from eight samples. The straight line has a slope of -2.

turned on. In order to investigate this unexpected result carefully, it was decided to expend most of the computing effort on the computation of the structure factor for lattices with $L=64$.

Starting from a semicircle state, each $R_1>0$ lattice was run for 40 960 passes, which is several times the apparent longitudinal relaxation time. Some of the $R_1=0$ lattices were run for only half this time, because the longitudinal relaxation time is shorter in this case, and the transverse relaxation is given by spin-wave theory. The values of $\langle\langle|\mathbf{M}|\rangle_{\mathbf{r}}\rangle_t$ were obtained by averaging over the last half of each run, sampling every 20 passes. For $L=64$, the magnetization was found by this procedure to be 0.435 16, 0.4018, 0.313, and 0.244 for $R_1=0$, 1/8, 1/4, and 3/8, respectively. The fluctuations in $\langle\langle|\mathbf{M}|\rangle_{\mathbf{r}}\rangle_t$ between runs become larger as R_1 increases, as does the time-averaged longitudinal susceptibility for a single run. Because only one initial state was used for each $p=1$ sample, we do not know if the variations in the time averages for different initial states of the same sample are as large as the variations between samples. The transverse susceptibility, obtained from the time-dependent fluctuations of $\langle\mathbf{M}\rangle_{\mathbf{r}}$ averaged over the last half of each run, becomes smaller as R_1 increases. For $R_1=3/8$ and $L=64$, $\langle\mathbf{M}\rangle_{\mathbf{r}}$ remains close to its initial direction for the duration of the run, and the transverse susceptibility is not much larger than the longitudinal susceptibility. This naturally implies that there are many local minima of the free energy, at least on the time scale of the simulation. For $R_1=1/8$ and 1/4 the direction of $\langle\mathbf{M}\rangle_{\mathbf{r}}$ may change substantially at first, but then it seems to settle into some local minimum of the free energy, although the transverse susceptibility remains large.

Results for the angle-averaged $S(\mathbf{k})$ for $L=64$ lattices with these strengths R_1 of the random $p=1$ field are shown on a log-log plot in Fig. 1. Each data set is an average of eight samples of the randomness, with one final state used for each sample. The data for the samples with no random fields approximately follow a $1/|\mathbf{k}|^2$ law, with an additional δ function at $\mathbf{k}=0$, which does not appear on the log-log plot, as predicted by spin-wave theory. As R_1 is increased, the weight of the peak is progressively pushed out to larger values of $|\mathbf{k}|$, with the sum rule on the integral over $\mathbf{k}$ of $S(\mathbf{k})$ being preserved. Because of the sum rule, the fluctuations in the different small $\mathbf{k}$ modes are strongly coupled. This makes it difficult to estimate the statistical error for a single mode. Suffice it to say that the fluctuations of $S(\mathbf{k})$ for a single $\mathbf{k}\neq0$ mode of a single sample are of about the same size as the average value for that mode.

For $R_1=1/8$, the slope on the log-log plot of $S(\mathbf{k})$ in the accessible small $|\mathbf{k}|$ region is approaching -3. Due to the sum rule, this indicates that there is no evidence for any LRO at this value of R_1, even though the value of $\langle\langle|\mathbf{M}|\rangle_{\mathbf{r}}\rangle_t$ is still not much reduced from its $R_1=0$ value at $L=64$. At $R_1=1/4$, $S(\mathbf{k})$ shows an apparent slope of -2.85 ± 0.05 for small $|\mathbf{k}|$ on the log-log plot. By $R_1=3/8$ the $L=64$ samples are showing multiple local minima of the free energy. This may be an indication that some correlation length has become comparable to the sample size, but $S(\mathbf{k})$ at $R_1=3/8$ can be fit at small $|\mathbf{k}|$ by a power law in $|\mathbf{k}|$ with an exponent, $(-2+\eta)$, of -2.63 ± 0.07. A QLRO phase with a continuously varying value of η has recently been found in a similar model by Emig, Bogner, and Nattermann.[28]

In order to distinguish clearly between an infinite ξ with a continuously varying exponent η and a finite ξ, we would need data for larger L or R_1 closer to 1/2. Either of these approaches would require a substantial increase in computing effort. If there really is an infinite ξ and an η which varies continuously as a function of R_1, then we would like to know if this behavior continues out to the maximum allowed value of R_1, and, if so, how η behaves near that point. If it were practical to perform simulations for larger values of L, we believe that we would see the appearance of many local minima for any nonzero allowed value of R_1.

To study the $p=2$ case, we concentrated on samples with $R_2=3/8$, which means that only 1/4 of the states were allowed at each site. Smaller values of R_2 give an $S(\mathbf{k})$ almost indistinguishable from the result for $R=0$, the model without randomness. Note that $R_2=3/8$ can be obtained from $R_1=3/8$ by removing an additional 3/8 of the allowed states at each site, and thus restoring a twofold symmetry. Because the random anisotropy constraints now cause most of the attempted moves to be immediately rejected, each sample was run twice as long as for $p=1$. Also, two semicircle states, differing in average orientation by $\pi/2$ from each other, were used for each sample as initial states.

The average magnetization for $L=64$ and $R_2=3/8$ was obtained by averaging over the last quarter of each run. The result was $\langle\langle|\mathbf{M}|\rangle_{\mathbf{r}}\rangle_t=0.43613$, slightly *larger* than the result for the $L=64$ system with $R=0$. The results obtained using the two different initial states for a given sample did not appear to be more similar to each other than results from two different samples. The direction of the magnetization rotated significantly during most runs, indicating that ξ is at least as large as L, and that the observed behavior is unlikely to be due to a failure of the system to relax. For one of these eight samples the final states of the two runs appeared to be in the

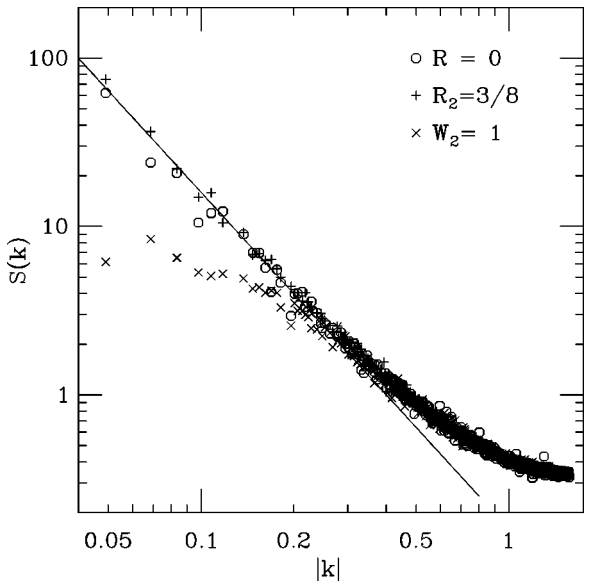

FIG. 2. Angle-averaged magnetic structure factor (per spin component) for the vortex-free XY model with random anisotropy on $64\times64\times64$ simple cubic lattices, log-log plot. Each data set shows an average of data from eight samples. The $R_2=3/8$ data set includes two states (with different initial conditions) per sample. The straight line is identical to the one shown in Fig. 1.

same local minimum. Studying smaller samples for much longer running times also gave no indication that the results were caused by insufficient relaxation.

Eight samples with maximal random anisotropy (only two allowed states at each site, labeled $W_2=1$ in Fig. 2) were also studied. In this case, with only two allowed states per spin, a Metropolis-type algorithm, for which spin flips allowed by the vortex-free constraint were made with probability 3/4, was used to improve the efficiency of the program. For $W_2=1$, the $L=64$ value of $\langle\langle|\mathbf{M}|\rangle_{\mathbf{r}}\rangle_t$ was 0.4662, and the orientation of $\langle\mathbf{M}\rangle_{\mathbf{r}}$ always remained close to its initial direction.

The structure factor for these $p=2$ cases, again averaged over eight samples, is shown in Fig. 2, along with the data for $R=0$. We see that at $L=64$ the structure factor for $R_2=3/8$ is not distinguishable from that of $R=0$. Although the data for $R_2=3/8$ appears to be slightly above the data for $R=0$ at small $|\mathbf{k}|$, this is a sampling artifact. The actual average of $\langle|\mathbf{M}|\rangle_{\mathbf{r}}$ for the 16 $R_2=3/8$ states used in constructing the figure is 0.4349, while for the 8 $R=0$ states it is 0.4357.

For $W_2=1$, ξ is approximately eight lattice spacings, and the line shape appears to be Lorentzian (plus the δ function at $\mathbf{k}=0$). In this case it appears that large L behavior is seen already for $L=16$. A sample with $W_2=1$ and $L=16$ was run for approximately 5×10^5 steps per spin. The system appeared to relax to equilibrium within the first 50 steps per spin, which was essentially the same as the relaxation time for the $L=64$ samples, and no transitions were seen out of the local minimum, which retained an $\langle\mathbf{M}\rangle_{\mathbf{r}}$ almost parallel to that of the initial semicircle state. The transverse susceptibility is approximately 15 times the longitudinal susceptibility.

V. DISCUSSION

The model we are using for our computer simulations is one in which all allowed states of the spins have the same energy. If the dynamical behavior is ergodic, then, by definition, it must be possible to get from any initial state to any final state. One might imagine therefore that this model, with single-spin flip dynamics, could have an ergodicity-breaking transition as a function of the strength of the randomness. That is, there could be a transition between having a connected phase space to having a phase space which is broken into many disconnected pieces.

It is easy to see, however, that any semicircle state can be connected to another semicircle state with an arbitrary choice of the semicircle, by single spin flips which do not violate the constraints. This remains true as long as R_1 or R_2 is less than 1/2. Therefore we cannot explain the result that for large values of R_1 or R_2 we find many local minima when L becomes large by a percolation transition in phase space. The breakdown of ergodicity is a true phase transition, because the transition rates between different local minima only go to zero in the infinite volume limit.

One should remember that the phase space available to an individual spin depends on how well aligned its neighbors are. In a semicircle state, the magnetization is $2/\pi$, about 0.636 62. Thus, when the system relaxes into a state with $\langle|\mathbf{M}|\rangle_{\mathbf{r}}<0.5$, the ability of individual spins to reorient by single-spin flips is greatly reduced, even though the entropy of the system as a whole has increased.

Entropy barriers[29] are just as effective as energy barriers in suppressing transitions between different minima. If the paths in phase space between different local minima must pass through intermediate states in which the value of $\langle|\mathbf{M}|\rangle_{\mathbf{r}}$ in a volume ξ^3 is close to 0.6, then the probability of making such transitions is suppressed by a factor exponential in the correlation volume.

The above estimate may be unduly pessimistic. For example, it may be enough to increase the local magnetization in a surface layer, so that the entropy barrier is only proportional to ξ^2. Nevertheless, the basic principle, that uncorrelated single-spin flips are not an efficient way to achieve large-scale reorientation of $\mathbf{M}$, is correct.

It would not be surprising if an alternative dynamics could be developed which flipped large clusters of spins simultaneously,[30,31] and was thus more effective in moving through phase space. Therefore we would like to check our results to see if they reflect true equilibrium behavior by developing such an algorithm. However, the results as they stand seem internally consistent, and they are also consistent with the other related results cited earlier.

For $p=2$, there is no instability of the LRO when the randomness is too weak to induce the creation of vortex lines. We remind the reader that when vortex lines are allowed, as for the strong random anisotropy limit of Eq. (2), the LRO appears to be unstable in three dimensions, and the low-temperature phase seems to have only QLRO.[9] The nature of the transitions between the LRO, QLRO and paramagnetic phases are clearly of great interest, but they cannot be explored within the vortex-free model.

It should be noted that the infinite vortex fugacity used in our model does not satisfy the smoothness conditions used in

the proof of Aizenman and Wehr.[4] Therefore our finding that the random $p=1$ field destroys the LRO in our model is an indication that the smoothness conditions can be relaxed in three dimensions. Of course, it does not follow from this that the smoothness conditions can be relaxed in four dimensions.

An alternative method[32,33] of removing the vortices is by placing a lower bound on the allowed values of $\cos(\theta_i-\theta_j)$ for all nearest neighbor pairs of i and j. This method directly violates the smoothness condition (4.5) of Aizenman and Wehr,[4] and is a more severe constraint than the vortex fugacity method used here. It is likely that this alternative method would produce results in qualitative agreement with those found here using the Kohring-Shrock-Wills method.

VI. CONCLUSION

In this work we have used Monte Carlo simulations to study a vortex-free XY model in three dimensions with random $p=1$ and $p=2$ fields. This toy model is intended to represent the effects of random pinning on uniaxial CDW's and SDW's. We have found that for CDW's the LRO should be destabilized by weak random pinning, but that for SDW's the LRO should survive. These conclusions are consistent with experiment. Our results for the $p=1$ case are consistent with the existence of a QLRO of the type discussed by Emig, Bogner, and Nattermann.[28]

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